Chapter 9
The Epistemic Role of Gestures: A Case Study on Networking of APC and AiC

Tommy Dreyfus, Cristina Sabena, Ivy Kidron, and Ferdinando Arzarello

Abstract In this case study, the epistemic role of gestures is considered empirically. The analysis of gestures is included into the AiC analysis of a small excerpt of the data from Chap. 2 by means of the notion of semiotic bundle, which forms a crucial component of the APC-space. For this purpose, APC and AiC are coordinated and then locally integrated in an asymmetric way.

Keywords Networking of theories • Gestures • Epistemic

9.1 Introduction

Chapter 6 deals with Abstraction in Context (AiC) as a theoretical framework for analyzing processes of constructing abstract mathematical knowledge by the so-called RBC analysis (recognizing, building-with, constructing). In that chapter, we explained why the AiC team, when presented with the tasks and the transcript

T. Dreyfus (✉)
Department of Mathematics Science and Technology Education, Joan and Jaime Constantiner School of Education, Tel Aviv University, P.O.B. 39040, Ramat Aviv, Tel Aviv 69978, Israel
e-mail: tommyd@post.tau.ac.il

C. Sabena
Dipartimento di Filosofia e Scienze dell'educazione, Università di Torino, Via Gaudenzio Ferrari, 9/11, 10124 Torino, Italy
e-mail: cristina.sabena@unito.it

I. Kidron
Department of Applied Mathematics, Jerusalem College of Technology, Havaad Haleumi Street, 21, Jerusalem 91160, Israel
e-mail: ivy@jct.ac.il

F. Arzarello
Dipartimento di Matematica, Università di Torino, Via Carlo Alberto, 10, 10123 Torino, Italy
e-mail: ferdinando.arzarello@unito.it

A. Bikner-Ahsbahs and S. Prediger (eds.), *Networking of Theories as a Research Practice in Mathematics Education*, Advances in Mathematics Education,
DOI 10.1007/978-3-319-05389-9_9,

illustrated in Chap. 2, immediately focused on Task 3. The main reason for this was that Task 3 offered the students an opportunity to construct new (to them) knowledge about notions they had never met before. These notions were specified in the a priori analysis carried out in Chap. 6. We remind the reader that the aim of the a priori analysis is to identify and formulate the constructs which (according to the researchers' judgment) the teacher intended the students to construct. We also note that in this chapter the AiC team relates to an RBC analysis of the knowledge-constructing processes with limited attention to different parts of the context (the use of the computer, the role of the teacher), which will be attended to in Chap. 10.

The most productive situation for an RBC analysis of individual students' constructing of knowledge tends to be students working in pairs because their discussions often provide the researcher with information on their thought processes. For Carlo and Giovanni, this was not the case. When attempting to carry out an RBC analysis of the students' work on Task 3, the AiC team found that their utterances were not many and often vague, and hence the data were too sparse to analyze and difficult to interpret.

This is where the contacts between the AiC team and the APC team became important. The APC team's analyses are multimodal. This multimodality includes, in particular, a focus on the learners' gestures, in addition to their verbal utterances. The role of gestures in APC is central. The AiC team learned from the APC team and their semiotic bundle (SB) analysis (Chap. 3) how to pay attention to gestures. The question arose, whether gestures could provide some of the data the AiC team lacked in order to carry out an effective RBC analysis, and how the RBC analysis might change as a consequence of taking gestures into account. In particular, this raises the issue of the epistemic function of gestures, and more specifically, whether and in what sense gestures can contribute to the construction of knowledge.

In order to examine these issues, the AiC team used the methodological experience of the APC team in interpreting gestures and adopted some of it. This was facilitated by the fact that, in some sense, the two teams start from rather close positions: both are fundamentally interested in student cognition (and additional aspects), and both employ a micro-analytical approach to data analysis. On first sight, one might therefore ask whether networking was even an issue. Were the two teams attempting integration or were they only trying to smooth out minimal differences? Were the differences indeed minimal? In fact, while both approaches have a strong socio-cognitive tenet, and while their micro-analytic methods of data analysis may be similar in grain size, the foci of the two teams are rather different: focus and interpretation depends on the researchers' interest and theoretical frame. The APC team focuses mainly on the semiotic resources observable in the classroom while students solve problems or discuss a mathematical task; hence the focus is on what they do, produce, and on their interactions (among themselves, or between them and a teacher). The observable semiotic resources include utterances, gestures, and inscriptions (utterances, graphs, sketches, formulas). In such a sense, they scrutinize also the role gestures may play in the formation of mathematical knowledge. Hence the communicative function of gestures is extremely important to APC

researchers in their SB analysis, as are phases of student–teacher interactions, as well as thinking tools (see Chap. 3).

AiC researchers, on the other hand, are primarily interested in the construction of knowledge. Hence, their focus is mainly on the learners, the teacher forming part of the context, and researchers consider gestures as relevant only insofar as they have an epistemic function in the construction of knowledge.

These differences lead to different research questions and different choices of data for analysis. What makes data interesting for AiC researchers and what makes data interesting for APC researchers? What can we say about the nature of data required, or at least desired, for analyzing gestures with SB (see Chap. 3) or with RBC (see Chap. 6)? First of all, and trivially, when the aim is to analyze gestures, only excerpts with gestures are relevant, and this already considerably restricts the choice of data. Secondly, the APC team favors excerpts in which gestures have a communicative function in the learning process; for the AiC team, on the other hand, the main criterion for the choice of excerpts with gestures is the potential for the emergence of new constructs. They are interested in the role gestures might have in the process of constructing knowledge; as pointed out in more detail below, the main function of such gestures is epistemic, and they may well be isolated in time, and made by a learner to and for him- or herself without social interaction. As a consequence, the situations of Tasks 1 and 2, which were intended as a preparatory phases for constructing knowledge, were of less interest to the AiC team, and the team focused on the situation of Task 3, that is, lines 249–379 in the transcript (see Appendix).

As a consequence of these different data requirements, and of the different foci of the teams, and in spite of the closeness, in some sense, of the two approaches, we were left with a very small intersection of data that could have formed a basis for parallel and then comparative or common analysis, as is often done in research that networks two theoretical frameworks (see, for example, Chaps. 11 and 12). Therefore, this chapter will be somewhat different from the subsequent case studies. It will relate to the two theoretical frameworks in an asymmetric way for two reasons: the dearth of data for parallel analysis as well as the asymmetrical aim of using one theoretical framework in order to enrich the analysis of the other one. In the following section, we present an attempt to integrate gesture analysis into the RBC analysis of knowledge construction in the situation of Task 3 (Sect. 9.2); this attempt to integrate gestures has been based on the SB methodology and carried out by the two teams together. We will then discuss methodological aspects of the RBC analysis that are related to this integration of gestures (Sect. 9.3), as well as theoretical consequences for AiC, and feedback to APC (Sect. 9.4); and we will end with a reflection on the process (Sect. 9.5).

The central aim of the chapter in the framework of this monograph is to show that one team (AiC) can exploit the experience of another team (APC) in order to explore how to improve its methodology, in this case how to incorporate gesture analysis according to SB into the RBC analysis, and how this team's theoretical framework can grow methodologically and theoretically in the process.

9.2 The Data and Their RBC Analysis

Our aim in this section is to demonstrate how data based on the students' gestures complement the data based on their utterances and how this combined data set allows carrying out an RBC analysis of the students' constructing actions while working on Task 3. We focus in particular on the second part of the students' work on this task (lines 302–349 in the transcript), because this is the excerpt that allows us to best demonstrate our analysis.

As described in Chap. 2 (Fig. 2.3), Task 3 had been designed by the teacher in order to give the students an opportunity to explore the exponential variation at both the local and the global level. Besides the graph of $y = a^x$, it contains the points $P(x;a^x)$, and $H(x+\Delta x;a^x)$; it was also supposed to contain the point $Q(x+\Delta x;a^{x+\Delta x})$, and the students related to this point as if it were there (see Fig. 2.4). It also contained, two sliders, whose variation allowed the students to modify, respectively, the increment Δx and the base a of the exponential. The computer screen configuration is shown in Fig. 9.1.

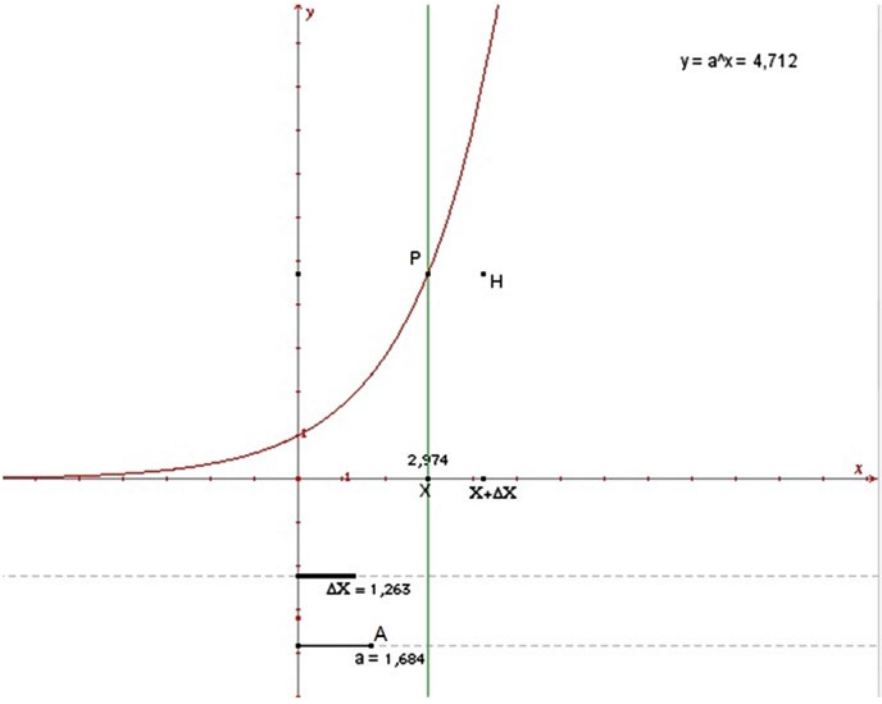


Fig. 9.1 The computer screen configuration of Task 3 (see Fig. 2.3)

9.2.1 *Narrative Summary of the Students' Work on Task 3*

As mentioned, we focus on the part of the activity in lines 249–379. We first give a narrative summary of what happened in this excerpt. The aim of the narrative summary is to serve as a frame of reference for the reader during the following micro-level analysis (Sects. 9.2.2 to 9.2.7). The narrative summary does not pretend to be an analysis or to be objective; we are aware that any view, even if it is descriptive, is influenced by the viewer's selections.

Episode 3a (lines 249–281). The students quickly observe that the point P and the base a (which they call "the rate of growth") can be varied; they also note that Δx can be varied, and identify Δx with PH.

Episode 3b (lines 282–324). Varying P, the students observe that HQ varies with P, and that as P moves to the left, HQ becomes small and the secant appears to become a tangent. They briefly and vaguely also comment on what happens as PQ gets small (298, 301) and mention the option of varying a, but then return to consider the effect of varying P. They also explore and comment on what happens for what they call "P near zero," by which they mean $y_P \to 0$. Now the teacher joins them, and participates in the conversation until almost the end of the lesson (until line 368). The teacher's participation is active – he not only asks questions but provides information.

Episode 3c (lines 325–343). The first issue discussed with the teacher is what happens as Δx becomes very small; while the students focus on the phenomenon that the line becomes (nearly) a tangent, the teacher keeps asking what information this provides them. Nested within this episode, the students recall, in a different formulation, that the (secant) line approximates the function better, the nearer "P is to zero" (lines 331–334).

Episode 3d (lines 344–353). Under the teacher's continued questioning and later his suggestion of the term "approximation," the students conclude that the exponential function can be approximated by a set of little tangent elements, each steeper than the preceding one.

Episode 3e (lines 354–367). The teacher guides a discussion establishing that the "growth percentage" or the ratio between a value and its successor (these are the teacher's expressions; the students repeat some of them) remains constant and that this is consistent with the growth rate being low. "The function crushes on the x-axis" (according to the teacher) when the values of the function are close to $y=0$ (for small x). The students repeat, in their own words, part of what the teacher says. The teacher leaves and the students begin to summarize what they are going to write: that the exponential function can be approximated by little straight line segments of increasing slope; that for small x, these straight-line segments are almost like a single (straight) line, and hence that "at the beginning," that is for $x \to -\infty$, the graph is similar to a line and has a constant rate of growth (366–369).

Episode 3f (lines 370–379). Finally, they turn to the question of what happens when a varies. They seem to keep P and Δx constant and observe that the area of the triangle PQH grows as a grows.

9.2.2 Lines 300–302: Behavior of the Tangent Line for Small x

In Episode 3b, the students have chosen and fixed a rather small value of Δx, and explore the behavior of the tangent line, focusing in particular on the case in which P is to the left of the origin, which they express as "P is small." (In the transcript, underlining designates the part of an utterance during which the speaker gestured.)

300	G	we can say that if P it's small, that is more like a tangent, it seems, if you take it much small
301	C	a single point
302	G	eh, it can <u>be approximated to one line</u>, with P very small, then instead as long as it increases…

Gesture in 302

A significant gesture occurs in line 302: Giovanni is positioning and slowly moving rightwards his left hand on the desk, as shown in the picture. He refers to "if P it's small" (line 300) or "with P very small" (line 302); this can be interpreted in two almost equivalent ways as x_P being small (close to $-\infty$) or as y_P being small (close to zero). It is a moot question which one he means. It is much more important what he would like to say about P being small: "that is, it seems rather like a tangent" (line 300), and "it can be approximated to a line" (line 302). He seems to find it difficult to express what he means in words; his left hand, positioned horizontally in front of the computer on the table, is an additional means of expression. We interpret the combination of his utterances and his gesture as expressing his image of what happens when P is small. Possibly, expressing his image also helps him construct a more definite image of what happens when P is small, namely that the graph of the exponential function is similar to a straight line and therefore well approximated by its tangent. Hence the gesture is non-redundant with respect to the student's words (in the sense of Kita 2000; see Chap. 3 for a discussion about the characterization of gestures).

During collaboration with the APC team, the AiC team learned to consider gestures such as the one made by Giovanni in line 302 in a manner similar to how they were used to look at verbal utterances: as expressing information. Here, the information expressed is that when P is small, the tangent line has an almost horizontal position. The AiC team also learned to distinguish between redundant and non-redundant gestures. The gesture in line 302 is non-redundant in the sense that it adds information beyond the one in the utterance. This additional point of view allowed us to interpret line 302 as a building-with action: Giovanni recognizes the notion of

tangent as relevant for the situation he is currently dealing with and builds-with previous constructs including "tangent" and a certain possibly rather vague notion of "approximation."

9.2.3 *Lines 308–313: Behavior of the Function for Large* x

Still in Episode 3b, Giovanni summarizes his insight from a different point of view:

308 G yes, if we move P we can see that the point, eh, sorry the HQ segment becomes smaller, it decreases…and this, the point QH, can you see?
309 C because P and Q have always the same distance
310 G yes
311 C ok, so ok, ok, so ok, because if it means that they increase, the more you move them over there, it increases very very much
312 G yes
313 C because it's an exponential function

Gesture in 311: C quickly moves the hand upwards to the right

In line 309, Carlo joins the action. The link between line 308 and 309 is not obvious even though Giovanni (in line 310) expresses agreement with Carlo (line 309). If the students had gestured here, the researchers might have had better access to the exchange. But in this instance, we were not so fortunate. Noticing that "P and Q have always the same distance" (line 309), Carlo considers the opposite end of the x-axis (line 311); he also gestures, waving his right hand with the pen in the air in a repeated upward movement to the right. While the gesture, because of its wavy nature, is not more definite than the verbal expression, gesture and speech mutually support each other: the words indicate that there is an increase; the gesture shows that it is on the right side of the screen and becoming larger. The AiC team learned from the APC team to consider such instances as a semiotic bundle of gesture and speech. Moreover, the gesture by Carlo is anticipatory with respect to his words. This fits the "information packaging hypothesis" proposed by Kita (2000) and the "growth point" model by McNeill (2005) (see Chap. 3 for a theoretical elaboration on gestures).

Together, words and gestures give a rather clear expression to the middle part of the intended construct C_{11} (see the a priori analysis in Chap. 6 for details): "As x grows (P moves to the right), the slope of the tangent grows (for $a > 1$)." The third part of C_{11} ["As x decreases (P moves to the left), the slope of the (secants and the) tangent decreases to zero (for $a > 1$)"] is implicit already in line 308 (and earlier in line 302); hence we interpret this excerpt as evidence that the students are in the process of constructing C_{11}.

9.2.4 *Line 316: Constructing the Dependence on* x *(C_{11})*

The next significant gesture occurs briefly afterwards, in line 316:

316 G eh, ok, when the P it's very close to the 0, the line that passes for Q and H represents [*begins gesturing on the desk by screenshot (a)*] more and more [*gesture in screenshot (b)*] the function… the smaller it is [gesture (*c*)]

Gestures in 316 (a) *(b)* *(c)*

When Giovanni gets to "represents more and more the function" (line 316), he puts his right hand on the table, next to the computer, with the thumb and index finger touching the table and approaching each other while the hand moves to the right (see screenshots in transcript line 316); he then repeats the same movement again. Giovanni looks at Carlo's face; Carlo looks at the screen; nobody looks at the gestures. We note that the gestures, though explicit, seem to be almost automatic, expressing, together with the words, Giovanni's thinking. Our interpretation follows.

APC interpretation. Giovanni's gesture repeats many times the small back-and-forth movement of index and thumb, while softly moving his hand towards the right. Through this catchment (according to McNeill et al. 2001, a catchment is recognizable when some gesture form features are seen to recur in at least two, not necessarily consecutive, gestures), he metaphorically expresses the limiting process of QH tending to zero. In this way, he is able to pictorially add more information to his words (P is very close to zero), showing that this gesture is non-redundant as well.

AiC interpretation, as enriched by the APC interpretation. "P is very close to zero" together with the movement of the hand to the right (although it "should" move to the left, physically it is much easier to move the right hand to the right than to the left) refers to the y-coordinate of P only, as its x-coordinate moves in the direction of $-\infty$. The movement of P, expressing that x approaches $-\infty$, is expressed only by the gesture, not in words – once again, the gesture is non-redundant. Simultaneously, the thumb represents the point Q and the index finger the point H, and as the hand moves to the right the index finger and the thumb metaphorically express the limiting process that QH becomes smaller and tends toward zero. In addition, the words, though not the gesture, express that Q is almost on the x-axis and hence the line through H and Q represents the graph of the function ever better.

Giovanni thus gave expression to his constructing the third part of C_{11}, namely the variation of the slope as P moves on the graph to the left (see Chap. 6 for details). Taking all of this together, we claim that the students as a pair have now constructed C_{11}, at least in some vague form that relates to fixed Δx and increasing or decreasing segment HQ rather than to the slope of the tangent (which is the ratio of these two quantities). In fact, this represents a variant $C_{11'}$ of C_{11}: "As P gets closer to $y = 0$, the exponential function can be approximated by the secant line." This vague form lacks some of the aspects of the intended construct, for example the fact that the (slope of the) tangent is obtained as a limit of a sequence of (slopes of) secants – the first part of C_{10} ["For any given P, that is, locally, as Δx tends to zero, the slope of the secant tends to the slope of the tangent; the slope of the secants and the tangent are all positive (for $a > 1$)"]. According to Davydov (1972/1990), it is typical and expected that constructs start from a vague form and then progressively become more elaborate and precise. Here, while C_{11} is already quite elaborate, it is still partial since C_{10} is absent, and the full form of C_{11}, as identified in the a priori analysis, relies on C_{10}.

9.2.5 Line 331: Completing the Co-construction of C_{11}

The teacher has joined the group, and Giovanni repeats his conjecture about what happens as "P becomes small":

331 G yes, look… [*pointing at the screen*] and then we have discovered also that the nearer P is to [*Carlo's gesture (a)*] y equal to zero, the more this line approximates [*gesture (b1) on the desk*] the [*gesture (b2) in the air*] function

Gestures in 331:

(a) Carlo's gesture accompanying Giovanni's statement in 331

(b1) Giovanni's gesture repeating the one from 316

(b2) Giovanni's gesture representing decrease (to the left)

The fact that the teacher joins them (line 324) gives the students a chance to repeat their finding about the behavior as "P becomes small," this time (line 331) in clearer words than before (line 316). Their explanation is supported by short gestures accompanying the utterance in line 331 by both students: Carlo gesturing a flat movement with his right hand (screenshot 331a), Giovanni repeating a shorter version of the gesture he had made already in line 316 (here screenshot 331b1), and then a gesture representing the function graph decreasing to the left (screenshot

331b2). The AiC team interprets the progressively more elaborate language as a sign of consolidation of the construct, in this case C_{11} and/or $C_{11'}$.

This interpretation by the AiC team, while acceptable, is incomplete. The APC team points to signs of a close collaboration between the two students. This is witnessed by the fact that the verbal statement in line 331 made by Giovanni is illustrated by Carlo's gesture in screenshot 331a: right hand suspended horizontally in front of him. This gesture is synchronous to Giovanni's words "the nearer P is to." Such synchrony within the semiotic bundle has been called an "interpersonal synchrony" (Sabena 2007). It may be interpreted as a sign of the fact that an APC-space has been built by the students through their common work in the problem-solving activity. This interpersonal synchrony shows that the constructing action has been a co-construction. The consideration of the gestures, in addition to and together with the verbal utterances, thus eliminates the vagueness of the AiC interpretation with respect to who has constructed C_{11}.

Giovanni's first gesture is a recurrent gesture, a catchment in McNeill's terms (McNeill et al. 2001). Following McNeill, the APC team interprets the catchment as a signal that this idea has been internalized by Giovanni and is recalled here in his reasoning. The AiC team interprets the recurrence as additional evidence for consolidation, supplementing the "progressively more elaborate language" criterion (Chap. 6).

9.2.6 Lines 335–343: The Limit as Δx Tends to 0

The teacher now takes the lead, giving the students little opportunity to independently construct their knowledge, and the AiC team little reason to perform an RBC analysis. The thoughts move quickly under the lead of the teacher. The teacher focuses the discussion on the transition as Δx tends to 0. The students had twice identified Δx with PH and briefly varied the slider determining this quantity in order to confirm the identification, but they had never commented on the effect of the varying Δx. Nevertheless:

335 G and moreover another thing, if the Delta x is very small…

337 G [*pointing at the screen*] the line becomes nearly a tan.., a tangent [*gesture*]

Gesture in 337: G holds the fingertips of the flat vertical left hand against the interior of the flat vertical right hand, while moving the right hand upward

339 G to the, to the function

340 T and so, it gives you some information about what? When the Delta x tends to become very very small, what kind of information do you get?

341	C	if the Delta x becomes small… it means that…[*looking at the screen, where Giovanni is moving something using the mouse*] the Delta x becomes <u>small</u> [*gesture*]	*Gesture in 341 (a): C is pointing with index and thumb (the "Delta gesture")* 
		when… when between P and Q… that is [*gesture*] the space decreases	*Gesture in 341 (b): C is moving his open hand vertically from the bottom upwards* 
342	T	oh sure, it is almost trivial, isn't it? Therefore he was saying that this line tends to become…	
343	G	[*gesture*] tangent. [*C nods*]	*Gesture in 343: C's anticipatory gesture: puts his hand in a horizontal position* 

The APC team provides the following analysis. Giovanni's gesture and words are mutually supporting each other. The gesture comes toward the end of line 337, when he says "nearly a tangent" and holds the fingertips of the flat vertical left hand against the interior of the flat vertical right hand, while moving the right hand upward. This gesture can be interpreted as showing in an iconic way how the secant becomes a tangent, and can therefore be interpreted as at least a partial construction of C_{10} by Giovanni.

During the gesture, but not before, Giovanni turns his head toward the teacher. Carlo, meanwhile, yawns (line 337) and seems uninterested. A little earlier he had asked whether he had to consider always the same distance between P and Q (line 327), and now, asked about what happens "when the Delta x tends to become very very small" (line 340), he answers in a circular way: "the Delta *x* becomes small when… when between P and Q… that is, the space decreases" (line 341). As can be observed along the whole transcript (see Appendix), Carlo has many difficulties in expressing himself thoroughly with verbal utterances. However, he often accompanies his vague words with gestures, which shed some light on his stream of thought. In line 341, he performs two gestures. The first one (screenshot a) is a sort of pinching gesture, performed with pointed index and thumb, and it indicates that the Delta x considered is small. The APC team has observed this gesture many times during the teaching experiment; it is performed both by the students and by the teacher. It has been called "Delta gesture" (Arzarello et al. 2009; Sabena 2007), since it usually appears co-timed with utterances referring to increments of the *x* or the *y* variables. It shows iconic features with respect to a segment in the Cartesian

plane and it is rooted in the students' activities with the finite differences of functions in the previous year. In this episode, the two fingers appear very close to each other, since the attention, as the teacher prompts (line 340), is directed to consider the Delta *x* as becoming small.

The second gesture performed by Carlo in the same fragmented sentence is constituted by his open right hand moving vertically from the bottom upwards (screenshot 341b). It refers iconically to the tangent line in the right part of the screen (as the outcome of making the Delta *x* tending to zero).

The teacher pushes towards the idea that the secant is becoming a tangent (line 342). The two students react to the teacher's prompt in two different ways: Giovanni (in line 343) using words, and Carlo using an iconic gesture (screenshot 343), which anticipates Giovanni's words (and which is not seen by Giovanni, who is looking at the screen).

9.2.7 Lines 344–349: The Limit as Δx Tends to 0

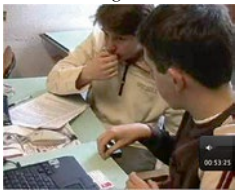
In Episode 3d starting with line 344, the teacher asks an open question: "and then what kind of information will it give you in this case?", which has an immediate effect on Giovanni – instead of explaining to the teacher what he already knows, he is now expanding his knowledge using verbal and gestural ways of expressing his thinking (lines 345, 347, and 349):

345 G ah, one can say [*gesture (a)*]… one can say that [*so far G has kept the gesture, while looking at it silently].*
[gesture (b)]
the exponential function <u>becomes</u> [*gesture (c)*]
very <u>little</u> [*gesture (d)*]
lines…

Gesture in 345 (a)

Further Gestures in 345:

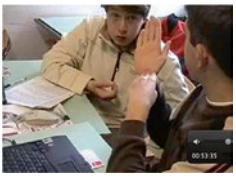
(b) G joins his fingers on the desk and traces a trait rightwards



(c)

(d) G moves his right hand little by little upwards

Giovanni's gestures sequence rightwards is repeated twice.

346	T	uh… it could be approximated to some small lines, which however…	
347	G	that is [*gesture (a)*], that…with increasing slopes [*gesture (b)*], that join together [*gesture (c)*] in a, that touch each other in a point [*gesture (d)*]	*Gesture in 347 (a): G's two-hands configuration* 

Further Gestures in 347:

(b) G's right hand moving upwards

(c) G's left hand touching the right palm

(d) G's left index touching the right palm

348	D	therefore you are imagining to approximate with many small segments	
349	G	well [*gesture (a)*],	*Gesture in 349 (a): initial phase of Giovanni's "Delta gesture"* 
		if you take it… I don't know, if you take it with a very large zoom… you can approximate it with many small lines [*gesture (b)*]	*Gesture in 349 (b): G final phase of the Delta gesture. The gesture has been kept during the whole sentence, a little larger and moved rightwards and upwards with higher slope (as before the right hand).* 

This episode has a greater degree of complexity than the previous ones because construction of new knowledge occurs during interaction between the students and the teacher, and because of the degree of complexity of the knowledge under consideration. Moreover, part of this episode (specifically the utterances in lines 344–347) was identified by both teams as central for their analysis of the learning process. Therefore, the collaboration of the two teams on this episode was more one of parallel or even common analysis than of APC ideas supporting the AiC analysis. For these reasons, our presentation intertwines the two analyses, while pointing to the origin of some of the interpretations in AiC or in APC.

Giovanni intends his gestures in line 345–349 to be seen by the teacher; this can be concluded from the orientation of his body. The gestures are part of Giovanni's discourse to the teacher; they are communicative (as opposed, for example, to his gestures in line 316).

We have already commented on Giovanni's catchments in lines 316 and 331 and have discussed their role in the consolidation of C_{11}/C_{11}'. Similarly, we have commented on the two catchments – the repetition by Giovanni of the gesture with the palm (screenshots 345a, c, d and 347a–d) and his repetition of the gesture with two pointed fingers (screenshots 345b and 349a, b) – saying that they support the consolidation process. In fact, McNeill describes catchments as a "thread of recurring gestural imagery" (McNeill 2005, p. 19): as such, they show how language and imagery can contribute to making sense of the mathematical concepts through their dialectic. Through the blending of imagistic and discursive aspects, catchments can contribute to making apparent the new concepts; namely, they have an epistemic function, because they contribute to constructing knowledge (this aspect is grasped and underlined by the teacher, in his comment to Giovanni's productions in line 348).

In Giovanni's gestures in line 345, the APC team recognizes two catchments, expressing two approximation processes: (i) on the left, namely that the function is approximated by the line $y = 0$ when x tends to $-\infty$; (ii) on the right, namely that small slices of the tangent approximate the function also in this case. They have an iconic feature, insofar as both represent some aspects of the relationship between the graph of the function and that of a line (resp. the line $y = 0$, and the tangent to the function's graph). But they have also a metaphoric aspect, which is expressed through the repetition of the gesture: the two catchments capture the limit process through the dynamic character conveyed by the repetition of the same gesture. In this sense catchments may indicate the epistemic character of gestures.

Specifically, Giovanni's second gesture in screenshot 345b repeats the gesture previously made in lines 316 and 331. The third gesture in screenshot 345c repeats the gesture previously made in line 337. This latter gesture is then elaborated in line 347, when the hands move vertically, representing the movement of the line along the exponential function (which can be observed in the DGS file).

The AiC team observes that the second gesture in screenshot 345b appears to be very similar to the earlier gestures but it occurs in a different context. This gesture seems to show a decreasing interval. Earlier (in line 316 and in line 331), the decreasing interval was QH. Here (in line 345) the decreasing interval is the interval

on which the exponential function is taken to be approximately straight. In other words, the gesture is now associated with the more general meaning "tends to zero." Hence the catchment expresses much more than repetition and more than consolidation – it expresses a generalization of the context in which Giovanni sees and applies the notion of convergence to zero.

Giovanni continues gesturing: he shows the tangent line, repeating his gesture from line 337. Both in words and gestures, there is first one tangent, and then many tangent bits. Hence, there is more than catchment here: in the second repetition, the meaning of many tangent bits is added to that of tangent. Giovanni refers to "the exponential function becomes very little lines" (line 345) and at the same time holds his right hand up, moving it in a way that is clearly not smooth and conveys quite well different secant or tangent bits at different places. This is a considerable mental jump that has been made by means of the hand – from a single tangent to a sequence of tangent segments that join together to approximate the exponential function. The gestures clearly express that there is a construction of knowledge. At the same time, the language is evolving and becoming more elaborate. It is difficult to tell whether his own hand movements or the teacher's "it could be approximated to some small lines" (line 346) allowed Giovanni to express himself more clearly in line 347, adding that with increasing slope the bits join together, and in line 349 that they approximate the function. Most probably, it was a combination of both. In any case, this is another instance where Giovanni at first lacked the words to express what he saw in his mind and hence another case of significant gestures supporting his construction of knowledge. Here Giovanni constructed C* ["The exponential function can be approximated by many small lines with an increasing slope that join together"], making the transition from the previous local construct – the geometric representation of the derivative is a tangent – to a global view of a continuous, piecewise linear approximation to the exponential function by joining together many small tangent line segments whose slope increases monotonically. This concludes the AiC analysis.

Meanwhile, the APC analysis continues: an analysis of this last segment of Episode 3d (lines 345–347), in view of theoretical notions proposed by McNeill, reveals another aspect of the way gestures can contribute to the production of abstraction in context. This segment is composed of three successive components:

(a) first a gesture from the desk to the air with the right hand (screenshot 345a);
(b) then the repetition of the gesture (pinching gesture on the table) of a previous set of catchments (which referred to the graph where $x<0$, but now the attention is in the part where $x>0$) (screenshot 345b);
(c) finally, the two hands are raised in the air keeping them with extended fingers touching each other: the left hand is moved to touch the right at a certain angle (screenshot 345c); the teacher echoes Giovanni's words and prompts for further elaboration (line 346); Giovanni repeats gestures (screenshots 347a–d).

Only towards the end of the episode is Giovanni able to express in words what he has intuited and represented in the second catchment. In the language of McNeill, the gesture in screenshot 345a can be interpreted as an index of a "growth point."

According to McNeill (2005), a growth point (GP) marks the starting point for the emergence of newsworthy information prior to its full articulation. A growth point combines both imagery and linguistic components in a dialectical way: "A GP contains opposite semiotic modes of meaning capture – instantaneous, global, non hierarchical imagery with temporally sequential, segmented, and hierarchical language" (McNeill 2005, p. 18). In a growth point, the two modes are simultaneously active in the mental experience of the speaker, creating a dialectic, and, therefore, a sort of instability. The process ends when the growth point "is unpacked into an increasingly well-formed, hence increasingly stable, structure on the static dimension" (ibid., p. 18). The unpacking of the growth point provides a resolution of the dialectic; this resolution is shown by a linguistic form, often accompanied by a gesture: "Images vary materially from no apparent gesture at all to elaborate multi-dimensional displays; but, hypothetically, imagery is ever present. What varies is the amount of materialization" (ibid., p. 18).

An unpacking of the gesture in screenshot 345a is given by the words that accompany the gestures in the screenshots 345c and 347d where Giovanni expresses the idea that small slices of the tangents approximate the graph of the function. The index finger is touching and almost pushing on the hand (screenshot 347d): the gesture expresses in a global way both "touch" and "point" (this second meaning is anticipated in the gesture, with respect to words). The growth point marks the starting point of this refinement process: Giovanni first recalls his previous idea on what happens on the left part of the graph with the pinching gesture (screenshot 347a), then changes his focus to the right part of the graph and uses his peripersonal space (i.e., the space being immediately around the body) in the air to represent his refined ideas about the tangent. Possibly the echoing and prompting words of the teacher encourage him to finally express his intuition in words. This concludes the APC analysis.

9.3 Comments on the Analysis

In the specific case at hand, our analysis of the constructing actions focused on the role of gestures in the construction of knowledge. From the semiotic bundle methodology, the AiC team learned that modes of expression tend to be strictly linked with each other, and that the interpretation of one of them is linked to the interpretation of the others. Specifically, we have paid particular attention to utterances that are accompanied by significant gestures and to gestures that invited utterances and helped learners to formulate their thoughts. According to the AiC perspective, gestures are significant if they do more than underline the importance of the speaker's words or point to a specific object that is intended by speech (e.g., "this"). In some cases (e.g., line 337) this function is mainly communicative; in other cases (e.g., lines 302 or 316) it includes the students' attempts to clarify thoughts to themselves, and hence to contribute to a constructing action: this may give the gesture a crucial role in the construction of knowledge. In still other cases

(e.g., line 347), gestures are communicative while contributing to a constructing action, and hence significant for the social construction of knowledge.

The role of gestures in the constructing process is a double role: on the one hand, gestures with an epistemic function support and possibly influence the constructing process by allowing the learner to realize the shape or movement or other spatial aspects of the object "under construction." Showing an aspect of the construct kinesthetically by means of a static or dynamic gesture may support the learner in mentally creating that construct. On the other hand, gestures may also raise the learner's, the teacher's, a peer's, or a researcher's awareness of the constructing process, thus obtaining a communicative function in addition to the epistemic one. Since gestures may be anticipatory while words appear only later in the constructing process, gestures may draw the attention of a teacher or peer to the learner's thinking. Similarly, gestures may help the researcher to identify and interpret the initiation, the end, and other features of the constructing process.

As a side remark, we note that our case study illustrates the methodological nature of data. The data we use here are based on recordings that were made during the relevant class period. These recordings include videotapes and students' written productions. However, the videotape is not the data, nor is its transcript; in fact, the original transcript included the words as spoken by the students but no mention of their gestures. In view of the expected analysis, we revised the transcript so as to include gestures (and we may have disregarded information that could be of interest to other researchers with other research aims). Hence, the videotape became data for us once we transcribed it with focus on verbalizations and gestures.

In order to detail the possible roles of gestures, we revisited some of the excerpts discussed above: while Giovanni gestures in line 302, Carlo seems to pay no attention to the gesture; and while Carlo gestures in line 311, he does not look at Giovanni, nor does Giovanni seem to even notice Carlo's gesture (he looks at the screen). We infer that, at this stage, the function of their gestures is to illustrate or clarify the mathematical objects and their behavior to themselves rather than to communicate to the other. This strengthens our argument that the gestures play a role in the construction of knowledge of the learner as he reflects without necessarily communicating with another person. Similarly, in line 316, we noted above that the gestures, though explicit, seem to be almost automatic, expressing, together with the words, Giovanni's thinking. Carlo did not look at the gestures and the gestures did not have a communicative function.

In line 316, some aspects of the mathematical situation are expressed by means of words and gestures, others by words only, and still others by gestures only. Neither the words alone, nor the gestures alone, would have been easy to interpret. It is in the multimodal combination that they lend us confidence that our interpretation is accurate. Hence, this is a case where gestures helped the researcher interpret the constructing process.

Repeating gestures have several functions and cases. From the AiC point of view, repeating a gesture such as Giovanni's repetition of his 316 gesture in line 331 and again in line 345 seems to indicate consolidation, which in this specific case now occurs with a communicative function that it did not have before. We surmise that gestures may also (and actually do in the present case) support the learner to more

firmly establish knowledge that is still fragile. However, we are much more fascinated by another repetition, namely Giovanni's repetition in line 337 of his 316 gesture. It is important to note that the 337 context is quite different from the 316 one. In line 316, Giovanni considered the vertical segment QH as x tends to $-\infty$; in other words, his gesture appears to relate to the process by which a (positive) difference of functional values becomes ever smaller. In line 337, on the other hand, he appears to relate to positive values of x and considers the interval in which the exponential function may be approximated by a straight line segment; the underlying idea is that the smaller this interval, the better the approximation, and his gesture relates to this interval, also a positive quantity, becoming ever smaller. While the constructs identified in the a priori analysis are quite different (and some do not explicitly appear in the a priori analysis because they seemed of minor importance to the researchers), Giovanni's gesture indicates the commonality of the two contexts, in both of which a positive quantity becomes ever smaller and potentially tends to zero. This is an unexpected (by the researchers) construct that demonstrates Giovanni's process of abstraction in a neat manner. Such "generalizing catchment" suggests that this gesture attached with the idea of a very small and decreasing interval might become a "standard gesture" for Giovanni in a diversity of contexts where the consideration of small intervals whose length decreases to 0 is relevant.

9.4 Insights from Networking

In the introduction (Sect. 9.1), we explained that only a very small intersection of the available data could have formed a basis for parallel and then comparative or common analysis by the APC and the AiC teams; this small intersection was the transcript lines 344–347. This in itself may be considered as a theoretical insight: in spite of the closeness of two theoretical frameworks, in this case APC and AiC, both of which are socio-cognitive and employ a micro-analytical approach to data analysis, the differences in focus may be such that the two teams tend to concentrate their analyses on different parts of the learning process. Moreover, even in episodes where gestures have an epistemic function, and therefore both approaches have something to say, it is not clear whether the two analyses can be integrated into a coherent picture of the episode. In order to make this issue more concrete, we present a comparative analysis of the transcript lines 344–347 under consideration, based on the interpretations presented in Sect. 9.2.7.

9.4.1 Comparative Analysis of Lines 344–347

The researchers of both teams agree in their analyses that Giovanni expresses, by his words and gestures, significant new understandings in lines 345 and 347. The APC team stresses that Giovanni expresses how he sees, at that specific moment,

the mathematical objects approximating the exponential function. In fact, the sequence of gestures by Giovanni tells us that he is imagining the exponential function as composed of (or "approximated by," as the teacher specifies) many little line segments. This sequence occurs after the teacher, supervising the group-work, asks what happens when Δx becomes very small. The APC analysis takes into account the teacher's intervention and didactic choices. In particular, we observe that what we can call the "didactic memory of the students" (in analogy to that of the teacher, studied by Brousseau and Centeno 1991) can play a role in the building of new knowledge and in linking it to the "didactical past." More precisely, some signs emerge from the past history of the students and help them in picturing and acting on the new situation. In fact, in the previous year they had used and shared the "Delta gesture" to indicate a difference of values of a function for different values of the x's that increased at a constant step (they were studying functions through the so-called finite difference method). This recollection that emerges from their didactic memory is used as a sign to represent the mathematical object and as a tool to enter the new situation: for example, from this sign and through its modifications Giovanni starts his reasoning about the properties of the function a^x. By considering the teacher's didactic choices – the approach to functions made via finite increments, analysis of the function behavior through the behavior of its tangent line (whose slope is easy to compute) – we may explain Giovanni's view of the function, and why the students thus "read" and "see" a function graph as composed of or as approximated by many little consecutive segments: they see the graphs/functions through their increments. Keeping the x-increment constant (as usually done in previous activities), it is the y-increment that expresses the increment of the function, and gives information on the slope. The almost omnipresence of the gesture with two fingers extended (the "Delta gesture"), which has been shared in the classroom since grade 9 in activities in which functions were studied using the finite differences technique, is thus related to this modality of seeing the functions.

The AiC team with its focus on the learners and their cognitive processes stresses the newness (to Giovanni) of viewing the exponential function as a sequence of little tangent lines. The two teams perceive these utterances with gestures as inserted differently in the flow of the students' activity. The AiC team connected it to the preceding focus in line 337 on the appearance of a tangent as Δx tends to 0, and the consequent view of the little lines as bits of tangents, whereas the APC team related it to the teacher's choices, mainly the choice of approximating a function by finite increments, in which the x-increment is first being kept constant, and later made tending to 0. For AiC, the teacher and the students' previous experiences are considered as an important part of the context that may influence the learner's process of construction of knowledge. Thus, while this choice imposed by the teacher (and ignored by the AiC analysis in this chapter but not in Chap. 10) can explain what the AiC team perceived as "a considerable mental jump from a single tangent to a sequence of tangent segments," the APC team uses the connection to the previous occupation with tangents to explain the nature of the line segments as seen by the student.

Similarly, different functions of the gestures are considered by the two teams: while the AiC team focused on the epistemic function of Giovanni's gestures in their potential contribution to the elaboration of his understanding, and the support they give to his increasing power to express himself better in words, the APC team focused on the communicative function of the gestures, Giovanni using them to transmit his understandings to the teacher. Neither the difference in the way the two teams – AiC and APC – see the excerpt as inserted in the flow of the activity, nor the difference in the way the two teams see the function of the gestures, leads to contradictions. Indeed, quite the contrary: they complement each other and point to failures in each team's analysis to grasp and describe the complexity in a more comprehensive way. In this sense, the analyses have been coordinated successfully. Together, the two analyses provide far deeper insight than each one separately. We point out that the additional understandings contributed by each analysis (and hence by each theoretical framework) to the other one happened in a case where the two approaches are similar in that they both consider evolving cognitive and social aspects of the situation by means of micro-analysis of an enhanced transcript (enhanced by a description of the gestures). It is thus not surprising that the analyses are compatible, but it is surprising that they are nevertheless so different and complement each other so extensively. Hence, we were led to the question of when a gesture is meaningful for APC and when it is meaningful for AiC; an outcome of this question is the notion of epistemic gesture to be discussed in the next subsection.

9.4.2 Epistemic Gestures

In our networking process, we considered a gesture to be significant if its epistemic function is to contribute to the construction of knowledge. We call such gestures *epistemic gestures*. Examples of epistemic gestures are those in lines 302, 311, and 316. In fact, we noticed in some of these instances that while Giovanni gestures (line 302), Carlo seems to pay no attention to the gesture; and while Carlo gestures (line 311), he does not look at Giovanni, nor does Giovanni seem to even notice Carlo's gesture (he looks at the screen). We infer that, at this stage, the function of the students' gestures is to illustrate or clarify to themselves the mathematical objects and their properties rather than to communicate to one another. This strengthens our argument that these gestures play a role in the construction of knowledge. Hence, these are epistemic gestures par excellence. The gestures analyzed later (those in lines 337 and 345) occurred during a conversation with the teacher and serve, at least in part, a communicative function, as has also been observed by the APC team; in such cases, it is more difficult to decide on the epistemic function of the gesture, but we venture the claim that at least the gestures in screenshot 345d, where Giovanni says "the exponential function becomes very little lines" while moving his right hand little by little upwards, serves both a communicative and an epistemic function in that it allows Giovanni to create, in his mind's eye, the image of the approximating sequence of tangent line elements.

We conclude that epistemic gestures may, but need not necessarily, serve a communicative function; using notions from AiC, one of their characteristics is that they form, often together with verbal expressions, an inseparable part of the students' reorganization of their knowledge into a new construct. This justifies the term "epistemic gestures."

To the APC team, the meaningfulness of a gesture emerges from two sources: (i) the relationships with the other signs in the semiotic bundle (for instance, a gesture may be genetic with respect to a written sign; or it can add meaning to co-occurring words); and (ii) with respect to the evolution of mathematical meanings in the activity. An example is the Delta gesture, which in this episode is associated with the local approximation of the function by means of little tangent lines. Thus, pointing gestures may also be important for the semiotic bundle, as well as repeated gestures (catchment), which may provide hints of the learners' line of thinking.

While these criteria are formulated in different terms and stress important aspects that have been neglected by the RBC analysis, they are fundamentally consonant with the AiC characterization: source (i) has appeared in a natural way also in the AiC description above since it is usually only in combination with verbal mode that a gesture can be identified as being epistemic. Source (ii) refers, just as AiC does, to the meaning students associate with the mathematical objects they deal with, but adds depth by stressing the evolutionary aspect of these meanings more explicitly than AiC. In summary, while there is a great deal of resonance between the ways AiC and APC consider the epistemic function of gestures, the two approaches mutually enrich each other and hence the interaction between the teams allowed us to deepen the analysis. The notion of epistemic gestures is an excellent example of this.

9.5 Reflection

Our networking process was driven by a common but somewhat vague research question (Q in Radford's 2008 triplet P, M, Q), what gestures can contribute to the process of constructing knowledge. This question obtained more definite forms as the work progressed: Can gestures have an epistemic function that supports the construction of knowledge? Can gestures that have an epistemic function be characterized? Can the specific epistemic function gestures play in a particular constructing process be identified?

The two teams strove to provide some elements to answer these questions, by analyzing chosen episodes, at first each team according to their own methodology, and then by combining the analyses (in terms of the networking landscape, see Chap. 8). Since this was not successful because of the differences in research questions and in choices of data for analysis, a need for coordinating arose. For this purpose, we were looking for excerpts from the transcripts that were of high interest to both teams.

The approaches of both teams are fundamentally interested in student cognition (and additional aspects), and both employ a micro-analytical approach to data analysis. Hence it was surprising that we faced difficulties in choosing excerpts for common analysis that could then be coordinated. These difficulties can be explained by differences in the underlying principles (P) of the two approaches:

- APC is most interested in the evolution of signs in the social interaction, which includes both the teacher as an intervening subject and the didactic choices of the teacher in the classroom (i.e., the social and cultural dimension, according to a Vygotskian perspective). In other words, the teacher and the didactic trajectory are considered as part of the analyzed elements.
- AiC is most interested in the evolution of meanings of single students or small groups of students, within a socio-constructivist perspective; the didactic choices and the teacher's interventions are considered as part of the context.

As a consequence, the result of the networking process consists mainly in an exchange at a methodological level (M) that led to a local integration of the semiotic bundle tool into the AiC methodology. The networking process did not progress beyond the methodological level, and we suggest that the differences between the principles (P) account for this as well.

As a result of the networking process, we found that there are gestures with an epistemic function and that some gestures that are relevant for analyzing the construction of knowledge belong to this category of epistemic gestures. In some cases, it was necessary to include epistemic gestures into the RBC analysis as potential epistemic actions; in other cases, including epistemic gestures as potential epistemic actions enriched the RBC analysis. This led to a broadening of AiC, some aspects of which will now be discussed. We do not distinguish between methodological and theoretical aspects because there is no clear borderline between methodology and theory in AiC (Hershkowitz 2009).

The very notion of the epistemic function of gestures obtained its importance through the networking process described in this chapter. This epistemic function is perceived somewhat differently by APC and by AiC. AiC tends to consider the epistemic function of a single gesture within the process of "thinking" or constructing or even of formulating. APC tends to consider the epistemic function of a sequence of gestures in the overall flow of ideas within the social interaction. Both approaches have gained from this an added point of view. The stress of the SB analysis on the evolutionary aspect of meanings led to an important benefit for AiC: looking at the meaningfulness of a gesture with respect to the evolution of mathematical meanings in the activity stresses the evolutionary aspect which is crucial for AiC.

As compared with earlier RBC analyses, the evidence we admitted and paid attention to in the present analysis was broader since gestures were considered as potential indicators of epistemic actions. There was also a change in the questions we were asking, such as: How and why did a student use gestures instead of or in addition to words? How did this help the student to form ideas? Were gestures repeated or modified along the constructing process? What thought processes may

be expressed by repeating gestures? Were gestures, like words, becoming more and more elaborate and clear? These were additional questions the AiC team asked, not questions that replaced previously asked questions. The analysis presented in Sect. 9.2 shows how the interpretations and answers to these additional questions enabled a deep analysis of the knowledge-constructing process with respect to both abstraction and consolidation.

In the analysis, gestures played an eminent role in deciding whether a constructing action occurred; repeated gestures, and in particular gestures repeated in a different context, had an especially important role expressing generalization. Gestures may have a distinct advantage over words in this respect since they may be repeated as is, whereas somewhat different words are likely to be chosen in a different context.

Repeated gestures have been interpreted as a sign for consolidation. We have discussed this in detail in the case of line 331 where the repeated gesture for a decreasing interval is accompanied by more precise language, a prime criterion for consolidation in previous research. While the language is becoming more precise, the gesture in its repetition becomes more evident. McNeill gives descriptions about catchment that are reminiscent of consolidation: the repeated gesture becomes more elaborate, more abstract with repetition (see the discussion above about catchments).

In some cases, AiC researchers encounter methodological problems in analyzing groups of students due to a dearth of information on particular students. Those examples of collaboration between the students in which the verbal statement made by one student is illustrated by the gesture of another student help the AiC researchers to better understand the interplay between the social and the cognitive dimensions. An analysis that takes into account the gestures highlights how social interaction, by means of coordination between the gestures of one student and the words of another student, enables the flow of ideas and the development of the constructing process. This comment might be especially useful in those cases in which the AiC researchers analyze the construction of knowledge of a group of learners and decide to consider the group as an entity.

We already mentioned that, according to Davydov, it is typical and expected that constructs start from a vague form and then progressively become more elaborate and precise. Indeed, the view of abstraction underlying AiC is based on Davydov's (1972/1990) ideas, according to which the process of abstraction starts from an undifferentiated and possibly vague initial notion, which is not necessarily internally and externally consistent. The development of abstraction proceeds by establishing an internal structure by means of links and results in a differentiated, structured, consistent entity. Reinforcing previously accepted fragile knowledge by means of gestures, especially when the learner lacks the words to express what he sees in his mind, is therefore consistent with the description of the genesis of abstraction as expressed by Davydov. By means of repeating the gestures the learner is able to further elaborate his previous fragile knowledge.

In spite of the asymmetry of the networking process described in this chapter, the interaction between the teams was not unidirectional. At the beginning of the networking process, the APC team had the strong conviction, based on gesture studies (e.g. by McNeill), that the gestures help the learner to think, and not only to

communicate. After the interaction with the AiC team and the networking process, the APC team could refine their claim by means of the more precise definition of "epistemic gesture." Possible links between McNeill's theory (2005) and AiC theory can be the following: the growth point can constitute the beginning of a building process, and the catchment can be a signal of the consolidation process.

While the analysis presented in this chapter is of a single case study, it raises several questions for further research and might well lead to theoretical developments about the role of gestures for processes of abstraction in the future. Questions that arise include: Are gestures important tools of constructing knowledge in other mathematical content areas, which ones, and to what extent? Do iconic and metaphoric gestures play different roles in the constructing process? Do gestures play an especially important role in processes of abstraction that are related to generalization such as happened in this analysis?

So what case of networking does this case study present? We noticed that the data described in Chap. 2 was insufficient for the AiC team to carry out an RBC analysis. The reason for this is that the data was collected within a different theoretical framework and for a different purpose than carrying out an analysis of knowledge construction. This led the AiC team to learn about APC and to expand their view profoundly. The influence of APC on AiC led to additional methodological tools, insights, questions, and results. In terms of Radford's (P, M, Q) triplet, the AiC team may not have established new principles, but we did ask new questions, and use new methods that led to results and insights. In terms of the networking landscape, we have been coordinating two analyses from different perspectives and then locally integrating them in an asymmetric way, leading from APC to AiC. This greatly enriched AiC but also provided insights to APC. In fact the APC group has been encouraged to study the epistemic function of gestures, which is a new idea for the group. Based on this, the group is now considering fresh aspects of gestures in mathematics (e.g. catchments and growth points, see McNeill 2005), which are considered in the literature of gestures in everyday conversation but are new for entering into the analysis of gestures in order to reveal interesting aspects of mathematical thinking.

This type of asymmetric networking may well be more easily and more broadly applicable by the wider research community than the deeper networking experiences to be presented in the following chapters, because cases where a research team is in need of additional theoretical ideas or methodological tools in order to understand phenomena are frequent. Our case study elucidates what might happen in such a case.

References

Arzarello, F., Paola, D., Robutti, O., & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics, 70*(2), 97–109.

Brousseau, G., & Centeno, J. (1991). Rôle de la mémoire didactique de l'enseignant. *Recherches en didactiques des mathématiques, 11*, 167–210.

Davydov, V. V. (1990). *Types of generalisation in instruction: Logical and psychological problems in the structuring of school curricula* (Soviet studies in mathematics education, Vol. 2; J. Kilpatrick, Ed., J. Teller, Trans.). Reston: National Council of Teachers of Mathematics. (Original work published 1972.)

Hershkowitz, R. (2009). Contour lines between a model as a theoretical framework and the same model as methodological tool. In B. B. Schwarz, T. Dreyfus & R. Hershkowitz (Eds.), *Transformation of knowledge through classroom interaction* (pp. 273–280). London: Routledge.

Kita, S. (2000). How representational gestures help speaking. In D. McNeill (Ed.), *Language and gesture: Window into thought and action* (pp. 162–185). Cambridge: Cambridge University Press.

McNeill, D. (2005). *Gesture and thought*. Chicago: University of Chicago Press.

McNeill, D., Quek, F., McCullough, K.-E., Duncan, S., Furuyama, N., Bryll, R., Ma, X.-F., & Ansari, R. (2001). Catchments, prosody and discourse. *Gesture, 1*(1), 9–33.

Radford, L. (2008). Connecting theories in mathematics education: Challenges and possibilities. *ZDM – The International Journal on Mathematics Education, 40*(2), 317–327.

Sabena, C. (2007). *Body and signs: A multimodal semiotic approach to teaching-learning processes in early calculus*. Dissertation, Università degli Studi di Torino.