

Chapter 8

Introduction to Networking: Networking Strategies and Their Background

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Abstract The chapter provides the language and strategies for networking already published in former *ZDM* issues and books. The concept of networking is clarified and the networking strategies and networking profiles are described. The five theoretical approaches from Chaps. 3, 4, 5, 6, and 7 are compared with respect to the concept of theories as a dynamic way of understanding through the triplet (system of principles, methodologies, set of paradigmatic questions). After that, case studies from Chaps. 9, 10, 11, and 12 are briefly introduced.

Keywords Networking of theories • Methodology • Networking strategies

The comparison of Chaps. 3, 4, 5, 6, and 7 in Part II of this book gave an example of what is meant by the abstract term “diversity of theories”. Five theoretical approaches were presented that differ not only in their key constructs, but also in their main questions, principles, methodologies, and the specificity of the results (Radford 2008, 2012). In Part II, the five theoretical approaches and their research practices were presented next to each other. However, the plurality of theoretical approaches can only become fruitful when different approaches and traditions *come into a dialogue*. For this purpose, different networking strategies have been specified (Prediger et al. 2008b) and applied in various projects. Reflection on these projects has offered interesting first contributions to a methodology of networking (Prediger et al. 2008a).

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According to Radford (2008), this networking process takes place in the so-called semiosphere, which – referring to Lotman – he describes as “an uneven multi-cultural space of meaning-making processes and understandings generated by individuals as they come to know and interact with each other” (Radford 2008, p. 318). Core elements of this cultural semiotic space of mathematics education research are theoretical approaches such as those presented in Part II. Cultural exchange within and between theories unfolds the diversity of theories and shapes the semiosphere’s dynamic nature through individuals as they participate in dialogical processes of meaning-making and exchange. Radford characterizes dialogue as the “door for entering the semiosphere” (Radford 2008, p. 318), but a dialogue between theories may also shape and support the development of the semiosphere. The case studies presented in Part III of this book, namely Chaps. 9, 10, 11, and 12 will give examples of such possible dialogues between theories.

This introductory chapter frames the case studies by embedding them into general methodological considerations. For this purpose, we briefly present the landscape of strategies for networking (Sect. 8.1) and discuss how Part II of the book contributes to making theoretical approaches understandable and comparable (Sect. 8.2). Section 8.3 will give an advance organizer for how the networking strategies will be applied in each case study. Section 8.4 presents a first attempt to classify the different aims and benefits of the case studies through the construct of profiles that will be refined later in Chap. 14.

By this structure, we intend to (1) make clear the meta-theoretical and methodological starting points of the case studies, and (2) give advance organizers for the case studies in Chaps. 9, 10, 11, and 12. In Part IV of this book, Chaps. 13, 14, and 15, we will reflect on what we have learnt from the case studies. This includes some refinements of the constructs offered in the present chapter.

8.1 Embedding: Landscape of Networking Strategies

By networking, we mean research practices that aim at creating a dialogue and establishing relationships between parts of theoretical approaches while respecting the identity of the different approaches (cf. Prediger et al. 2008b; Bikner-Ahsbahs and Prediger 2010; Bikner-Ahsbahs 2010).

Given this working definition, there are still many different ways and degrees to bring theoretical approaches into dialogue. For systematizing and reflecting these ways in a conceptual framework, a landscape of networking strategies has been specified that allows distinguishing between different degrees of integration (Prediger et al. 2008b). In this landscape (Fig. 8.1), the strategies *ignoring* other theories and *unifying* theories in a global way serve as the poles on a scale for the degree of integration. Whereas *ignoring* is often guided by a pure relativism concerning theories considered as arbitrary and isolated, the call for a *global unification* is led by the idea of having one unique theory (that Dreyfus 2006 compared to the grand unified theory of which many physicists dream), both being extreme positions.

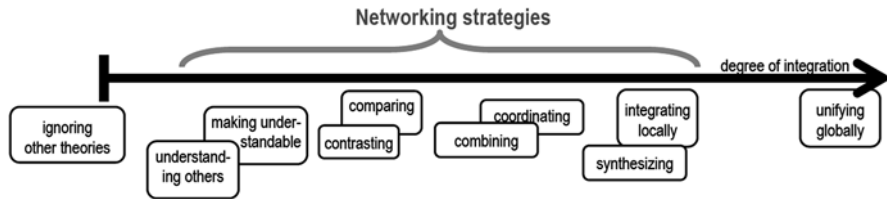


Fig. 8.1 A landscape of strategies for connecting theoretical approaches (Prediger et al. 2008b)

Based on the position that theories are not isolated but can learn from each other, the focus lies on intermediate strategies for finding connections as far as possible (but not further) which can be placed in between the two extremes on the scale in Fig. 8.1. All these intermediate strategies are called networking strategies: “networking strategies are those connecting strategies that respect on the one hand the pluralism and/or modularity of autonomous theoretical approaches but are on the other hand concerned with reducing the unconnected multiplicity of theoretical approaches in the scientific discipline” (Prediger et al. 2008b, p. 170).

In a first approximation, the networking strategies were ordered with respect to the degree of integration of the theories in question. The strategies are structured in pairs: *understanding* and *making understandable*; *comparing* and *contrasting*; *combining* and *coordinating*; and *integrating locally* and *synthesizing*:

- Every attempt to connect theoretical approaches provides the practical experience that it is not trivial to *understand* theories that have been developed in unfamiliar research practices. Hence, all inter-theoretical communication and especially all attempts to connect and apply theories and research results must start with the hard work of *understanding others* and, reciprocally, with *making the own theory understandable*. For understanding a theory, its interplay with the research practices are crucial. Understanding hence refers to all Radford’s (2008, 2012) constituents: not only key constructs, but also principles, questions, methodology, and results.
- The most widely used pair of networking strategies is *comparing* and *contrasting* theoretical approaches. Comparing and contrasting only differ in degree, not in substance. Whereas comparing refers to similarities and differences in a more general way of perceiving theoretical components, contrasting is more focused on extracting typical differences. By comparing and contrasting, the specificity of theories and their possible connections and limitations can be made more visible: strong similarities are points for linking and strong differences can make the individual strengths of the theories visible.
- Whereas the strategies of comparing and contrasting are mostly used for a better understanding of typical characteristics of theories and theoretical approaches in view of further developing theories, the strategies of *combining* and *coordinating* are mostly used for a networked understanding of an empirical phenomenon or a piece of data. Following the idea of triangulation, combining and coordinating means looking at the same phenomenon from different theoretical perspectives

as a method for deepening insights into the phenomenon. The distinction between combining and coordinating is drawn according to the degree of integration of theory elements with respect to their compatibility. *Combining* theoretical approaches does not necessitate the complete compatibility of the theoretical approaches under consideration. Even theories with conflicting basic assumptions can be combined in order to get a multi-faceted insight into an empirical phenomenon in view. In contrast, we use the word *coordinating* when a conceptual framework (in the sense of Eisenhart 1991) is built by fitting together elements from different theories for making sense of an empirical phenomenon. A conceptual framework is not a new theoretical approach but a pragmatic bricolage for the purpose of understanding empirical phenomena.

- Whereas the strategies of combining and coordinating mainly aim at deeper insights into an empirical phenomenon, the strategies of *synthesizing* and *integrating locally* are focused on the development of theories by putting together a small number of theoretical approaches into a new framework. We make a gradual distinction between the two related strategies which this time refers to the degree of symmetry of the involved theoretical approaches. The notion *synthesizing* is used when two (or more) equally stable theories are connected in such a way that a new piece of theory emerges. But often, the theories' scope and degree of development is not symmetric, and there are only some constructs or aspects of one theory integrated into an already more elaborate theory or converted and elaborated into another one. This integration should not be mistaken as *unifying totally*, which is why we emphasized the "locally" in the strategy's name *integrating locally*. We call a local integration symmetric if a concept at the border of two theories is worked out and integrated into both theoretical approaches. The latter may be further developed and result in synthesizing.

Of course, the practical work of applying these strategies is more complicated than the model with its strict distinctions made for analytical reasons. Most researchers apply more than one strategy at once (as we do in Part III of this volume, see Sect. 8.4), and an exact topology cannot be given since the degree of integration always depends on the concrete realizations and networking methods. However, the landscape still serves as a useful approximation towards a conceptual framework for discussing and reflecting research practices of networking and their preconditions. It also provides a frame that can describe the development of the networking process (Bikner-Ahsbabs et al. 2010). In the long term, it may help in approaching methodological considerations for connecting theories.

Prediger et al. (2008a) tried to give an overview of many different methods that can be useful for supporting processes of networking, for example:

- cross-experimentation,
- initiate parallel processes of conceptualizing the same problem into different research problems
- convert a problem taken from one approach into a new approach
- interpret the use and role of a notion in two approaches
- parallel analysis,

- compare theories with respect to their articulation in research on the same topic with different focus and data,
- analyze the same empirical phenomena with different approaches.

This book reports on networking practices that started with the last-mentioned method. As we will see, the initial exercise of analyzing the same video led to other methods of networking, and, in this way, the initial exercise allowed a further elaboration of networking methodologies, that is, reflection on the methods, strategies, limits, and benefits (see Chaps. 13, 14, and 15).

8.2 Making Understandable and Comparing Five Theoretical Approaches

Chapters 3, 4, 5, 6, and 7 in Part II of this book can be read as the authors' attempts to make five theoretical approaches understandable. As these chapters have shown, theoretical approaches cannot be explained by their key constructs alone. Understanding a theory means to understand their articulation in research practices which comprise many implicit assumptions. The reference to the same video of Carlo, Giovanni, and the exponential function (presented in Chap. 2) facilitates making explicit some of the implicit aspects of the theoretical approaches.

The applied theoretical approach and the corresponding research practices do not only shape the conceptualization of phenomena, but also influence what counts as relevant questions, analyzable units of data, and adequate methods to answer the questions. However, it was remarkable that although the task was to analyze given (mostly alien) data, three out of the five approaches (TDS, ATD, and AiC) also referred to the *design* of learning situations and tasks, hence included constructive next to the descriptive considerations as a core element in the research and theory formation. The different priorities for designing learning arrangements seem to have shaped also the typical questions posed in the different theoretical approaches and the methodologies for answering them. This observation exemplifies the fact that the design practices are interconnected not only with the research practices but also with the theoretical approach.

The five analyses of the same video now allow a first comparison of the different theoretical approaches:

- *Size*: TDS and ATD are mature theories with a long tradition and large research communities contributing to their development; these theoretical approaches provide many complex key constructs which have evolved over time. In contrast, AiC, IDS, and APC are younger and more local theories, developed for specific purposes and applied in smaller communities.
- *Questions*: Whereas AiC mainly focuses on the learning of the students (in context), APC and IDS mainly focus on the interaction between teacher and students. In contrast, the systemic and epistemological perspective of TDS orients its questions around the functioning of the complex didactical systems and the search

for fundamental situations, and ATD on different institutional settings and their constraints. For AiC, TDS, and ATD, the research questions are deeply connected to different design practices which are typical for their scientific work.

- *Kinds and units of data*: Depending on the different typical research questions, some theories could immediately start an analysis when having only one video, while others needed more information on the curriculum background, teachers' intentions, etc. before having a suitable unit of analysis. These experiences show that "data" does not exist independently from the theoretical approach; rather, every theoretical approach shapes the kind of data constructed for conceptualizing empirical phenomena.
- *Methodical principles*: The AiC, APC, and IDS teams conduct micro-analyses of learning processes of different kinds. The AiC team executes an a priori analysis to capture the expectations of the designer with respect to the intended constructions and a posteriori analyses to learn from the data what additionally has to be taken into account. The IDS team reconstructs social interactions, epistemic processes, and value attribution and aims at aggregating data to build ideal types. The APC team focuses on the semiotic bundle and its synchronic and diachronic analyses in order to disclose multimodal relationships. The TDS team and the ATD team match different methods; for both, design plays an important theory-driven role. The TDS view encompasses epistemologically conducting a priori analyses of the a-didactical potential of the situations and a posteriori analysis with theoretical reflections including characteristics of the didactic contract. ATD considers praxeologies on different institutional levels taking constraints into account. Hence, the methods and methodologies are deeply related to conceptual and procedural tools the theories offer.
- *Objects*: Theories bring specific areas into focus and at the same time leave others aside, namely the focus on abstraction (AiC), on specifically fruitful situations in classrooms with a potential for learning with interest (IDS), on semiotic resources in classrooms (APC), on the epistemological potential of didactical situations (TDS), and on the anthropological nature of human activities in institutions (ATD). Even when using the same data sets, objects and their areas of attention reflect the diversity of theories.

These first aspects of comparison show that it is the concrete analysis of one set of videos that facilitates the access to the design (of teaching and learning arrangements) and research practices connected to the theories.

For networking these different approaches and their research practices towards a higher degree of integration, further networking strategies have been applied in four case studies. By these case studies, we intend to contribute to the overall methodological question of how to deal with the diversity of theories (Question 3 in Chap. 1), here refined to Question 3':

How can we network different theoretical approaches, that is, what methods, strategies, and meta-theoretical constructs are needed for creating a dialogue and establishing relationships between parts of theoretical approaches while respecting the identity of the different approaches? What can we learn from networking practices empirically, theoretically, and methodologically and where are the limits?

The Networking Group decided not to conduct only abstract discussions on these questions but treat them as empirical (second order) research questions. So we involved ourselves in four case studies of concrete research practices which were supposed to give local answers to these big questions. Among all the different attempts of networking the Networking Theories Group has experimented with (see Chap. 15), this was the most fruitful one for the group's methodological long-term aim: understanding and reflection on strands and issues of networking practices.

8.3 Outlook on the Four Case Studies for Networking

The four case studies in the following Chaps. 9, 10, 11, and 12 each use different networking strategies, each with respect to selected aspects of two or three out of the presented five theoretical approaches. We describe them briefly here, on the one hand as first concretization of the strategies presented in Sect. 8.1, on the other hand as an advance organizer for the core chapters of the book:

- In *Chap. 9* (Case study of the epistemic role of gestures – networking between APC and AiC), the two analyses of a video scene are coordinated with respect to the epistemic role that gestures play in the epistemic process. Gestures played a prominent role in the theoretical construct of the Semiotic Bundle in APC. AiC has learned from APC how to systematically engage in gesture analyses, and has hence locally integrated one aspect of the methodology. In this way, the concept of epistemic gestures emerged. This new concept is an example where the asymmetric local integration on the methodological level of networking led to enrichments of both theories, namely by raising new questions and developing a new concept without touching the principles.
- In *Chap. 10* (Case study of context/milieu – networking between AiC, TDS, and ATD), the networking process started from the common vision that learning and teaching processes depend on the context in which they develop. The idea of context is conceptualized differently by the three theories. A broader notion of the idea of context could be elaborated by comparing the three complex theoretical key constructs of context, milieu, and the media-milieu dialectic. This comparison of related but not equal constructs revealed a deeper theoretical understanding of the key architecture of the three theoretical approaches and the use of data served for illustration and as a base for theoretical reflection. The networking strategy of contrasting allowed the insightful showing of limits of the theoretical approaches and the nature of concepts within their theories.
- *Chapter 11* (Case study of the epistemological gap – networking between APC and IDS) starts by comparing the analyses of the same scene in the video with seemingly contradictory results. By trying to *coordinate* the analyses and to harmonize these contradictory results, the new concept of epistemological gap emerged and was included into both theories. This chapter thus provides an example of the networking process of local integration.

- *Chapter 12* (Case study of the Topaze effect – networking between IDS and TDS): In the first spontaneous data analysis, each group reconstructs different phenomena in the video. The case study *compares and contrasts* two of them within a cyclic networking process of analyzing separately, sharing the results, reflecting on the process, re-analyzing the data, etc. The attempts to *combine* the analysis and the results led to a deeper understanding of the episode and the theoretical constructs involved on the one hand and on the other hand to providing deepened insights into the character of the two conceptualized phenomena and the common empirical idea that the two phenomena try to capture.

8.4 Networking with Different Profiles

When the discussion on networking practices started in the CERME working groups, there immediately arose a need not only to distinguish between different networking strategies but also to distinguish the networking practices by complementary starting points and aims with respect to theoretical and empirical considerations (Bikner-Ahsbahs et al. 2010, p. 164).

The first attempt to draw this distinction resulted in specifying two dichotomic profiles: prototypically, networking practices with a bottom-up profile start from empirical data or phenomena and aim at a deeper understanding of these data or phenomena. In contrast, a top-down profile mostly starts from theoretical considerations and aims at theoretical progress (Arzarello et al. 2008). Although we briefly classify the four case studies here with respect to these prototypical profiles, the reflection on them in Chap. 14 will show that, in reality, both profiles appear in each case, only with different priorities.

The case study on gestures (Chap. 9) consequently follows empirical aims, namely understanding the role of gestures in the video of Carlo, Giovanni, and the exponential function. Two different approaches are coordinated in order to gain insights into the empirical situation.

The case study on the Topaze effect (Chap. 12) also started with the aim of deepening understanding of an empirical phenomenon. In these cases, networking also fulfills the classical purpose of triangulation of data analysis by two theoretical approaches. But in the case of the Topaze effect, the networking went beyond the empirical phenomenon and contributed to a further development of the theories: seeing through different theoretical lenses obliged the researchers to rework the concept of the Topaze effect that had been taken for granted within TDS and to rethink the boundaries of the Funnel pattern as described by Bauersfeld (1978). This development left the theories' principles unchanged. Realizing limitations in the theoretical approaches motivated a sharpening of theoretical constructs *within* these theories, not *between*.

The case study on the epistemological gap (Chap. 11) took the seemingly contradictory results of analyses of the same data as a starting point and through elaborating their understanding revealed a new concept that could be integrated

into both theories. This case study shows the relevance of replication studies and the coordination of results.

In contrast to these bottom-up profiles (starting from the data), the case study on context/milieu (Chap. 10) provides an example for a top-down profile. It starts from three strong theories and compares and contrasts one of the most complex constructs of each. This comparison contributed to making the theories more explicit, especially for AiC, but it also revealed a common aspect all the three theories share.

The chapters are ordered according to their mutual dependency. However, the order with respect to complexity would have definitely placed Chap. 10 as the last one.

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