Chapter 7
Introduction to the Theory of Interest-Dense Situations (IDS)

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Abstract The chapter briefly introduces the theory of Interest-Dense Situations (IDS) by referring to the data from Chap. 2. IDS provides a frame for how interest-dense situations and their epistemic and interest supporting character are shaped through social interactions in mathematics classes distinguishing three levels: the social interactions and how the participants are involved, the dynamic of the epistemic processes, and the attribution of mathematical value.

Keywords Theories • Theory of interest-dense situations

7.1 Theory of Interest-Dense Situations: An Overview

The development of the Theory of Interest-Dense Situations[1] began around the millennium with the assumption that, in mathematics classrooms, the social situation plays an important role in the question as to whether learning with interest is possible or not. This theory was formulated to determine how to build situations with the potential to support learning mathematics with interest in everyday classrooms. The need for this theory came from the lack of knowledge of how to do so.

Interest research, conducted primarily by educational psychologists, had shown that the impact of interest on learning is especially fruitful (Krapp 1992, 2004; Prenzel

[1] The development of this theory is described in detail in Bikner-Ahsbahs (2005).

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1998; Schiefele and Csikszentmihalyi 1995; Schiefele and Schreyer 1994; Csikszentmihalyi and Schiefele 1993). An intervention study in physics classrooms even demonstrated that the usual decay of individual students' interest (Hoffmann and Häußler 1998) could be halted. But the new settings could not be transferred to mathematics. The only advice for mathematics classrooms by Bauer (1988) concluded that mathematics teachers should employ a wide range of approaches in order to give every child the chance to learn with interest. Research concentrating only on individual interest did not seem helpful for establishing teaching criteria for everyday lessons, and various researchers began to realize the important role of the social contexts in class (Baumert et al. 1998, p. 327; Gardner 1998, p. 41; Renninger 1998, p. 229; Deci 1992, p. 45). However, the social dimension of interest development had been neither conceptualized as a learning theory nor empirically investigated. This lack of knowledge resulted in the need to know more about what situations in everyday classrooms have the potential to facilitate learning with interest, specifically in mathematics. Such knowledge was sought by first carefully considering concepts of individual interest from the point of view of social interactions. Doing so sparked a paradigm shift in looking at interest-based learning as a specific kind of social interaction.

Two conceptualizations of individual interest offered starting points for a paradigm shift: (1) interest seen as a person–object relation (Krapp 1992, 2004; Schiefele et al. 1979) foregrounded the content; and (2) situational interest (Mitchell 1993) is determined by situational conditions. The connection of both concepts to self-determination theory (Deci 1998), which argues that interest arises from the experience of competence, autonomy, and social relatedness, thus provided indicators of how to promote learning with interest in class.

As a person–object relation, interest is observable through *actions which are directed towards the acquisition of new insights, connected with positive emotions, and self-intentional; that is, the reasons for the actions are the objects of interest themselves.* This kind of sustained individual interest can emerge out of situational interest (Hidi and Renninger 2006; Mitchell 1993) which is supported by situational conditions but which could disappear if the conditions change. Situational interest can be maintained if students become deeply involved in an activity and experience its content as meaningful (Mitchell 1993). The paradigm shift occurred through a changing emphasis from interest as an individual concept towards a more collective concept created through social interactions.

7.1.1 Principles

The main ideas of the previously described conceptualization of interest are taken as sensitizing concepts to define the key concept of interest-dense situations (Bikner-Ahsbahs 2005). Interest-dense situations are particularly fruitful epistemic situations which can occur in everyday mathematics courses when the learners work cooperatively and intensely to advance their own and their peers' ideas (*involvement*), construct further and deeper mathematical knowledge (*dynamic of the epistemic process*), and highly value mathematical objects or methods (*attribution* *of mathematical value*). These situations are considered as interest-dense because their underlying epistemic processes encourage students to be more attentive and engaged, thus leading to dense social interactions. Engaged learners indicate situational interest when they become deeply involved in and mark the mathematical constructions as meaningful. In this sense, situational interest can be regarded as a pattern of participation in interest-dense situations.

The approach refers to a specific kind of social constructivism (e.g. Jungwirth 2003; Krummheuer 2000; Steinbring 1998, 1999; Bauersfeld 1993) as its *background theory* (Mason and Waywood 1996). Its basic philosophy reflects Weber's (1921, 1922) view that understanding the social world requires understanding people's actions. The background theory also builds on s*ymbolic interactionism* (Blumer 1969) that has further developed Weber's view. Blumer starts from the fundamental assumption that people act according to their interpretations which are a result of, and can change during, social interactions. Learning mathematics is regarded as a process of constructing mathematical knowledge within social interactions, and individuals may co-construct knowledge by participating in and contributing to these constructions.

7.1.2 Questions

The theory of interest-dense situations is a foreground theory with a middle range scope (Mason and Waywood 1996), situated in the background theoretical framework of interpretative research on teaching and learning. Researchers in this field examine and seek to answer three paradigmatic questions: How are interest-dense situations shaped in various teaching and learning situations? What conditions nurture or hinder the emergence of these situations? How is situational interest supported and maintained? In the development of this theory so far, the teacher has played a central role, and data collection has been limited to a single sixth-grade class (Bikner-Ahsbahs 2005) and to primary school students (Stefan 2012), still narrowing its applicability. However, the theory's scope could be expanded by investigating further situations and contents concerning the three paradigmatic questions, for example processes such as proof and argumentation, interactions at different ages, contribution of signs, and technology. To do so, the theory might need to be broadened by theoretically generating new phenomena and concepts.

7.1.3 Methodology and Key Constructs

IDS-methodology entails the principles and key constructs as tools to investigate interest-dense situations, especially their epistemic processes, but it also has to be open-minded towards the idiosyncratic conditions of mathematics classrooms and how they contribute to build and stabilize IDS and, hence, support situational interest. To avoid subsumption and to adapt to the classroom features, its methodology is based on the principle of reconstruction. Data are gathered according to the concept of theoretical sampling (Strauss 1994; Glaser and Strauss 1967), which calls for cyclical theory-driven data collection and analysis. The principles of symbolic interactions guide reconstructive data analysis to answer a specific question by reinterpreting the interpretations of those involved. Understanding of relevant social interactions then yields understanding of a situation. Since we observe phenomena in everyday classrooms, we take the ethnomethodological view that society is reconstructed in daily life and that actions indicate why people act in certain ways (reflexivity assumption of ethnomethodology; see Garfinkel 2003). On this basis, regularities in classroom interactions are empirically reconstructed on three methodological levels stepwise deepening insight: based on *individual involvement in social interactions* (level 1), *the dynamic of the epistemic process* (level 2) is investigated and gained insights are deepened by analyzing *the attribution of mathematical value* (level 3).

These reconstructions demand enough data for identifying and idealizing key features on the three levels for systematically constructing ideal types (Bikner-Ahsbahs 2003; Gerhard 1986) which, according to Weber (1922, p. 190), yields theoretical insight: ideal types characterize specific features in an idealized way and act as tools to build theories or further theories by re-analyzing existing and new data (Bikner-Ahsbahs 2003, p. 212; 2015).

The first of four steps is the basis of building ideal types (ibid. 2003, p. 215; ibid. 2015). It follows the rules of analysis for interpretative teaching and learning research (Jungwirth 2003). Within this methodologically controlled reinterpretation, we systematically utilize the three levels of notions of utterance (Beck and Maier 1994; Austin 1975). The *locutionary level* is the content level of what is actually said. The *illocutionary level* is that of social relations indicated by how something is expressed and how actions and interactions with others take place. The decision to act at all belongs to this level. The *perlocutionary level* concerns the intended and factual impacts of the individuals' contributions.

We now turn to key constructs describing the emergence of IDS. In line with the theory's principles and methodology they first have been developed empirically based and were then used to construct ideal types.

7.1.3.1 Individual Involvement in Social Interaction Structures

Within social interaction, the individual involvement can be characterized by the participants' orientations with respect to teachers' expectations concerning the mathematical content. In classrooms, a teacher normally has aims and therefore expects the students to produce a specific kind of mathematical meaning. However, he/she may either behave *steered by* his/her own *expectations* leading the students to produce what is expected; or *steered by situations* trying to understand the students' epistemic processes. Within the flow of social interaction, the students also may either behave *dependent* on the teacher's *expectation* trying to produce what

Table 7.1 Merging students' and teachers' behavior

students' behavior / **teacher's behavior**	***expectation-dependent*** (reproduces the teacher's expectations of content)	***expectation-independent*** (reconstructs own meaning)
steered by expectations (expects factual answers from students)	*expectation-dominant*	
steered by situations (tries hard to understand students' constructions of meaning)		*expectation-recessive*

the teacher wants to hear, or they behave *independently* of these *expectations* following their own line of thought.

In mathematics classrooms two ideal types of interaction structures that merge the different kinds of individual involvement may be approximately observed. In Table 7.1, the behavior of students and teachers is described according to how they nurture or hinder the arousal of interest-dense situations. If *expectation-independent student behavior* and *situationally steered teacher behavior* mix, an *expectation-recessive interaction structure* emerges in which both teacher and learners concentrate on and support processes of constructing mathematical meaning independently of the teacher's expectations. *It nurtures the emergence of interest-dense situations*. This interaction structure is fragile because the given mathematical situation and current constructions of mathematical knowledge are the only features that allow orientation for students and teacher. The *expectation-dominant interaction structure* appears if the teacher and students are guided by the teacher's content-specific expectations towards a task. It is more stable but hinders the emergence of interest-dense situation because the teacher guides the students in such a way that they produce exactly what the teacher wants to hear, while the students try to figure out what the teacher wants to hear. If an expectation-dominant interaction structure occurs within an epistemic process the emergence of an interest-dense situation is deeply disturbed. The two remaining fields neither represent interaction structures nor do they address IDS; they even may lead to conflicting situations.

7.1.3.2 The GCSt[2] Model of Epistemic Actions

It is a characteristic of interest-dense situations that they entail fruitful epistemic processes within an expectation-recessive interaction structure. These processes are built through three central collective actions executed within social interactions: gathering and connecting mathematical meanings, and seeing structures. Gathering meanings refers to collecting bits of mathematical meaning that are similar with

[2] The actions of gathering, connecting and structure seeing are collective in the sense that they are built by social interactions.

respect to solving the posed problem. Connecting meanings happens if a limited number of collected bits of meaning are interconnected or linked to other meanings. If there are sufficient gathering and connecting actions then structures can be seen, that is, a system of relationships for which many examples can be found. Structure-seeing is absolutely necessary for a learning situation to even be considered interest-dense. Once the epistemic process is reconstructed by these three actions, it is represented with symbols that give an overview of the whole process (see Fig. 7.2 in Sect. 7.2.2).

Because of the expectation-recessive interaction structure, teacher and students orient themselves towards the epistemic process leading to structure-seeing in various ways. Material may first be gathered; this is the foundation for making connections, and after that, structure-seeing can occur. Such a process can also arrive at structure-seeing if gathering and connecting activities are intertwined. Meanwhile we can explain how a general epistemic need and situational interest mutually further each other (Kidron et al. 2011) and, thus, nurture the epistemic process.

7.1.3.3 Types of Producing Valuable Mathematical Ideas

During the epistemic process, a system of mathematical values is shared among teacher and students directing and supporting the joint epistemic process to produce mathematically substantial ideas. This system is based on an implicit agreement: the students follow this system of values in order to produce mathematically valuable ideas and the teacher assists them. This way different production types are constituted; for example in a competition of ideas mathematical patterns are created or in a quality inspection the validity and significance of a fact is examined thoroughly. Students who participate in producing a mathematically valuable idea that is appreciated by others may identify themselves with that idea, create authorship and agency. The teacher supports this process not only by valuing highly those ideas, but also by accepting fuzzy explanations at the beginning. The students can attach their individual meanings to it and advance the process of interaction by making the expressions in question more precise; the teacher accepts and supports this process of clarification for example by explicitly offering terms to support students' expressing (Bikner-Ahsbahs 2004).

7.1.3.4 Further Methodological Considerations

The theory of interest-dense situations is a social constructivist theory that cannot say much about cognitive processes of individuals and does not provide tools for epistemological analyses. It is a theory for classrooms addressing general and specific features. For example, we assume that, if interest-dense situations occur at all in everyday lessons, every mathematics classroom shapes its own specific types of epistemic processes leading to the emergence of interest-dense situations. As a general tool, the GCSt model helps to investigate them and to represent their process structure (see Figs. 7.1 and 7.2 from Sect. 7.2.2.2). Epistemic structures and production types already gained may provide sensitivity for specific conditions that foster or hinder the emergence of IDS in the single given classroom, but this will not always be possible. For applying the key constructs to another classroom further condition might have to be theorized and included, too. The analysis of the video data in the next section will give an idea of how theory expansion of IDS takes place, how its methodology in the use of methods and techniques are applied even if the data do not meet all criteria of IDS methodology.

7.2 Illustrating the Theory of IDS Through Analysis of the Video of Carlo, Giovanni, and the Exponential Function

In the following sections we will first show how the framework is broadened to make IDS applicable to this episode. We will then pose questions that will be answered by analyses of data concerning the three methodological levels: individual involvement in social interactions, dynamic of the epistemic process, and attribution of mathematical value.

7.2.1 Use of the Theoretical Framework

The given data shaped by three subsequent episodes differs substantially from the classroom material used so far; namely, most of it consists of group-work with computers and without the – normally very relevant – teacher. Thus, in order to address the empirical material in the episodes of Tasks 1–3, it is first necessary to modify IDS in order to broaden its scope and make it applicable to the given data set.

7.2.1.1 Widening the Methodological and Theoretical Background

Social interactions are built around objects, such as computers, with mathematical concepts or visual on-screen representations. The two students in the episode will use the computer to construct knowledge objects, of which there are different kinds. According to Knorr Cetina (1999) there are two types of knowledge objects. Intrinsic knowledge objects, such as the exponential function in the video of Carlo and Giovanni (see Sect. 2.1.3), are imperfect. That is, they lack completeness because they are not fully understood or there is something further to learn about them; this lack of completeness drives the need to know more about them, which leads to involvement in meaningful epistemic processes. Hence, these intrinsic objects, which are shared in the group, have the potential to initiate interest-dense situations and to contribute to the formation of interest. Extrinsic objects, such as tools like the computer mouse, are ready for use. If they are being used in the epistemic process, they normally become only visible if they pose obstacles or disturbances.

The two students, the dynamic geometry files, and the worksheet together shape a social object-related group. The epistemic process within the social interaction refers to the exponential function as a shared intrinsic knowledge object which appears to be incomplete to the students, thus encouraging them to learn more about it. Extrinsic objects can be technical or material objects that are ready for use.

7.2.1.2 Research Questions in the Light of IDS

Even though the emergence of interest-dense situations is the focus of this analysis and this chapter, the research questions should be extended to cover other constituents of the specific situation and confirm whether the extended theoretical framework fits the data:

1. How does this group act on mathematical objects? Do the students collectively construct mathematical knowledge through social interactions? How are the students involved?
2. Which are the intrinsic and extrinsic knowledge objects? Are there epistemic patterns due to the specific constituents in this situation?
3. Can the episodes be regarded as interest-dense situations? Are there conditions that foster or hinder the emergence of an interest-dense situation?

The results of our analysis are presented according to these questions.

7.2.2 Initial Data Analysis

7.2.2.1 Individual Involvement: Analysis of the Social Interaction in the Group

In the video on Tasks 1 and 2, Carlo and Giovanni (see Sect. 2.1) refer to the same objects in their activities. They both construct knowledge about exponential functions through their use of the computer and by referring to the images on the screen. However, the activity is distributed. Generally, Carlo gives Giovanni instructions to do something with the computer (lines 9 and 11), and Giovanni (abbreviated by "G") does what Carlo (abbreviated by "C") wants him to do (line 12) – but this is not always the case (for full transcript, see Appendix):

9 C: yes… modify also the measure unit of the y-axis, that is, you put, instead of 2.7, you put another thing
10 G: the y-axis?
11 C: yes… what have you done?
12 G: oh, I have moved it, I have put it larger like so, as you can see…ok
13 C: but you see that, that is, you must modify 2.7 […]

The computer reacts to the students' input through visible signs – drawings, animations, numbers, algebraic expressions. Both students interpret these signs on the screen in their order of appearance. Giovanni describes what can be seen on the screen most of the time (line 48).

47 C: go towards the negative ones
48 G: when it arrives to minus, at 2.7, it goes, it goes in 0… because then you see when it arrives in 0, you can continue to move, but it remains always on the 0

The roles of the students are quite stable, as shown in the following excerpt, in which Carlo reads the worksheet and writes down the results. Giovanni is in contact with the computer. Even if Giovanni does something on his own, he still regards it as a collective activity:

54 G: to −1 it does not yet go on the 0, wait! **Let us** [*emphasis added by the authors*] go, a little bit more −2, 0, 3, 3… more or less towards the 6

The students interact by building their interpretations on each other's. This is shown through their use of the same words (lines 173–174, 177–178, 182–184), their completion of each other's sentences (lines 185–186), or their references to each other's statements (lines 178–179, 185–186):

173 C: you try to put it a little more low… so… you try with 1… you look: with 1 it's a line
174 G: with 1, it's a line
175 C: we expected this
176 G: uuh
177 C: instead, if it's less than 1, also…
178 G: with a less than 1…
179 C: we expected this so
…

Then, it goes on:

182 G: ehh, this is x, and this P's y
183 C: that is the x
184 G: that is the x
185 C: so you can see
186 G: yes, yes… it never touch the zero, it doesn't touch

The signs on the screen also indicate knowledge objects, on which both students focus. Commenting too much does not seem to be worthwhile. Verbal interaction can be reduced, and long pauses appear. Deictic expressions indicate which aspects the students are examining (lines 183, 184). Carlo even says “you can see” (line 185), while Giovanni describes what he sees. Both negotiate what they are seeing at that moment; their social interaction is built on commonly perceived objects.

In their further work on Task 2, and more intensively in their work on Task 3, the discourse becomes denser; pauses nearly disappear. From line 325 on, the situation changes fundamentally as the teacher joins the group. The computer screen becomes just a tool for representations to which the students and the teacher refer only if necessary. The social interactions between students and teacher become even more intense.

7.2.2.2 Analyses on the Epistemic Level and the Level of Attribution of Mathematical Value

In order to be able to reconstruct typical regularities about how interest-dense situations are fostered or hindered in this class, we would need many more episodes. Since this group acts as a unit to construct mathematical meaning socially, the epistemic action model (GCSt model) can be applied to reconstruct the epistemic process of the abovementioned three episodes. The result of the whole analysis is represented symbolically in Fig. 7.2, its legend in Fig. 7.1.

In the video on Task 1 (see first line of Fig. 7.2), initiations and gathering are the main epistemic actions through which the students explore the dynamic nature of the exponential function by experimenting with the DGS file.

In their work on Task 2, the students learn to change the base of the exponential functions through connecting actions and again familiarize themselves with this

Initiations		Gathering meaning	•	Structure Seeing		Structure seeing & justifying, verifying, proving	
Opposing		Connecting meaning		Structure seeing & making concrete			
Gesture	G	intrinsic objects	io	Extrinsic object	eo		

Fig. 7.1 Symbols for the compressed process diagram

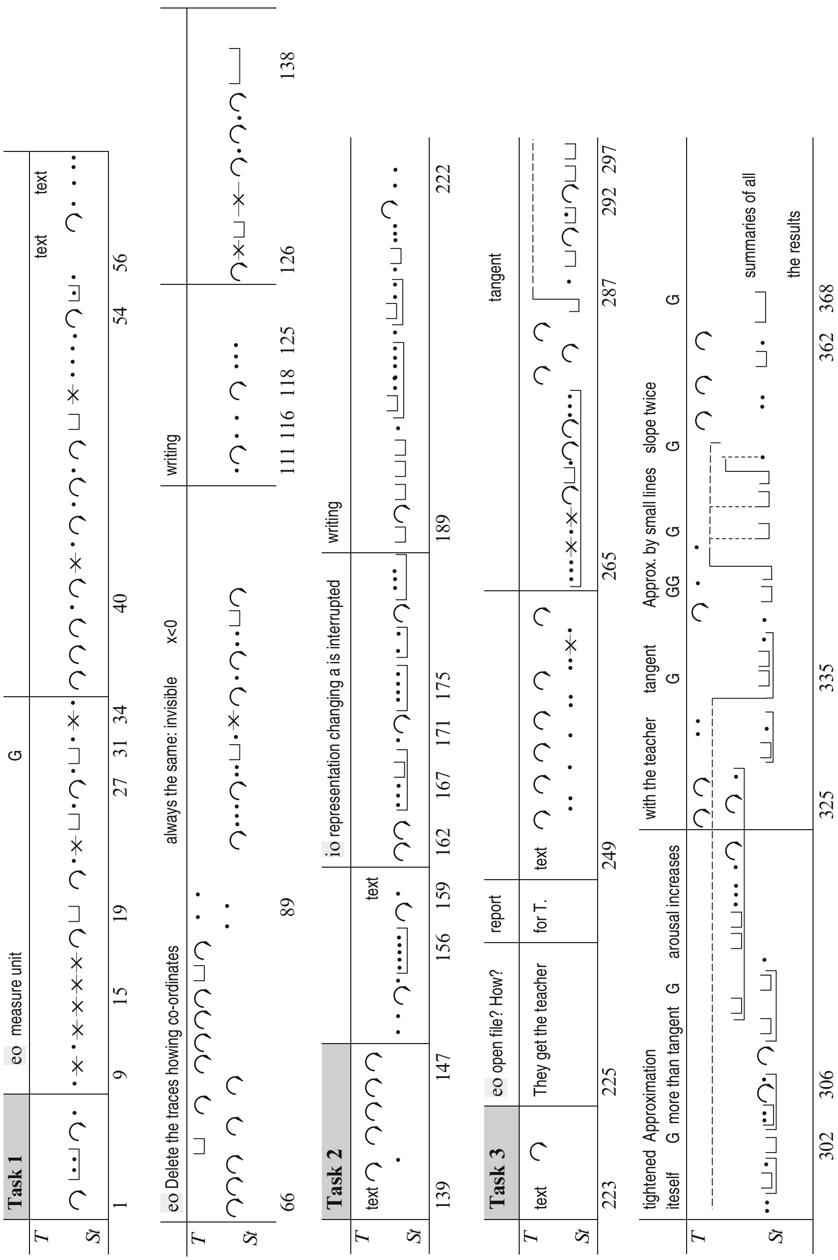

Fig. 7.2 Compressed process diagramme

more complex mathematical situation. Even when gathering takes place here, it becomes part of the connecting actions (Fig. 7.2). As before, the students begin with experimenting and observing. In line 287, the situation changes:

287	C:	look it… slowly… slowly it seems that… I do not know, like, saying tangent
288	G:	eh… yes
289	C:	it seems that it touches it, let's go, let's go, let's go
290	G:	eh, yes… here
291	C:	slowly… slowly
292	C:	it's tangent

Carlo expresses a hypothesis about a certain structure that he sees (line 287), naming it *tangent* and Giovanni agrees (line 288). This is proven through slowly testing the process with the computer. Carlo seems to show increased situational interest (line 287–292) while the computer shows a dynamic situation in which the mathematical idea can be proven. At the end, Carlo sees its structure: "it's tangent" (line 292). The students generate a testing situation through making connections, thus the structure is hypothesized, tested, and labeled.

At the end of the work on Task 3, the teacher joins the group, and the role of the computer in this process changes (327–354). Since the social interactions now take the form of a direct discourse, we will use the three levels of notions of utterances for a discourse analysis.

In line 327, Carlo asks the teacher a question. The teacher does not answer, but instead repeats the principal words as a question: "always the same distance?" (line 328). On the illocutionary level, this gives the students the chance to explore and think again. They then realize that the distance between P and Q[3] is not constant; it may decrease if Q approaches P. In line 331, Giovanni describes the asymptotic behavior of the function: "[…] the nearer P is to y equal to zero, the more this line approximates the function". The reaction of the teacher builds on the previous utterance in trying to understand: "therefore you approach it enough" (line 332); "yes": Giovanni feels accepted and understood (333). In line 334 ("when a function stretches to crush itself on the x-axis"), the teacher adds another idea, but Giovanni on the locutionary level follows the idea of a *nearly tangent* through approximation (line 335–337). The teacher does not disturb this structure-seeing process, but supports it by saying "yes" (336) and then admiring: "uh" (338). Thus, on the illocutionary level, the student's contribution is respected as valuable. In line 340, again, the teacher asks for additional information, directing the students' ideas towards reasoning (on the perlocutionary level): "and so, it gives you some information about what? When the Delta x tends to become very very small, what kind of information do you get?" (line 340). The teacher is successful, since Carlo responds, "if the Delta x becomes small… it means that… the Delta x becomes small when… when between P and Q… that is, the space decreases" (line 341). The teacher strengthens this view: "oh sure, it is almost trivial, isn't it? […]" (line 342) indicating this on the illocutionary level to be important. At the same time he makes clear to look at what is obvious.

[3]Through the points P and Q of the graph of the exponential function a secant is drawn.

The teacher continues, including "tends to" as an additional idea (line 342). Giovanni finishes the teacher's sentence using the word "tangent" (line 343). This means that the epistemological views of the teacher and the students are similar (see Chap. 11). Giovanni is able to think with the teacher. *Tends to* makes sense in Giovanni's train of thought; therefore, he is able to integrate this idea. Again the teacher builds on the utterances before this: "and then what kind of information will it give you in this case?" (line 344). On the perlocutionary level, he asks the students to continue deepening to reason within their own view. Giovanni follows this directive metaphorically: "ah, one can say… one can say that the exponential function becomes very little lines…" (line 345). Again the teacher uses the student's words and changes the direction slightly by adding the idea of approximation: "uh… it could be approximated to some small lines, which however…" (line 346). Giovanni takes this utterance as an invitation (perlocutionary level), and now does nearly the same. He also builds his answer on the utterances of the previous conversation and adds a new perspective of observations about the touch point and the slopes: "that is, that… with increasing slopes, that join together in a, that touch each other in a point" (line 347). The teacher tries to get a better understanding of the students' aim: "therefore you are imagining to approximate with many small segments" (line 348). On the perlocutionary level, this causes an explanation that shows a deeper understanding, adding again another idea – the idea of zooming in and approximating the graph: "well, if you take it… I don't know, if you take it with a very large zoom… you can approximate it with many small lines" (line 349). The teacher now appears to be interested in the students' thoughts, which once again encourages the students to look more deeply and initiates a discussion on the rate of change.

In this situation, the teacher and the students shape an expectation-recessive interaction structure. The teacher's guidance is done implicitly; mathematical value attribution deepens the students' epistemic processes and leads to structure-seeing. As additional results, the following patterns may be relevant for nurturing or hindering the emergence of IDS in the classroom:

1. Learning about an intrinsic object is interrupted when an extrinsic object as an obstacle occurs.
2. Intrinsic obstacles do not necessarily induce deep learning processes between peers. From lines 162 to 167, the students observe that representing the graphs is interrupted when the base becomes negative. Since the students agree that this happens because the graph simply gets too high, they feel content. A deepening of learning does not take place, and an interest-dense situation does not emerge.
3. Cyclic patterns appear in which the computer file is used as a tool to experiment: hypothesizing, testing, observing, describing, re-hypothesizing and evaluating, etc.
4. The students get used to the first DGS file by gathering meaning and get familiar with functions by connecting actions through the second DGS file.
5. Shorthand constructions of meanings change into more complex processes of connecting meanings when the students start to write down their results together.
6. When constructing meaning becomes more complex, interactions between the students become more intense. Gestures indicate that they get nervous. Structure-seeing begins in line 287.

7. Before the discourse with the teacher, constructing meaning takes place around what is happening on the computer screen. When the teacher joins the epistemic process, the computer is no longer the source of knowledge. Structure-seeing occurs within a more complex discourse, and the frequency of using gestures to support statements increases (see the case study on gestures in Chap. 9).

By and large, these patterns indicate that the computer is helpful for gathering and connecting meanings, and the teacher is helpful for deepening insight.

7.2.2.3 Emergence of Interest-Dense Situations

There are indications suggesting that an interest-dense situation arises in Task 3 through the three-step task design. The students do not follow the teacher's expectations. They find their own ways of constructing mathematical meaning. In Task 3, students are pushed further to construct mathematical concepts in an epistemic process that prompts structure-seeing. The students themselves do not value the mathematical concepts that they gained explicitly, but the worksheets they complete do. The students are asked to find things that were unexpected, interesting, and so on. This means that the task evaluates as interesting what the students find out beforehand. At the end, the teacher accepts the results and demands that students deepen them by praising the students' comments and using them to steer their conversation and reasoning.

7.2.3 Need for Extended Data and Analysis

The data set has only been analyzed according to how, in these episodes, the emergence of one interest-dense situation is fostered. The second video on Task 3 offers additional data that shows an extra episode in which the emergence of an interest-dense situation is hindered (Chaps. 11 and 12). If additional data of the class were available, the results obtained could be taken as hypotheses to be further investigated towards an overview about how IDS are shaped in this class.

Due to the reflexivity assumption of ethnomethodology, the teacher's interview is not needed, as everything that is relevant is assumed to be activated during and indicated in the lessons.

7.3 Conclusion

The theoretical framework's scope could be widened so that the IDS methodology could be applied to the given features of the data set. Even though further data analysis is needed to reconstruct regularities about how interest-dense situations in this classroom are shaped, we may hypothesize that a three-step design of the task is particularly fruitful. In this, gathering is the main first action, in which students become familiar with the digital worksheet; connecting is the main part of the second step; and structure-seeing is supported in the third by placing the specific concept to be explored into a computer worksheet. This last hypothesis is related to initiating interest formation through task text: "Describe what is interesting, what is expected and unexpected; share your impressions with the other; explore; give arguments and justifications; and write your results down." In this way, the students' results are valued beforehand, initiating involvement in meaningful epistemic processes. In addition, the digital worksheets are designed to test hypotheses, giving the students control over their results so that they can take responsibility for their own learning processes. Extrinsic knowledge objects are quickly removed so that they hardly disturb the flow of ideas. Finally, in the interest-dense situation of Task 3, when the students are ready to learn, the teacher joins by steering the conversation to deepen students' insights implicitly. Our impression is that there is a whole system of features that foster the emergence of interest-dense situations, not just one.

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