Chapter 6
Introduction to Abstraction in Context (AiC)

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Abstract The chapter briefly introduces the theoretical framework of Abstraction in Context (AiC) by referring to the data from Chap. 2. AiC provides a model of nested epistemic actions for investigating, at a micro-analytic level, learning processes which lead to new (to the learner) constructs (concepts, strategies, …). AiC posits three phases: the need for a new construct, the emergence of the new construct, and its consolidation.

Keywords Theories • Abstraction in context • Epistemic actions

6.1 Abstraction in Context – An Overview

Abstraction in Context (AiC) has been developed over the past 15 years with the purpose of providing a theoretical and methodological approach for researching, at the micro-level, learning processes in which learners construct deep structural mathematical knowledge. Theoretically, AiC attempts to bridge between cognitive and situated theories of abstraction, as well as between constructivist and activity oriented approaches. Methodologically, AiC proposes tools that allow the researcher to infer learners' thought processes. Since we can only give an overview of AiC in the limited space available here, we refer the interested reader to more

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detailed treatments of the theory (Schwarz et al. 2009), the methodology (Dreyfus et al. 2015), and their relationship (Hershkowitz 2009) that have recently been given elsewhere.

AiC is a theoretical framework rather than a full-fledged theory, because its strength lies in suitably choosing and interpolating between elements from cognitive and situated approaches as well as activity theoretical and constructivist elements, and in the development of methodological tools that take these varied aspects into account.

6.1.1 Principles

6.1.1.1 Focus on Abstraction

Understanding the processes by which students construct abstract mathematical knowledge is a central concern of research in mathematics education. In schools, abstraction may occur in a variety of curricular frameworks, classroom environments, and social contexts. The attention to such a variety of contexts requires a hybrid reference to theoretical forefathers that belong to different traditions, Freudenthal and Davydov. Freudenthal (1991) describes what mathematicians have in mind when they think of abstraction. He has brought forward some of the most important insights to mathematics education in general, and to mathematical abstraction in particular. These insights led his collaborators to the idea of "vertical mathematization." Vertical mathematization is a process by which learners reorganize previous mathematical constructs within mathematics and by mathematical means in such a manner that a new abstract construct emerges. In vertical reorganization, previous constructs serve as building blocks in the process of constructing. Often these building blocks are not only reorganized but also integrated and interwoven, thus adding a layer of depth to the learner's knowledge, and giving expression to the composite nature of mathematics.

Davydov was one of the most prominent followers of the historical cultural theory of human development initiated by Vygotsky. For Davydov (1990), scientific knowledge is not a simple expansion or generalization of people's everyday experience. It requires the cultivation of particular ways of thinking, which permit the internal connections of ideas and their essence to emerge; the essence of the ideas and their connections then, in turn, enrich reality. According to Davydov's "method of ascent to the concrete," abstraction starts from an initial, simple, undeveloped and vague first form, which often lacks consistency. The development of abstraction proceeds from analysis, at the initial stage of the abstraction, to synthesis. It ends with a consistent and elaborated form, to which the essence of the ideas and their connections lend concreteness. Hence, it does not proceed from concrete to abstract but from an undeveloped to a developed form.

AiC adopts vertical mathematization and ascent to the concrete as the essential characteristics of processes of abstraction. It investigates how these processes occur in a specific learning environment, a particular social context, and a given curricular context. Giest (2005) points out that Activity Theory is most suitable for this since it proposes an adequate framework for considering processes that are fundamentally cognitive while taking social and other contextual aspects into account. In Activity Theory, individual actions occur in context and make sense only within the activity in which they take place. The kinds of actions that are relevant to abstraction are epistemic actions – actions that pertain to the knowing of the participants and that are observable by participants and researchers. In addition, outcomes of previous activities naturally turn to artifacts in further ones, a feature which is crucial to tracing the genesis and the development of abstraction through a succession of activities that might form part of a curriculum.

As researchers in the tradition of Freudenthal, we are a priori attentive to certain constructs afforded by the activities we observe. In tune with Davydov and a cultural-historical theory of development, we also look at other constructs that may emerge from classroom activities. This is well expressed by Kidron and Monaghan (2009) when dealing with the need that pushes students to engage in abstraction, a need which emerges from a suitable design and from an initial vagueness in which the learner stands:

> … the learners' need for new knowledge is inherent to the task design but this need is an important stage of the process of abstraction and must precede the constructing process, the vertical reorganization of prior existing constructs. This need for a new construct permits the link between the past knowledge and the future construction. Without the Davydovian analysis, this need, which must precede the constructing process, could be viewed naively and mechanically, but with Davydov's dialectic analysis the abstraction proceeds from an initial unrefined first form to a final coherent construct in a two-way relationship between the concrete and the abstract – the learner needs the knowledge to make sense of a situation. At the moment when a learner realizes the need for a new construct, the learner already has an initial vague form of the future construct as a result of prior knowledge. Realizing the need for the new construct, the learner enters a second stage in which s/he is ready to build with her/his prior knowledge in order to develop the initial form to a consistent and elaborate higher form, the new construct, which provides a scientific explanation of the reality. (Kidron and Monaghan 2009, pp. 86–87)

Hence we postulate that the genesis of an abstraction passes through a three-stage process: the need for a new construct, the emergence of the new construct, and the consolidation of that construct.

6.1.1.2 Focus on Context

The C in AiC stands for context. According to AiC, processes of abstraction are inseparable from the context in which they occur. Therefore, it was unavoidable to mention context already in the previous subsection. For AiC, the focus is on the students' processes of construction of knowledge. The "context" integrates any piece of the present and past environment that can influence the individual processes of construction of knowledge. As we show in Chap. 10 on context/milieu there are different approaches towards "context" in different didactic cultures. For AiC, artifacts are conceived as a part of the context. In another theory which privileges the cultural and social dimensions, artifacts are constituents of mathematical activity. For AiC, context has many components. One of them is the social context, often including peers or a teacher; another is the historical context, which refers to the students' prior experiences in learning mathematics; a third is the learning context, which includes, among others, curricular factors, socio-mathematical norms, and technological tools. In any specific activity, tasks given to the students are an essential part of the context.

Chapter 10 of this book deals in depth with the role of context in processes of abstraction. In order to avoid repetitions, we therefore keep this subsection very short and only mention that the context situates processes of abstraction for the learners, while allowing the researcher to focus on the learners' cognitive actions in the given context or situation. Hence, context is the notion that allows AiC to bridge between a cognitive and a situated approach.

6.1.2 Questions

AiC was developed in response to a question that arose in the framework of a research-based curriculum development project (Hershkowitz et al. 2002), namely for what mathematical concepts and strategies students achieved in-depth understanding and retained them in the long term. Hence, the design of task sequences lies at the origin of the questions asked by the originators of AiC, and remains one of their concerns. The research questions AiC attempts to answer include:

- Given a sequence of tasks, what are the intended constructs – the mathematical methods, concepts, and strategies – that the designers intended the students to construct when carrying out the task-based activities? How are these intended constructs structured, how are they related to each other, and how are they based on previous constructs?
- For each of the intended constructs, how did the students go about actually constructing it, and how does each student's construct compare with the intended one? Is it partial, and in what sense?
- Did the students construct alternative or non-intended constructs? Which ones?
- What was the origin for the students' motivation to construct; from where did their need for a new construct originate?
- Which previous constructs were used and consolidated during the constructing process?
- What were the characteristics of the constructing process? Was it sudden or prolonged, continuous or interrupted? Were several constructing processes developing in parallel? If so, how did they interact and influence each other (see, for example, Dreyfus and Kidron 2006)?
- What role did contextual factors play in the process? For example, did groups of students co-construct, and if so, were the group members' constructs compatible in the sense that they can continue co-constructing in the following tasks?

- Did technological tools play a role in the constructing processes, and what role?
- What can we learn from the students' constructing processes about the design of the activities, in particular their micro-design?

As indicated by the last question, one of the aims is to improve the design of sequences of activities, in particular their micro-design. Micro-design includes all local aspects of design from the choice of a particular real-life setting for a task and the potential mathematical limitations imposed by that setting, via the degree of openness of a task, the balance between its qualitative and quantitative aspects, and the degree to which students are encouraged to justify their decision and actions, down to a specific choice of words or a specific formulation of a question.

6.1.3 Key Theoretical Constructs and Methodology

Theory and methodology are closely intertwined in AiC (Hershkowitz 2009). Therefore we cannot describe the key theoretical constructs of AiC, the epistemic actions, without also describing the key methodological aspects of AiC, as the methodology's main purpose is the identification of students' epistemic actions. It is important to point out the dynamic character of the theory: the analyses to identify abstraction processes through the unveiling of its epistemic actions not only helped in the understanding of learners' cognitive processes, the theory as well as the methodology underwent successive refinements (Kidron and Dreyfus 2010a, b; Dreyfus and Kidron 2006). The more technical aspects of the methodology are described elsewhere (Dreyfus et al. 2015).

6.1.3.1 The Dynamically Nested Epistemic Actions Model

The central theoretical construct of AiC is a theoretical-methodological model, according to which the emergence of a new construct is described and analyzed by means of three observable epistemic actions: recognizing (R), building-with (B), and constructing (C). Recognizing refers to the learner seeing the relevance of a specific previous knowledge construct to the problem at hand. Building-with comprises the combination of recognized constructs, in order to achieve a localized goal such as the actualization of a strategy, a justification, or the solution of a problem. The model suggests constructing as the central epistemic action of mathematical abstraction. Constructing consists of assembling and integrating previous constructs by vertical mathematization to produce a new construct. It refers to the first time the new construct is expressed or used by the learner. This definition of constructing does not imply that the learner has acquired the new construct once and forever; the learner may not even be fully aware of the new construct, and the learner's construct is often fragile and context-dependent. Constructing does not refer to the construct becoming freely and flexibly available to the learner: becoming freely and flexibly available pertains to consolidation.

Consolidation is a never-ending process through which a student becomes aware of his or her constructs, the use of the constructs becomes more immediate and self-evident, the student's confidence in using the construct increases, the student demonstrates more and more flexibility in using the construct (Dreyfus and Tsamir 2004), and the student's language when referring to the construct becomes progressively more elaborate. Consolidation of a construct is likely to occur whenever a construct that emerged in one activity is built-with in further activities. These further activities may lead to new constructs. Hence consolidation connects successive constructing processes and is closely related to the design of sequences of activities.

In processes of abstraction, the epistemic actions are nested. C-actions depend on R- and B-actions; the R- and B-actions are the building blocks of the C-action. At the same time, the C-action is more than the collection of all R- and B-actions that contribute to the C-action, in the same sense as the whole is more than the sum of its parts. The C-action draws its power from the mathematical connections, which link these building blocks and make them into a single whole unity. It is in this sense that we say that R- and B-actions are constitutive of and nested in the C-action. Similarly, R-actions are nested within B-actions since building-with a previous construct necessitates recognizing this construct, at least implicitly. Moreover, a lower level C-action may be nested in a more global one, if the former is made for the sake of the latter. Hence, we named the model the dynamically nested epistemic actions model of abstraction in context, more simply the RBC-model, or RBC+C model using the second C in order to point at the important role of consolidation. The RBC-model is the theoretical and micro-analytic lens through which we observe and analyze the dynamics of abstraction in context.

6.1.3.2 A Priori and a Posteriori Analyses

As part of the AiC methodology, an effort is made to foresee trajectories of students' learning: an a priori analysis of the activities (Ron et al. 2010) is carried out before data are collected. Early contacts with the TDS team have reinforced our habit to systematically carry out a priori analyses. Assumptions are first made about the previous knowledge of the students, about constructs they are expected to have acquired during earlier activities (and which may be more or less available to them). Then the question is asked what knowledge constructs are required to deal with each task and to complete it to the designer's or teacher's satisfaction; we also ask what constructs might be helpful but not necessary to deal with the task. We are particularly interested in constructs that have not been relevant in previous activities carried out by the same students. It is our working assumption that the new constructs that emerge for the students when dealing with the task are closely linked to the intended ones. The intended constructs are of course to be distinguished from what students actually construct during the activities, although a close correspondence between intended constructs and learners' actual constructs may be expected if the design and the a priori analysis are adapted to the learner.

The a priori analysis has a considerable influence on the a posteriori RBC-analysis of the data collected, usually by audio and video recordings, from students carrying out the activities. Therefore, we give an operational definition for each intended construct, which fixes under what circumstances the researcher will say that a student is using or expressing a construct that corresponds to the intended one. One aim of the a priori analysis is to focus, at least initially, the researchers' attention on the intended constructs, while keeping an open mind for possible alternative or unintended constructs during the ensuing a posteriori RBC micro-analysis of students' knowledge-constructing processes.

6.2 Illustrating Abstraction in Context in the Case of Carlo, Giovanni, and the Exponential Function

The aim of this section is to illustrate the main notions of AiC as introduced above by means of an excerpt from the work of Carlo and Giovanni. However, for reasons to be explained below, this aim can only be partly realized.

6.2.1 A Priori Analysis

As usual in AiC research, we begin with an a priori analysis. Chapter 4 includes an a priori analysis for Tasks 1 and 2, carried out by the TDS team. They identified nine constructs C_1–C_9, and we assume that had we carried out such an analysis, we would have ended up with a similar list of constructs. We therefore adopt their analysis, and continue here with an a priori analysis of Task 3. Task 3 is very open, and therefore there are not many detailed indications about the constructs that might have been intended by the designer and/or teacher. However, given the quantities that can be varied in the Dynamic Geometry Software (Fig. 2.3, Chap. 2) and the instructions given in the task which relate to this variation, we propose the following constructs as those which the designer/teacher probably intended the students to construct:

- C_{10} For any given P, that is, locally, as Δx tends to zero, the slope of the secant tends to the slope of the tangent; the slope of the secants and the tangent are all positive (for $a>1$).
- C_{11} As P moves on the graph, the slopes of the corresponding secants (and hence the slope of the tangent) vary. As x grows (P moves to the right), the slope of the tangent grows (for $a>1$). As x decreases (P moves to the left), the slope of the (secants and the) tangent decreases to zero (for $a>1$).

C_{12} As a increases, the slope of the secant (for given x, P) increases (and consequently the slope of the tangent increases as well). As a decreases towards 1, the slope of the secant decreases towards 0. As a becomes smaller than 1, the slope of the secant (and consequently of the tangent) becomes negative; the function is decreasing rather than increasing. The parts of C_{10} that depend on $a > 1$ have to be adapted for $a < 1$.

We stress that these are the constructs that we as AiC researchers found in our a priori analysis. They are not necessarily identical to what the teacher in fact intended, and they may, of course, be different from what the students actually constructed when working on the task.

In the present case, we learned from the answers of the teacher as reported in Sect. 2.2.2 that the intended constructs resulting from our a priori analysis are compatible with the declarations of the teacher, and that according to the teacher they are within reach of the students, given the previous knowledge of the class and the socio-mathematical norms that are characteristic for the class, such as explorations that favor the production of conjectures and should motivate their validation as well as argumentation in support of conjectures (see the answer to question 1 in Sect. 2.2.2).

We further note that these intended constructs give general formulations and properties of the resulting constructs in the specific case of the exponential function. From the teacher's answers (Sect. 2.2.2), we know that this activity was given as preparation before the notion of derivative had been formally introduced: "The worksheet […] is situated […] before the formal approach to the concept of derivative of a polynomial function. […] The activity intends to clarify the principal features of increasing behaviours and of exponential functions. In particular, it intends to explain the reason why at the increasing of x an exponential of base greater than 1 will increase, definitively, more than any other polynomial function of x, whatever grade of the polynomial. In the project, exponential functions and sequences are used to cope with problem situations coming out from exponential models" (the teacher's answer to question 17 in Sect. 2.2.2). So again, our "guess" was confirmed after the event.

We note finally that as researchers we should always expect students to develop other constructs than the ones provided by the a priori analysis. Here especially, because of the open formulation of the task, we may expect constructs different from C_{10}, C_{11}, and C_{12} to emerge for the students. Examples of such "other" constructs in the present case are the following:

C_{11}' As P gets closer to $y = 0$, the function can be approximated by the secant line.

C* The exponential function can be approximated by many small lines with an increasing slope that join together.

The first of these has been called C_{11}' because it is a complementary construct to (the second part of) C_{11}. On the other hand, C* constitutes a transition from a local to a global view: a construct that seems rather independent of the constructs C_{10}, C_{11}, and C_{12} which were identified a priori; it has therefore been assigned a separate notation. The alternative constructs C_{11}' and C* will play a role in the a posteriori analysis below.

6.2.2 Need for Extension of Data

In AiC, we focus on particular kinds of curricula (see Schwarz et al. 2009) and within these, on tasks with a high potential for supporting the construction of knowledge that is new to the learner. This requires the elaboration of sequences of activities that offer the students opportunities to learn well defined mathematical ideas, for example the notion of integral as an accumulating quantity; or that order is relevant when rolling two dice and therefore getting a 1 and a 4 is twice as likely as getting two 4's, etc. It also requires the elaboration of further activities to apply these ideas as tools in familiar contexts or as tools in contexts that necessitate the elaboration of new ideas. What is common to all these learning aims is that they include adding new connections between students' previous knowledge, hence adding depth to the students' understanding and integrating their knowledge in ways not available to them before. In brief, the design intends to create a didactical sequence aimed at vertical reorganization of students' knowledge.

Most of the tasks that the two students in the analyzed video, Giovanni and Carlo, were asked to work on are not of this kind. These tasks require more phenomenological observation than explanations of the phenomena. For example, Tasks 3a and 3b ask the students to describe the phenomena that occur as Δx tends to zero; these tasks do not require any kind of justification. This suggests that the students had previously experienced the limiting process and were now asked to recall it, and possibly reconstruct it in the case of a new example they may not have dealt with yet; from the point of view of AiC, no new construction was required but the students were offered an opportunity to consolidate some of their previous constructs. In Task 3, the students were asked to "Describe briefly the figure, moving first P, then Δx (changing its length), then A; write briefly your observations on the sheet." This formulation suggests that the students had never explored before what happens when varying the parameters x, Δx, and a, and hence that the teacher intended that, in the course of this exploration, his students would meet situations they had never met before. This would offer the students an opportunity to construct new (to them) knowledge but as long as the requirement is descriptive rather than explanatory or connective, this new knowledge is simply an addition to existing knowledge and no need for vertical reorganization would arise. Even in tasks with more potential, such as studying the effect of changing x on the slope of the linear function that best approximates the function $y=a^x$, the stress in the task formulation is on *how* rather than on *why*. This may have served the teacher's plans: it may have provided a common background for the class to use as basis for a teacher-led whole-class discussion in the next lesson. However, such tasks focusing on phenomenological descriptions are not where we can observe the type of knowledge construction in which AiC researchers are primarily interested. This knowledge construction may then happen during the whole-class discussion. In fact, due to the excellent preparation the students were given, it is likely to happen, but we do not have data about this. Therefore, an AiC analysis of most of the data we have is inappropriate and unlikely to yield results about processes of constructing new knowledge by vertical reorganization.

6.2.3 A Posteriori Analysis

We present here our attempt at analyzing the preceding part of the students' work on Task 3, namely transcript lines 249–301 (see Appendix). Unsurprisingly, we will not be able to identify any constructing actions.

The students start on Task 3 in line 249. Until line 281, they identify parts of the situation on the screen. Only in line 281 do they finally read the task. Until then, a main issue in the discussion focuses on identifying the segment PH with Δx. This identification is not a mathematical relationship but a given of the task. The students need to make this identification in order to get access to the situation, but this is not an epistemic action providing them insight into mathematical connections or relationships. It is a preparatory action and is of interest to us only as such.

In what follows, the students make purely phenomenological observations of what happens as one of the parameters varies, in accord with what they were asked to do in the task. They first seem to vary P; they seem to observe phenomena but do not draw any conclusions; all they say is that QH changes as a consequence of changing P. This could potentially have led to insights such as "the slope changes"; "the slope of the secant changes"; "the slope of the tangent changes"; "the derivative changes" – all depending on the preparation of the students and the requirements of the task. Had the students reached such insights, we would have claimed that they recognized some of their previous constructs as relevant to the present situation, and possibly that they built-with them a dynamic image. But the task does not require such insights and the students' utterances do not indicate such insights. Our interpretation of these utterances is that the students' thinking did not include such insights.

Then, in line 285, Carlo seems to refer to the fact that Giovanni now changes the Δx instead of P. This leads to a mathematically more significant observation that might later become useful, namely that "it approaches slowly … slowly … a tangent" (lines 287, 289, 291, 292). From the point of view of AiC, we might identify this as recognizing a previous construct (tangent) as relevant in the current context. This recognizing action might then act as a seed for a subsequent constructing action, possibly of C_{10}. The role of such seeds for later constructing actions has been discussed elsewhere (Kidron et al. 2010).

Several more observations are made subsequently, namely what happens when Δx increases (lines 294–296) or what is the quality of the tangent approximation when P moves to the left or to the right (lines 298–301). The students correctly observe that as P moves to the left, the approximation is better than when P moves to the right. These observations later become relevant. However, at this stage they are cumulative. They do not require nor provoke any vertical reorganization. They are not used for a purpose like solving a problem or justifying a mathematical relationship, and therefore no building-with actions occur. They do not even qualify as recognizing actions since such an epistemic action, as defined above, implies that the students recognize a specific previously constructed mathematical concept or strategy.

As a consequence of what we wrote in the previous section, the tasks given to Giovanni and Carlo were such that only in very few excerpts of the protocol might an RBC analysis be expected to yield constructing actions; moreover, these excerpts are all concentrated in lines 302–347, and will be analyzed in Chaps. 9 and 10 because they are the same data on which two different networking processes are described in these chapters. Readers who would like to see an RBC analysis that is independent of the data used in this monograph, and demonstrates how such an analysis works in a case where it is appropriate, are referred to the literature, for example to Dreyfus et al. (2015), which focuses on the methodology.

In this chapter, we gave a brief introduction to Abstraction in Context (AiC). We described our view of abstraction as it is grounded in the work of Freudenthal and Davydov, and the notion of context as it is pertinent for AiC. We introduced the idea of epistemic action as it emerges from Activity Theory and the dynamically nested epistemic actions model, which is the key theoretical construct underlying our methodology. In the second part of this chapter, we attempted to demonstrate our methodology using the data of Carlo and Giovanni, and explained why this attempt was only partially successful. We are aware that the present description has its limitations and refer the reader to longer and deeper descriptions available in the literature (Dreyfus et al. 2015, and references therein; Schwarz et al. 2009).

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