

Chapter 15

Reflection on Networking Through the Praxeological Lens

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Abstract In this chapter, the Anthropological Theory of the Didactic (ATD) is given a different status, its lenses and constructs being used for reflecting on the networking enterprise itself. For this purpose, the notion of praxeology first introduced for modeling mathematical and didactic activities is extended to research practices. This extension leads us to consider that the proper level for addressing networking issues is in fact the level of research praxeologies, and to reflect on the collaborative work carried out by the different teams and its outcomes in the light of this perspective. Along the way, we also rely on other constructs, and especially on the ideas of milieu and media-milieu dialectics.

Keywords Networking of theories • Research praxeology • Methodology

In the previous chapters, the Anthropological Theory of the Didactic (ATD) introduced in Chap. 5 has been just one of the theories involved in the networking process of the Networking Theories Group. In this chapter, we give it a different status, using its lenses and constructs for reflecting on the networking enterprise itself, following ideas initially presented in Artigue et al. (2011a). For this purpose, the notion of praxeology first introduced for modeling mathematical and didactic activities is extended to research theories and practices. This extension leads us to consider that the proper level for addressing networking issues is in fact the level

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of research praxeologies, and to reflect on the collaborative work carried out by the different teams and its outcomes in the light of this perspective. Along the way, we also rely on other constructs, and especially on the ideas of milieu and media-milieu dialectics (see Chaps. 4, 5, and 10).

15.1 Introduction

As explained above, in the previous chapters the Anthropological Theory of the Didactic (ATD) has just been one of the theories involved in the networking process engaged around the video provided to the group by the Italian team. It was not given a particular status, and the Networking Theories Group (networking group or simply group in the following) used constructs in some sense neutral with respect to the different theories for organizing the presentation of the different approaches, and for situating its networking efforts. It used for instance the categorization proposed by Radford in terms of Principles, Questions, and Methodologies for introducing the different theoretical approaches, and systematically referred to the scale of networking processes proposed in Prediger et al. (2008) for situating achievements in the four case studies. However, for the authors of the present chapter, the idea progressively emerged that this theory could provide useful tools for approaching the idea of networking itself, and for analyzing the networking efforts of the group and their outcomes.

Why this idea? In ATD, as explained in Chap. 5, mathematical and didactic practices are modeled in terms of praxeologies. A basic assumption in the theory is that this notion of praxeology can be productively used for modeling any forms of human practice, not just those attached to the production or dissemination of mathematical knowledge. If we take this assumption seriously, it should also be possible and productive to model our research practices in such a way, and especially those developed for achieving networking goals. When adopting such a position, immediately many questions emerge: How to express research practices through the (task, technique, technology, and theory) filter imposed by the model of praxeologies? What changes in perspectives does it induce? There is no doubt, for instance, that the fact that in ATD theories are embedded in praxeologies and not treated as autonomous entities leads to questioning of the nature of the networking enterprise itself. What does it mean exactly to network “theoretical frameworks”? Can this idea make sense without considering the whole research praxeologies of which these theoretical frameworks are part? What exactly have the teams involved in the networking group networked? Can such a perspective help in understanding the potential and limitations of the work undertaken, identifying and organizing its outcomes, designing more effective networking practices? What challenges does it raise?

These questions have paved the way for the reflection we have developed and that we invite the reader to share with us in this chapter. In the next section, we will extend the notion of praxeology to research praxeologies, insisting on the dynamic

character of these objects and the crucial role that didactic phenomena play in these dynamics. Then we will use this extension to reflect on the collaborative work carried out by the different teams and its outcomes.

15.2 From Theoretical Approaches to Research Praxeologies

Theories are often presented in a static way as a structured network of concepts (see for instance Niss 2007). In this book, we have adopted a dynamic and operational vision by referring to Radford's elaboration in terms of principles, methodologies, and paradigmatic questions (Radford 2008). Considering theories as elements of research praxeologies is also adopting a pragmatic and dynamic vision of theories, trying to make clear how they inform and shape the practical research work, and conversely how they progressively emerge from it and integrate its results. In this section, we first introduce how research practices can be interpreted in terms of praxeologies, then discuss the connection between their practical and theoretical blocks, emphasizing the bridging role played by didactical phenomena, and illustrating our discourse by some examples taken from previous chapters.

15.2.1 What Is a Research Praxeology?

As any other praxeology, research praxeologies are composed of an amalgam of pieces that can be described by a set of four elements $[T/\tau/\theta/\Theta]$. The pair $[T/\tau]$ corresponds to the practice (or know-how) of research, with the *types of tasks* T that are approached and the *techniques* τ used to carry them out. We can consider that, at its core, the types of tasks are mainly composed of the research questions and problems approached. Formulating a problem, looking for appropriate *milieus*, organizing the experimental work, putting it into practice, gathering data, analyzing it, relating the observations to other investigations and previous results, discussing and evaluating the results obtained, etc. are examples of different types of tasks carried out in a research project. However, a research practice contains much more other action: presenting a result obtained at a local seminar, giving a talk at an international conference, preparing a funding application to the national government, reviewing a paper for a journal, supervising a PhD project, etc. These should thus be considered as part of research praxeologies and it must be clear that, in their own way, they also contribute to the development of the different theoretical approaches. The *techniques* correspond to the different possible "ways of doing" that can be used to carry out a task of a given type, with usually many slight variations and sometimes strong differences between them. When some of these techniques acquire a rather stable, systematic and justified form, we usually talk about "research methods" or "methodologies", as each team of the networking group has tried to present in Part II of the book.

The block $[\theta/\Theta]$ of research praxeologies forms the *technological-theoretical* discourse used to describe, justify, and interpret both the research practice and the

results obtained. The first component, the *technology* θ is the first level of description, explanation, and justification of the practice. It includes the methodological discourse used for explaining and justifying the choices made in terms of research methods.¹ It also provides a preliminary description of the results obtained, before they integrate the theory, once their stability has been proved and they can be considered as basic assumptions. The *theory* Θ is a second level of justification of the practice. It is made up of the main principles, notions, and properties that are considered as unquestionable. It is interesting to see, in the chapters of Part II, how this basic discourse can vary from one framework to another as they are based on different primary terms: “interest-dense situations” in IDS, “semiotic bundle” in APC, “epistemic actions” or “context” in AiC, “praxeologies” in ATD, “didactic and a-didactic situations” in TDS. There are many other elements of the theory Θ that remain implicit in each framework, for instance the priority given by AiC and APC to the students’ constructions of knowledge, while TDS and ATD initially focus on the institutional construction of knowledge; the focus of AiC, TDS, and ATD on the epistemic dimension of teaching and learning activities; the reasons for choosing a given type of empirical data; etc.

These first chapters of the book presenting the main theoretical frameworks also show to what extent the practical and theoretical blocks of praxeologies are mutually dependent. For instance, the presentation of TDS makes clear that this theory orients research questions towards the study of didactic systems, not towards the cognitive functioning of individual learners. In contrast, AiC orients research questions towards the understanding of such a cognitive functioning, and in it the didactic systems to which the individual learners belong are taken as elements of the context. Each type of question generates its own research tasks. Quite often, researchers rely on familiar techniques for solving these tasks, but research also leads to the creation of specific methodologies (techniques and associated technological didactic discourses). As mentioned in Chap. 4, for instance, the methodology of didactical engineering emerged in TDS and since the 1980s it has played a crucial role in TDS research praxeologies.

The results obtained and their theoretical exploitation, thus the theoretical block of praxeologies, are in turn shaped by the research tasks articulated and the techniques used for carrying them out. For instance, the solving of research tasks oriented by ATD will not lead to the identification of “epistemic actions” in the sense of AiC; and, reciprocally, the solving of research tasks oriented by AiC will not lead to the identification of the constraints conditioning the ecology of mathematical knowledge in a given institutional context in the sense of ATD.

Such interdependence of the different elements of research praxeologies leads us to conjecture that the networking of theories should be approached at the level of *research praxeologies*, and that, for being productive, the methodologies developed for such networking should allow researchers to consider both the practical and theoretical block of research praxeologies and their

¹The term “methodology” usually denotes both the research methods or ‘techniques’ and the discourse developed around these methods.

interactions. The language used for expressing and supporting these networking practices is not neutral from this perspective. It must allow researchers to share the know-how of research praxeologies. If focused only on theories, it may reinforce the risk of underestimating the crucial role played by the practical block of research praxeologies. Up to what point did the networking group limit this risk, and how?

The presentation of ATD (see Chap. 5) also makes clear that the progression of knowledge goes along with the progressive structuration of praxeologies: point-wise praxeologies, characterized by a precise type of task and technique, organized into local structures sharing a same technological discourse, and at a next level into regional structures sharing some theory. Within this perspective, theoretical networking should oblige researchers to situate themselves at a regional level, considering that each piece of theory shelters a diversity of point and local research praxeologies. This is not an easy condition to satisfy, considering the constraints to which research projects are submitted. Up to what point have the different networking strategies allowed the networking group to address this difficulty, with what consequences? Another point is that all theoretical frameworks involved do not have the same size, in other words the same level of regionality. For instance, IDS and AiC are much more local than ATD and TDS. How has this affected the networking enterprise and its results?

15.2.2 *The Dynamic Dimension of Praxeologies*

Research praxeologies, as any other praxeological form, are living entities that evolve and change, which affects at the same time their four components and their interactions. The evolution of the practical block $[T/\tau]$ produces new theoretical needs that make the theoretical block $[\Theta/\theta]$ progress and, reciprocally, the evolution of concepts, interpretations, or ways of thinking and the emergence of new results lead to the construction of new techniques and the formulation of new problems. In this dynamic, the two-level structure of the theoretical block of praxeologies has an important functionality. As said before, the *technological* discourse (θ) produces a first description, explanation, and justification of the research tasks approached (the *questions*, in the model provided by Radford (2008), the techniques used to approach them, and the first *results* obtained by this work). The *theoretical* discourse (Θ), as a second level of justification, contains the basic notions, conceptualizations, and principles used in the technological discourse and in the practical block. In most praxeologies, this second level is mainly implicit: it is made of the “folk knowledge” everybody uses without being conscious of it. In research praxeologies, it is important to make it explicit in order to control the assumptions made and to make them evolve if necessary. It is, however, a very stable hard core (in the sense of Lakatos 1978) of regional research praxeologies. The *technological level* of justification thus plays the “transactional” role of including the first results obtained in the practical block as preliminary descriptions of regular facts and phenomena, then transferring

the most robust of these results to the theoretical block in the form of new principles to adopt and new germs of methodologies and problems.

The notion of didactic transposition in ATD can be a good example of this transactional role of the “technology” between the practical elements of research praxeologies (types of tasks and techniques used to approach them) and the theory. At the beginning, the process of didactic transposition was obtained as a result of the analysis of different mathematical school contents, to show that the mathematical knowledge that is taught at school can be questioned and compared to the scholar knowledge where it comes from and that legitimates its introduction at school. It thus appeared as a *result* of the investigations carried out, the description and explanation of a regularity observed, an element of the *technological* discourse. It was the explanation of a (hypothetical) phenomenon. Then new types of problems started to be raised (*new types of tasks*) using this process: how the didactic transposition of some given school content is carried out, how it affects the conditions of its teaching, what happens when the didactic transposition is interrupted, etc. After some research about the transposition of different contents, it became an assumption made in ATD (and also TDS) that any content involved in any teaching and learning process comes from a didactic transposition process, an assumption giving rise to a new *theoretical* ingredient. This result is no longer questioned; on the contrary, it leads to new research *techniques*, those of analyzing the taught contents, looking for the way they are described as “knowledge to be taught”, and tracing their evolution from the scholar institutions to the school ones.

As in any other scientific discipline, and depending on the maturity of the field, research praxeologies can appear as different kinds of amalgams, more or less organized. It is the historical development of the field that helps structure these praxeological amalgams, making them more coherent and easier to diffuse, according to different didactic and institutional transposition processes that we are starting to know better. It seems reasonable to conjecture that while each of the didactic perspectives studied in this book can be considered as mature, this is not the case for the research praxeologies that the networking enterprise caused to emerge on top of these. Networking tasks have been articulated and germs of techniques developed, but, at this stage, these are certainly more craft techniques than well described and acknowledged research methodologies; the theoretical block of these praxeologies is still in an emergent state. This makes a dynamic vision of research praxeologies all the more important here.

15.2.3 The Role of Phenomena in the Dynamics of Research Praxeologies

In Artigue et al. (2011a, b), we argue that, for understanding the dynamics of research praxeologies, specific attention should be paid to the notion of didactic phenomenon, due to its emergence at the interface between the practical and theoretical blocks of research praxeologies: “In a first approach, we can characterise

didactic phenomena as empirical facts or regularities that are raised through the study of research problems. Some of these phenomena can enrich the initial theoretical frame to produce new interpretations and new techniques or research methodologies, while others remain at the level of the “results obtained” and are reinvested to formulate new problems or to propose new diagnosis and practice-development tools, thus enriching the *technology*” (ibid., p. 2383).

In Chap. 12, for instance, three didactic phenomena are considered in the analysis of video-2: the Topaze effect, the funnel pattern, and the semiotic game. These three phenomena are incorporated in theoretical frameworks, respectively in TDS, Bauersfeld’s Interactionism, and APC, and detached from the particular research praxeologies where they emerged. The Topaze effect is part of the theory of didactical contract in TDS, and identified as one of the didactic effects of the paradoxical nature of the didactic contract. The idea of semiotic game has been incorporated into APC and, beyond its theoretical status, it has become a didactical technique helping teachers align students’ utterances with institutionalized forms of knowledge. Through the associated processes, these phenomena have been objectified and decontextualized, which explains why we could so easily invoke them for interpreting the video-2 episode. We can say that in both cases the *technological level* of the TDS and APC research praxeologies have evolved, even if the main principles and conceptualizations (the *theory*) remain stable.

15.3 Analyzing Networking Through the Praxeological Lens

For analyzing the networking enterprise through the praxeological lens, we first consider the tasks and techniques which have been developed along the project. We then come to the knowledge produced in terms of networking by solving these tasks. In terms of praxeologies, we thus study the emergence and dynamics of networking praxeologies, from their practical block to their theoretical block. Within such a praxeological perspective, all components are equally important and the lessons from this project involve all of them.

15.3.1 *The Practical Block of Networking Praxeologies*

15.3.1.1 **Starting from a Technical Artifact: The Videotape of Two Students at Work**

From the beginning of the project, the idea that its realization would require the sharing of a common object of study was clear to the different researchers involved in the networking group. The Italian team proposed to use a video associated with one of its projects, and the proposal was accepted. It seemed to the involved researchers that a video, while certainly influenced by the particular project at

stake, its theoretical background, and the questions addressed, was an object open enough for starting a productive networking enterprise. However, at that time, the group did not discuss in depth the reasons that could have made this video a “good transactional object.” Its choice was partly one of convenience: taking an object already there made it possible to start the project immediately, exploring the networking potential of this object. Its limitations would certainly help select or develop more appropriate objects if needed. In fact, the video was a technical artifact inscribed in an APC research praxeology, and much more shaped by APC than the group initially imagined:

- the session was designed by a teacher-researcher working in close collaboration with the Italian colleagues;
- the Italian team was especially interested in the role of components of the semiotic bundle, and this had strongly influenced the way the students’ activity and exchanges, as well as the interaction between the students and the teacher, were captured;
- the complementary information the Italian team thought necessary to give us was influenced by what they looked for in the data, and the information they needed for securing their interpretations.

In addition, the session had taken place in an educational system and culture that most of the members of the networking group were not very familiar with. However, as evidenced by the previous chapters, through the tasks designed around this artifact and the techniques developed, the group succeeded in transforming it into a transactional object and part of a productive milieu for its networking activities and emerging praxeologies. Analyzing the whole process through the praxeological lens thus led to investigating how tasks and techniques were progressively created, and what can be learnt from this activity in terms of networking praxeologies.

A first task spontaneously emerged: the different teams should analyze the video, each one with its specific theoretical lens. However, the networking project required anticipating and organizing the communication between the different analyses. This was achieved through a system of common questions, and through different techniques, progressively built. Two especially productive elements resulted. First, the difficulties the teams all had in using their technological and theoretical tools for developing the analysis from the video and the contextual information provided by the Italian team. This observation led to a first productive common question: each team was asked to identify exactly what it missed for carrying out the analysis of the video. It was also asked to make clear why it felt this limitation so problematic, and to connect the invoked reasons to the principles, questions, and methodologies specific to its approach. The answers to this question and their comparison played a key role in situating the different theoretical approaches with respect to each other, and understanding the respective lenses they used for approaching the “real world” and the influence of these lenses on their research practices. From that phase also resulted a questionnaire for the teacher-researcher. His answers, accompanied by a second short episode (video-2), complemented the material milieu the networking group was interacting with.

15.3.1.2 The Evolution of Milieus and Tasks

The second productive element came from these additional data: the description by the teacher of his didactic use of semiotic games in the answers to the questionnaire (cf. Chap. 2). For a diversity of reasons, all teams noticed this element. Once again, a specific task and a new process of study were built around this element, which transformed it into a transactional object. The technique used was the following. First, the TDS team was asked to associate a question with this element. The question, articulated in the TDS theoretical discourse, was about the possible relationship between semiotic games and a phenomenon of limitation of the a-didactic milieu. Each team was then asked to re-formulate this question within its own theoretical discourse before trying to answer it. Re-formulations, the work carried out in answering the resulting questions, and the answers eventually provided were then exchanged and discussed; new questions emerged, leading to work at the level of theoretical constructs and phenomena, and to progress in the networking enterprise. For instance, the use of video-2 for making sense of the teacher's discourse around semiotic games led to the case study reported in Chap. 12, in which the possible connections between the ideas of Topaze effect and funnel pattern were systematically investigated. More globally, each case study involving a few teams around the study of specific questions is the result of such a process.

A retrospective look at the whole enterprise shows this regular move from the contact with the initial then complemented milieu, to research questions and tasks collaboratively negotiated to exploit this milieu. These tasks organize the work of each team and pave the way towards productive exchanges around this work. In a second phase, these tasks and the work carried out for working them out become a new shared milieu with which the teams interact for answering questions and tasks situated at a more meta-didactic level. One of the first examples of such a move, not reported in this book, was the moment when, from the observation that all analyses of video-1 paid specific attention to the social dimension of the learning process, it was decided to clarify the ways this attention was expressed and theoretically instrumented in the different discourses, and compare them (Kidron et al. 2008). Chapter 10 on context, milieu, and media-milieu dialectic in fact obeyed a similar logic. It is worth noticing that, in these two cases, the move to a meta-didactic level had also as a consequence that the teams involved were obliged to take into account their respective theories at a regional level.

15.3.1.3 Some Less Successful Attempts

If we take seriously the needs of the networking enterprise in terms of contact with the range of research praxeologies associated with a given technological-theoretical block, there is no doubt that the initial milieu and its extensions mentioned above have evident limitations. It only allows approaching the research praxeologies of the different teams very partially. Retrospectively, we interpret a task proposed by Ken Ruthven at one of our first meetings as an attempt to overcome these limitations. The task had no link with the videos. It proposed to question our respective theoretical approaches through

the way we would transform a teacher question into a research question. The starting point was thus an object external to the different research praxeologies involved, but it came from an empirical system shared by all of us: the profession of mathematics teacher in a European country. The example selected was the following:

How is it that some students can learn to tackle a particular type of mathematical problem successfully (as shown by their performance in the class), but be unable to do so two weeks or months later? What strategies can the teacher use to reduce the likelihood of this occurring?

Answer this question along the following guidelines and write 2 to 4 pages:

- (a) How do you – a priori – answer this question and what are your basic assumptions?
- (b) How do you transform the raised problem into a research question starting from the question above?
- (c) What is your research design?
- (d) What type of results would you expect?

All teams answered these questions, which were also proposed to the researchers involved in the Theory Working Group at the 5th Conference of the European Society for Research in Mathematics Education, and the eight responses received were presented and discussed at the conference (Prediger and Ruthven 2007). However, within the networking group, the task was no further exploited. Retrospectively, we see two reasons for this. On the one hand, the task started from an observation shared by all of us in our respective educational contexts, but it was too disconnected from the work we were engaged in for not being perceived as an artificial exercise; on the other hand, the initial milieu for this task did not offer sufficient potential of retro-action for dealing with the heterogeneity of the answers provided. Enriching the initial milieu would have been thus necessary for developing a productive networking activity. However, at that time, our understanding of the conditions to be satisfied for initiating productive networking praxeologies was not developed enough. This track was abandoned.

This was also the case for an initial attempt made at connecting directly our respective principles and key concepts through a system of conceptual maps. We worked on this task at one of our first meetings but did not find the results very convincing and gave up. Retrospectively, this attempt that was not further developed confirms our vision that connecting theories and concepts cannot be achieved without involving strategies that allow researchers to situate these within research praxeologies, and create appropriate milieus for that. At this starting stage of the networking, working at the level of the theory was only useful to point out the differences between the approaches, without helping in the mutual understanding of each other's visions and the searching for commonalities to promote collaborative analyses.

15.3.1.4 General Comments

We will not enter further into these attempts, but they must not be omitted from this retrospective reflection. They show that, in this new area of research, praxeologies are in a state of emergence. Tasks and techniques for solving them, that is to say appropriate methodologies, cannot be simply borrowed from the practical blocks of the research praxeologies familiar to us. In particular, the constitution of milieus and the

organization of appropriate media-milieu dialectics likely to produce knowledge regarding networking are not obvious. Drawing the lessons from this particular networking enterprise imposes thus to precisely look at the tasks successively created along the process and the milieus arranged for these, not only at the results obtained. We conjecture that an important reason for the success of this project is that the networking tasks designed made it possible to overcome the limitation of an approach focused on the theories themselves. The anchoring of tasks in the analysis of two videos helped the teams engage some practical blocks of their respective research praxeologies and consider them as objects of study. The succession of tasks taking into account the questions progressively emerging from this study, and the associated evolution of the milieus with which the researchers interacted, played also a crucial role for addressing the different components of research praxeologies and their dialectic interactions. Another essential point is the way the different researchers contributed themselves to the milieu. Compared with networking efforts carried out by a single researcher, this networking enterprise engaged researchers with different backgrounds and theoretical expertise. This expertise contributed to the antagonist dimension of the milieus at stake. In most of the tasks collectively designed, researchers acting as elements of the milieu offered resistance to the interpretations or claims that other teams could propose; they obliged them to make visible implicit assumptions and arguments, naturalized in their research praxeologies. This antagonist role was reinforced by the fact that many researchers were not really familiar with the other theoretical approaches involved.

Beyond the level of tasks and milieus, the techniques used in the networking process were a combination of familiar research techniques and specific techniques used for carrying out the collaborative work planned. For instance, as made clear in the different chapters of the book, each team used its own techniques for analyzing the videos and the complementary material. Reading these analyses, one can grasp the technical diversity at stake, despite the limitation of the material involved, the essential pieces of it being a 1-hour video showing two students working essentially in an autonomous way, and a very short video complementing it. The specific techniques used for collaborative work included those usual in collaborative scientific work: presentations and discussions, group work on specific issues and collective reports, co-writing of texts, both in regular face-to-face meetings and at a distance. However, the evolution of tasks went along with an evolution in the organization of all these ingredients, the collaborative work taking a cyclic nature: formulation of a question, team work on this question, exchange and comparison of the work developed and its outcomes, reflection on its networking potential, new questions, etc. And, at the end, a systematic reflective stance with the interpretation of the whole process in terms of the ordered structure of networking processes. As shown by the case studies, the generating questions were of a different nature: from questions directly emerging from the analysis of the data as in Chap. 12 already mentioned, to more general questions such as in Chap. 10 in which the aim of the case study is to understand how three of the theoretical approaches involved, AiC, TDS, and ATD, take in charge the idea of context. However, one characteristic of the technical work developed in the case studies is its anchoring in the data shared by the networking group, and especially the two videos.

15.3.2 The Theoretical Block of Networking Praxeologies

A retrospective analysis of the networking enterprise through the praxeological lens must go beyond the practical dimension of networking praxeologies and consider their theoretical block. The current emerging state of these networking praxeologies does not make this an easy task: the technological and theoretical discourses are not fully articulated. However, as pointed out in the introductory chapter of the book, there is no doubt that this networking enterprise relies on theoretical principles. For instance, it considers theoretical diversity as a normal state of the field of mathematics education, not a sign of some scientific immaturity. It adopts a dynamic and functional vision of theories. These principles are expressed using a language and references familiar to the community of mathematics education. Along the development of the project, some aspects of a theoretical discourse progressively consolidated and became more specific. One example is provided by the differentiation between different forms of networking and their ordering along a networking axis. The networking group has systematically used this structure for situating its networking efforts and their outcomes, as attested by the different case studies, and this technological tool resulted in being useful. Another example is the more recent idea of networking profile introduced in Chap. 8.

Creating categories and hierarchies is often a first step in the development of a theoretical discourse. These constructions confirm thus that networking praxeologies are emerging. For approaching their theoretical block, it is certainly appropriate to consider the interface between the theoretical and practical block, the place where results emerge which can contribute to the development of a technological discourse and contribute to the praxeological dynamics. A first point to be mentioned is that the results of the networking work go beyond networking. As evidenced by several case studies, the tasks designed and the way they were carried out questioned the different theoretical approaches involved, not just their possible connections. A typical example is provided by Chap. 12, in which the interpretation of the same episode by three different phenomena led to a process of deconstruction–reconstruction of these phenomena, the reconstruction being influenced by the contact established among them. Even when there is no such process of deconstruction–reconstruction, each case study has as a result a deepening of the understanding of each theoretical approach by the researchers already experts in it. This could have been anticipated. In this long-term process, each theoretical approach, except APC, has been questioned on its capacity to make sense of data shaped by another educational and didactic culture; the interpretations each team provided have been systematically confronted with alternative views strongly defended by their authors; theoretical constructions have been challenged by researchers who did not understand them but wanted to make sense of them and of their potential.

However, whatever is the interest of such progression in the understanding of our own or other theories, what was expected were results in terms of networking. As shown by the different chapters, the project has produced such results, and they cover the different levels of the landscape for networking strategies mentioned above. This is not the place for listing them here. In line with the praxeological lens

we adopt in this chapter, we prefer to focus on the way these results may support the emergence of a proper technological discourse.

Let us give an example, considering once again Chap. 12. In this chapter a connection is established between the Topaze effect and the funnel pattern. This is achieved through the following process. First, each phenomenon is situated within its theoretical environment and precisely described. Then the functional proximity between the two phenomena is made clear: the two of them are identified as ways of maintaining the fiction that learning has occurred when the conditions for such learning do not exist; this makes it possible to subsume the two phenomena under a common umbrella. However, the analysis makes clear how the characteristics of each theoretical approach shape the way this fiction is expressed, giving complementary insights on it. By doing so, the analysis makes visible the strength and limitation of each approach. The whole process results thus in an original technological discourse having clear networking characteristics.

The work carried out shows other possible formats for the emergence of a technological discourse attached to networking praxeologies. Without having the ambition of identifying all of them, we would like to give another example, considering Chap. 11. In this case, the two research praxeologies at stake are APC and IDS. When considering a given episode from a short video excerpt, they raise the common question (or research task) of how to explain a hypothetical failure of the teacher–students interaction. Then the technological elements provided by each approach as possible explanations appear to be contradictory. A common work starts which remains at the technological level: there is no contest of the basic principles of each frame nor of the type of methodologies used (both at the theory level). The final result is an enrichment of both technologies by a new emerging concept, that of “epistemological gap”. We can forecast that, if the concept remains productive and robust in its use for approaching new tasks and in instrumenting new techniques, it could become a basic notion of the praxeology and enter its theory. What is sure is that the development also affects the practical block as the new analysis provided would lead to the raising of new problematic questions and the development of both analytical techniques. As the authors pointed out, this special case of networking praxeologies is certainly made possible by the proximity of their theoretical principles: view on data, unit of analysis, orientation, etc.

As a counterexample, a case of success and failure of networking can be mentioned referring to Chap. 9 on the epistemic role of gestures. A quite similar theoretical proximity between AiC and APC (at least at the level of the unit of analysis) enables both approaches to be enriched by the other – inclusion of the gesture analyses in AiC and of the epistemic dimension in APC. However, an attempt to include the ATD team in the networking initially failed due to the difficulties of the ATD researchers in combining their analysis with those of the AiC and APC teams. In the ATD theory, gestures are part of the praxeologies and, thus, of the knowledge that is to be taught and learnt and of the didactic strategies used to do so. This distance from the AiC and APC assumptions about the mediator role of gestures acted as a barrier for the integration of the ATD analysis in the common work.

15.4 Conclusion

Analyzing this networking enterprise through the praxeological lens makes clear that, within this project, specific networking praxeologies have been developed. Considering the questions raised in the introduction, and the risk of underestimating the crucial role played by the practical block of research praxeologies, there is no doubt that this risk has been avoided. If, during the first meetings, some attempts were made at connecting directly the different theoretical approaches through descriptions and maps trying to link the main concepts of each theory, quickly the strategies evolved to tasks allowing the researchers to mobilize both the practical and the theoretical block of their research praxeologies and make the whole praxeologies the object of joint study. This is certainly one reason for the success of the enterprise that the praxeological lens helps identify. What also contributed to the success of the enterprise was the fact that these research praxeologies were not considered as isolated objects, but were engaged in the solving of common questions around a shared set of data. One can observe here an evident proximity with the strategy developed in the European project ReMath, whose networking ambition was also clear regarding the semiotic potential of digital technologies. In ReMath, indeed a system of cross-experimentations was developed, common questions articulated about these cross-experimentations, and case studies carried out (Artigue et al. 2009). Common questions addressed and case studies are thus common ingredients of the two projects. In ReMath, however, cross-experimentations played a crucial role in the networking praxeologies developed. Each team was asked to experiment with two digital tools: one familiar, because produced by the team itself; and the other alien, because produced by another team from another country with a different theoretical background. The case studies focused thus on the comparison of the two pairs of experimentations of the same digital tool. Networking praxeologies were thus different, but the two projects shared the same vision of theories as dynamic and functional objects. ReMath also had the vision that networking could only be achieved through the production of specific tasks allowing making visible how theoretical concerns impacted the design of digital tools and their didactic use. The cross-experimentation process was one of the techniques used for making visible the tacit part of design and research practices. The techniques used in our project are certainly less demanding from an experimental perspective, but, in some sense, the limitation of the experimental constraints has allowed the focusing of the work on the progressive definition of tasks and constitution of milieus making us able to maximize the profit that could be taken from the limited corpus of data used. And the long term of this project with no external limit in time made this progression possible.

Such characteristics contrast with many earlier efforts made at networking theoretical frameworks, even if the word networking was not used. For instance, the Special Issue of *Educational Studies in Mathematics* (Zan et al. 2006) results from a Research Forum at the 28th PME conference, held in 2004 in Bergen, and considers the diversity of theoretical frameworks used in research on affect in

mathematics education. As expressed by the editors, a special feature of the Special Issue is “to show how different frameworks can help in interpreting and intervening in students’ learning processes, through the analysis of an empirical account of a particular student’s solving of a mathematical problem in the classroom” (Zan et al. 2006, p. 118). However, in the six articles constituting the issue, the place attached to the analysis of this empirical account is very limited, and the different analyses are just juxtaposed.

In spite of the lessons provided by this experience, it however remains a very limited experience. Only a tiny part of the praxeological complexity of the research frames has been involved in the networking process. The networking tasks presented in this book, built around the collaborative study of a particular set of data, cannot engage the entire set of questions where each of the research praxeologies can show its potential, as well as its limitations. This is true for the five theories involved but is especially obvious for “big theories” such as ATD and TDS.

In the introduction of this chapter, we also raised the issue of the different sizes of the theory involved. In fact, this networking experience shows that differences in size are not necessarily an obstacle to networking processes, when adequate points of contact between theories are identified. For instance, Chap. 10 involves three theories of very different size: AiC, TDS, and ATD. As shown in Chap. 10, the networking process was associated in that case with a progressive extension of the perspective from the cognitive and individual perspective underlying AiC to the institutional perspective underlying ATD. TDS acted as an intermediate level, which on the one hand could be connected to AiC through its cognitive roots and vision of learning as an adaptation process, and on the other hand was connected to ATD through its systemic perspective and vision of learning also as an acculturation process. Moreover, the possibility of connection between AiC and TDS-ATD, already connected for decades within the French didactics community, was reinforced by a shared concern with the epistemology of the discipline. This shared concern was for instance made clear by the convergence between the a priori analysis made by AiC and TDS researchers.

Another fundamental element of the networking technology, its description and justification, is what we can call its “didactic component”. The main condition for networking to develop is the diffusion of research praxeologies among the community of researchers – a diffusion that is not just an acknowledgement of what is done in the different frames, their specificities, differences, and commonalities, but a high degree of comprehension at all levels of the research praxeologies. What we have called the “dialogue” between research praxeologies (Artigue et al. 2011a, b; Trigueros et al. 2011), the condition for researchers from different approaches to work together, needs special teaching, learning, and study conditions of the problems raised by the others, the methodologies used, the notions used to interpret the work done, and the kind of results obtained. It clearly appears at this point that the very reading of the others’ productions (papers, communications, informal analysis, teaching productions, etc.) is far from being enough to enable fruitful dialogues to develop. The craftsmanship dimension of research needs people meeting face to

face, seeing the other carry out the research analyses, questioning and explaining the research gestures observed, trying to imitate the practice of others before fully understanding it. The results obtained in terms of research production are maybe not necessarily relevant; they are, however, absolutely crucial for the personal share of these research implicit skills and competences. The workshop activities that are not shown in this book, the walks, meals, informal discussions, the share of failure experiences, as well as some successes, are also part of networking praxeologies and should not be underestimated. The humility, modesty, patience, generosity of the participants – especially those with a deeper research background – are part of the conditions that should integrate a networking praxeology to make it effective. In fact, such practices are not new. They are normal ingredients of researchers' activity each time their work involves different communities, all the more different disciplines. What is new, however, is to take them as objects of study, to investigate their particular characteristics and ecology, to understand their dynamics and try to make them more effective, to identify their outcomes, and to share the resulting knowledge with the research community at large. For that purpose, ATD can be a useful lens.

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