Chapter 14
Networking as Research Practices: Methodological Lessons Learnt from the Case Studies

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Abstract The methodological reflection on the case studies from Chaps. 9, 10, 11, and 12 starts with elaborating the data–phenomena distinction by which the role of data and phenomena in empirical networking practices can be grasped deeply. The gradual distinction between more empirical and more conceptualized phenomena clarifies the status of empirical situations, data, and theory and how these are linked to phenomena. Looking at the case studies, data–phenomena distinction is referred to the networking strategies and the monitoring role of research questions. The chapter finishes with summarizing potential empirical, theoretical, and methodological benefits of networking practices.

Keywords Networking of theories • Methodology • Role of data • Role of phenomena

The authors of Chaps. 9, 10, 11, and 12 have already reflected on the difficulties and gains of their networking practices within each of the chapters. In this chapter, we present the methodological lessons learnt from a more general perspective. We start in Sect. 14.1 with the role of data and phenomena and discuss in Sect. 14.2 the empirical and theoretical benefits of networking practices between theoretical approaches.

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14.1 Lessons Learnt on the Role of Data and Phenomena

14.1.1 Looking Back

The specific idea of starting the networking in 2006 was to challenge the researchers with the demand to analyze alien data (i.e., data coming from a study in another research frame). The choice of data contributed to producing an "antagonist" milieu for the research groups they had to deal with. In fact, since data were given and were not fully suitable for analysis, this caused different reactions:

- using experiences of previous studies with digital technologies for developing a hypothetical a priori analysis of the situation to enrich the understanding of the data at hand (by the TDS-team in Chap. 4);
- including the teaching material of the whole course into the analysis (the strategy of the ATD-team in Chap. 5);
- using just a part that is suitable enough (the strategy of the AiC team in Chap. 6);
- enlarging the theoretical frame in order to apply the theory to the given data (as decided by the IDS-team in Chap. 7);
- including additional foci into the given data to take into account additional aspects into the transcript (by the APC and AiC teams in the case of epistemic gestures in Chap. 9).

The important role of data became even more visible in two specific needs that the different teams expressed: the need for further data that fitted better to the usual analysis; and the need for reflecting about the role of data as a link between theory and empirical issues, here the video episode. The first need resulted in collecting additional data and conducting the case studies of networking as presented in Part III of the book. The second need was already present in the presentation of the theories in Part II and was focused on in more detail in some case studies, for example in the case of epistemic gestures (Chap. 9).

The important role of data also appeared in other networking activities conducted by the Networking Theories Group which are documented outside this book. In one networking activity, researchers from different theoretical approaches translated a common problem in classrooms to different research questions and sketched possible research designs (Prediger 2008). While comparing the different research questions and designs, the different kinds of desire for data became apparent. In another activity, one research question posed by the TDS team was translated into research questions of the other teams and led to distinguishing between problems and phenomena (see for example Artigue et al. 2011). In many networking practices, the issue of insufficient data is apparent. In the case study on the epistemic role of gestures (Chap. 9), the authors even wondered why the two teams – that methodologically have a lot of features in common – experienced difficulties in selecting a common piece of data.

The problem of inadequate data in networking practices can be understood more deeply by distinguishing the notions of data and phenomenon, as discussed in the following section.

14.1.2 The Data–Phenomenon Distinction

According to Knipping and Müller-Hill (2013), qualitative research in mathematics education should follow the principle of clearly distinguishing data and phenomena where data are the means to identify and investigate phenomena in mathematics education:

> As a practical and methodological consequence of a clear conceptual distinction between data and phenomena, a large amount of research effort has to be spent to face the resulting problem of detecting a genuine phenomenon rather than some artefact of the experimental setting. (Knipping and Müller-Hill 2013, p. 3)

In their paper, Knipping and Müller-Hill refer to the work of Bogen and Woodward in philosophy of science who describe data as being "idiosyncratic to particular experimental contexts" (1988, p. 317), whereas "phenomena, by contrast, are not idiosyncratic to specific contexts. We [Bogen and Woodwad] expect phenomena have stable repeatable characteristics which will be detectable by means of a variety of different procedures which may yield quite different kinds of data" (ibid., p. 317).

If we accept Bogen and Woodward's definition of phenomena for mathematics education, then phenomena are constructed by human beings who realize these stable and [repeated or] repeatable characteristics in mathematics education as an instance of a more general pattern. Given that mathematics education is still a young discipline, its phenomena often are not very clear or even not well known. Therefore, research in this field is not only conducted to investigate phenomena but also to identify, disclose, and describe phenomena. Some phenomena can easily be detected and shared based on common-sense knowledge of mathematics education without any strong theoretical foundation; others need more profound conceptualizations and sophisticated methodological and methodical arrangements to make them accessible for the human perceptual system. For example, one member of the Networking Theoreis Group described the following phenomenon: "in one situation a child may be able solve a specific task but later the same child is not able to solve it anymore" (problem and succeeding networking activities documented in Prediger 2008). The research teams separately translated the description of this phenomenon into research questions and developed a research design for its investigation. Through this translation, the phenomenon was conceptualized in different nuances by the different theoretical frameworks. Hence, the phenomenon changed its status. In the first case we talk about an *empirical phenomenon*, and in the latter about a *conceptualized phenomenon*, although of course no phenomenon can be perceived completely independently of the theoretical approach or even from simple pre-assumptions. That is why we understand the distinction between more empirical and more conceptual phenomena as a gradual one according to the degree in which the theory guides the conceptualization of the phenomenon. Figure 14.1 roughly sketches these connections between data, more empirical and more conceptualized phenomena in the interplay between theory and reality (here, concretely, episodes of teaching and learning) which can, on the one hand, be perceived as an interplay between particular and general, but also (as a second dimension) between the more vague to the more theoretically focused and structured perspective that allows us to see connections.

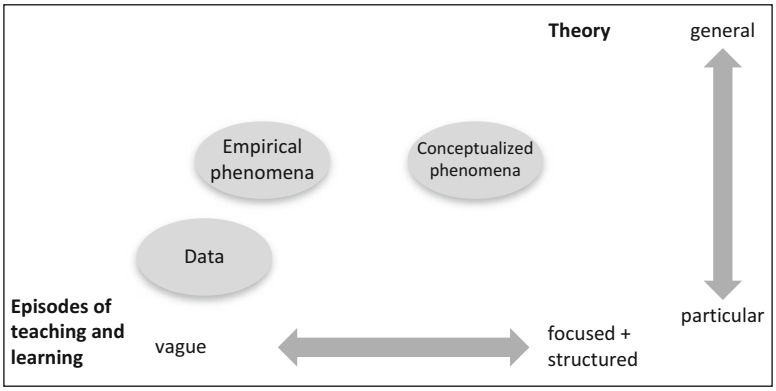

Fig. 14.1 Rough localization of data, empirical phenomena, and conceptualized phenomena for each theoretical approach

In contrast to data, phenomena are not directly perceivable, as they are constructions. However, data themselves are not relevant; for research they are only interesting as means to "constitute evidence for the existence of phenomena" (Woodward 1989, p. 394). The reason for underlying problems with data in the networking of theories is not the data themselves but the kinds of phenomena that the research teams normally are used to identifying and investigating through data. The given video of Carlo and Giovanni struggling with the exponential function and its transcription were taken from a larger study on the introduction to variation and calculus in the first years of secondary school (see Chap. 2). The role of different semiotic resources, including gestures and embodied ones, was of great importance for the APC team in this study. The semiotic bundle notion and the semiotic game phenomenon were built up along with the video analysis of this and other episodes in the project. Although these kinds of phenomena were not relevant in the other approaches, the other teams attempted to analyze the alien data by taking them as a constitutive means for identifying and analyzing one or more home phenomena that were not intended by the APC team. The teams strongly experienced that the given data were only partly appropriate for this endeavor.

In contrast to our terminology of more empirical and more conceptualized phenomena, for Artigue et al. (2011) phenomena only exist in research contexts as a result of theory-driven investigations; hence, all phenomena are regarded as already being conceptualized:

> In a first approach, we can characterise didactic phenomena as empirical facts, regularities that arise through the study of research problems. Some of these phenomena enrich the initial theoretical framework to produce new interpretations and techniques or research methodologies, while others remain at the level of "results obtained" and are reinvested to formulate new problems or to propose new diagnostic and practice-development tools. (Artigue et al. 2011, p. 2383)

In the quotation, Artigue et al. (2011) also address the degree of conceptualizing phenomena that may be a result of specific research and lead to new problems, products, etc., but also to theoretical concepts whose status is determined by the relationship to the other concepts and principles of the theory. This is in line with the gradual distinction between empirical and conceptual phenomena.

In the case of the Topaze effect (Chap. 12), two conceptualized phenomena which had been the results of previous research within two different theoretical frames were networked. Through this networking process the two conceptualized phenomena and their theoretical status were strengthened and further conceptualized. This revealed a "too early naturalization of a phenomenon" as a new phenomenon in the culture of mathematics education and led to deepening insight into the theories' blind spots but also to uncovering the empirical phenomenon of the fiction "that teaching [in the two phenomena] has led to learning" as the underlying idea which both teams agreed upon.

Taking the view of Bogen and Woodward, data are means to identify more general phenomena and investigate claims about phenomena within theories. For example, the AiC team (in Chap. 9) broadened their notion of what constitutes data in that they admitted gestures to be data. In the second stage the AiC team identified that gestures may shape part of the constructing process, hence conceptualized this phenomenon resulting in the term "epistemic gesture." The AiC team stated, "As compared with earlier RBC analyses, the evidence we admitted and paid attention to in the present analysis was broader since gestures were considered as potential indicators of epistemic actions" (Sect. 9.5). In our view, it is not the data themselves that provide evidence for a phenomenon but the way data are used to provide evidence for the constitution of phenomena, the way they are freed from their complexity and "their highly irregular coincidences" (Bogen and Woodward 1988, p. 326), the way they are analyzed and interpreted, and this is determined by the theory's methodology and principles. Bogen and Woodward also emphasize, "Often the characteristics which data must have to be useful as evidence can only be purchased at the cost of tolerating a great deal of complexity and idiosyncrasy in the causal processes which produce data" (ibid., p. 319). This by-product of producing data also holds in research practices in mathematics education and it explains why alien theorists might be able to find some evidence of their home phenomena in given data, as has been shown in Chaps. 3, 4, 5, 6, and 7 of Part II and the case studies in Chaps. 9, 10, 11, and 12 of Part III.

14.1.3 Relating Research Questions to the Data–Phenomena Distinction

In the case study on epistemic gestures between APC and AiC (Chap. 9), the AiC team started to integrate gestures into their methodology of studying epistemic processes of constructing knowledge and asked "whether and in what sense gestures can

contribute to the construction of knowledge" (Sect. 9.1). In spite of the integration of a common type of data into the case study, research questions were different in the two approaches. An outcome of the two data analyses and their comparison was the discovery of an interesting phenomenon, namely that some gestures supporting the constructing process were used without the producer and his peer looking at them. The discovery of this phenomenon raised dialectic questions and pushed the networking process further. The deeper analysis on the epistemic function of these gestures and the succeeding common, combined research process about the epistemic role of gestures in the knowledge construction processes resulted in the notion of an epistemic gesture. This combined research process demanded choosing *common data for analysis.* The difficulties of finding such data required intense work about the idea of epistemic gesture and the role of gesture for AiC and was analytically focused on a very small piece of the given video for which the transcript had to be refined and enriched. Only after this step did the AiC team state, "the videotape became data for us once we transcribed it with focus on verbalizations and gestures" (Sect. 9.3). In contrast to this way of approaching data, the APC team normally starts from observing the video and not from interpreting the transcript.

This step of modifying the data with respect to the phenomenon under question shows how data are made in different research practices from the same video, addressing the epistemic role of gestures in the AiC approach and the communicative function of gestures in the APC approach. The dialectic between the teams' questions and data usage helped to detect the new phenomenon, and reflecting the different views helped to clarify the process of conceptualization: the AiC team sees the epistemic function of gestures in single gestures which contribute to constructing knowledge; the APC team identifies the epistemic function in sequences of gestures as part of the semiotic bundle. The epistemic function of gestures here seems to be a concept at the boundary of the two approaches which allows establishing a locally integrated methodology but no local integration on the level of principles. The principles are not close enough.

In contrast to the case study on gestures, in the case of context, milieu, and media-milieu dialectic (Chap. 10) the researchers were able to choose a common piece of data which was much larger and not explicitly reported on. Data were not chosen for common analyses but *to refer to for separate analyses* that served as the basis for comparing and contrasting the role of the three concepts in their home theoretical background. In this case study of networking, data served as a common reference pool, but not as a resource for research itself. The questions reflect this role of data in the networking process, since:

> TDS researchers might ask what milieu the teacher is making available to the students and how she is managing its evolution in order to establish a meaningful connection with the mathematical knowledge aimed at. AiC researchers might ask how the teacher's intervention influences the students' construction process as described by means of the RBC epistemic actions. ATD researchers in their turn might ask what responsibilities the teacher and the students are assuming in the media-milieu dialectics and what conditions enable them to manage it. (Sect. 10.3)

Questions for analyses in AiC stress the epistemic process itself, whereas researchers in TDS and ATD ask how this process is made possible. Already these questions indicate an interesting point in networking: researchers were able to build on ideas and results of the other analyses in a complementary way. This made them identify *epistemological sensitivity* as an underlying proximity in their respective approaches. That means, the three theories share the aim to understand the epistemological nature of the episode, while, at the same time, each of the three theories accesses data in its own ways. The teams pose different questions concerning contextual influence. Through comparing and contrasting these questions, researchers tried to elucidate the ways in which the three theoretical approaches address the issue of contextual dependence of teaching and learning processes through their concepts. Context in AiC is everything that does not belong to the epistemic process itself, but does influence the construction of mathematical knowledge. The milieu is, for TDS, the main concept describing the environment with which the learner interacts in order to produce a mathematical piece of knowledge. The dialectic of media-milieu clarifies the dynamic nature of the milieu being changed by media. The three concepts are accessed by different data or different foci on data in a complementary way sharing *epistemological sensitivity*, which facilitated establishing connections and reflecting on them. The researchers claim that such proximity seems to be crucial for undertaking the networking practice between their theories. However, we see it as an open question how the networking could function without such proximity.

In the case of epistemological gap (Chap. 11), the data were not a problem at the beginning. Both teams were able to use the same video, namely the extra video starting after Task 3 (see Appendix for a complete transcript), as a common piece of data for separate analyses answering home questions. But the ways in which the data reflected the core questions were different. The IDS team focused more on the discourse whereas the APC team focused on the gesture–speech interplay. Questions in the networking process were very interesting because they directed the attention of the two teams. The first question was: Which of the two results are more suitable for understanding the episode? This made the two teams reconsider the raw data to refine the utterances, include the students' protocols, and produce a written transcript in which gesture pictures and speech intonation were included. In this case, the data was reworked for a more common analysis. This first step led to a refinement of the concept of semiotic game and raised another question: What is the deeper reason why Giovanni reduces to be engaged in such a short situation? This question brought the idea of the epistemological gap as a vague idea to the fore. During the following months, both teams took this episode as a prototype represented by the refined data set that evidenced the phenomenon of an epistemological gap. Its mechanism was still only vaguely understood. Since the two theories did not offer an appropriate theoretical frame to conceptualize this empirical phenomenon, a literature review was conducted. Concepts and results from research on personal epistemology could be included into the two approaches, leading to a process of conceptualizing the phenomenon of epistemological gap and clarifying the mechanism of it. In this way, a local integration of a new construct at the boundary of both theories has emerged and connected the two approaches within the semiosphere (Radford 2008;

cf. Sect. 8.1 in this book). This case of epistemological gap does not only demonstrate the data–phenomena distinction but also the gradual difference between a more empirical phenomenon and a more conceptualized phenomenon. The latter is part of a theory while the former is a construction which may still appear more pretheoretical and less elaborate.

The previous reflections on research questions and their relations to the data–phenomena distinction show that questions, explicitly or implicitly posed, may guide researchers' attention in research practices and their mediating between data and phenomena. In networking processes, often dialectic research questions from different approaches mediate the comparing and contrasting of theories. The resulting synthesized common questions seem to support processes of coordinating which may lead to a local integration.

14.1.4 Relating Networking Strategies to the Data–Phenomena Distinction

The four case studies show that processes of networking may lead to uncovering an underlying proximity or even an empirical phenomenon underlying the theories' concepts. They also show that the researchers are often unaware of these proximities at the beginning of a networking process but they can be achieved as a result. Since common proximities or empirical phenomena allow for complementary views, they may be a starting point for the networking strategy of coordinating. If such a common empirical phenomenon is first uncovered, it may be vague at the beginning, like the epistemological gap, but further investigated in a process of coordinating showing how far networking processes can reach. By a process of conceptualizing, the empirical phenomenon changes its character and status, and may be worked out and finally conceptualized. In this way, the empirical phenomenon turns into a conceptual phenomenon that then belongs to the theoretical approach and may finally result in a local integration, as in the case of epistemological gap (Chap. 11).

The case study of networking on the Topaze effect (Chap. 12) started with two such conceptualized phenomena to which different questions directed the separate analyses. Through networking, the underlying common empirical phenomenon was able to be uncovered and at the same time the nature of both conceptualized phenomena as being limit concepts was clarified. This case and the case of context-media-milieu (Chap. 10) showed the fruitfulness of the networking strategy of comparing and contrasting, even without further degrees of integration. This was different in the cases of the epistemological gap and of the epistemic gesture: the networking strategy of coordinating encompassed conceptualizing a phenomenon and even led to local integration of new constructs in both cases.

Empirical phenomena – even if different theories share them – may be elaborated differently in different theories, bringing to the fore complementary views. The other way round, a shared empirical phenomenon may be hidden in concepts of different theories but can be uncovered through networking processes. In both cases, the

networking strategies of comparing and contrasting are especially fruitful for revealing the complementary nature of differently conceptualized phenomena.

By these methodological reflections, the role of data for the networking practices is also clarified. The networking practice also depends on the kind of data used. As long as data are used separately and modified with respect to each theoretical approach, the networking practice may reach the stage of combining because researchers stay within their home theoretical approach. As soon as common questions are investigated, the choice of common data may become difficult within this strategy because the home theories look at different empirical phenomena and possibly slightly different data. At this stage, the strategy of coordinating may help to overcome difficulties. The intermediate strategy of coordinating seems to be that of transforming separate views towards a more integrating view on the empirical phenomena. At this stage the phenomena may also change their nature, from a more empirical towards a conceptual status, leading finally to local integration.

14.2 Lessons Learnt on the Empirical and Theoretical Benefits of Networking Between Theoretical Approaches

What can we generally gain from networking of theoretical approaches? We discuss our methodological considerations on different benefits in two steps: in Sect. 14.2.1, we summarize possible *empirical* benefits and *theoretical* benefits; and in Sect. 14.2.2 we show the strong interdependence between both.

14.2.1 Empirical and Theoretical Benefits from Networking Practices

Considering the same empirical material from different theoretical lenses is not a new research practice; it has often been applied by many researchers in terms of theory or *perspective triangulation* (e.g., Schoenfeld 2002). The notion *perspective triangulation* was introduced by Denzin (1970), and was presented by him (together with method triangulation, data triangulation, or investigator triangulation) as research practices for increasing validity of an empirical analysis. During the last 40 years, though, it became evident that a systematic triangulation of theoretical lenses often does not offer increased validity: if different theoretical lenses capture different aspects of research objects or conceptualize the research objects in different ways, their results are not comparable. However, additional theoretical views mostly focus on additional and complementary aspects which altogether deepen and broaden the understanding of an empirical situation and thus shape "triangulation as a research strategy" for increasing research quality (cf., e.g., Flick 2007, p. 20ff.).

In this sense, the networking practices as presented in Chaps. 9, 10, 11, and 12 might be perceived as practices of classical perspective triangulation, seeing the substantial empirical benefits received by complementary insights into complex empirical phenomena. However, we put emphasis on the fact that the presented cases of networking go beyond perspective triangulation in three aspects:

1. *Empirical benefits:* Sometimes, perspective triangulation is naïvely discussed as a practice of "different theoretical lenses for the *same* data." Different theoretical approaches rarely deal with the same data since data is constructed within a theoretical frame; this point was extensively discussed in Sect. 14.1. Instead of a simple perspective triangulation on the same empirical material, our networking practices enhanced the empirical benefits by enlarging and reshaping data while connecting the approaches.
2. *Theoretical benefits:* As was argued by the data–phenomenon distinction in Sect. 14.1, networking activities do not only aim at a deeper understanding of empirical phenomena, as will be discussed below.
3. *Methodological benefits:* The methodological reflection of possibilities, benefits, and limits constantly accompanies the dialogue between theoretical approaches. In this sense, networking practices also aim at increased methodological awareness.

Coming back to the benefits for the theoretical approaches themselves, networking of theories can facilitate the development of theories in four directions (Bikner-Ahsbahs and Prediger 2010):

(a) *Explicitness:* Starting from the claim that a theory should make its background theories and its underlying philosophical base (especially its epistemological and methodological foundations) as explicit as possible, the maturity of a theory can be measured by the degree of its explicitness: the more implicit assumptions are explicitly stated and the more parts of the philosophical base shape explicit parts of the background theory, the more we would consider the theory to be *mature*. A step towards such a development took place in the case of the Topaze effect through uncovering blind spots and some limitations of the theories (Chap. 12).
(b) *Empirical scope:* Formal theories have a large empirical scope. They characterize empirical phenomena in a global way and often cannot exactly be concretized through empirical examples (Lamnek 1995, p. 123). On the other hand, contextualized and local theories have a limited scope but their statements can more easily be made concrete by the empirical content (see Krummheuer and Brandt 2001, p. 199). This proximity to empirical phenomena makes contextualized theories a suitable background to guide practice in schools. However, developing local theories in order to *enlarge their empirical scope* can be an important direction for theory development. This happened for example to AiC in Chap. 9.

(c) *Stability:* Stability is a long-term aim for theory development on a longer time scale. A new theory might be a bit fragile because its concepts and the relationships among its key concepts are still in progress, for example in IDS. Through networking with other theories, IDS concepts proved to be fruitful (Chaps. 11 and 12), its principles could be strengthened (Chaps. 7 and 12), and the disclosure of empirical phenomena (common to other approaches) unfolded its complementary view on specific empirical phenomena (Chaps. 11 and 12).

(d) *Connectivity:* Science is characterized by argumentation and interconnectedness, as Fischer (e.g., 1993) emphasizes. This can, for example, be realized by establishing relationships through linking theories, by declaring commonalities and differences. Hence, establishing *argumentative connectivity* is another important direction for the development of theories. This direction has been touched on in all case studies since argumentative connectivity is an intrinsic feature of networking practices in general.

14.2.2 Interdependences Between Empirical and Theoretical Progress

Although the discourse on different networking profiles (see Chap. 8, following Arzarello et al. 2008) might suggest that networking practices either aim at theoretical or empirical benefits, our case studies show that both can often be connected since the development of empirical analysis, conceptualized phenomena, and theoretical constructs often interdepend.

These interdependences are also highly connected to the role of results. Radford (2012) added research results to his triplet (questions, methodology, principles) for describing theory as a fourth component: research results as the source for the dynamic development of theories. New results may enlarge the amount of phenomena that can be investigated and the number of key constructs. However, they also may have an impact at least on enlarging and understanding more deeply the home methodologies, paradigmatic questions, and also principles. In networking practices, results play an important role in understanding more deeply what networking approaches, their principles, methodologies, and questions mean, too. The four case studies gave examples that networking may:

1. uncover underlying empirical phenomena that later can be investigated and yield new constructs within the theories or at the border of them (Chaps. 9 and 11);
2. yield new constructs at the border of theoretical cultures. According to Lotman (1990, p. 134), the new dynamic of cultural development comes from the periphery, therefore concepts at the boundary of theories may lead to new research directions,

integrating theoretical views or providing complementary or supplementary considerations (Chaps. 9 and 11);
3. lead to clarifying methodological aspects such as the role of data and phenomena in the networking research (Chap. 9);
4. build new networking methodologies such as cross-methodologies including cross-data collection, cross-task design, cross-experimentation, and cross-analyses which all have a cyclic pattern of interconnected research actions followed by an exchange that leads to a refinement of the research actions etc. (for example Chaps. 10 and 11);
5. strengthen the understanding of theories by clarifying their foci, what also is taken as relevant, what is left aside, and finally identifying blind spots and thus making assumptions more explicit (Chaps. 10 and 12);
6. produce results about implicit practices in research cultures such as the naturalization of phenomena within a research culture (Chap. 12).

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