Chapter 13 Beyond the Official Academic Stage. Dialogic Intermezzo

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Abstract The chapter goes beyond the official academic stage and presents the researchers' personal experiences during their networking practices and their reflections on these processes. These experiences and reflections were collected by individually arranged interviews and then assembled.

Keyword Networking of theories

13.1 The Story of This Networking Project

In this book, the products of the Networking Theories Group are presented in ways in which readers expect researchers to write: from the perspective drawn from established theories (Part II), from the networking point of view with several theories on a common research interest (Part III), and from a research methodological point of view that reflects upon the research processes in the networking activities (Part IV, following this chapter).

Throughout the work in the Networking Theories Group, the researchers went through individual stages of experiences with the networking of theories. This chapter presents an attempt to tell their stories about this networking experience. It should be viewed as the result of a somewhat journalistic approach: the chapter is organized via critical questions written in italics below, which serve as guides. These questions were asked to five researchers from the five teams and theories involved in the book. Given that it was difficult to string together material from approximately 8 h of interviews in excerpts of direct speech, the responses are summarized and the questions slightly altered.

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In the interviews, the expression "home theory" turned out to be useful when referring to the theoretical approach with which a researcher usually works. As the relationship between the home theory and other theoretical approaches is central in the networking process, it is used here occasionally.

13.2 The Initiation of This Networking Project

Mathematics education is still a nascent scientific discipline. Why did you start the networking project at this stage? Wasn't there enough work left to do on your home theory?

The Networking Theories Group started to work after the Congress of European Research on Mathematics Education (CERME) conference in 2005. In a panel and in a working group, intensive discussions on theoretical perspectives and paradigms had taken place. All interviewees had noticed limitations of their home theories or the need to integrate approaches and results of other theories within their own research. Because the scholars were internationally cross-linked, some of them felt the need to get to know other theories in a depth that is hard to achieve to a satisfactory degree simply by reading other scholars' papers and listening to some talks. The group included representatives of both established and relatively new theoretical approaches.

All participants were convinced that connecting the theoretical approaches was important to further develop mathematics education, even though most of them said retrospectively that they had no experience in doing so before the project, and that the notion of "networking" emerged slowly. However, the Theory of Didactical Situations and the Anthropological Theory of the Didactic are different in their history, as the second emerged as a development of the first, and a mutual dialogue between them has occurred from scratch.

As empirical researchers at heart, the members of the Networking Theories Group agreed from the beginning that common work on empirical data was important. Whereas different theoretical approaches can be worked out from an abstract point of view, words on this "meta" level can be misleading. To make sure that the scholars were talking about the same thing while networking of theories, the group decided to work on data. Besides, the histories of the theories involved in this project have in common that they were shaped by and developed for the examination of empirical data. It appeared, therefore, natural to initiate networking processes in the context of data analyses.

Everybody reads others' papers or attends talks. Isn't this already a sort of networking experience?

No, this is actually only a small part of it. The group started first by introducing the theories to one another (understanding others and making theirs understandable). Looking back, this encounter was considered to be necessary but not sufficient by

all interviewees. Some scholars described these first meetings as the beginning of their networking. They presented their own views with those of the others, such as by comparing and contrasting views on a theoretical level. Others considered the readings of the video analyses from different theoretical perspectives (Part II in this book) as the beginning of their networking. From the point of view of several interviewees, the networking of theories started at the moment when joint research questions were worked out or even with the common process of writing on research papers. All agreed that the potential scope of networking activities is very limited unless it leads to joint research.

13.3 The Work in This Networking Project

What was the main motivation for examining the episodes of Carlo, Giovanni, and the exponential function? Was it just an exercise for networking theories, or did it deserve interest in its own right?

The researchers recalled different primary motivations for working on the videos. The AiC team (working on the theory of epistemic actions of abstraction in context) noticed well before participating in this project that the comprehension of gestures in their research might be useful. At the same time, the APC team (working with the semiotic bundle construct) was interested in studying the context of epistemic actions. The video of Carlo and Giovanni provided a good opportunity for both groups to work together.

The Topaze effect case study (cf. Chap. 12) was also initiated by the analyses of different home theories. Understanding the three competing interpretations of the same episode was the challenge for this group. It is a remarkable feature of networking dynamics that the question as to whether there is a Topaze effect was brought up by the APC team. The Topaze effect and the funnel pattern, which were both early considerations for the episode, were hypotheses that served as driving forces.

These patterns were identified in the first analyses and discussions. It was striking that the same episode could be interpreted as a semiotic game, on the one hand, and as a Topaze effect or a funnel pattern, on the other hand. It soon became clear that a deeper analysis involving the corresponding home theories would be needed. This, finally, led to certain theoretical clarifications for these notions within the respective home theories.

Every theory has developed its own way to work with data over the years. Isn't this a major obstacle to the networking of theories?

The networking activity started from the assumption that even if the data collection was shaped by the inscription in APC, there was space for analysis of this data using other theoretical approaches. Looking back, the problems were rather underestimated at the beginning.

The data in a research context is made for research purposes and this is shaped by the theoretical approaches. Theories also require a certain type of data. For instance, in terms of extent versus detail, different theories require different extents and different details. The intense connection between data and theoretical frames is highlighted by the experiences of the need for different data and for the knowledge of phenomena that are typical for certain theoretical constructs. For instance, talking about a Topaze effect only makes sense using the Theory of Didactical Situations. Similarly, the funnel pattern is shaped within the interactionist perspective of Bauersfeld (1993), underlying the IDS. At the beginning of the work of the networking teams, some researchers reported that they worked with notions from other theories, such as the Topaze effect, on a somewhat metaphorical level. When the discussion became deeper, the specialists' points of view were important, and a thorough theoretical penetration of these notions was deemed necessary (cf. Chap. 12).

Some scholars asked themselves, to what extent did the theoretical background of the involved teacher have an impact on the video, and how should this be dealt with in the analysis. Domingo is a teacher-researcher who works with the semiotic bundle. In particular, he is trained in employing gestures in classroom situations. One of the researchers of the other teams described the beginning of her analysis in the following manner: "First, we found it strange how Domingo acts." The design of the instructional tasks in the video is less appropriate for certain teams than for others. For instance, the tasks posed by Domingo were rather explorative and, as such, not specific enough for AiC in the sense that the knowledge that was intended to be constructed could not easily be identified. For the involvement of the APC group in the project, on the other hand, it was helpful to have a teacher whose approach stimulated the use of gestures and whose video ensured that the gestures were captured.

For some networking teams, it was difficult to identify sufficient manageable data on an episode (cf. the documentation in Chaps. 2, 3, 4, 5, 6, and 7 with initial and extended data). More than usual, it was necessary to agree to certain compromises concerning the data. For some theories, for instance, it is everyday business to consider how a teacher acts in a certain situation; if this question becomes central, it is difficult for theories such as the ATD that do not consider actions at all. Furthermore, the AiC theory does not describe the role of a teacher in a prominent way: it considers the teacher as a part of the context, which is a very flexible construct to handle.

Being already experienced in the home theory at the data-recording stage normally allows the researchers to identify potential problems and fix some of them right away. However, this will remain a challenge in networking theories where scholars need to satisfy the needs of the involved theoretical approaches both in the design process and while recording data. Additionally, even if this were achieved, the data needs would still change when being utilized in other approaches (cf. Chap. 14 for a detailed methodological discussion on the role of data).

13.4 Looking Back on This Networking Project

It is not a new thing in (mathematics) education that different theoretical approaches are employed to consider the same situation. Schoenfeld (2002) describes how to deal with results from different theoretical backgrounds in the triangulation method. Before the start of this networking project, Hannula et al. (2004) considered, for instance, four different frameworks for affect to "evaluate these frameworks from different perspectives." Is networking theories much ado about nothing new?

All interviewees believe that this networking project went beyond what is described as triangulation. The researchers reported on a couple of experiences that they had had before these networking practices and that went beyond the method of triangulation. The main point seemed to be that networking theories influenced the view on the involved theories or even influenced the theories themselves (cf. Sect. 14.2.1 for a discussion on the differences from triangulation).

Scholars who use networking practices question theoretical approaches and the values that come with them but strengthen them in the end. For the APC team, it was difficult when their result that the semiotic game was successful for the extra video starting after Task 3 was confronted by other analyses that pointed out epistemic differences in the discussion between the teacher and the students. Apart from the obvious research question regarding whether these results would be reconcilable, the other teams' first analyses were a blow to the positive values attributed to a "successful" semiotic game. In the end, the theoretical clarifications proved to be necessary, but they confirmed the initial analyses. In the case of APC, this process put the underlying values into the perspective of the other theories by the introduction of the "epistemological gap": the positive attribution to the statement that even if the semiotic game works this does not yet imply that the underlying epistemic actions reflect this in the intended manner (cf. Chap. 11).

Are there hands-on effects of networking practices?

Networking of theories helps to make theoretical notions more precise; in fact, this even applies to highly developed theories, such as TDS. Its legacy includes a variety of phenomena in the teaching and learning of mathematics that are shaped by the theory and examined empirically within this approach, such as the Topaze effect. Confronted with the data examined here, it still became necessary to go back to the theoretical roots of this effect and to make the definition of this phenomenon more precise.

Networking of theoretical approaches pushes the involved approaches to their limits. In this project, this was almost a permanent experience for the Anthropological Theory of Didactics (ATD). The networking project pushed the researchers to conduct analyses that were motivated by an unusual point of view. For instance, the ATD team normally does not initially focus on the teacher and her or his actions unless the teaching and learning process is described. For the AiC, the teacher only appears as part of the context, which can be given more or less importance in a flexible manner. Other theoretical approaches frequently work on understanding the teacher's role. For the ATD team, the unusual work of the material caused a constant feeling of not being at home, but their members still regard this experience as an interesting and rewarding one.

There is no doubt that networking of theories enriches research practices or, in terms of ATD, research praxeologies (see Chap. 15 and Artigue et al. 2011). The construct of research praxeologies offered an approach to better understand and manage the interaction between researchers, which was underestimated before the networking theories project began. In a similar way, the repertoire of the Theory of Didactical Situations was especially rewarding for the networkers, as it provided some elements to describe the common research strategies used. The a priori analvsis was often mentioned as an example of what will influence research in the future. The notion of milieu was a complex, albeit very inspiring, tool, which helps the researcher to understand the necessity for networking strategies to have a shared empirical set of objects available for the analysis (cf. Chap. 10). Additionally, it provides a rich collection of studied situations and phenomena that several groups are interested in. The anthropological approach to localizing institutional constraints of teaching and learning and to describing their praxeologies is also named by all interviewees as inspiring, even if the theoretical link – for example, to the Theory of Interest-Dense Situations – is not clear.

Networking of theories creates a networking spirit in everyone involved. By knowing the limits of the home theory better, the networkers unanimously report experiencing a networking attitude in their normal research. "I could do networking alone now," one of the researchers said, adding, "but it would be very risky."

What was important in order to get the networking of theories to work?

All of the networkers emphasized the role that the people in the group played in getting the networking going. There was no overall strategic master plan; instead, the commitment of several researchers led at certain stages to strategies that were worked out in Prediger et al. (2008) (cf. Chap. 8). The approaches varied in the first stage; then, later, the strategies used for networking were still diverse, but the scholars had more of a feeling that they had been able to build tasks and methods for the networking work. In this way, some interviewees recall the work as similar to their usual research at that particular stage.

Everyone's motivation was needed to maintain interest in reaching this point, but the basic motivation differed in the subjective views of the researchers. Some scholars were interested in the comparison of theories, in general. For them, it was like an exercise in networking theories. Some scholars were interested in the particular elements of other theories. Others were motivated by the differences in the explanations in the preliminary analyses, as in the analyses of APC and IDS, for example.

All of the researchers underlined the importance of the personal and social dimensions of networking. Especially in times with no progress, people who push

for continuing the project are necessary. The interviewees stressed the importance of a working cooperation- and confidence-based atmosphere that allows for considering problems several times from different points of view.

Are any two theories on the teaching and learning of mathematics suitable for networking?

Certainly, some obstacles were identified while the research teams defined their networking case studies. Early in this discussion, the metaphor of a lens was utilized to describe how the various theories could be used to analyze different "grain sizes" in the data. The APC and ATD approaches are at the extreme ends of the resolution of the lens, even if they both explicitly attribute an important role to the semiotic dimension of mathematical and didactic activities, as shown in Arzarello et al. (2008).

Corresponding grain sizes are neither necessary nor sufficient to do networking together, however. In the networking of theories, most researchers worked on different data sources than those that they usually treat. The usual grain sizes differ in this area, i.e. the different approaches have different units of analysis. The APC construct semiotic bundle typically addresses distinct and short pieces of the data: because a one-second gesture may help to explain the result of a longer working process, this also can be of interest to other theories, even though they would not go down to such short grain sizes. Slightly bigger grain sizes are used to determine whether a classroom episode can be qualified as interest-dense in the IDS approach. Even though interest-dense situations typically can last for a couple of minutes or even up to a whole lesson, the methods used in this theoretical approach address different grain sizes, including very short ones, such as a gesture or a sign.

One of the researchers hypothesized that a useful feature for making networking successful is a common interest in phenomena of comparable timescales. This is meant differently from the aspect of grain sizes. For example, it is traditionally not considered primarily relevant in ATD to determine whether a learning process of a couple of minutes is interest-dense or of another quality, unless the process is not questioned and located within a broader teaching and learning project. However, because questions on this timescale are the main motivation of IDS, it is difficult to work out a research question of relevance to both theories (hence, questions are considered to be an important component of theories, cf. Chap. 1). The representatives of both theories have thought independently on the reasons for this difficulty and have come to similar explanations. They believe that if the overlap of both research interests and possible grain sizes is small, it is difficult to participate in a joint networking effort though, for example, the Theory of Interest-Dense Situations can use insights gained from ATD. Similarly, even though the need for different data remains, ATD could more easily find common research questions with semiotic or distinct epistemic approaches if they address what is learnt before considering the question of how this is achieved.

13.5 Looking into the Future: What's Next?

Has the investigation of the video been an exercise that is now completed, or would some of the results be worth considering in more detail?

Some results of the project show that the analyses were not merely exercises for the members of the Networking Theories Group and that the underlying research questions on the video episode warrant interest in their own right. Nevertheless, it would be very challenging to produce data that make it possible to involve all theories of the group together. Because no teacher would produce a Topaze effect deliberately, it is difficult to investigate how it is constituted by social interactions in the classroom. Besides, producing an example of a veritable Topaze effect would not fundamentally add to the theoretical knowledge because this phenomenon is situated as an idealized limit concept.

A learning situation with gestures seems more likely to be producible. To obtain this, the idea was brought up to start from Domingo's course design, picking out certain key features and asking teachers to adapt these in their design of a course. This could enable the researchers to study the semiotic game under the conditions in which an epistemic gap occurs on a broader empirical basis.

Can research designs be thought of to understand more about networking?

The challenge for a follow-up design would be - again - to find a learning situation that fits the aims of several research groups. Surely, it would remain difficult to find empirical material that is both interesting and usable for every theory involved.

Although several ideas for a networking design appeared during the networking process, it is not clear which theories could be utilized to fully exploit the knowledge that the researchers gained in terms of networking praxeologies and on which one can, thus, rely for developing a vision of possible research dynamics in the area. On the other hand, the networking praxeologies should prove fruitful in new contexts and research questions, and previous research projects might be better understood with the help of these. This should also provide new insights into research processes of networking theories.

References

- Artigue, M., Bosch, M., & Gascón, J. (2011). Research praxeologies and networking theories. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), *Proceedings of the seventh Congress of the European Society for Research in Mathematics Education CERME* 7 (pp. 2381–2390). Rzeszów: University of Rzeszów.
- Arzarello, F., Bosch, M., Gascón, J., & Sabena, C. (2008). The ostensive dimension through the lenses of two didactic approaches. ZDM – The International Journal on Mathematics Education, 40(2), 179–188.

- Bauersfeld, H. (1993). Theoretical perspectives on interaction in the mathematical classroom. In R. Biehler, R. W. Scholz, R. Sträßer, & B. Winkelmann (Eds.), *Didactics of mathematics as a scientific discipline* (pp. 133–146). Dordrecht: Kluwer.
- Hannula, M., Evans, J., Philippou, G., & Zan, R. (2004). Affect in mathematics education Exploring theoretical frameworks. Research forum. International Group for the Psychology of Mathematics Education. www.emis.de/proceedings/PME28/RF/RF001.pdf. Accessed 2 Dec 2013.
- Prediger, S., Bikner-Ahsbahs, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches – First steps towards a conceptual framework. ZDM – The International Journal on Mathematics Education, 40(2), 165–178.
- Schoenfeld, A. H. (2002). Research methods in (mathematics) education. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 435–488). Mahwah: Erlbaum.