Beyond the Standard Model Phenomenology and the ElectroWeak Symmetry Breaking

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Abstract This report contains a review of the motivations and main directions in the construction of models beyond the standard model. The lectures follow the role played by symmetries in model building as a guiding line. After a review of the symmetries of the Brout-Englert-Higgs sector of the standard model, and the role of spin in the naturalness argument, a pedagogical introduction to supersymmetry and extra dimensions is given, both being seen as an extension of Poincarè symmetries of space-time.

1 Why Do We Need to Go Beyond the Standard Model (BSM)?

The Standard Model of Particle Physics (SM for brevity) was proposed in the 1960s and early 1970s by Sheldon Glashow [26], Stephen Weinberg [46] and Abdus Salam to describe, in terms of fundamental degrees of freedom (or particles), the theory of electro-weak interactions first proposed by Enrico Fermi in the 1930s. The effective 4-fermion interactions in Fermi's theory are replaced by the presence of massive gauge bosons, the charged $W^{\pm}$ and the neutral Z: while the W is required to explain the beta decay of atoms, the Z is a prediction of the gauge structure introduced in the model and its discovery in 1984 was a crucial validation of the theory. The model can be divided in the following sectors:

– $\mathrm{SU}(3)_c$ gauge group (colour), that describes strong interactions (QCD);
– $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ that describes electro-weak interactions, it is spontaneously broken to an exact $\mathrm{U}(1)_{em}$ gauge group describing electromagnetic interactions (QED);

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E. Gardi et al. (eds.), *LHC Phenomenology*, Scottish Graduate Series,
DOI 10.1007/978-3-319-05362-2_3

- The Brout-Englert-Higgs (BEH) sector, that describes the breaking of the electro-weak symmetry (EWSB) by means of a scalar field that develops a vacuum expectation value;
- Fermionic matter: the model contains three generations of fermions, each one consisting of two quarks (that describe the baryonic matter, like pions and nucleons), a charged lepton (electron) and a neutrino;
- Yukawa interactions between fermions and the Higgs sector that give mass to the fermions and are responsible for flavour mixing.

The Standard Model is therefore composed by sectors, or building blocks, which are closely related to each other and their non trivial interplay is essential for the success of the model to describe phenomena observed at various energies. What glues together and inter-links these building blocks? The apparently complex structure of the Standard Model is based on symmetries! Gauge symmetries describe the behaviour and interactions of vector bosons, the force carriers. The generation of masses is based on a mechanism of spontaneous symmetry breaking: the theory is invariant under the $SU(2)_L \times U(1)_Y$ gauge symmetry, which forbids masses for both the gauge bosons and fermions in the SM. However the vacuum of the scalar BEH field is not invariant! The field content of each generation of fermions is fixed by the cancellation of gauge anomalies in the theory: all gauge symmetries of the SM, in fact, play a role in determining the number and charges of fields in a single generation. A different choice from the one observed in nature would invalidate the theory by breaking explicitly one or more gauge symmetries.

The presence of three generations is not fully understood yet. Nevertheless, three is the minimal number of generations that allows for non-trivial phases in the Yukawa sector. Such phases break the invariance under the CP symmetry (Charge conjugation and Parity), and this realisation was worth the Nobel prize for Kobayashi and Maskawa [33]. Furthermore, the structure of Yukawa couplings, which is determined by the quantum numbers of the fermions, ensures the presence of two unbroken global symmetries – Baryon and Lepton number conservation. They are accidental symmetries, in the sense that they have not been used in the construction of the model. It is well known that both Baryon and Lepton symmetries are broken by non-perturbative effects, however the combination $B - L$ is exactly conserved, and this is enough to ensure the stability of the proton.

The interactions of the SM have been tested very accurately in the last 30 years, mainly by LEP (Large Electron Positron collider) at CERN and the proton-antiproton collider Tevatron at Fermilab. The data show in fact an amazing agreement with the SM predictions (including quantum loop effects), with some measurements agreeing up to a part in 1,000 [5]. As of today, the sector of the theory which is the least experimentally known is the EWSB sector, i.e. the physics associated with the BEH field. In the Standard Model, this sector is described in terms of a complex scalar doublet of $SU(2)_L$. After the EWSB, 3 of the degrees of freedom of this field are eaten by the massive gauge bosons $W^{\pm}$ and Z providing them with a longitudinal polarisation, while the fourth is a physical scalar, the Higgs boson.[1] The Higgs boson has eluded all experimental efforts to discover it,[2] until July 2012 when the LHC collaborations CMS and ATLAS announced the discovery of a new resonance at a mass of 125 GeV [1, 14], that has similar properties as the Higgs boson (see Bill Murray's contribution in these proceedings). This discovery seems to complete the validation of the Standard Model, as all of the predicted particles and interactions have now been observed. So, why do we still need to talk about New Physics?

1.1 Evidences and Hints of New Physics (BSM)

There are several evidences and hints that seem to suggest the presence of New Physics: they can be roughly divided into two classes. On one hand, there are experimental results (**EXP**) that cannot be explained within the Standard Model. On the other, purely theoretical considerations (**TH**) do not allow us to accept the SM as the ultimate theory. Here it follows a brief incomplete list:

- **EXP**: *neutrino masses.* The observation and patterns of neutrino oscillations [38] suggest that at least two of the three neutrino species have mass, and from cosmological observations we know that the mass scale has to be very small (sub-eV). The masses can be added by simply extending the SM with three right-handed singlet neutrinos. See-saw mechanisms [45] would hint that the mass of the new states is at the $10^{3\div 13}$ GeV level.
- **EXP**: *Dark Matter in the Universe.* Twenty-three percent of the total mass of our universe [2, 31] is made of non-baryonic and non-luminous matter, which is not accounted for in the SM. This observation is also supported by astrophysical observations: rotation curves of disk galaxies, gravitational micro-lensing of galaxy clusters and large structure formation models. A particle interpretation would suggest a weakly interacting particle with a mass of $\sim 100 \div 1{,}000$ GeV. This new particle must be neutral and stable (on cosmological time scales).
- **EXP**: *Baryon asymmetry in the Universe.* The Universe is populated by baryons, however the number of anti-baryons is very scarce. To explain this, the mechanism of baryogenesis has been proposed: it requires three conditions formulated by Sakharov [41], namely that baryon symmetry is broken (by anomalies in

[1] There has been a dispute on the proper name this boson should have. In the author's view, Peter Higgs was the first to explicitly mention the existence and properties of such particle [29, 30], therefore it deserves its name. On the other hand, the mechanism of EWSB by a vacuum expectation value has been applied to gauge symmetries by Brout and Englert in an earlier paper [24], and also in a publication by Guralnik, Hagen and Kibble later the same year [28]. For this reasons, the scalar field and mechanism are dubbed BEH, while the scalar particle is named Higgs boson in these proceedings.

[2] In a 1976 paper [23], John Ellis and collaborators pointed out how difficult it is to discover such a state, to the point of discouraging any experimental effort in such directions. Fortunately, their advice has not been followed.

the SM), that the model violates CP (Yukawa interactions in the SM) and that there is a non-thermal process (the EW phase transition in the SM). However, in the SM, the amount of CP violation in the quark sector is not enough to explain the baryon density at present days by many orders of magnitude.
- **TH/EXP**: *gauge coupling unification.* Running the SM gauge couplings at high energies, their values tend to converge to the same value at $\sim 10^{15\div 17}$ GeV. A new unified gauge theory (GUT) may be present at such energy scales [39].
- **TH**: *quantum gravity.* Classical gravity, well described by general relativity, should break down at energy scales close to the Planck mass $M_{Pl} \sim 10^{19}$ GeV, where quantum effects should arise. The SM is necessarily invalidated at such energy. This scale can be lowered in models where the fundamental Planck scale is lower (for instance, models with extra dimensions [9]).
- **TH**: *Higgs mass (electro-weak scale) instability.* The Higgs mass is sensitive to quadratically divergent radiative corrections, thus it is unstable. In physical terms, new states that couple to the Higgs will generate corrections to its mass which are proportional to the mass of the new state. As the mass of the Higgs cannot exceed $\sim$1 TeV (we now know that it may be 125 GeV), such states must be at the TeV scale in order to avoid large cancellations with the tree level mass (naturalness argument).

All the entries in this non-exhaustive list point to the presence of new phenomena or new particles at a given scale. It is striking that only two entries require new particles at of below the TeV scale (which is accessible at the LHC): a weakly interacting particle candidate for Dark Matter and the naturalness argument on the Higgs mass. Other cases can also be lowered to the TeV scale: TeV scale see saw in the case of neutrino masses, accelerated unification or gravity in extra dimensions (TeV scale Black Holes); however, this is not required! In these lectures we will be mainly interested in the naturalness argument which involves the BEH sector of the Standard Model (and, sometimes, Dark Matter). Before starting our journey in the landscape of physics Beyond the Standard Model (BSM), it is important to point out what the naturalness argument really is: it is a theoretical prejudice against the Standard Model as the fundamental theory of particle physics. In fact, the Standard Model is based upon a renormalisable lagrangian, therefore divergences can be consistently reabsorbed in tree level parameters (the BEH field mass in this case) and, no matter how large they are, they leave no trace in observable quantities! The naturalness argument bases its power on the assumption that the SM is an effective model valid up to a cutoff, i.e. a high energy scale above which the model must be replaced by a more fundamental theory or where new particles should be included to complete the model. The truth told, we do know that above the Planck scale a theory of quantum gravity must replace the SM! One may conclude that a natural Higgs boson must have a mass close to the Planck scale (and thus goes the W and the Z bosons), unless a protection mechanism is at work. Protection mechanisms in particle physics are called **symmetries**. A central point in these lectures will be the role of symmetries in models of physics Beyond the Standard Model.

2 The EWSB Sector, and the Role of Symmetries

In the SM, the electro-weak symmetry breaking (EWSB) is *described* by the Brout-Englert-Higgs scalar field. Such field is a complex scalar (spin-0), which transforms as a doublet under the $SU(2)_L$ symmetry and has hypercharge $1/2$ (and no colour). The most general renormalisable action for the BEH scalar ϕ is:

$$\mathscr{S}_{BEH} = \int d^4x \; (D^\mu \phi)^\dagger D_\mu \phi - m_\phi^2 \phi^\dagger \phi - \frac{\lambda}{2} (\phi^\dagger \phi)^2 \,; \tag{1}$$

where

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi_0 \end{pmatrix} \tag{2}$$

is the doublet in components, and $\phi^\dagger \phi = \varphi^- \varphi^+ + \varphi_0^* \varphi_0$. The above action is invariant under local $SU(2)_L \times U(1)_Y$ transformations, provided that the covariant derivative D_μ contains the proper gauge fields

$$D_\mu \phi = \left(\partial_\mu - i g \sum_{a=1}^{3} W_\mu^a t^a - i g' \frac{1}{2} B_\mu \right) \phi \,. \tag{3}$$

You should be already very familiar with the fact that, for $m_\phi^2 < 0$, the potential for the BEH field in Eq. (1) has minima with $\langle \phi \rangle \neq 0$. In fact, the equation of motion for a static ϕ (i.e. such that $\partial_\mu \phi = 0$) is:

$$\left(-m_\phi^2 - \lambda \phi^\dagger \phi \right) \phi = 0 \,. \tag{4}$$

The solutions are $\langle \phi \rangle = 0$, which is a maximum, and

$$\langle \phi^\dagger \phi \rangle = - \frac{m_\phi^2}{\lambda} \tag{5}$$

which is a minimum of the potential. The theory is still invariant under $SU(2)_L \times U(1)_Y$ gauge symmetries: in fact both the action in Eq. (1) and the minimum condition in Eq. (5) are invariant, thus we can use $SU(2)_L \times U(1)_Y$ transformations to write the solution as

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \,, \quad v^2 = -2 \frac{m_\phi^2}{\lambda} \,. \tag{6}$$

The explicit solution in Eq. (6) is NOT invariant under the gauge symmetries, thus, when plugged back into Eq. (1), it will give mass to 3 of the gauge bosons:

$$m_{W^\pm}^2 = \frac{g^2}{4} v^2\,, \qquad m_Z^2 = \frac{g^2 + g'^2}{4} v^2\,. \tag{7}$$

Therefore, we can say that it is not the condition $m_\phi^2 < 0$ that breaks the symmetry, but the choice of an explicit form of the non-trivial vacuum solution for the BEH field. Now, 3 out of the 4 degrees of freedom in the BEH field are eaten up by the massive $W^\pm$ and Z to play the role of the longitudinal polarisation of the massive vector field (which is absent in the massless limit), and the remaining one appears in the spectrum as a massive physical scalar:

$$\phi = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}\,, \tag{8}$$

where the field h in this expansion is the Higgs boson, and it has a mass

$$m_h^2 = -2 m_\phi^2 = \lambda v^2\,. \tag{9}$$

Why is this a mere *description* of the EWSB? The mechanism relies on the fact that the mass has the wrong sign, i.e. $m_\phi^2 < 0$, however there is no explanation of what the origin of such negative mass is! Explaining the origin of the negative mass square can be addressed in BSM models.

2.1 Symmetries: Exposed and Hidden Ones

Let's have a closer look at the BEH action in Eq. (1), reported below:

$$\mathscr{S}_{BEH} = \int d^4x\ (D^\mu \phi)^\dagger D_\mu \phi - m_\phi^2 \phi^\dagger \phi - \frac{\lambda}{2} (\phi^\dagger \phi)^2\,. \tag{10}$$

This action was built based on invariance under the SM gauge symmetries: it is the most general gauge invariant and renormalisable lagrangian for a scalar field that transforms as a doublet of SU(2). Is it there any other symmetry that we missed?

There are three kinds of symmetries:

1. Symmetries of space time: Poincaré (rotations, translations, Lorentz boosts) and C, P and T.[3]
2. Gauge (local) symmetries: $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ (the colour symmetry $\mathrm{SU}(3)_c$ plays no role here).

[3]C P and T are discrete symmetries of space-time: C stands for charge inversion, P for spatial parity and T for time inversion. All field theories are, by construction, CPT invariant, i.e. invariant under a combination of the three discrete symmetries.

3. Accidental (global) symmetries.
4. Discrete symmetries.

The action in Eq. (10) contains a very important hidden (and approximate) global symmetry: the **custodial symmetry**. Let's for a moment ignore gauge interactions: if we do so, the action only depends on the combination $\phi^\dagger\phi$ (and $(\partial^\mu\phi^\dagger)\partial_\mu\phi$). This element can be written as

$$\phi^\dagger\phi = \varphi^-\varphi^+ + \varphi_0^*\varphi_0 = \frac{1}{2}\left(\varphi_1^2 + \varphi_2^2 + \varphi_3^3 + \varphi_4^2\right) \tag{11}$$

where $\varphi^+ = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}$ and $\varphi_0 = \frac{\varphi_3 + i\varphi_4}{\sqrt{2}}$ are decomposed in terms of real and imaginary parts. If we define a real 4-component field

$$\tilde{\Phi} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix}, \tag{12}$$

the action in Eq. (10) can be re-written as

$$\mathscr{S}_{BEH} = \int d^4x \, \frac{1}{2}(\partial^\mu\tilde{\Phi}^T)\partial_\mu\tilde{\Phi} - \frac{1}{2}m_\phi^2\tilde{\Phi}^T\tilde{\Phi} - \frac{\lambda}{8}(\tilde{\Phi}^T\tilde{\Phi})^2 \,. \tag{13}$$

Now, it is evident that the action is invariant under an orthogonal rotation of the 4-component real vector $\tilde{\Phi}$, i.e. the action is invariant under SO(4) $\sim$ SU(2)×SU(2)! This hidden global symmetry is at the origin of the so-called custodial symmetry!

The 4-dimensional rotation group SO(4) is equivalent to SU(2)×SU(2): the latter structure becomes evident if we rewrite the BEH field as a 2×2 matrix in the following way:

$$\Phi = \begin{pmatrix} \varphi_0^* & \varphi^+ \\ -\varphi^- & \varphi_0 \end{pmatrix}. \tag{14}$$

This field transforms as a bi-doublet under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, i.e.

$$\Phi \to U_L \Phi U_R^\dagger$$

where $U_{L/R}$ are two independent SU(2) transformations. The action in Eq. (10) can now be re-written as

$$\mathscr{S}_{EBH} = \int d^4x \, \frac{1}{2}\mathrm{Tr}(\partial^\mu\Phi^\dagger)\partial_\mu\Phi - \frac{1}{2}m_\phi^2\mathrm{Tr}\Phi^\dagger\Phi - \frac{\lambda}{8}(\mathrm{Tr}\Phi^\dagger\Phi)^2 \,, \tag{15}$$

where $\mathrm{Tr}\Phi^\dagger\Phi$ is the matricial trace. This latter lagrangian is explicitly invariant under a $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ transformation:

$$\mathrm{Tr}\Phi^\dagger\Phi \to \mathrm{Tr}U_R\Phi^\dagger U_L^\dagger U_L\Phi U_R^\dagger = \mathrm{Tr}\Phi^\dagger\Phi U_R^\dagger U_R = \mathrm{Tr}\Phi^\dagger\Phi\,. \tag{16}$$

In this notation, the vacuum solution in Eq. (6) can be written as

$$\langle\Phi\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\,. \tag{17}$$

This vacuum is invariant under a SU(2) symmetry, defined as the symmetry for which $U_L = U_R$:

$$\langle\Phi\rangle \to \langle U_L\Phi U_R^\dagger\rangle = \frac{v}{\sqrt{2}}U_L U_R^\dagger = \langle\Phi\rangle\,. \tag{18}$$

In other words, the Higgs vacuum is breaking $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R \to \mathrm{SU}(2)_D$! The preserved $\mathrm{SU}(2)_D$ is the custodial symmetry. This global symmetry is very important in the SM: in fact, it protects the relative values of the W and Z mass against radiative corrections. To understand this fact, we need to include gauge interactions, that were put aside at the beginning of the section: the SM gauge group is not $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$, however the $\mathrm{SU}(2)_L$ is same while the $\mathrm{U}(1)_Y$ can be identified with the diagonal generator of $\mathrm{SU}(2)_R$. In other words, the $\mathrm{SU}(2)_R$ is only partially gauged, and this fact explicitly violates it. If we imagine a world where both SU(2) were gauged, then the BEH vacuum solution would contain 3 massless gauge bosons (corresponding to the $\mathrm{SU}(2)_D$) and 3 massive ones with equal mass:

$$m'_W = m'_Z = \frac{\sqrt{g^2+g'^2}v}{2}\,. \tag{19}$$

In the SM, the $\mathrm{SU}(2)_R$ is missing its charged gauge bosons: therefore, the massless W disappears, and the mass of the massive one would be given by the above formula, without the g'. This structure of the W and Z mass is therefore dictated by the global symmetries of the BEH field, and this approximate symmetry will tend to protect the ratio of the two masses from large corrections.

This fact allows to define a parameter

$$\rho = \frac{m_W^2}{m_Z^2\cos^2\theta_W}\,, \qquad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}\,. \tag{20}$$

At three level, ρ is exactly 1. As the custodial symmetry is not an exact symmetry in the SM, radiative corrections tend to spoil the equality, however, due to the custodial symmetry, the corrections to ρ are of the order of 10^{-3}! Keeping such corrections small in models of new physics is a big challenge!

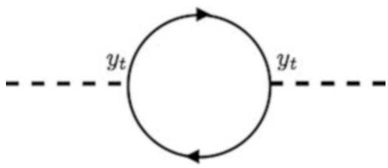

Fig. 1 Top loop contribution to the Higgs mass

2.2 Radiative Stability

The Higgs mass (and thus the electro-weak scale) suffers from divergent radiative corrections. To better understand this statement, let's closely look at the contribution from the top quark. In the SM, fermion masses are generated via Yukawa interactions. For the top

$$\mathscr{L}_{top} = -y_t \bar{Q}\phi t_R + h.c. \tag{21}$$

where $\bar{Q} = (\bar{t}_L, \bar{b}_L)$ is the left-handed $SU(2)_L$ doublet and t_R the right-handed singlet. This interaction will generate a top mass $m_{top} = \frac{y_t v}{\sqrt{2}}$, once the BEH field is replaced by its vacuum solution. Similar interactions can be added to generate a mass for the bottom, and for the other quarks and leptons. In the following, we will focus uniquely on the top because it is the only quark with large coupling to the Higgs: in fact, numerically $y_t \sim 1$.

The above Yukawa interaction will also contribute to the ϕ mass via the loop diagram in Fig. 1:

$$\begin{aligned}
-i\delta m_\phi^2 &= -3(iy_t)^2 \int \frac{d^4k}{(2\pi)^4}\, \mathrm{Tr}\frac{ik^\mu\gamma_\mu}{k^2}\frac{1+\gamma^5}{2}\frac{ik^\nu\gamma_\nu}{k^2}\frac{1-\gamma^5}{2} \\
&= -\frac{3y_t^2}{2(2\pi)^4}\int d^4k\, \frac{\mathrm{Tr}k^\mu\gamma_\mu k^\nu\gamma_\nu(1-\gamma^5)}{k^4} \\
&= -\frac{3y_t^2}{2(2\pi)^4}\int d^4k\, 4\frac{1}{k^2} \\
&= \frac{3y_t^2}{8\pi^4}\, i2\pi^2 \int k_E dk_E \\
&= i\frac{3y_t^2}{8\pi^2}\int dk_E^2
\end{aligned} \tag{22}$$

The last integral is clearly divergent. One way to regularise the divergence is to add a hard cutoff to the integral, i.e. $\int dk_E^2 \to \int_0^{\Lambda^2} dk_E^2 = \Lambda^2$. Thus, the corrected ϕ mass would be

$$m_\phi^2\Big|_{1-\text{loop}} = m_\phi^2 - \frac{3y_t^2}{8\pi^2}\Lambda^2\,. \tag{23}$$

Equation (23) contains two intriguing features: first of all, the contribution of the top loop is negative, thus this may be the source of the EWSB which was apparently missing in the standard model. In other words, if we start with vanishing BEH mass, $m_\phi^2 = 0$, the top quantum corrections will generate a negative contribution. The second feature is related to the size of such a contribution: in fact, the Higgs boson mass is related to the BEH mass, and we now know that the Higgs is light with a mass of 125 GeV, therefore one can estimate the value the cutoff Λ should have:

$$m_h^2 = -2m_\phi^2 = \frac{3y_t^2}{4\pi^2}\Lambda^2 = (125\ \text{GeV})^2 \Rightarrow \Lambda = 450\,\text{GeV}\,. \tag{24}$$

This implies that the divergent loop should be cut off by New Physics at a scale not far from the TeV, which is many orders of magnitude below the natural scale where we would have expected the SM to lose its validity: the Planck mass 10^{19} GeV. This offset of 16 orders of magnitude is called the **hierarchy problem**. There is a "weaker" hierarchy problem, dubbed **Little hierarchy problem** [10]: if we play at adding generic contributions from New Physics at a scale Λ_N in the form of higher dimensional operators, they will necessarily induce corrections to well measured quantities in the SM, like the ρ parameter. All in all, the precision constraints would require $\Lambda_N > 5 \div 10\,\text{TeV}$: why is this scale one order of magnitude larger than the required cutoff in the BEH mass term? The above considerations are at the root of the naturalness argument, arguably the strongest motivation for the presence of New Physics around the TeV scale.

Why is the scalar mass so sensitive to the cutoff of the theory? The ultimate reason is related so symmetries! Did you know that the responsible is the spin, i.e. rotation symmetry? In fact, for particles with non-zero spin, like spin-1 vector bosons and spin-$\frac{1}{2}$ fermions, the mass is protected against quadratically-divergent corrections, hence no hierarchy problem arises.

2.2.1 Spin 1

Spin 1 particles have three possible spin configurations. In the case of a massless vector boson, the lagrangian is invariant under gauge symmetries, and one can use a gauge transformation to remove one of the degrees of freedom. Physically, this corresponds to the fact that the photon has only two circular polarisations, but no longitudinal modes. Adding a non-zero mass to the vector, gauge symmetries are broken and the vector regains its lost polarisation. Thus there is a fundamental difference between massless and massive vector bosons.

Even for massive vectors, loop corrections to the masses are protected. In fact, for large momenta running in the loop (near the cutoff), the mass of the vector can be neglected, thus a gauge symmetry is restored. The restored gauge symmetry makes the loop correction vanish near the cutoff.

2.2.2 Spin 1/2

Spin 1/2 particles have two spin configurations. For fermions, a mass term connects left-handed to right-handed chiralities. For massless fermions, the two polarisations are independent, thus they can be considered as physically distinct fields. This fact increases the number of symmetries of the system!

Again, in the limit of small mass, the two polarisation decouple and the loop correction must vanish!

2.2.3 Spin 0

No such argument applies to scalar fields, which have a single spin configuration!

2.3 Symmetries and New Physics

The naturalness argument requires the presence of a symmetry (protection mechanism) to shield the Higgs mass from quadratically divergent loop corrections. We have seen that masses can be protected by the presence of spin, or more specifically by chirality in the case of fermions and gauge symmetries in the case of vectors. Ultimately, every new physics scenario tries to apply these two symmetries on the Higgs boson!

– **Spin-0 related to spin-1/2**: is there a symmetry that relates a scalar to a spin-1/2 particle? If so, the chirality will protect the fermion mass, and our new symmetry will project the protection on the scalar!

  (a) *Supersymmetry*: it extends the Poincaré algebra to include a symmetry between particles with different spins.

– **Replace a spin-0 with spin-1/2**: can we trade the scalar Higgs for fermions? QCD does it: it is a theory based on quarks and vector gluons, however, due to the strong interacting regime, fermions form mesons which are scalars.

  (b) *Technicolour/Composite Higgs models* [15, 34]: the Higgs emerges as a composite state (meson) of new fermions bound together by a new strong interaction.

– **Replace a spin-0 with a spin-1 (vector)**: can we use gauge symmetries to protect the Higgs, i.e. embed the BEH scalar into a gauge boson?

  (c) *Extra dimensions* [8]: in models with extra spacial dimensions, a vector boson has more then three polarisations. The extra polarisations appear, from the 4-dimensional point of view, as scalars, however they are secretly part of a vector and thus protected by extra dimensional gauge symmetries.

– **A special spin-0, global symmetries**: can global symmetries have any role in protecting the Higgs boson?

(d) *Little Higgs models* [42]: the Higgs arises as the Goldstone boson of a spontaneously broken global symmetry. The global symmetry is not exact, thus the Higgs develops a mass, however the model can be engineered to have finite one-loop corrections. It does not work beyond one loop!

3 Supersymmetry: A Fermion-Boson Symmetry

Supersymmetry is a symmetry that relates fermions and bosons to each other. It is useful to address the naturalness problem because it can associate the BEH scalar with a fermion: as the two partners share the same physical properties, the chiral symmetry which protects the fermion mass will also protect the scalar partner mass.

Let's consider a spin-0 quantum state $|s>$: supersymmetry can be thought of in terms of an operator Q, which transforms the scalar state into a spin-1/2 state $|f>$:

$$Q|s>=|f>\ . \tag{25}$$

In order for the equation to respect rotational invariance, the operator Q must carry spin-1/2, thus transforming as a spinor under Lorentz transformations: it is a fermionic operator. The minimal spin-1/2 representation of the Lorentz group is a Weyl fermion, i.e. a 2-component chiral fermion. One can however construct supersymmetric theories with any number of chiral generators Q^i (extended supersymmetry). Q is a fermion, therefore it will respect anti-commutation relations:

$$\{Q,\bar{Q}\} = -2\sigma^\mu p_\mu\,, \quad \{Q,Q\} = 0\,, \quad \{\bar{Q},\bar{Q}\} = 0\,. \tag{26}$$

Furthermore, being a spin-1/2 object, it has the following commutation properties with the position and momentum operators:

$$[Q, p_\mu] = 0\,, \qquad [\bar{Q}, p_\mu] = 0\,. \tag{27}$$

So far, we have only used the transformation properties of a spinor under space-time symmetries. What we obtained is that we can define a closed algebra including the usual Poincaré algebra, extended by the addition of the fermionic operator Q. In this sense supersymmetry is an extension of space-time symmetries.

3.1 How to Construct a Supersymmetric Quantum Field Theory?

The most straightforward way would be to write down a theory containing a scalar field φ and a chiral fermion χ, corresponding to the two related quantum states, and then derive the transformation properties of the fields under the operator Q. This procedure, however, turns out to be quite lengthy.

A shortcut is offered by the previous observation that the operator Q can be formally included into the Poincaré algebra: this fact leads to the introduction of *superfields* [35, 44]. Like space-time co-ordinates x^μ correspond to the momentum operators p^μ, we can think of associating co-ordinates θ (and $\bar{\theta}$) to the supersymmetric operators Q (and its conjugate $\bar{Q}$). What we are doing, therefore, is to enlarge space-time by adding some "funny" new co-ordinates: in fact, Q being a spinor implies that θ and $\bar{\theta}$ must be anti-commuting co-ordinates (or Grassmann variables). In some sense, they are spinors themselves, thus they carry an index labelling its two components (in the following we will often omit the spinor indices, assuming that they are always properly contracted). A field living in the superspace $\{x^\mu, \theta\}$, a superfield, is therefore simply a function of the extended set of co-ordinates x^μ, θ and $\bar{\theta}$. For a "scalar" superfield:

$$S(x^\mu, \theta, \bar{\theta})\,. \tag{28}$$

One can also define functions that transform non-trivially under Lorentz transformations: for instance, a "spinor" superfield W^α or a "vector" superfield W^μ, etc. Note that the Lorentz properties of the superfields do not have anything to do with the spin of the fields they encode! The next step would be to understand how a superfield transforms under supersymmetric transformations generated by Q.

Now, θ is a special kind of co-ordinate because it anti-commutes with itself and has two independent components: therefore, powers of θ^n with $n > 2$ must vanish. This is due to the fact that when contracting more than two identical spinors, at least two of them must have their spin aligned and such configuration is forbidden by Pauli's exclusion principle. This means that any superfield can be expanded in a finite series in powers of the super-coordinate θ. The most general expansion for our "scalar" superfield reads:

$$\begin{aligned} S(x^\mu, \theta, \bar{\theta}) = a + \theta\xi + \bar{\theta}\bar{\chi} + \theta\theta b + \bar{\theta}\bar{\theta} c + \\ \bar{\theta}\bar{\sigma}^\mu \theta v_\mu + \bar{\theta}\bar{\theta}\theta\eta + \theta\theta\bar{\theta}\bar{\zeta} + \theta\theta\bar{\theta}\bar{\theta} d\,, \end{aligned} \tag{29}$$

where σ^μ are, as usual, the Pauli matrices. The coefficients of the expansion are standard fields, functions of x^μ only: a, b, c, and d are scalar (spin-0) fields; ξ, χ, η and ζ are chiral fermions and v_μ is a vector. However, one should define some more minimal representations of the supersymmetric algebra, i.e. superfields that have less independent components than the general expansion. This selection is similar to the definition of spins: even though a spinor in 4 dimensions has 4 components, the minimal representation is a 2-component (chiral) Weyl fermion. The minimal superfield is the *chiral superfield* Φ, defined as

$$\Phi(y^\mu, \theta) = \varphi(y^\mu) + \sqrt{2}\,\theta\chi(y^\mu) + \theta\theta F(y^\mu)\,, \tag{30}$$

where $y^\mu = x^\mu + i\bar{\theta}\bar{\sigma}^\mu\theta$. Note that Φ only depends on $\bar{\theta}$ implicitly via y^μ. The definition of Φ can be formally extracted from the supersymmetric transformation properties of superfields, however the formal treatment is beyond the scope of these lectures. Note also that Φ contains a scalar field φ, a 2-components spinor χ, and an extra field F, whose function will be clear shortly.

The next step is to write an action for the superfield: in addition to the integral over the usual space-time, we need to integrate over the super-coordinate θ. There are, in this case, two possible ways of integrating:

$$\int d^2\theta\,, \qquad \text{and} \qquad \int d^2\theta d^2\bar{\theta}\,, \tag{31}$$

where $\bar{\theta}$ is the hermitian conjugate of θ. Another consequence of the fermionic nature of θ is that the only non-vanishing integrals are:

$$\int d^2\theta\ \theta\theta = 1\,, \quad \int d^2\theta d^2\bar{\theta}\ \theta\theta\bar{\theta}\bar{\theta} = 1\,, \tag{32}$$

where the two integrals are normalised to 1 for convenience. As we can always expand any function of θ and $\bar{\theta}$ in a finite series of terms, the integral definition is such that $\int d^2\theta$ selects the term of the expansion proportional to $\theta\theta$, i.e.

$$\int d^2\theta\ S(x^\mu, \theta, \bar{\theta}) = b + \bar{\theta}\zeta + \bar{\theta}\bar{\theta}d\,, \tag{33}$$

while

$$\int d^2\theta\, d^2\bar{\theta}\ S(x^\mu, \theta, \bar{\theta}) = d\,. \tag{34}$$

Here we chose to integrate over a "scalar" superfield because we want the Action to be invariant under Lorentz transformations.

Finally, we need to define a supersymmetric action, which contains an integration over the super-coordinates and is invariant under supersymmetric transformations. There are two possibilities, and they are both important in the definition of supersymmetric theories. On one hand, one can integrate over the whole superspace any "scalar" superfield (note that products of superfields are also superfields, being functions of θ and $\bar{\theta}$):

$$\mathscr{S}_1 = \int d^4x \int d^2\theta d^2\bar{\theta}\ S(x^\mu, \theta, \bar{\theta})\,. \tag{35}$$

As already noted, this selects the $\theta\theta\bar{\theta}\bar{\theta}$ term of the expansion (D-term): you can check that under supersymmetric transformations, such term only picks up total derivatives, which vanish once integrated over the whole space-time. This is enough to prove that the definition $\mathscr{S}_1$ is a good supersymmetric Action. What are the dimensions in mass of such a term? From the expansion in Eq. (29), as scalar fields like a have dimension 1 and fermion fields like ξ have dimension $3/2$, we can deduce that θ has dimension $-1/2$. Following our normalisation of the integrals, $d\theta^2$ must have dimension 1. The action is a pure number, therefore the superfield (or combination of superfields) we integrate over in $\mathscr{S}_1$ must have dimension 2 in mass: this fact will play an important role in defining a renormalisable supersymmetric action.

The second possibility is to integrate over $d^2\theta$ a chiral superfield after setting $\bar{\theta} = 0$, i.e.

$$\mathscr{S}_2 = \int d^4x \int d^2\theta \, \Phi(x^\mu, \theta) + h.c.. \tag{36}$$

This selects the F-term in the expansion in Eq. (30), which can be shown to also transform with a total derivative under supersymmetric transformations. Counting dimensions, the chiral superfield Φ (or product of chiral superfields) must have dimension in mass 3 for the action $\mathscr{S}_2$ to be a pure number.

3.1.1 Supersymmetric Action for a Chiral Superfield

Let's consider a chiral superfield Φ, and try to build an Action for its field components. The full expansion of Φ reads

$$\begin{aligned}\Phi(y^\mu, \theta) = \varphi + \sqrt{2}\theta\chi + \theta\theta F - i\bar{\theta}\bar{\sigma}^\mu\theta\partial_\mu\varphi + \\ \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\chi - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial^\mu\partial_\mu\varphi \,,\end{aligned} \tag{37}$$

where all the fields φ, χ and F are intended to be functions of x^μ, and the $\bar{\theta}$ dependence is explicit.

Let's first use $\mathscr{S}_1$: the first attempt would be to integrate over a single chiral superfield

$$\int d^4x \int d^2\theta d^2\bar{\theta} \, \Phi(y^\mu, \theta) = -\int d^4x \, \frac{1}{4}\partial^\mu\partial_\mu\varphi \; = 0 \,, \tag{38}$$

because the $\theta\theta\bar{\theta}\bar{\theta}$ term is a total derivative. The same would be true for any chiral superfield, including products of two or more superfields. As a second try, we can integrate over the product of a chiral superfield with its complex conjugate (which is not a chiral superfield):

$$\int d^4x \int d^2\theta d^2\bar{\theta} \, \Phi^\dagger\Phi = \int d^4x \, (\partial^\mu\varphi)^\dagger\partial_\mu\varphi - i\,\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi + F^*F \,. \tag{39}$$

This looks like the kinetic term for a scalar field φ and a fermion χ. The extra field F does not have any derivative, thus it is not a dynamic field (auxiliary field), and it can be easily integrated out. Note also that this is the lowest dimensional object we can write down, and the only one which is renormalisable, because it saturates the dimension of the integrand (2) and therefore its coefficient is a pure number. Adding the product of more than two superfields would induce higher dimensional operators, because we would need to divide by a mass scale to compensate for the dimension in mass of the extra superfields. As a summary, we discovered that the action $\mathscr{S}_1$ can be used to define kinetic terms for the field coefficients of a chiral superfields, plus a static F field (auxiliary field) which can be integrated out.

To use $\mathscr{S}_2$, we need chiral superfields: after setting $\bar{\theta} = 0$, their expansion simplifies

$$\Phi(x^\mu, \theta) = \varphi + \sqrt{2}\,\theta\chi + \theta\theta F\,. \tag{40}$$

The most general action will therefore be

$$\int d^4x \int d^2\theta \left[\frac{1}{2}\mu\ \Phi\Phi + \frac{1}{3}y\ \Phi\Phi\Phi\right], \tag{41}$$

where we have kept only the normalisable interaction (and neglected a linear term). Here μ has dimension of mass, while y is a pure number. The integral selects the terms in the expansion proportional to $\theta\theta$. There are two possibilities: either we take an F component from one superfield and the scalar ones from the remaining ones, or we select a fermion from two superfields, and scalars from the remaining ones:

$$\begin{aligned} &\int d^4x\ \int d^2\theta\ \left[\tfrac{1}{2}\mu\ \Phi\Phi + \tfrac{1}{3}y\ \Phi\Phi\Phi + h.c.\right] \\ &\qquad = \int d^4x\ \left[F\left(\mu\varphi + y\varphi^2\right) - \left(\tfrac{1}{2}\mu + y\varphi\right)\chi\chi + h.c.\right]. \end{aligned} \tag{42}$$

This action term generates a mass for the fermion, an interaction of fermions with the scalar, and an interactions of the F term with scalars. The F terms is not dynamical, as it does not have derivatives, therefore we can use the equations of motion to calculate it in terms of the other fields, and eliminate it from the action. Putting together this term with the kinetic terms in Eq. (39), the equation of motion for the auxiliary field F reads

$$F^* + \left(\mu\varphi + y\varphi^2\right) = 0\,. \tag{43}$$

Therefore

$$F^*F + F\left(\mu\varphi + y\varphi^2\right) + F^*\left(\mu\varphi + y\varphi^2\right)^* = -\left|\mu\varphi + y\varphi^2\right|^2 = -F^*F\,; \tag{44}$$

where the first term comes from the kinetic action, and the last two from the superpotential action in Eq. (42) and its complex conjugate. Putting all the pieces together, we obtain the following action for the scalar φ and the fermion χ fields:

$$\mathscr{S}_\Phi = \int d^4x \left[(\partial^\mu \varphi)^\dagger \partial_\mu \varphi - i\bar{\chi}\bar{\sigma}^\mu \partial_\mu \chi - \left|\mu\varphi + y\varphi^2\right|^2 - \left(\frac{1}{2}\mu + y\varphi\right)\chi\chi + h.c.\right]. \tag{45}$$

Note that both the complex scalar φ and the chiral fermion have mass $m_\varphi = m_\chi = \mu$: this is the first consequence of supersymmetry.

3.1.2 General Expressions

The derivation above can be extended to a generic number of superfields. If we have N chiral superfields Φ_i, the most general action can be written as

$$\mathscr{S} = \int d^4x \left\{ \int d^2\theta d^2\,\bar{\theta}\,\Phi_i^\dagger \Phi_i + \int d^2\theta\; W(\Phi_i) + h.c. \right\}\,, \tag{46}$$

where the *superpotential* W is a function of the superfields (if renormalisable, it can contain up to trilinear terms). Besides the kinetic terms for scalars and fermions, the action will contain the following interactions:

$$\mathscr{S}_W = \int d^4x \left[-\sum_i \left| \frac{\partial W(\varphi)}{\partial \varphi_i} \right|^2 - \frac{1}{2} \sum_{i,j} \frac{\partial^2 W(\varphi)}{\partial \varphi_i \partial \varphi_j} \chi^i \chi^j \; + h.c. \right]\,, \tag{47}$$

where the first term (the scalar potential) comes from the integration of the F terms.

4 Minimal Supersymmetric Standard Model

Now that we are familiar with the construction of supersymmetric actions with the use of superfields, we can attempt to construct a supersymmetric action that contains the Standard Model one. A chiral superfield contains a Weyl spinor and a complex scalar: it is therefore an ideal candidate to describe the SM fermions and their super-partners. Recall that the Standard Model is indeed build in terms of Weyl spinors (chiral fermions), because left- and right-handed components of the same fermion transform differently under the electro-weak gauge symmetry.

Vector bosons, on the other hand, cannot be expressed in terms of chiral superfields. One way to deal with them is to define another close representation of the supersymmetric algebra: the *real superfield* $V(x^\mu, \theta, \bar{\theta})$ which is a general "scalar" superfield with the constraint

$$V = V^\dagger\,.$$

The superfield V is often called *vector superfield* for the simple reason that the vector coefficient v^μ in the expansion (29) is not projected out. However, we are still using a "scalar" superfield to construct it! One can use the same actions $\mathscr{S}_1$ and $\mathscr{S}_2$ to define a proper action for the components of the real superfield (which turn out to be a vector v^μ and a chiral fermion λ), and use them to implement gauge interactions. This construction will be left out of this series of lectures, and we refer the reader to Ref. [35, 44] for more details.

4.1 Supersymmetry in Action: Naturalness of the Top Loop

Let's start from the problem that motivated the introduction of supersymmetry in the first place, i.e. the naturalness of the Higgs mass which is endangered, in the SM, by the divergent loop contribution of the top Yukawa coupling (among others). We will first consider the supersymmetric version of the top Yukawa:

$$\mathscr{S}_{top} = \int d^4x \, \left[-y_t \chi_Q \varphi_H \chi_{t_R} + h.c. \right] , \tag{48}$$

where we have re-written the interaction in Eq. (21) in terms of Weyl spinors: χ_Q for the left-handed doublet and χ_{t_R} for the right-handed singlet. The BEH field is called here φ_H to distinguish it from the scalar super-partners of the tops. A supersymmetric version must contain three chiral superfields which have the same quantum numbers as the SM fields:

$$\Phi_H = \varphi_H + \sqrt{2}\theta\chi_H + \theta\theta F_H , \tag{49}$$

$$\Phi_Q = \varphi_Q + \sqrt{2}\theta\chi_Q + \theta\theta F_Q , \tag{50}$$

$$\Phi_{t_R} = \varphi_{t_R} + \sqrt{2}\theta\chi_{t_R} + \theta\theta F_{t_R} . \tag{51}$$

The most general superpotential one can write down is:

$$W = y_t \Phi_Q \Phi_H \Phi_{t_R} . \tag{52}$$

This is the only term, compatible with the quantum numbers (colour, hypercharge and SU(2) properties) of the superfields, and being renormalisable. Using the derivation in Eq. (47), the supersymmetric interactions generated by such a term are:

$$\mathscr{S}_{susy-top} = \int d^4x \Big[- y_t \left(\varphi_H \chi_Q \chi_{t_R} + \varphi_Q \chi_H \chi_{t_R} + \varphi_{t_R} \chi_Q \chi_H + h.c. \right) \\ -y_t^2 \left(\varphi_Q^* \varphi_H^* \, \varphi_Q \varphi_H + \varphi_H^* \varphi_{t_R}^* \, \varphi_H \varphi_{t_R} + \varphi_Q^* \varphi_{t_R}^* \, \varphi_Q \varphi_{t_R} \right) \Big]. \tag{53}$$

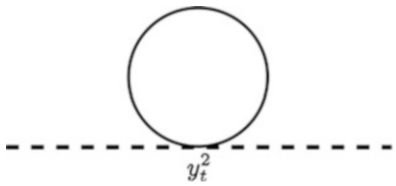

Fig. 2 Stop loop contribution to the Higgs mass

The first term is exactly the SM top Yukawa coupling, thus this superpotential is a proper supersymmetric extension of the top Yukawa sector. The BEH field φ_H has two additional 4-scalar interactions with φ_Q and φ_{t_R}. Such interactions, will contribute to the loop corrections to the Higgs mass, see Fig. 2. Each loop will contribute (here we assign a mass m to the scalar top partners):

$$\begin{aligned} -i\delta m_\phi^2 &= -3iy_t^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-m^2} \\ &= \frac{3y_t^2}{16\pi^4} 2\pi^2 i \int k_E^3 dk_E \frac{1}{-k_E^2-m^2} \\ &= -\frac{3y_t^2}{16\pi^2} \int dk_E^2 \frac{k_E^2}{k_E^2+m^2} \\ &= -\frac{3y_t^2}{16\pi^2} \int_0^{\Lambda^2} dk_E^2 \left(1-\frac{m^2}{k_E^2+m^2}\right) \\ &= -\frac{3y_t^2}{16\pi^2} \left(\Lambda^2 - m^2 \log\frac{\Lambda^2+m^2}{m^2}\right). \end{aligned} \tag{54}$$

Summing the contribution of the two scalar tops (stops) with the top one in Eq. (22), the quadratically divergent term cancels out, and we are left with log divergent terms

$$\delta m_\phi^2 = -\frac{3y_t^2}{16\pi^2} \left(m_Q^2 \log\frac{\Lambda^2}{m_Q^2} + m_{t_R}^2 \log\frac{\Lambda^2}{m_{t_R}^2}\right). \tag{55}$$

Note here that the two masses m_Q and m_{t_R} we arbitrarily assigned to the stops are not supersymmetric: in fact, the fermionic top is still massless because its mass can only be generated by the EWSB. On the other hand, masses for the scalar tops are allowed by gauge invariance. We will comment on the origin and significance of such terms later on: for now notice that in the case of exact supersymmetry (i.e. $m_Q = m_{t_R} = 0$) the loop correction to the BEH field mass would be exactly zero.

Table 1 Chiral superfield content of the MSSM. For completeness, we also add their Baryon and Lepton numbers B and L

	Label	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	3B	L	$(-1)^{3(B-L)}$
r-h electron	e_R	**1**	**1**	1	0	−1	−
l-h leptons	L	**1**	**2**	$-1/2$	0	1	−
r-h up quark	u_R	**3**	**1**	$-2/3$	−1	0	−
r-h down quark	d_R	**3**	**1**	1/3	−1	0	−
l-h quarks	Q	**3**	**2**	1/6	1	0	−
Higgs (up)	H_u	**1**	**2**	1/2	0	0	+
Higgs (down)	H_d	**1**	**2**	$-1/2$	0	0	+

4.2 The MSSM

In a minimal supersymmetric version of the Standard Model [35], besides supersymmetric gauge interactions that we have not described here, we need to promote each SM field to a chiral superfield (listed in Table 1). Note the presence of two Higgs doublets, H_u and H_d with opposite hypercharges. If we had only one Higgs, say H_u, we would only be able to write Yukawa interactions for up quarks:

$$W_u = y_u \Phi_{H_u} \Phi_Q \Phi_{u_R} \, . \tag{56}$$

The other Yukawas in the SM should be written in terms of $\varphi^*_{H_u}$, which is contained in H_u^* (which is not a chiral superfield!). Thus, in order to preserve supersymmetry, we are forced to introduce a second Higgs doublet, with opposite sign hypercharge that will couple to down-type fermions:

$$W_d = y_d \Phi_{H_d} \Phi_Q \Phi_{d_R} + y_e \Phi_{H_d} \Phi_L \Phi_{e_R} \, . \tag{57}$$

We can also add a bilinear in the two Higgs superfields:

$$W_H = \mu \Phi_{H_u} \Phi_{H_d} \, , \tag{58}$$

which will generate a mass for the two Higgs scalars $m_{H_u} = m_{H_d} = \mu$.

There is another reason why two Higgses are needed: in the SM, a complete generation of fermions is anomaly free. In supersymmetry, the Higgs superfield will contain a new fermion doublet, the superpartner of the Higgs. The presence of a single higgsino would generate anomalies: the role of the second Higgs is therefore to cancel such anomalies. The three terms listed here should be the only superpotential terms, because they generate the needed fermion yukawa couplings and a mass for the Higgses. However this is not the case generically.

4.2.1 Troubleshooting 1: Unwanted Superpotential Terms (the Need for *R*-Parity, and Dark Matter)

In addition to the Yukawa interactions, the superfield content in Table 1 allows for many more "dangerous" terms to be added. The game here is to add all combinations of two or three superfields that can lead to a term invariant under all gauge symmetries. For instance, one can add an operator made of three quark singlets:

$$\Phi_{u_R}\Phi_{d_R}\Phi_{d_R}\,; \tag{59}$$

an operator of this kind would be forbidden in the SM because it contains three fermions.

Also, Φ_L and Φ_{H_d} have exactly the same quantum numbers, thus they can be interchanged:

$$\Phi_{H_d}\Phi_{H_d}\Phi_{e_R}\,, \quad \ldots \tag{60}$$

What symmetries of the SM are violated by such superpotential terms?

The first kind will violate Baryon number (the operator has a net baryon number -1, like an anti-neutron); the second kind violates lepton number. Such terms are very dangerous because, among other things, they can mediate the proton decay. Recall that both Baryon and Lepton number conservation are an accidental consequence of the matter content of the SM. In supersymmetric extensions of the SM, such an accident does not occur. The basic reason for this is that in the MSSM, both the fermions and the Higgs boson are traded with "scalar" superfields, therefore Baryon and Lepton numbers, which are a relic of the flavour symmetry in the SM, are completely removed.

At this point there are two options: one may either assume that the "unwanted" terms are small for some unknown reason, in order to satisfy the bounds. The other route is to add a symmetry that would forbid such terms: this symmetry necessarily is a shadow of Baryon and Lepton symmetries, and goes under the name of **R-parity**.

4.2.2 R-Parity and Dark Matter

The solution is to somehow impose Baryon and Lepton number conservation by hand. We therefore impose a Z_2 parity on the superfields, defined as:

$$P_M = (-1)^{3(B-L)}\,; \tag{61}$$

it is called *matter parity*, and it is defined in terms of $B-L$ because this combination is anomaly free in the SM and thus it is guaranteed to be conserved. Requiring the superpotential to be even under matter parity eliminates all the unwanted terms.

We can further elaborate this symmetry: any action term must contain an even number of fermions, so we can redefine matter parity by adding an extra "-1" for fermions, without affecting the allowed interaction terms:

$$P_R = (-1)^{3(B-L)+2s}\,, \tag{62}$$

where s is the spin of the field. This parity now acts on the field components of the superfields (thus it is not compatible with supersymmetry), and it is called *R-parity*. Note that now scalars and bosons in the same superfield have opposite R-parity; furthermore, all SM states (matter fermions and scalar Higgses) have R-parity $+1$, while the supersymetric partners (squarks, sleptons and higgsinos) have R-parity -1. This implies that the lightest supersymmetric particle is stable, because it cannot decay into SM states only!

Can it play the role of Dark Matter [11]?

A few comments are in order here: as we have seen, R-parity is needed to save the MSSM from fast proton decays, so it is added by hand on the supersymmetric action. While the presence of a stable Dark Matter candidate is a consequence of imposing R-parity, it should not be considered as a prediction of supersymmetry itself. In fact, the most general supersymmetric MSSM would not contain any dark matter candidate! Supersymmetry also contains an internal symmetry, called R-symmetry, which is a phase transformation of the θ co-ordinates. It should be stressed here that this R-symmetry has nothing to do with R-parity, but it is simply related to the fact that two fermions are always needed to write a Lorentz invariant interaction term.

Note also that R-parity is not exactly equivalent to imposing Baryon and Lepton conservation: in fact, it is a parity, therefore only terms with an odd Baryon or Lepton number are forbidden. For instance, a Majorana mass for neutrinos, which would violate lepton number by two units, is allowed. Thus the typical neutrino mass mechanisms (small Yukawa couplings with a singlet, see-saw, etc.) can be implemented in the MSSM.

4.2.3 Troubleshooting 2: Supersymmetry Cannot Be Exact! Where Do We Expect the Superpartners?

Another problem is that supersymmetry is not an exact symmetry, because it would predict that SM states and their partners have the same mass (we are pretty confident that there are no scalar electrons around!).

One way to break supersymmetry without spoiling its nice properties (mainly the cancellation of divergences), is to add only "mass terms", i.e. couplings with a positive mass dimension: the reason behind is that at high energies, well above the supersymmetry breaking mass scales, supersymmetry is restored, thus the divergences still cancel out! This principle is called *soft supersymmetry breaking*. We should also be careful not to violate R-parity if we want a Dark Matter candidate in the model. The allowed terms are therefore:

– Higgs mass terms: $-m^2_{H_u}\varphi^*_{H_u}\varphi_{H_u} - m^2_{H_d}\varphi^*_{H_d}\varphi_{H_d}$;
– Scalar quark and lepton masses: $-\tilde{m}^2_Q\varphi^*_Q\varphi_Q - \tilde{m}^2_{t_R}\varphi^*_{t_R}\varphi_{t_R} + \ldots$;
– Trilinear scalar couplings (in the same form as Yukawa couplings): $A\varphi_{H_u}\varphi_Q\varphi_{t_R} + \ldots$;
– Gaugino masses (masses for the fermion partners of gauge bosons).

Note that a huge number of soft supersymmetry breaking terms can be added to the MSSM (more that 120!). In order to study the phenomenology, one needs to make simplifying assumptions or develop a mechanism of supersymmetry breaking.

We will not go into the details of the supersymmetry breaking mechanisms, however a few comments are in order:

– If supersymmetry is spontaneously broken, then the Goldstone theorem (extended to fermionic symmetries) would predict the presence of a massless fermionic field (called *goldstino*). Such object presents a challenge for the phenomenology: one way out is to promote supersymmetry to a local symmetry (*supergravity*), where the goldstino is eaten by the spin-3/2 partner of the graviton to give rise to a massive spin-3/2 state (*gravitino*). In some models, it is the gravitino that plays the role of the Dark Matter, being the lightest supersymmetric (thus R-parity odd) state.
– The most popular supersymmetry breaking mechanisms are *m-SUGRA*, *AMSB* and *GMSB*: m-SUGRA is based on a naive assumption that supergravity would couple equally to all states of a given spin, therefore if gravity breaks supersymmetry then all particles with same spin would receive the same supersymmetry breaking mass. In AMSB (Anomaly Mediated Supersymmetry Breaking), it is gravitational anomalies that break supersymmetry: this is therefore a realistic model of breaking mediated by gravitational interactions. In GMSB (Gauge Mediated SB), the basic assumption is that supersymmetry breaking (which may be generated dynamically *á la* Technicolour) is mediated to the MSSM via gauge interactions. Such models have very different predictions on the low energy spectrum of the MSSM.

One important question, however, independent on the origin of supersymmetry breaking is the value of the scale where the masses of the superpartners should be. The handle we have on this question is the naturalness argument. In fact, the partner masses enter the BEH field mass via loop corrections, like the ones generated by the top and stop loops:

$$m^2_{BEH} \sim \mu^2 - \frac{3y_t^2}{16\pi^2}\left(m^2_Q \log\frac{\Lambda^2}{m^2_Q} + m^2_{t_R}\log\frac{\Lambda^2}{m^2_{t_R}}\right). \qquad (63)$$

This formula is very important for the MSSM phenomenology. On one hand, we need the stop loop to give a contribution not far from the measured value of the Higgs mass: this observation would tell us that the masses m_Q and m_{t_R} should not be too far from the TeV scale. Another noteworthy element is the presence of the μ

term, which is a supersymmetric coupling, and which gives a positive mass to the BEH field: in order for the model to feature EWSB, we would need μ to be smaller than the top loop contribution. So the question is: why should a supersymetric coupling, which can take any value between 0 to the Planck mass, know about the scale of supersymmetry breaking? This question is called the μ-problem. One possible solution is to extend the MSSM with a singlet field, and use it to generate a μ-term via its vacuum expectation value [25]: this model is called NMSSM.

5 Extra Dimensions: Warm Up

Extra dimensions are also a simple extension of the space-time symmetries, by extending the usual 4-dimensional space-time with the addition of extra space dimensions. This idea was first proposed at the beginning of last century in an attempt to unify QED with General Relativity. While this project failed, extra dimensions appeared again when string theory was formulated: in order to have a mathematically consistent theory, string theory should be formulated on least 10 dimensions. This dimensions are certainly not part of the space we can normally probe (as you can check with your own eyes), so their effects must be somehow hidden and appear only above a certain energy scale. Initially string theory aimed at a consistent formulation of quantum gravity, therefore the scale where their effect would appear was assumed to be the Planck scale. In the early 1990s, I. Antoniadis and K. Benakli [6, 7] realised that extra dimensions may actually appear at much lower scales, possibly accessible to colliders, thus they ignited an active research program aimed at exploring the phenomenological implications of and new mechanisms available in extra dimensional space-time.

The first issue to be addressed is the mechanism to hide the extra dimensions at high scale: the simplest way is to postulate that the extra space is compact, another possibility is to modify the geometry of the space so that extra energy is needed for particles to propagate in the curved background. In the following we will see an example of both.

5.1 A 5D Scalar Field

Before detailing some of the models of BSM based on extra dimensions, we need to understand how their effect would appear at low energy. We can start our exploration with a scalar (spin-0) field with a single extra space co-ordinate: the action is simply extended to

$$\mathscr{S}_s = \int d^5x \; (\partial^M \Phi)^\dagger \partial_M \Phi - M^2 \Phi^\dagger \Phi \,, \tag{64}$$

where $M = \mu, 5$ labels the 5 directions in space-time (the derivative is promoted to a 5-vector), and

$$\Phi = \Phi(x^\mu, x_5)\,. \tag{65}$$

Here we are also assuming that the metric on the 5D space is an extension of Minkowski (flat space). From the above action we can derive the usual Klein-Gordon equation of motion:

$$-\partial^M \partial_M \Phi - M^2 \Phi = -\partial^\mu \partial_\mu \Phi + \partial_5^2 \Phi - M^2 \Phi = 0\,. \tag{66}$$

The simplest compact space is a circle, i.e. a space where we impose periodic conditions on the fields:

$$\Phi(x^\mu, x_5 + 2\pi R) = e^{i\alpha_\Phi} \Phi(x^\mu, x_5)\,; \tag{67}$$

in general a non-zero phase α_Φ (Scherk-Schwarz phase) may be imposed, for simplicity here we will only consider periodic fields and we will set $\alpha_\Phi = 0$. If we want to go to momentum space, along the visible directions the usual Fourier transform applies; on the other hand, along x_5 we need to Fourier expand in a series of functions (the domain of the function of x_5 is finite!). Therefore, one can rewrite the scalar field as

$$\Phi(x^\mu, x_5) = \int \frac{d^4 p}{(2\pi)^4}\, e^{i p_\mu x^\mu} \sum_n f_n(x_5) \varphi_n(p^\mu)\,; \tag{68}$$

where p^μ is the usual 4D momentum, f_n is a complete set of functions on the compact extra space (*wave functions*), and the "coefficients" $\varphi_n(p^\mu)$ can be interpreted as 4D fields (*Kaluza Klein modes*). Plugging this expansion in the equation of motion, we obtain a set of equations for f_n:

$$(p^2 - M^2) f_n - \partial_5^2 f_n = 0 \tag{69}$$

whose solutions are

$$\sin x_5 \sqrt{p^2 - M^2}\,, \qquad \cos x_5 \sqrt{p^2 - M^2}\,. \tag{70}$$

The periodicity implies that

$$\sqrt{p^2 - M^2} = n/R \tag{71}$$

where n is positive integer. Now p is the usual 4 dimensional momentum, therefore we can interpret p^2 as the 4D mass of the 4D field, and the above equation yields

$$p^2 = m_n^2 = \frac{n^2}{R^2} + M^2 = n^2 m_{KK}^2 + M^2\,, \quad m_{KK} = 1/R\,. \tag{72}$$

The complete expansion of the field is then (where we have properly normalised the wave functions f_n)

$$\Phi(p^\mu, x^5) = \frac{1}{2\pi R}\varphi_0 + \sum_{n=1}^{\infty} \frac{\cos n x_5/R}{\pi R}\varphi_{n,c} + \sum_{n=1}^{\infty} \frac{\sin n x_5/R}{\pi R}\varphi_{n,s}\,, \tag{73}$$

with effective 4D action

$$\mathscr{S}_s = \int d^4x \Big[(\partial^\mu \varphi_0)^\dagger \partial_\mu \varphi_0 + \\ + \sum_n (\partial^\mu \varphi_{n,c/s})^\dagger \partial_\mu \varphi_{n,c/s} - (M^2 + n^2 m_{KK}^2) \varphi_{n,c/s}^\dagger \varphi_{n,c/s} \Big]\,. \tag{74}$$

5.2 Orbifold

Starting from the circle, more spaces can be defined by using the symmetries of the circle itself: one can in fact identify points mapped one into the other by such symmetry. For instance, the circle is invariant under a mirror symmetry with respect to any diameter: $x_5 \to -x_5$. If a circle is defined for $x_5 \subset [-\pi R, \pi R)$, then the mirror symmetry identifies positive and negative points. The resulting space (the interval) is defined on $x_5 \subset [0, \pi R]$.

On the fields, the orbifold projection means that each field must satisfy:

$$\Phi(p^\mu, -x_5) = \pm \Phi(p^\mu, x_5)\,. \tag{75}$$

Each field is characterised by a sign choice; the wave functions that do not respect the transformation properties are then removed.

$$\Phi^+ = \frac{1}{2\pi R}\varphi_0 + \sum_{n=1}^{\infty} \frac{\cos n x_5/R}{\pi R}\varphi_{n,c}\,; \tag{76}$$

$$\Phi^- = \sum_{n=1}^{\infty} \frac{\sin n x_5/R}{\pi R}\varphi_{n,s}\,. \tag{77}$$

Note that the massless $n = 0$ mode is only present for Φ^+; both choices have a tower of massive states with the same mass but different wave functions.

5.3 A 5D Vector (Gauge) Field

The action for a gauge (vector) boson can again be obtained by a simple extension of the 4D case. A vector field must first be generalised to a 5-vector: $A_M = \{A_\mu, A_5\}$. For an abelian gauge group the action is:

$$\begin{aligned}\mathscr{S}_{gauge} &= \int d^5x \left[-\frac{1}{4}F_{MN}F^{MN}\right] \qquad \text{[here} \quad F_{MN}=\partial_M A_N - \partial_M A_N] \\ &= \int d^5x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}F_{\mu 5}F_5^{\mu}\right] \\ &= \int d^5x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\partial_\mu A_5 \partial^\mu A_5 + \frac{1}{2}\partial_5 A^\mu \partial_5 A^\mu - \partial_\mu A_5 \partial_5 A^\mu\right]. \end{aligned} \tag{78}$$

The $\mu 5$ term generates a mixing between the 4D vector components A_μ and the 4D scalar term A_5: this is similar to the mixing we obtain in the SM between the massive vectors and the Goldstone components of the BEH field. To simplify the equations, we can add a "gauge fixing" term to the action, which is a total derivative that can cancel out the mixing term and decouple the vector and the scalar. The extra dimensional R_ξ gauge fixing term is then (similar to the ones used in the SM):

$$\mathscr{S}_{GF} = \int d^5x \left[-\frac{1}{2\xi}\left(\partial_\mu A^\mu - \xi \partial_5 A^5\right)^2\right]. \tag{79}$$

The "gauge fixed" action now reads:

$$\begin{aligned}\mathscr{S}_{gauge+GF} = \int d^5x \Bigg[&-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}\partial_5 A^\mu \partial_5 A^\mu \\ &+\frac{1}{2}\partial_\mu A_5 \partial^\mu A_5 - \frac{\xi}{2}(\partial_5 A^5)^2\Bigg]; \end{aligned} \tag{80}$$

where A_5 features an action similar to a scalar field.

5.3.1 Vector A_μ

From the action in Eq. (80), the equation of motion for the vector part is

$$\partial^\mu F_{\mu\nu} + \frac{1}{\xi}\partial_\nu \partial^\mu A_\mu - \partial_5^2 A_\nu = 0\,. \tag{81}$$

We can Fourier transform and expand the field as before

$$A_\mu(x^\nu, x_5) = \int \frac{d^4p}{(2\pi)^4}\, e^{ip_\alpha x^\alpha} \sum_n f_n(x_5)\, A_\mu^n(p^\nu)\,; \tag{82}$$

and, assuming that the 4D fields A^n_μ satisfy the usual 4D equation of motion in ξ gauge for a massive state,

$$\partial^\mu F_{\mu\nu} + \frac{1}{\xi}\partial_\nu \partial^\mu A_\mu = -p^2 A_\nu\,. \tag{83}$$

we have the following equation for the wave functions

$$(p^2 + \partial_5^2) f_n = 0\,, \tag{84}$$

which is the same as in the scalar case (but with $M = 0$). The final KK expansion is therefore analogous to the scalar one. The spectrum contains one massless gauge boson (thus in 4D gauge symmetries are respected), and a tower of massive states. Where do the massive states get the longitudinal polarisation, as there is no BEH field here?

5.3.2 Scalar (and the Extra Dimension "BEH" Mechanism)

The equation of motion for the A_5 scalar reads:

$$(\partial_\mu \partial^\mu - \xi \partial_5^2) A_5 = 0\,, \tag{85}$$

which is similar to the one for a 5D scalar, with the exception of the parameter ξ. After the usual Fourier expansion, the equation for the wave functions is:

$$\left(\frac{p^2}{\xi} + \partial_5^2\right) f_n = 0\,, \tag{86}$$

thus the expansion is the same as above, except for the substitution $p^2 \to \frac{p^2}{\xi}$.

The masses will therefore be

$$m_n^2 = \xi n^2 m_{KK}^2\,, \tag{87}$$

which look like the masses of a Goldstone boson in the "BEH" mechanism in ξ-gauge. Note that the only mode whose mass is independent on ξ is the zero mode $n = 0$. The massive states can be decoupled in the limit $\xi \to \infty$ (Unitary gauge): what we learn, therefore, is that the massive modes of the scalar polarisation A_5 are the Goldstone bosons eaten up by the massive vectors! Compact extra dimensions have, therefore, a build-in BEH mechanism that gives mass to the Kaluza-Klein tower of states. This fact has been used to construct "Higgsless" models [19, 20], where the W and Z are identified with Kaluza-Klein states that pick up their mass without the need for a scalar BEH field (and no Higgs boson).

5.3.3 Gauge Invariance

The 5D action is invariant under a generalised gauge transformation:

$$A_M \to A_M + ig\partial_M \alpha(x^\mu, x_5)\,. \tag{88}$$

The local gauge parameter α must satisfy the same properties as the gauge field A_M, thus it also is a periodic function of x_5. We can therefore Fourier expand both A_μ and α, and write down 4D gauge transformations for each KK mode:

$$A_\mu^n \to A_\mu^n + ig\partial_\mu \alpha^n(x^\mu)\,. \tag{89}$$

Naively, we would expect the presence of an infinite number of gauge groups, however, as shown in the mass spectrum, the extra polarisation "spontaneously breaks" the gauge invariance associated with the massive modes; only the 4D gauge invariance of the massless mode is (explicitly) preserved.

Caveat: the Fourier expansion of the gauge transformation properties is a bit naive, one should really consider the gauge transformation on 5D fields!

5.3.4 Orbifold

We can now extend the analysis to orbifolds. As before, the field must be associated with a parity under the orbifold symmetry. However, the parities of the A_μ and A_5 components are related to each other by the fact that they belong to a vector! So if the orbifold symmetry is $x_5 \to -x_5$ (change sign to the 5th component of the vector but not to the other 4), the parity assignment for the 5D vector must be

$$A_\mu(-x_5) = \pm A_\mu(x_5)\,, \qquad A_5(-x_5) = \mp A_5(x_5)\,, \tag{90}$$

in other words their parity must be opposite! Recall that A_M must be a vector, i.e. share the same transformation properties as space-time co-ordinates, because it appears in the covariant derivative: $D_M = \partial_M - igA_M$.

For a $+$ vector, the scalar is $-$: in this case, the vector contains a massless zero mode and massive vectors (with *cos* wave function), while the scalars only contain a tower of Goldstone bosons (with wave function *sin*).

For a $-$ vector, the scalar is $+$: now the vectors only contain a tower of massive states (*sin*), while the scalars contain a physical massless scalar and a tower of Goldstone bosons (*cos*).

Note that the number of massive states always matches, so that the massive "scalars" always provide for the extra polarisation of the massive vectors. Also that for $-$ vectors, the 4D gauge symmetry is broken, as signalled by the absence of a massless vector in the KK expansion! However, a massless scalar is present! The breaking of gauge symmetries is however constrained: the proper definition is that the transformation of the A_μ under the orbifold projection is: $A_\mu(-x_5) = G(A_\mu(x_5))$, where G is a gauge transformation. Therefore, one can only break a gauge group $\mathscr{G}$ to a subgroup $\mathscr{H}$ of same rank. As a consequence, U(1) symmetries cannot be broken.

5.4 A 5D Fermion

The Dirac Gamma matrices must be generalised to 5D, i.e. we need to define a set of 5 (not 4) anti-commuting matrices. The natural choice is to promote γ^5 to the role of the gamma matrix for the 5th direction. The minimal spinor is now a 4-component one, and it is not possible to define chiral projections. The action is

$$\mathscr{S}_f = \int d^5x \; i\bar{\Psi}\Gamma^M \partial_M \Psi - m\bar{\Psi}\Psi \,, \tag{91}$$

where the 5D fermion can be described in terms of two 2-component Weyl fermions:

$$\Psi = \begin{pmatrix} \chi \\ \bar{\eta} \end{pmatrix} . \tag{92}$$

In terms of Weyl fermions, the action reads

$$\mathscr{S}_f = \int d^5x \;\; -i\bar{\chi}\bar{\sigma}^\mu \partial_\mu \chi - i\eta\sigma^\mu \partial_\mu \bar{\eta} - \bar{\chi}\partial_5\bar{\eta} + \eta\partial_5\chi + m(\bar{\chi}\bar{\eta} + \eta\chi) \,; \tag{93}$$

from which we can derive the following equations of motion

$$-\, i\bar{\sigma}^\mu \partial_\mu \chi - \partial_5\bar{\eta} + m\bar{\eta} = 0 \,, \tag{94}$$

$$-i\sigma^\mu \partial_\mu \bar{\eta} + \partial_5\chi + m\chi = 0 \,. \tag{95}$$

The KK decomposition is in the form

$$\chi = \sum_n g_n(x_5)\chi_n(x^\mu) \,, \quad \bar{\eta} = \sum_n f_n(x_5)\bar{\eta}(x^\mu) \,, \tag{96}$$

where χ_n and $\bar{\eta}_n$ are usual 4D Weyl spinors.

The usual procedure can be followed: we can plug the expansions in the equations of motion, use the 4D equations of motion to replace derivatives with the 4D momenta and combine the two equations. We obtain that both f_n and g_n must satisfy the same equations of motion as a massive scalar field [21].

Note that on a circle, both chiral fields η and χ have a massless mode! In order to have a massless spectrum that corresponds to the SM fermions, we need to remove one or the other in order to have a chiral spectrum!

5.4.1 Orbifold and Chirality

The orbifold symmetry changes sign to x_5: in order for the kinetic term to be invariant, the parities of χ and $\bar{\eta}$ must be opposite. This is clear for the form of the $\chi\partial_5\eta$ terms in the action, which would be odd (and removed by the projection) otherwise. This implies that only one of the two chiralities will have a zero model.

The massive modes of the two chiralities will be combined to form a massive Dirac fermion. The orbifold is thus an essential ingredient for Model Building! Note also that the mass term is forbidden exactly for the same reason.

5.4.2 Odd Mass Terms

Another possibility is to assume that the mass term is odd under the orbifold symmetry: this is not entirely inconsistent, because the fundamental domain of the orbifold is an interval where the mass is uniform. So, let's force the presence of a mass term!

The most obvious problem we encounter is that the mass term would like to couple the two zero modes to form a Dirac fermion of mass m, however one of the two chiralities is removed.

If we remove the η chirality, the equations of motion for the zero mode in Eq. (95) reduce to:

$$\partial_5 g_0 + m g_0 = 0\,, \qquad g_0(x_5) \sim e^{-m x_5}\,. \tag{97}$$

The wave function of a left-handed mode, therefore, is exponentially localised toward the $x_5 = 0$ boundary of the space (for $m > 0$)

For right-handed zero modes

$$-\partial_5 f_0 + m f_0 = 0\,, \qquad f_0(x_5) \sim e^{+m x_5}\,; \tag{98}$$

thus it is localised toward the other boundary.

This trick allows us to localise the massless modes toward one or the other boundary: this feature has been used to build models where the large hierarchies between fermion masses in the SM (the up quarks weights few MeV, while the top fares 175.000 MeV!) is explained in terms of their exponential localisation [27].

6 First Model: Gauge-Higgs Unification in Flat Space

The goal of this class of models is to embed the BEH field in a gauge symmetry, by promoting it to an A_5 component of a bulk gauge boson; in order to have couplings between the Higgs and the electro-weak gauge bosons, the $SU(2)_L \times U(1)_Y$ gauge bosons and the Higgs should be unified into a single gauge group G. Such a group must then be broken down to the SM one, therefore we will need to work with an orbifold compactification (this is also required by the chirality of SM fermions!). The group G must fulfil the following requirements:

- G must contain at least $3_{SU(2)} + 1_{U(1)} + 4_H = 8$ generators;
- At the level of zero modes, only SU(2)$_L$×U(1)$_Y$ must survive, i.e. the orbifold must break $G \to$SU(2)×U(1);
- Breaking a gauge group corresponds to assign a parity + for the unbroken generators, and − for the broken ones. This must be done in a consistent way, i.e. a gauge boson can be mapped into itself up to a (global) gauge transformation:

  $$A_\mu(-x_5) = U A_\mu(x_5) U^\dagger ,$$

  where U is a gauge transformation of G. In particular, this preserves the rank of the original group G and the rank of the preserved gauge group: to obtain the SM, therefore, G must have rank-2 [18];
- For the scalars, at zero mode level, a doublet of SU(2)$_L$ with non-zero hypercharge should survive.

An attractive possibility is to use G =SU(3) [12, 43]: it has rank 2, exactly 8 generators and it can be broken to SU(2)×U(1) with

$$U = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{99}$$

written in terms of 3 × 3 matrix generators. The parity assignments of the gauge components will therefore be:

$$\begin{pmatrix} + & + & - \\ + & + & - \\ - & - & + \end{pmatrix} . \tag{100}$$

The 2 × 2 block corresponds to SU(2) generators, the + in the lower corner to a U(1) generators, finally the 4 components with parity − will provide the BEH field candidate (A_5 has opposite parities from A_μ), as they transform like a doublet under SU(2).

6.1 Spectrum

The spectrum of vector bosons will contain SU(2) gauge bosons $W^\pm$ and W^3, which contain a zero mode and a tower of massive modes; a U(1) gauge boson B with same spectrum as the SU(2) ones; two charged gauge bosons, with the same quantum numbers as the BEH field, $C^{\pm}$ and D^0, that have no zero mode and just a tower of massive modes. They are embedded in the SU(3) structure as:

$$A_\mu = \begin{pmatrix} \frac{1}{2}W^3_\mu - \frac{1}{\sqrt{12}}B_\mu & \frac{1}{\sqrt{2}}W^+_\mu & \frac{1}{\sqrt{2}}C^+_\mu \\ \frac{1}{\sqrt{2}}W^-_\mu & -\frac{1}{2}W^3_\mu - \frac{1}{\sqrt{12}}B_\mu & \frac{1}{\sqrt{2}}D^0_\mu \\ \frac{1}{\sqrt{2}}C^-_\mu & \frac{1}{\sqrt{2}}D^0_\mu & \frac{2}{\sqrt{12}}B_\mu \end{pmatrix} . \tag{101}$$

The scalar sector will only contain a massless doublet of $SU(2)_L$ (the KK tower of massive scalars are "eaten" by the massive gauge bosons), that will play the role of the BEH field, embedded in SU(3) as:

$$A_5 = \begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{2}}\phi^+ \\ 0 & 0 & \frac{1}{\sqrt{2}}\phi_0 \\ \frac{1}{\sqrt{2}}\phi^- & \frac{1}{\sqrt{2}}\phi_0^* & 0 \end{pmatrix} . \tag{102}$$

At tree level, the BEH field will not have any potential, because its interactions can only come from the gauge boson action:

$$\mathscr{S} = \int d^5x \, \left[-\tfrac{1}{2} \mathrm{Tr}\, F_{MN} F^{MN} \right] ,$$
$$\text{where} \quad F_{MN} = \partial_M A_N - \partial_N A_M + g(A_M A_N - A_N A_M) \, . \tag{103}$$

No A_5^2 nor A_5^4 terms are present in this action! So, the potential for the Higgs is generated at one loop. We expect it to be **finite**, because the tree level action does not contain a counter-term either for the mass or quartic coupling! Note that this is true at all orders in perturbation theory!

6.2 Potential Issues

- The Higgs field is a gauge boson, so it couples to all particles with strength dictated by the gauge coupling g. What about fermion masses? To obtain masses below m_W, we can use the mass trick to localise the light quarks towards the two boundaries of the space, in order to reduce the overlap to the Higgs.
- How about the top mass? This is a crucial issue, as the localisation can only suppress the couplings with respect to g. One may use gauge group factors to enhance the coupling (as in Ref. [12]), violate Lorentz invariance in the extra space explicitly [36] or curve the space [15, 32].
- How about the Higgs mass? The potential is one-loop generated, so the mass should be rather small. The precise value depends on the details of the model.

6.3 The BEH Potential in Gauge-Higgs Unification

The BEH potential is generated completely at one loop. Only the zero mode will be sensitive to the eventual negative mass, thus the vacuum solution must be independent on the extra co-ordinate x_5. This implies that no tree level mixing with the heavy gauge bosons will be generated! The reason is that modes with different mass have orthogonal wave functions.

6.3.1 The Hosotani Mechanism

Let's assume that the BEH field does develop a vacuum expectation value that breaks $SU(2)_L \times U(1)_Y \to U(1)_{em}$: the vacuum will have the SU(3) embedding

$$\langle A_5 \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \langle \phi_0 \rangle \\ 0 & \frac{1}{\sqrt{2}} \langle \phi_0 \rangle & 0 \end{pmatrix} . \tag{104}$$

For compact spaces allowing for a Scherk-Schwarz phase, it is always possible to find a gauge transformation $\Omega(x_5)$ such that

$$\langle A'_5 \rangle = \Omega(x_5) \langle A_5 \rangle \Omega^\dagger(x_5) + \frac{i}{g} \left(\partial_5 \Omega(x_5) \right) \Omega^\dagger(x_5) = 0\,; \tag{105}$$

so that the Higgs VEV disappears from the action. In fact, for

$$\Omega = e^{i \alpha x_5 \lambda_v}\,, \tag{106}$$

where λ_v is the SU(3) generator the vacuum solution is aligned with (in this case λ_6 of SU(3)) and α is a constant, Eq. (105) gives

$$\langle A_5 \rangle + \frac{i}{g} \frac{i\alpha}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = 0\,, \quad \Rightarrow \quad \alpha = \sqrt{2} g \langle \phi_0 \rangle\,. \tag{107}$$

The same transformation must be applied to the gauge vectors:

$$A'_\mu = \Omega(x_5) A_\mu \Omega^\dagger(x_5)\,. \tag{108}$$

What does it change in the theory? The action is invariant, however the periodicity condition on the field A'_μ is different from before:

$$\begin{aligned} A'_\mu(x_5 + 2\pi R) &= \Omega(x_5 + 2\pi R) A_\mu(x_5) \Omega^\dagger(x_5 + 2\pi R) \\ &= \Omega(x_5 + 2\pi R) \Omega^\dagger(x_5) A'_\mu(x_5) \Omega(x_5) \Omega^\dagger(x_5 + 2\pi R)\,, \end{aligned} \tag{109}$$

where

$$\Omega(x_5 + 2\pi R)\Omega^\dagger(x_5) = e^{i2\pi R\alpha\lambda_6}\,. \tag{110}$$

The periodicity condition in the new gauge basis, therefore, contains a non-zero Scherk-Schwarz phase for different components of the SU(3) gauge fields. Note that this re-definition can be interpreted as a modification of the boundary conditions only in orbifolds that do allow for Scherk-Schwarz phases, like the one we considered here. The modified periodicity conditions will also affect the spectrum for the massive gauge bosons and will depend on α (which is proportional to the BEH vacuum solution!). For the $W^\pm$ bosons, the new masses will read:

$$m_n^{W^\pm} = \frac{n+\alpha}{R}\,, \qquad m_{W^\pm} = \frac{\alpha}{R}. \tag{111}$$

As expected the "zero mode" has now acquired a mass proportional to α. The numerical value of α will therefore determine the relation between the SM W mass and the KK mass $m_{KK} = 1/R$.

6.3.2 Numerical Results

The calculation of the potential is rather complex. We know the spectrum as a function of the BEH vacuum α, we can use the Weinberg-Coleman potential:

$$V_{eff}(\alpha) = \pm\frac{1}{2}\sum_i \int \frac{d^4p}{(2\pi)^4}\log[p^2 + M_i^2(\alpha)]\,. \tag{112}$$

Some details of the calculation can be found in [8]. Different fields contained in the theory will give different contributions to the potential. As an example, we report here a potential computed in the framework of the model in Ref. [12], based on SU(3): besides the gauge sector, the model contains a set of bulk fermion fields where the top is embedded into, and a set of massive bulk fermions associated with the light generations which are kept massive by use of twisted boundary conditions. An example of numerical potential, as a function of the BEH vacuum α, can be found in Fig. 3: it's interesting to notice that the contribution of the gauge bosons (red/dashed) and of light fermions (green/dot-dashed) have minima at $\alpha \sim \langle\varphi_0\rangle = 0$, while it is the contribution of the top loops (blue/solid) that generates a non trivial vacuum. From the potential we can also calculate the Higgs boson mass, which is proportional to the second derivative of the potential. The results can be found in Ref. [12]: numerically, the Higgs mass is always fairly small (below 150 GeV). Furthermore, the measured value can be obtained for small values of the parameter $\alpha \sim 1/20$, from which we can extract the expected value of the Kaluza-Klein resonance masses: $m_{KK} \sim 20 \cdot m_W \sim 1.6$ TeV.

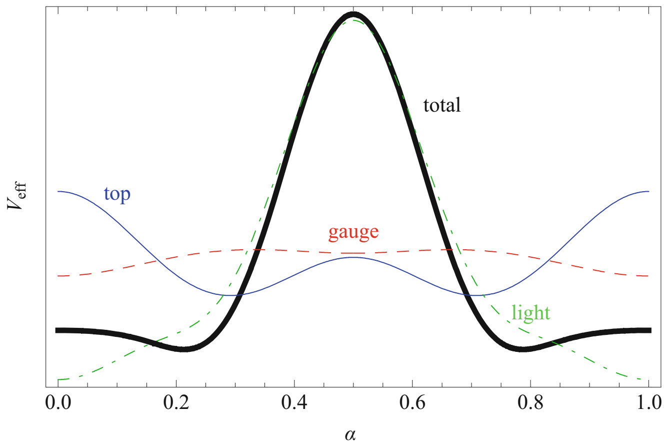

Fig. 3 One-loop Higgs potential for the model in [12]: the *thick line* is the total contribution, the *dashed* (*red*) one the contribution of gauge fields, while in *solid* (*blue*) the contributions of the *top* and in *dot-dashed* (*green*) the light generations. Overall, the potential has a local minimum for small values of $\alpha \sim 0.2$

7 Second Model: A Minimal Composite Higgs, or Gauge-Higgs Unification in Warped Space

A warped extra dimension (or Randall-Sundrum space [40]) has been widely studied, because it can fairly easily generate hierarchies between mass scales. In models of Gauge-Higgs, it offers two main advantages: it automatically enhances both the Higgs and the top mass.

The difference between flat and warped space is the metric: the simple Minkowski metric in flat space is replaced by

$$ds^2 = e^{-2x_5/R} dx^\mu dx_\mu - dx_5^2\,, \quad x_5 \subset [0, L]\,. \tag{113}$$

This metric has an interesting property, conformal invariance, which is more evident if we rewrite it in terms of $z = Re^{x_5/R}$:

$$ds^2 = \left(\frac{R}{z}\right)^2 (dx^\mu dx_\mu - dz^2)\,, \quad z \subset [R, R' = Re^{L/R}]\,. \tag{114}$$

An increase in the value of z by a factor of ξ (i.e. $z \to \xi \cdot z$) can be compensated by an analogous rescaling of $x_\mu \to \xi \cdot x_\mu$ to leave ds^2 invariant. A simple and intuitive way to think about this symmetry is the following: changing z is equivalent to moving along the extra co-ordinate, which is then linked by the symmetry to a rescaling of the size (and therefore of the energy) of physical 4-dimensional systems. Different positions in the extra space correspond, therefore, to different energy scales. One can choose the positions of the two end-points of the space so that $R^{-1} \sim M_{Pl}$ and $(R')^{-1} \sim 1\,\text{TeV}$: moving from the boundary at $z = R$ (Planck brane) to the $z = R'$ one (TeV brane) will rescale energy scales from the Planck scale down to the TeV. This symmetry is therefore considered to provide a solution of the hierarchy problem between the electro-weak and the Planck scales. Note that the length of the interval is $L = R \log R'/R$.

A gauge boson in the warped space will have an action [22]

$$\begin{aligned}\mathscr{S}_{gauge} &= -\tfrac{1}{4}\int d^4x\, dz\, \left(\tfrac{R}{z}\right)^5 F_{MN}F^{MN} \\ &= -\tfrac{1}{4}\int d^4x\, dz\, \left(\tfrac{R}{z}\right)(F_{\mu\nu}F^{\mu\nu} - 2F_{\mu z}F_z^{\mu})\,. \end{aligned} \quad (115)$$

The factors of R/z come from the metric. As in the flat case, a gauge fixing term is added to remove A_μ-A_5 mixing:

$$\mathscr{S}_{GF} = -\frac{1}{2\xi}\int d^4x\, dz\, \left(\frac{R}{z}\right)(\partial^\mu A_\mu - \xi z \partial_z (A_5/z))^2\,. \quad (116)$$

The equation of motion for the wave function of a vector is

$$z\partial_z\left(\frac{1}{z}\partial_z f_n\right) + m_n^2 f_n = 0\,, \quad (117)$$

whose solutions can be expressed in terms of Bessel functions of the first and second kind:

$$f_n = z\,(A\,J_1(m_n z) + B\,Y_1(m_n z))\,. \quad (118)$$

For the scalars, the equation of motion reads:

$$\partial_z\left(z\partial_z\left(\frac{A_5}{z}\right)\right) + \frac{m_n^2}{\xi}A_5 = 0\,. \quad (119)$$

As before, massive mode are Goldstone bosons eaten by the massive vectors, while for the zero mode

$$A_5 \sim z\,. \quad (120)$$

7.1 Custodial Symmetry?

We may want to try constructing a SU(3) model: however this is not acceptable in warped space. The difference with respect to the flat case is that the BEH vacuum depends linearly on the extra co-ordinate, thus mixing between various KK modes is possibly generated by it. In the flat case:

$$\langle A_5 \rangle W_n^+ W_m^0 \sim \langle A_5 \rangle \int f_n(x_5) f_m(x_5) = 0 \tag{121}$$

because the vacuum is a constant and two wave functions are orthogonal. In the warped case, the vacuum is not constant as $\langle A_5 \rangle \sim z$, therefore tree level corrections to the electro-weak precision measurements are usually generated, in particular to the ρ parameter. As we learned at the beginning of the lectures, the ρ parameters is protected against large corrections by the *custodial symmetry*: what we need here is therefore a way to implement the custodial symmetry in Gauge-Higgs models. The simplest way is to use SO(5) instead of SU(3) as the starting gauge group [3], because it contains SO(4) as a subgroup, and the coset space (which will embed the BEH field) is a bi-doublet of SO(4) – thus it has the correct custodial symmetry properties as the BEH field in the Standard Model. The extended gauge structure is then broken on the boundaries of the extra dimensions in the following way:

- On the TeV brane, we can break SO(5) → SO(4): the SO(4) ~ SU(2) × SU(2) contains the desired custodial symmetry (the breaking of this symmetry will be achieved via the BEH vacuum, like in the SM).
- The generators of SO(5) that do not belong to the unbroken subgroup SO(4) form a 4 of SO(4), like the BEH field in the SM!
- On the Planck brane, we break SO(5) → SU(2) × U(1), so that only the SM gauge invariance is preserved at the level of zero modes. As it is a subgroup of the unbroken SO(4), only the SU(2) × U(1) gauge bosons have zero modes, as desired.
- For the scalars, only the BEH has a zero mode.

This structure of the symmetry breaking is enough to ensure that the values of the W and Z mass respect the SM relations at tree level (thus $\rho = 1$ at tree level). This symmetry structure can also be used to protect couplings of the Z boson to light fermions against large corrections [4].

7.2 AdS/CFT

The presence of a conformal symmetry in the metric suggests a correspondence between models in warped space (anti de Sitter) and strongly interacting conformal theories in 4 dimensions. The correspondence goes as follows:

- Fields and symmetries on the Planck brane correspond to the elementary sector of the theory (like the photon in QED+QCD);

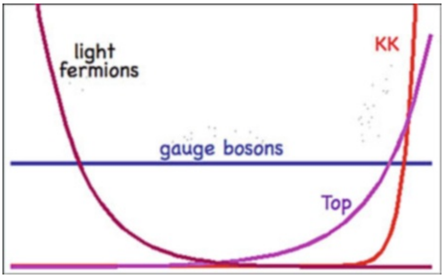

Fig. 4 Wave functions in the warped model for gauge bosons, light fermions, tops and KK modes, showing their localisation: the Planck brane is on the right, the TeV brane on the left. The Higgs is also moderately localised towards the TeV brane

- Fields in the bulk correspond to operators (bound states) of the conformal sector, the TeV brane breaks the conformal invariance and generates a mass gap (tower of meson resonances, where the mass gap corresponds to confinement);
- Symmetries in the bulk and on the TeV brane correspond to global symmetries of the strong sector (so, our strong sector is invariant under SO(5) which is spontaneously broken to SO(4)).

This model can therefore be seen as the SM (the Planck brane is invariant under the SM gauge group) coupled to a conformal sector which is invariant under a global SO(4) (that generates the custodial symmetry!). The BEH field is localised towards the TeV brane (as its wave function grown with z), therefore it is part of the strong sector and its vacuum solution is generated dynamically by the strongly interacting sector. Note that this mechanism is very similar to Technicolour, with the addition of the conformal symmetry. The correspondence has the status of a conjecture, without a formal mathematical proof: nevertheless, by studying the properties expected for a conformal strongly interacting sector and the properties of the fields in the warped space, one can define an accurate dictionary that relates the properties of the two sectors, both in the bosonic [17] and the fermionic sectors [13, 16].

The properties of all the fields depend on their localisation in the extra space: the cartoon in Fig. 4 shows the typical scenario. Gauge bosons have a flat profile (due to gauge invariance), while the Higgs is moderately localised toward the TeV brane thus revealing its composite origin. Light fermions, like leptons, light quarks and the right-handed bottom, are localised toward the Planck brane: they correspond to mostly elementary fields, and the localisation suppresses their overlap with the Higgs thus generating the hierarchies in the fermion mass spectrum. The top is localised toward the TeV brane, thus it is a mostly composite state: its localisation enhances the overlap with the Higgs, thus it makes possible to achieve masses larger than m_W, even though the coupling is of the order of the gauge couplings. The Higgs, being localised toward the TeV brane, is also a composite state! All the massive resonances (KK) are also strongly localised to the TeV brane, thus showing their composite nature.

7.3 Higgs Potential and Mass

The calculation of the Higgs potential proceeds as in the flat space, however the calculations are complicated by the presence of Bessel functions. Nevertheless, the result will look very similar to the Gauge-Higgs models in flat space, detailed in the previous section: the typical Higgs mass will range between $60 < m_H < 140$ GeV, thus the measured value can be easily obtained. More details on the impact of the Higgs mass measurements on this class of models can be found in [37].

8 Final Words

Many decades have passed between the first formulation of the Standard Model as a candidate to describe phenomena involving elementary particles, and now the last predicted particle has been discovered, the Higgs boson. This is, however, an open door rather than a lowering curtain for the field, because many questions are still waiting for answers: above all, the discovery of a scalar field is a materialisation of the naturalness problem. Despite the negative results of the LHC in the quest for New Physics, the last word is far from being spoken: new physics may reveal itself in ways that are difficult to disentangle from the abundant background in hadron colliders, and much more work is required from both the theoretical and experimental sides to finally pull the correct string and reveal the New Standard Model.

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