Chapter 2
Constructivism

2.1 Chapter Overview

In this chapter, I will provide a brief summary of Dummett's constructivist program. My central topics, the meaning of the logical constants and the admissibility of logical laws, lie at the heart of a grand philosophical system, in which Dummett deftly strings together philosophical insights about language, logic and metaphysics. In some cases, it is quite impossible to understand his arguments about logical consequence without having at least a general idea of the outline of the whole program. This chapter aims to give such an outline.

The exegetical efforts needed to bring the central features of Dummett's difficult and extensive writing into a more or less streamlined form are considerable. I have no doubt that for many of the substantial claims I attribute to Dummett, some dissenting quote or other of his could be found. I believe that I give a fair and charitable version, but other equally charitable versions might look very different, especially if they lay more weight on other works than I do.[1]

A further point of this chapter, and in fact the one I shall address first, will be to fix some important terminology.

2.2 Constructivism

The first terminological item that deserves immediate comment is "constructivism" itself. Dummett, as we shall see, takes much inspiration from constructive mathematics and intuitionism. His contention is that the intuitionistic ideas, especially about the revision of the logical laws, can be translated to empirical discourse as well and help in giving a general account of meaning.

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[1] For me, the central sources are his essays "TRUTH," "What Is A Theory Of Meaning (II)" (WTM) and the book "The Logical Basis of Metaphysics" (LBM).

Intuitionism will be discussed in detail in the next chapter, but here is already a short summary of the position: There are no objectively existing mathematical entities, such as numbers or sets. Mathematics is therefore not about such abstract entities; rather, it is about mental *constructions*. Numbers and the like are products of the human mind, they are constructed by us.

It is not quite clear that Dummett's constructivism about empirical statements entails that the objects such statements talk about are constructed by us in a similar way. Calling his position "constructivism" is thus a slightly problematic (though standard) choice.

For reasons that we will soon come to see, a better choice would have been "verificationism." However, I need to reserve that term to contrast certain constructive theories that take verifications as the central concept in a semantic theory with others that take falsifications to be more important. Predictably enough, I will talk about "falsificationism" in the latter case. Verificationism and falsificationism will be specific examples of constructive theories, and it would be too confusing to use the first of these terms ambiguously. Bulky neologisms such as "veri/falsificationism" are ruled out for aesthetic reasons.

Maybe most commonly, Dummett's position is referred to as "anti-realism." However, to the uninitiated, it must be completely mystifying why a philosophical position on meaning and logic should bear such a name. Clearly, it is an apt name for a metaphysical doctrine.

Indeed, in a way the resolution of metaphysical questions might be seen as the ultimate goal of Dummett's thoughts about language and logic. It is his surprising and revolutionary idea that the only viable way to reach such a resolution leads through these seemingly unconnected topics.

The exact way in which language, logic and metaphysics are supposed to be related is quite complicated. I'll give an outline of these connections throughout the next sections. However, the metaphysical upshot of the constructive theories of meaning and logic is not my main concern in this book. Therefore, I will try to avoid the term "anti-realism" as a generic tag of Dummettian contentions. I will use it when metaphysical assumptions feature heavily in a line of argument, or when metaphysical consequences are more important than details about language or logic.

Here is a list of the terms I just mentioned, and what I will take these terms to mean. I only give you short slogans that aim to hit the core of the notions I have in mind. The exact meaning of these slogans will become clear in the course of this chapter.

CONSTRUCTIVISM   The semantic values of statements must be, at least in principle, epistemically accessible. I will discuss two main kinds of constructivism: Verificationism and falsificationism.

VERIFICATIONISM   A constructivism that takes the core of the semantic values to be a positive notion, namely verification conditions. Mathematical proofs are one particular species of verification, so that I will make it clear when I want to talk about empirical verifications only.

FALSIFICATIONISM   A constructivism that takes the core of the semantic values to be a negative notion: *Falsification* conditions. Falsificationism in this sense is a relatively unknown species of constructivism, and it is one of the aims of this thesis to supply a fuller description. However, falsificationism will not make much of an appearance until Part II; the discussion in this first part is mostly about the better known verificationistic variations of constructivism.

INTUITIONISM   By this, I will mostly mean the verificationistic theory of mathematical statements that I will present in more detail in the next chapter. Unlike the three items above, I will understand intuitionism to be tied to a very specific constructive logic, viz. intuitionistic logic. If the context makes it clear that mathematics is not specifically at issue, figures such as "the intuitionist" will simply be adherents of intuitionistic logic.

ANTI- REALISM   Constructivism augmented by the claim that substantial metaphysical insights can be gained from the study of semantic values and logical laws.

So much for early terminological distinctions. Now let us immediately move on to the connections between constructivism and metaphysics alluded to in the last paragraphs.

2.3 Language, Logic, Metaphysics

Anti-realism, as is hard not to guess, is opposed to realism. Traditionally, under labels such as *idealism* and *platonism*, adherents of these stances had argued about what exists and what fails to exist. They fought over numbers, stones, mental states, quarks, and many things beside. Eventually, positivistic philosophers tired of these seemingly futile discussions and came to view them as quibblings over meaningless pseudo-questions. Their verdict was that philosophy had better turn its attention to the workings of language, in order to make sure that such meaningless propositions should immediately be debunked in the future.

One of Dummett's most spectacular ideas takes issue with this development: Yes, we should indeed concentrate on getting our philosophy of language right. Firstly, simply because language is important and interesting in its own right. But secondly, because linguistic insight will not *eradicate* metaphysical questions. Rather, such insight will make it possible to *answer* those questions.

However, it is not possible to answer those questions as they stand. The way they are phrased is, in Dummett's view, too *metaphorical*. About the specific example of mathematics, he writes:

> [W]e have here two metaphors: the platonist compares the mathematician with the astronomer, the geographer or the explorer, the intuitionist compares him with the sculptor or the imaginative writer; and neither comparison seems very apt (TOE, p. xxv).

Rather than asking whether a mathematician is like an astronomer or like a sculptor, that is, whether or not numbers exist on their own accord, Dummett asks what it is

that makes our statements about numbers *true* or *false.* Or, if we are reluctant to talk about truth, what makes our statements *correct* or *incorrect.*

Realism, often relativized to a given area of discourse (mathematics, particle physics, ethics, etc.), holds that what makes the statements in this area of discourse true or false (numbers, quarks, moral facts) exists independently of anyone talking or thinking about it. Anti-realism, on the other hand, claims that the truth or falsity of those statements do depend on us and our abilities to *recognize* their truth values.

The first hard problem for those who are, unlike me, mainly occupied with the metaphysical questions is to decide whether this reformulation really captures the core of the traditional debates.[2]

Granting that Dummett indeed has hit on such a common core, the important next step is to focus on a particular question about the distribution of truth and falsity over the statements of the language. The question is this: Can we assume that every statement is either true or false? If we answer "Yes, we can," then we are assuming the principle of bivalence:

BIVALENCE Every statement is determinately either true or false.

Dummett claims that if we assume this principle, then we are realists about the area of discourse in question.[3]

In order to explain why bivalence should be doubted by an anti-realist, I will have to delve a bit deeper into the exact nature of the dependence of truth values on speakers and hearers that I mentioned above. The anti-realism that Dummett has in mind is not a relativistic doctrine of the "your truth is not my truth" kind, where anyone is free to attribute truth and falsity to statements in ways that are only constrained by their imagination.[4] The dependence is much more of a positivistic ilk: The anti-realist holds that a statement cannot be true unless we are, at least in principle, able to come to know of its truth. In mathematics, "to come to know of its truth" means to find a proof of a statement; in empirical contexts, it means to *verify* the statement.

This particular way in which truth values depend on us language users now has the following consequence: If we cannot assume that we can recognize the truth or falsity of every statement, then we cannot assume bivalence, either. And indeed this is an assumption that we should be reluctant to make if the talk we engage in is sufficiently sophisticated.

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[2] See the introduction of Wright (1993) for detailed discussion on these and other matters that are only skirted here.

[3] "Realists about the area of discourse..." is a bit vague; the reason for this is that it is not always the existence of objects that is at stake, for example in discussions about the reality of the past or the future. In any case, even though Dummett often writes as if the case for or against realism has to be negotiated for each area of discourse separately, the arguments he brings forth are for the most extremely general. Therefore, I'll drop the relativization to a specific area of discourse until further notice.

[4] cf. Braver (2007) for a comparison between Dummett's and more liberal and anarchic conceptions of Anti-realism.

2.4 Decidability

To see why Dummett thinks bivalence cannot be assumed by a constructivist, we have to understand his notion of an undecidable statement. If it were not for undecidable statements, says Dummett, we would have no reason to doubt the validity of the principle of bivalence.

For the blueprint of the argument, we turn, once again, to mathematics. However, we immediately run into a problem: As we will see presently, Dummett's idea of decidability does not readily correspond to other conceptions of decidability found elsewhere in mathematics or logic, and what he tells us about his notion is far from perfectly clear. The best discussion I've seen of how to interpret Dummett in this respect is Shieh (1998), and this section draws heavily on that paper.

For Dummett, a mathematical statement is decidable if either we have already proved it (or its negation), or else if we know of a straightforward and sure-fire way of getting a proof of it or its negation. For the latter option, it is not enough to know of a method that will, in fact, decide the question; we need also to *know* that it will decide the issue, although we need not know whether the result will be positive or negative.

What makes this reading quite hard to come by is that it opens up the following possibility: Something that we are today able to prove might not have been decidable, even *in principle*, in the past. That is because in the past we might not have had the proof, nor any knowledge of how to construct it.

As was just noted, this conception has little to do with the standard idea of decidability. Consider our saying that propositional logic is decidable, whereas predicate logic is not. This has nothing to do with our *finding* decision methods. Propositional logic has always been decidable, and predicate logic will always remain undecidable in the classical sense. The question here hinges on the existence or non-existence of an algorithm, not on whether we know of that algorithm or not.

I will, however, be using the term in the way Dummett is using it. Here is an example to further illustrate this constructive sense of decidability. One fine Monday, an eminent mathematician wonders whether there are seventy[5] consecutive 7s in the decimal expansion of π. Although the mathematicians have computed π to a considerable length, they have not come across such a series yet. On the other hand, no one knows of a method of proving that such a series could not occur. The mathematician is thus dealing with an undecidable statement.

Now let us suppose that our mathematician, having no better idea how to tackle the problem, sets out to compute more and more decimal places of π. On Thursday, she comes across seventy consecutive 7s. She has obviously proven the statement she was wondering about, unless she made a mistake in the calculation. Therefore, it is equally obvious that the statement is now decidable. But this does *not* change anything about the fact that the statement was *undecidable* on Monday. The mathematician has not found out that the statement was decidable all along.

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[5] The example is adapted from Brouwer, only that he used *seven* 7s. These have in the meantime been found in the decimal expansion of π. As far as I know, seventy 7s have not yet turned up.

This idea of undecidability appears to be a very strict one, then. This seems especially true in view of the idealizations the constructivists are ready to make by talking about provability or constructibility *in principle*. Such talk is necessary because the constructivist would not like to be confined to what he has *actually* constructed. For example, if no one had ever thought about the number 347867536893243, we still would like to count it as one of the entities that are available to us, because we have a straightforward way of constructing it out of the basic building blocks of mathematics. Likewise, the question whether this number is prime or not is decidable, because we know of algorithms that we could employ here.

Even if the number should be so large that it would take us impossibly long to write it down, we would like to gloss over that fact and push that impossibility aside with the remark that we are only dealing with constructibility "in principle." Even more strikingly, the intuitionists have no problem with extending the range of "in principle constructible" entities to the transfinite.

But then, one might ask, why is the case different with the construction of the sequence of the decimal expansion of π? Surely, we would like to say that the question was "in principle" decidable even on Monday! The idealization involved here is, after all, a much lighter one than the one involved in getting us to a number so large we could never hope to write it down in all our lifetimes: all the mathematician had to do was calculate for three more days.

This reasoning is mistaken because our mathematician had no *guarantee* that she would come across seventy 7s, no matter how long she would have searched. The problem is not that the seventy 7s might have been so far down the line that it would have taken an impossibly long time to compute that far. The crucial point is rather that she could not have rested assured that she would hit the seventy 7s, no matter how much time she was willing and able to spend on the search.

A good way of putting the matter is this: The notion of a decidable mathematical statement is, in Dummett's use, a completely *epistemic* concept. Its extension solely depends on the state of the art in mathematics.

In addition to that, we will see in the next chapter that those constructivists who adopt intuitionistic logic will not be able to say that there are *absolutely* undecidable statements, i.e., statements which we will never be able to prove and never be able to disprove.[6] Undecidability is thus for the intuitionist always *pro tempore*, that is, the most an intuitionist can mean when she says that a statement is undecidable is that *at the present moment* there is no known decision procedure.

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[6] There is surely evidence in Dummett's writings that he sometimes understands "*in principle* undecidable" in this sense. But on balance, I think he more often uses the phrase "in principle" in the sense outlined above. See for example WTM p. 45, where he talks about "sentences which are, in practice or even in principle, decidable, that is, for which a speaker has some effective procedure which will, in a finite time, put him into a position in which he can recognize whether or not the condition for the truth of the sentence is satisfied."

2.5 Undecidability, Bivalence, and the Law of Excluded Middle

In dealing with an undecidable statement, we have nothing that guarantees that we can come to know of its being true or false. But lacking such a guarantee, a constructivist does not feel entitled to claim that the statement is either true or false. It can only be so if there is a way to find out whether it is true or false, and we have no guarantee that there is such a way.

This talk of guarantees sounds a bit stiff; a constructivist might rather want to say that there may be statements for which we can find no proof or disproof.[7] But, as I just mentioned above, the intuitionist cannot refer to such absolutely undecidable statements, even if he wanted to. This problem (if it is a genuine problem) is caused by the way in which intuitionists explain negations, as we'll see in Sect. 3.6.

This might well be a problem for Dummett also. As I mentioned before, Dummett thinks that anti-realists should turn away from classical logic and toward intuitionistic logic.

To give up bivalence means already to give up an important part of classical logic. The classical truth table semantics of the logical connectives presupposes bivalence. There *might* be an alternative story that does not invoke bivalence and still motivates all inferences of classical logic.[8] The much more likely outcome, however, is that without bivalence, some inferences will lose their validity.

This is exactly what happens in intuitionistic logic, which will come under closer scrutiny in the next chapter. However, let me already point to some characteristic features of this logic.

The first and most famous feature is the failure[9] of the Law of Excluded Middle (LEM):

$$\text{LEM} \vdash A \vee \neg A$$

Even those unacquainted with intuitionistic doctrine will have no trouble seeing why this law must be dubious if bivalence cannot be assumed. If there is a problem at all, it might be to appreciate that there is a *difference* between bivalence and the LEM at all. But there is: Bivalence is a doctrine about semantic values, while the LEM records that, as a matter of logic, "A or not A" will always hold.

In general, it is important to keep semantic principles and logical laws apart; I will try to aid the reader by writing the names of semantic principles such as "bivalence" in lower case letters and letting the names of logical laws start with upper case letters. Also, I will use abbreviations such as "LEM" only for logical laws.

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[7] I use "disproof" to mean the proof of the negation of a statement.

[8] Ian Rumfitt has suggested such a strategy in Rumfitt (2007).

[9] Like most propositional non-classical logics, intuitionistic logic is strictly weaker than classical logic in that classical logic validates all intuitionistic inferences. Non-classical logics are often described in an impressionistic way by pointing out which kind of classical inferences are not supported, as in this section.

Marking the difference between semantic principle and logical law does not change the fact that the connection between bivalence and LEM seems perfectly straightforward.[10] Such lucid plausibility can hardly be claimed for the second most famous characteristic feature of intuitionistic logic, the failure of Double Negation Elimination (DNE):

$$\text{DNE} \quad \neg\neg A \vdash A$$

To understand this failure, one needs to understand the way in which negation is explained in intuitionism. Without this explanation (for which you will have to wait until the next chapter), the impossibility to infer "A" from "not not A" must seem quite bizarre. To make matters even more puzzling, consider that Double Negation Introduction (DNI)

$$\text{DNI} \quad A \vdash \neg\neg A$$

is actually *valid* in intuitionistic logic, as is the following (nameless) inference:

$$* \quad \neg\neg\neg A \vdash \neg A$$

clearly, there is something very strange going on with double negations in intuitionistic logic. This can also be seen by considering the following inference, called the Law of Excluded Third (LET). Unlike LEM, this law is, again, *valid* in Intuitionistic logic.

$$\text{LET} \quad \vdash \neg\neg(A \lor \neg A)$$

Just as LEM corresponds to bivalence (i.e., "every statement is either true or false" [11]), this logical law corresponds to the semantic principle that Dummett calls *tertium non datur*: "No statement is neither true nor false." [12]

And just as Dummett wants to deny LEM and accept LET, he wants to deny bivalence and accept tertium non datur. That is, he is committing himself to the claim that no statement is neither true nor false, but not to the claim that every statement is either true or false. As he acknowledged at one point, "this confused some readers."[13] I dare say it continues to do so.

As I said, all this will become somewhat more perspicuous once the intuitionistic explanation of negation is in place. However, even then the mysteries will not disperse

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[10] To spell it out, the connection is established by an appeal to the disquotational scheme

"*A*" is true iff *A*,

the principle that a disjunction is true iff at least one of the disjuncts is and the principle that $\neg A$ is true iff *A* is false, and the semantic demand that no statement may have more than one truth value. None of these principles is completely uncontroversial, but they surely are intuitively plausible.

[11] TOE, p.XX.

[12] ibid.

[13] ibid.

completely. I will eventually, in the last part of this book, come to suggest constructive logics in which all double negation laws hold.[14]

2.6 Where to Start?

Instead of getting any further into the details of the logical systems that lie ahead, let us zoom out again to get the big picture of Dummett's philosophy back into view. Until now, we have seen the connection he forged between positions in metaphysics (realism vs. anti-realism), philosophy of language (bivalence or not) and logic (classical vs. intuitionistic). If this connection is really as stable as he claims, then conclusive arguments for one of these positions may lead to solutions in the other areas. But where exactly is it most likely that such conclusive arguments may be found?

As I said above, Dummett agrees with his positivistic predecessors that the metaphysical dispute as traditionally conducted shows little promise of providing a secure first foothold. This debate only presents us "with alternative pictures. The need to choose between these pictures seems very compelling; but the non-pictorial content of the pictures is unclear".[15]

Maybe we should start with logical considerations, then. However, as purely formal and uninterpreted systems, there seems little wrong with either classical or intuitionistic logic. It is the interpretation of the logical vocabulary that, if anything, will make a significant dent in this discussion. In other words, the question is what the logical constants might mean.[16]

The natural place to start, then, is the analysis of language; indeed, Dummett holds that the analysis of language supplies the base to all of philosophy:

> [T]he theory of meaning is the fundamental part of philosophy which underlies all the others. Because philosophy has, as its first if not its only task, the analysis of meanings, and because, the deeper such analysis goes, the more it is dependent upon a correct general account of meaning, a model for what the understanding of an expression consists in, the theory of meaning, which is the search for such a model, is the foundation for all philosophy (...) (Dummett 1973, p. 669).

This quote comes from Dummett's famous book on Frege's philosophy of language. He has appropriated many of Frege's insights, such as the idea that words only have meaning as parts of whole statements and that the meaning of utterances in other moods is derivative of utterances in the assertoric mood.

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[14] The status of LEM and LET will then depend on whether we are dealing with a verificationistic or a falsificationistic theory. In the verificationistic case, both will fail, and in the falsificationistic case, both will hold.

[15] LBM, p.10

[16] There *are* very important proof theoretic arguments for intuitionistic logic. These arguments, however, are also seen as concerning the *meanings* of the logical constants. Indeed, the field of study these arguments fall into is now known as *proof theoretic semantics*. Important as this field is, I will not consider it further.

The central questions for Dummett then are these: What do our assertions mean? What is it to understand a statement, what is it for it to be true? If these really are the fundamental questions of *all* of philosophy, then they are certainly also the questions we should start with in the slightly more circumscribed investigation of language, logic, and metaphysics.

2.7 Truth Conditions

A widely accepted way of specifying the meaning of statements is to equate them with their *truth conditions*. To understand a statement is to apprehend under which circumstances it would be true.

One may think that this strategy would seem unpalatable to the Dummettian constructivist, especially in view of Dummett's earlier work. There, he advised to steer clear of truth conditions in the theory of meaning and to employ *assertibility* conditions instead. I will come back to this anon; let us for now concentrate on the later[17] position, in which Dummett offers that all parties can agree on the use of truth conditions, but that their differences will become apparent when they spell out the properties of the truth conditions alluded to.

For the realist, these truth conditions will be independent of any means to come to know them. Their notion of truth is *epistemically unconstrained*.

The Dummettian constructivist will object to such notions of truth and truth conditions. However, the objection will not turn on the pictorial and unclear content of his metaphysical assumptions. Rather, the constructivist claims that the use of epistemically unconstrained truth conditions gives an unsatisfactory account of meaning.

2.8 Meaning as Use

Here is a sketch of Dummett's line of argument against epistemically unconstrained truth conditions.

How, he asks, can such a truth condition be constitutive of the meaning of a statement that people are supposed to be able to use competently? If the obtaining of a truth condition is systematically beyond my grasp, how could I possibly come to understand it?

Take a non-mathematical example, adapted with slight modification from Dummett: Suppose that a certain chap, Jones, has up to now been leading a sedate life

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[17] The talk of "earlier" and "later" is useful, but somewhat problematic. The change from assertibility to truth conditions in Dummett's thinking is not an abrupt one; rather, one sees a steady increase in the statistical likelihood of his talking about truth conditions, with the tipping point probably somewhere in the late 1970s. More on that in Sect. 2.9.

without encountering any particular perils. Now consider the statement "Jones is brave." If we assume bivalence, then this statement is true or false.

But we cannot *tell* whether Jones is brave or not until we watch him getting himself into a dangerous situation. Only then will his bravery or cowardice become apparent. Of course, there *might* be brain structures that predispose him already now to respond bravely in face of danger. Even if this were so, as of now we do not know how to find out about bravery by checking brain scans. If we assume that we are not in a position of putting Jones to the test by menacing him somehow, then we have no way to find out about the truth or falsity of the statement "Jones is brave."

Still, if we assume bivalence, the described circumstance is either among the conditions for the truth of that statement, or else among the conditions under which "Jones is not brave" is true.

If we accept this, Dummett argues, we cannot fully grasp the meaning of our statements. The meaning of the statement is made up of truth conditions, the present circumstance is one in which either "Jones is brave" or "Jones is not brave" is true, but we cannot know which. If we were grasping the meanings of the two statements fully, then we would have to be able to tell which statement is true under the present conditions.

But if we cannot be sure that we can grasp all aspects of the meaning of a statement, then the question becomes this: How are those aspects that are beyond our grasp helping to explain how linguistic exchanges function?

According to Dummett, it makes no sense to have a concept of meaning that has features that are not needed to describe linguistic behavior. This behavior is regulated by our notions of *correct assertibility*. If two speakers agree about all the ways in which a statement can correctly (and incorrectly) be used, then they attach the same meaning to the statement.[18] An aspect of a truth condition that is firmly out of our epistemic range cannot make a contribution to an agreement on whether a given statement was used correctly or not, and has consequently no place in a theory of meaning.

The above is Dummett's take on the oft-repeated Wittgensteinian insight that "meaning is use": If there are aspects of meaning that we can never be sure speakers grasp, no matter their linguistic behavior, then our meanings are too finely grained. In that case, our "theory of meaning is left unconnected with the practical ability of which it was supposed to be a theoretic representation."[19] A full grasp of meaning must be able to be demonstrated or *manifested* in one's use.

Epistemically unconstrained truth conditions fail this requirement of manifestability. The matter, says Dummett, stands differently if truth conditions are epistemically *constrained*. If it is always in principle possible to come to know them, then a manifestation of understanding will always be possible.

Take Jones again. What shows that you understand what "Jones is brave" means is that you recognize that certain behavior in a dangerous situation is brave. That is,

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[18] And this is the reason for his earlier view that assertibility conditions suffice for specifying the meaning of a statement.

[19] WTM, p.71

you know what would verify the statement: A certain type of behavior in a dangerous situation.

If we want to explain meaning in terms of truth conditions, then the concept of truth should not contain more than what is needed: A statement is true iff it can be verified, and to know the meaning of a statement is to know the conditions under which we would consider it verified. Such a theory of meaning, Dummett claims, is in a much better position to explain the role meaning plays not only in language *use* but also in language *acquisition*.

That the latter is a problem for the realist can be seen if we ask how we can learn to speak if the correct meaning of the statements we are presented with is beyond our cognitive reach. We learn to speak and to understand language by being trained in circumstances that we are able to recognize, not in circumstances that go beyond our epistemic grasp. It is then quite unconceivable how we come to know any parts of meaning that treat on unrecognizable features of reality.

These then are the famous twin challenges that Dummett brings forth against realistic semantic theories: The manifestation challenge and the acquisition challenge. He has expressed these challenges many times, often with subtle differences in the presentation. [20]

Furthermore, there is an extensive literature on the question how successful these challenges really are.[21] I will not even attempt an overview of the ramifications of this discussion. Instead, I will now come back to the above-mentioned switch from assertibility conditions to truth conditions.

2.9 Correct Assertibility or Truth?

In this section, I'll try to track, rather roughly, Dummett's meanderings back and forth between two core tenets:

1. The content of an assertion is given by those circumstances in which an assertion of it would be correct. The concept of truth is not really needed in a theory of meaning.
2. The content of an assertion is given by its truth conditions. The notion of truth is epistemically constrained. To make a correct assertion is simply to assert something true.

For Dummett, the general trend over the years has been to move away from position 1 and toward position 2. However, he often went back and forth between these positions, sometimes marking the difference, sometimes not. To convey a sense of this development, here are some salient quotes:

First up, a quote from his famous early paper, "TRUTH" (1959):

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[20] Apart from the sources I cited, Dummett (1975) offers a crisp presentation of the two arguments.

[21] One might start an exploration here: Hale (1999), Miller (2002), Miller (2003), Rosenkranz (2002), Tennant (2002) and Wright (1993).

[W]e should abandon the notion of truth and falsity altogether. (...) We no longer explain the sense of a statement by stipulating its truth-value in terms of the truth-values of its constituents, but by stipulating when it may be asserted in terms of the conditions under which its constituents may be asserted (TRUTH, p.17).

The reprint of this essay is preceded by an extensive comment in the preface of TOE (1978). This comment seems to mark a decisive move from position 1 to position 2:

On the way of putting it I adopted, one first proposes explaining meaning, not in terms of truth, but in terms of the condition for correct assertion, and then declares that, for statements whose meaning is so explained, the only admissible notion of truth will be one under which a statement is true when and only when we are able to arrive at a position in which we may correctly assert it. But, in that case, it would have been better first to state the restriction on the application of 'true', and then to have held that the meaning of a statement is given by the condition for it to be true in this, restricted, sense of 'true'. (...) Thus I should now be inclined to say that, under any theory of meaning whatever (...) we can represent the meaning (...) of a sentence as given by the condition for it to be true, on some appropriate way of construing 'true': the problem is not whether meaning is to be explained in terms of truth-conditions, but of what notion of truth is admissible (TOE, p.xxii).

Although this passage suggests a clear break with the old position, one will often find passages in later writings of Dummett that sound more like the original proposal, such as this passage from LBM (1993):

It is plain that any account of the practice of assertion will supply us with a general notion of the correctness and incorrectness of assertions. The root notion of truth is then that a sentence is true just in case, if uttered assertorically, it would have served to make a correct assertion. (...)

Thus the content of an assertion is taken as determined by the condition for it to be correct, and this in turn is identified with the condition for the sentence to be true: [W]e know what has been asserted when we know in what case the assertion is correct (LBM, p. 165–166).

In sum, it seems fair to say that Dummett thinks that there is a very close connection between truth and correct assertibility. However, the above quotes show that he is rather shaky on the question which of them has priority in an account of meaning. [22]

In addition, we have seen that he usually sees those two notions closely tied to the notion of verifiability: An assertion of a statement is correct and the asserted statement is true iff it is verifiable.

Nonetheless, there are also considerations of his under which the triad of truth, assertibility, and verifiability becomes undone in various ways, some of which I am going to discuss in quite some detail in the following chapters. In Sect. 3.8, I will discuss more thoroughly what the close tie between verifiability and truth brings us. Much later, in Part II, we will see what happens if correct assertibility is tied, not to verifiability, but to *non-falsifiability*. In that case, it will turn out that the close tie between correct assertibility and truth might not be plausibly upheld.

Whenever correctness and truth come apart, I will in principle stick with the earlier, i.e., the first of the two tenets at the beginning of this section: Essential to a grasp

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[22] Cf. Kirkham (1995), p. 248 ff. and Green (2001), p. 24. for further discussion and more sample quotes.

of meaning is a grasp of correct assertibility conditions. From that basis, however, I will keep an interested eye on what might be said about truth, e.g., whether it might still be argued to correspond to correct assertibility in some way, whether it should be altogether abandoned by a constructivist, etc. However, until the falsificationistic theories of Part II come along, the identification of truth and correct assertibility is less controversial (though still far from unproblematic).

2.10 Meaning Theory, Theory of Meaning and Semantic Theory

Let us take stock: Up to now we have a rough idea of how the constructivist conceives of meaning. I have also, though not yet in any detail, told you that under this conception some classical laws of logic might become problematic.

This section aims to spell out the transition from a given theory of meaning, whether constructive or not, to a notion of logical consequence more clearly. This will be the general blueprint of what will happen in many of the following chapters, starting with the very next one, in which the case for intuitionistic logic is examined.

To get started, we need once again to clear up some Dummettian terminology that could otherwise generate confusion:

- A MEANING THEORY for a particular language records all that a speaker needs to know, whether explicitly or implicitly, in order to be considered a competent speaker of that language. In contrast,
- A THEORY OF MEANING gives the general form in which a meaning theory has to be presented. "The task of a theory of meaning is to give an account of how language functions, in other words, to explain what, in general, is effected by the utterance of a sentence in the presence of hearers who know the language to which it belongs."[23]
- A SEMANTIC THEORY. This is a theory of how the correct assertibility (or the truth) of a statement is determined by its semantic value, and how its semantic value depends on the semantic values of its parts.

The idea of semantic values and their composition basically goes back to Frege. The semantic theory will have to answer questions about the reference of singular terms and the like. As we are mainly interested in questions of logic (and are going to concentrate on the propositional case), the question that we need to concentrate on is this: How do logically complex statements receive their semantic values? Assuming compositionality, how does the semantic value of a logically complex statement depend on the values of the constituent statements? For example, if we know the

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[23] LBM, p.21. Maybe "theory of language use" would have been a better label. Not only easier to discern from "meaning theory," it would have made the rest of this quote sound less puzzling: "The notion of meaning itself need not, therefore, play an important role in a theory of meaning." (ibid.) It might not, if there were no tight connection between meaning and use. However, we have seen in Sect. 2.8 that Dummett thinks that such a tight connection does hold.

value of *A* and the value of *B*, how do we get to the value of "*A* and *B*," "If *A*, then *B*" and so on?

This is the point at which it will be decided whether our theory is a constructive one or not. If we choose semantic values that are epistemically accessible (e.g., proof conditions, verification conditions, falsification conditions), then we are setting up a constructive semantics.

Once we have found out the semantic value of a statement, the next task of the semantic theory is to tell us whether the statement is correctly assertible/true in a given state of affairs or not. With this information, we can finally move on to a conception of logical consequence: An inference will be valid if it never fails to transmit correct assertibility/truth from premises to conclusions.

Let us see by way of example how the realistic semantic theory that gives the meaning of the classical logical constants works. It is as well known as it is simple. There is no mystery in how the semantic value of a statement relates to truth or falsity: The semantic value of a statement simply *is* its truth value.

The determination of the truth value of a complex statement is completely taken care of by the truth tables of classical propositional logic and the assumption that every statement takes exactly one of the truth values "True" and "False." The truth value of negations and disjunctions can be computed with the aid of the following tables:

¬	
T	F
F	T

∨	T	F
T	T	T
F	T	F

Given the usual definitions of the other connectives and, crucially, the assumption of bivalence, this is enough to give us full classical logic.

We have seen that in a constructive semantic theory, on the other hand, no assumption of bivalence can be made. Consequently, the constructivist claims, the account of the semantic values of complex statements and of logical consequence will have to be a different one.

Dummett's prime example of a semantic theory that is more adequate than the classical one is, once again, the account of mathematical statements in Brouwer's intuitionism: Such a statement is correctly assertible/true only if it is provable, and we cannot assume the correct assertibility/truth of a theorem if we cannot give a proof, or at least a method of constructing such a proof. The meanings of the logically complex statements are in turn given in terms of proofs of the constituent statements. In the next chapter, I will introduce two semantic theories that have been argued to be suitable for constructive theories. These will be the Brouwer-Heyting-Kolmogorov (BHK) interpretation[24] and the more formal Kripke semantics[25] for intuitionistic logic. The first spells out what a proof of a complex statement is by telling us, for example, that a proof of a conjunction is composed of two proofs, one for each

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[24] see Sect. 3.6.
[25] see Sect. 3.7.

conjunct. The latter is a version of Kripke's well-known possible worlds semantics
for modal logic.

2.11 Revisionism versus Eclecticism

But again, before getting into the details, let me try to get the big picture back into
view. A meaning theory will contain all that a speaker must know in order to qualify as
a competent speaker of a specific language. A theory of meaning outlines the general
features of such specific meaning theories and is informed by the underlying semantic
theory. The rules and laws of logic, finally, are validated by that semantic theory.

Given all this, we can now address the following question: How can a logical
practice be criticized and revised by a theory of meaning that purports to be based on
the idea that "meaning is use"? [26] If meaning is use, then isn't any use of statements
that people make in a systematic way bound to be correct? In particular, where
this "use" is the drawing of inferences, isn't each and every inference a community
accepts going to be constitutive of the meaning of the logical vocabulary and therefore
immune from revision? Isn't it enough to point to the fact that we are happy to infer
A from $\neg\neg A$ to establish the validity of that inference?

Here is Dummett's answer, couched in the terms we have introduced over the
last sections: Suppose that the semantic theory that backs up a logic arises from a
theory of meaning that can be criticized on systematic grounds, such as the arguments
from manifestation and acquisition I sketched above. In that case, that criticism is
transmitted, via the semantic theory, to the logic itself. Then, even if it might be argued
that the logic is well entrenched, in that the speakers themselves will not hesitate to
draw inferences in accordance with its rules, this practice can be criticized by the
theoretician. The practice on the whole, they will contend, is incoherent, and should
be revised so as not to lead us astray in our reasoning.

The next task will then be to supply a package of semantic theory and logic that
is better equipped to answer the worries that have been raised about the theory of
meaning. Presumably (I am not following anything Dummett explicitly says here),
if it should occur that there are several candidates, some principle of *minimum muti-*
lation should lead our choosing. A logic that is based on a sound linguistic basis and
preserves more of the inferences people are apt to draw should then be preferred to
one that suggests more bizarre revisions. (I stress this because I believe I can later
present an alternative to intuitionistic logic that, among other virtues, is closer to the
inferences we would normally like to draw.)

But is revision inevitable as soon as we find that a constructive logic can be
underwritten in the way sketched above?

According to Dummett,

> [i]ntuitionism (...) raises two philosophical questions.

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[26] Dummett is fully aware that at this point he can't claim to be following Wittgenstein any more,
cf. LBM p. xi.

> 1. Do intuitionists succeed in conferring a coherent meaning on the expressions used in intuitionistic mathematics, and, in particular, on the logical constants?
>
> 2. Is there a ground for thinking that classical mathematicians *fail* to confer an intelligible meaning on logical constants, and on mathematical expressions in general, as they use them? (EoI, p. 251).

Only a positive answer to *both* questions will lead to a revision. Nonetheless, Dummett, somewhat unenthusiastically, acknowledges that one *might* answer the first question positively and the second negatively. That is, one might say that the meaning of the connectives are, in the same area of discourse, equally intelligible under the intuitionistic and the classical interpretation.

Granted, such "eclecticism"[27] is of no great interest to anyone who hopes to find answers to metaphysical questions. The only route from the meaning of logical constants to anti-realistic metaphysical conclusions is to hold that constructive logicians give coherent meaning to its constants, while classical logicians *do not*.

But logic and its philosophy are of interest in itself, even if its study does not entail metaphysical conclusions. Let us try to get an idea of what it might be like to accept the intelligibility of both intuitionistic and classical logic.

The most obvious way in which such an eclectic position could be construed is as a pluralism of languages, much in the spirit of Carnap's principle of tolerance.[28] The logical constants of classical logic and those of intuitionistic logic are two distinct sets of constants. One natural thing to say is that one employs the intuitionistic language when one is concerned with the transmission of *provability* or *verifiability*, while one turns to the classical language when epistemically unconstrained *truth* is at issue.

If this policy is adopted, then an important question will be on what side assertibility will come down: Will an assertion be correct iff it is true, will it be correct iff it is verifiable or something else altogether? Here, we obviously see one of the cases in which Dummett's firm connection between truth, verifiability, and assertibility could dissolve.

A new kind of logical eclecticism that offers a way of viewing *both* intuitionistic and classical logicians as concerned with truth and using the same language has been suggested by J.C. Beall and G. Restall.[29] They call their new position *logical pluralism*.

Their thesis is that both intuitionistic and classical inferences are valid iff they preserve truth in every possible case. The notion of a "case" is intentionally under-specified. It is in precisifying this notion that the different logics come about. If a case is understood to be a complete and consistent *world*, we get classical logic.

If, on the other hand, we consider cases to be *constructions*, the resulting logic will turn out to be intuitionistic logic. A construction is very much what I will call

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[27] EOI, p. 250

[28] Carnap (1959)

[29] Beall and Restall (2006)

a "stage of investigation" when I discuss the Kripke semantics of intuitionistic logic in Sect. 3.7.[30]

According to Beall and Restall, what kind of "case" we take the present circumstance to be is a question of our interest. The different cases do not generally represent different areas of discourse.

2.12 Metaphysical Conclusions

Whether the eclecticist chooses the traditional route of Carnapian tolerance or the modernized version of Beall and Restall, the following seems hard to evade: In logic, there will be no morals, and very little metaphysics. Presumably, the eclecticist who endorses both classical and intuitionistic logic as intelligible will simply be committed to the same metaphysical upshots[31] as the monogamous classicist.

I am more concerned with the first of the questions Dummett raised at the beginning of the last section than the second one. That is, I am more interested in the question whether intuitionistic logic is really backed by a constructivist theory of meaning than in the question whether a realistic theory of meaning is coherent. As we have just seen that this might raise worries about the metaphysical fruits such an enterprise might bear, let me end this chapter by making my policy in this respect clear.

The connection between language, logic, and metaphysics is fascinating and puzzling in equal measures. I believe that much of the enduring appeal that has led many to study Dummett's very difficult writings lies in his bold reconception of metaphysical problems. Nonetheless, metaphysics is not the main goal of my inquiry, and I will not be perturbed if useful metaphysical upshots become unlikely.

I believe that it is ambition enough to spell out a constructive way to give meaning to the logical vocabulary and to investigate which logical laws are thereby underwritten. The metaphysical *consequences*, then, will not be pursued with much zeal, if at all.

On the other hand, if an argument requires strong realistic *assumptions* (such as Prawitz's appeal to an abstract realm of self-subsisting proofs that we shall meet in Sect. 3.8), then I will point those out and proceed under the presumption that a Dummettian constructivist would shy away from making such assumptions.

Closely tied to this policy is something I mentioned above: When there will be cases in which correct assertibility and truth drift apart, I will keep my main focus on correct assertibility. Oftentimes, it will in such cases be quite unclear whether an acceptable notion of truth is available at all; I will often do not much more than flag

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[30] Beall and Restall offer yet a third way to specify what a case is. A case might be a *situation*, something which might be incomplete and even inconsistent. This will give them a *relevant* logic, First Degree Entailment (FDE). I will say more about relevance and FDE in Chap. 3.

[31] If any. In fact, Beall and Restall argue that anti-realists can make use of classical logic (Beall and Restall 2006, p. 46 ff); their argument, however, has nothing much to do with their pluralism.

this worry and keep my attention on correct assertibility, whereas a metaphysician would undoubtedly not rest until the question whether a distinct notion of truth is available. After all, it seems clear that when truth and correct assertibility do not come to the same thing, any metaphysical conclusions should be drawn from the former, not the latter.

2.13 Chapter Summary

In this chapter, I have given a short survey of Dummett's expansive philosophical program. I tried to make the connections visible (and at least somewhat plausible) that Dummett sees between the analysis of language, the theory of logical consequence, and the most basic and important metaphysical questions. I showed how, based on a Wittgensteinian conception of meaning as determined by language use, Dummett thinks that certain semantic theories are unsuitable. We have seen that there is considerable uncertainty whether Dummett would prefer to think of meanings as conditions for correct assertibility or simply truth. If it is the latter, he would insist that the notion of truth that needs be employed has to be epistemically constrained to meet his conditions of manifestability and acquisition; hence, we are led toward a constructivistic theory of meaning. And such a constructivistic theory will, if thought through to the end, result in logical revision, or at the very least in a (metaphysically unexciting) form of eclecticism. As for a particular logic that we should embrace, Dummett campaigns for intuitionistic logic, which we will investigate in a more systematic manner in the next chapter.