

Chapter 11

Summary

In this last part, I would like to collect some of the main results of this work and make some general philosophical observations about them.

11.1 Resume

I presented the material in three large parts entitled BACKGROUND, FALSIFICATIONS and LOGICS.

Starting out on the background, I tried to give a concise but useful characterization of what the most important aspects of Dummett's constructivist program are. This included a larger overview of his wide-ranging aims, such as building a base from which it would be feasible to tackle metaphysical problems, primarily disputes between realists and anti-realists of various stripes. Though these results are fascinating, I concentrate on the basis he sets up for these considerations: The constructivist theory of language and logic.

The next chapter was devoted to the intuitionistic program, which incorporates the most important ideas Dummett builds on. I described the philosophical motivation and the early developments of intuitionistic logic and showed how the semantics of intuitionistic logic can be seen as a paradigm for Dummett's setup. However, not each and every one of the various semantics for intuitionistic logic will serve the Dummettian purpose. I argued that a Kripke semantics, supplemented by the classic BHK explanation, has some chance of being up to the task. I strove to give quite a thorough introduction to these matters, as the logical systems I introduce in the third part essentially draw on them.

A third chapter was needed to get some basics on gappy and glutty semantics clear. The concept of paraconsistency was explained and given some motivation. Importantly, I stressed that paraconsistency does not have to come bundled with dialetheism, the metaphysical doctrine that there are true contradictions. The alternative view that

paraconsistency results from assertible truth value gaps, analetheism, was presented as well.

The second part contains most of the exegetical work in this book. I try to collect and make sense of the things Dummett says about the role of falsifications in a semantical theory. To get some order into matters, I discerned five different stages, which were ordered in a pyramidal diagram. This diagram was supposed to facilitate orientation in the third part and to point out the axes along which the different positions could vary.

Vertically, with increasing height, the amount of interaction between verifications and falsifications increased. At the lowest level, there was no such interaction, simply because only one central notion was allowed (either verification or falsification).

At the intermediary level, there was one central notion that was responsible for the assertoric content of a statement, i.e., for determining under which circumstances an assertion is correct. However, there was a mixture of verifications and falsifications employed to determine the contribution the sense of a statement made to the sense of a complex statement containing it.

Finally, the apex of the pyramid was where both concepts enjoyed full interaction.

The left–right axis of the pyramid marks two different answers to the question: What makes an assertion correct? On the left side, we find the *verificationistic* theories, in which the answer is this: An assertion is correct iff it is verifiable. On the right hand side, however, the answer is that an assertion is correct iff it is unfalsifiable. I call theories that give this answer *falsificationistic*.

As this defining thought of falsificationism is rather little developed in Dummett's writings, even though it takes a central place in some of his most important works, I spend the second chapter of this part on it. I quickly come to argue that falsificationism as a general theory, applicable to all areas of discourse, does not seem very plausible. However, I try to find some examples that make the proposal plausible. The most clear-cut example here is taste talk, and generally areas of discourse in which faultless disagreements are possible.

The third part of the book, entitled LOGICS, finally addresses the question which concrete effects on logical principles the introduction of falsifications brings. I start with the one logic that we find in Dummett's own exposition, dual intuitionistic logic. This logic is supposed to underwrite a pure falsificationism, where all meanings are given in terms of falsification conditions only. Although an interesting approach, ultimately I find it unsatisfying, as it cannot give a plausible account of logical vocabulary.

I proceed in the next chapter to what I call expanded verificationism, where both verifications and falsifications can be employed to determine the meaning of the logical constants. I found that the logic that most naturally flows from this setup is one of the Nelson logics, N_3 (I dismiss the gluttony variation N_4 early on). I discuss some alterations on the Nelson conditional, as the failure of contraposition and modus tollens are a bit unsatisfying. However, in the end, I chose to stick with the original account.

Although I could not claim a complete knock out victory of N_3 over intuitionistic logic, I must say that the superiority of the Nelson account over the intuitionistic one

is among the claims I make that I feel strongest about. I think that constructivists would do very well to pine for revision along Nelson lines rather than intuitionistic ones, both because of the theoretical reasons given above and simply because the proposed changes would, I believe, appear to the uninitiated much less bizarre and impractical.

In the next chapter, I show how the Nelson ideas can be adjusted to fit the falsificationistic requirements. Given what has been established up to that point, the characterization of the logic is straightforward. I call the logic that I thereby create N_{3f} . However, there is a problem in getting people to speak coherently. What I mean by this is that even if assertions are correct as long as they are not falsifiable, we should not have people contradicting themselves. I propose a strategy to deal with this problem, and I show how to implement it formally.

In the last chapter of the third part, I discuss the strategies that mix verifications and falsifications without restraint. I call those strategies the *hybrid strategies*. The chapter starts out by defusing certain worries Dummett had about such a strategy. Then I play through some possible approaches, and I end up liking the following account best: An assertion is judged either according to the verificationistic norm or according to the falsificationistic one. Which one of the two it will be is decided by the distribution of the *burden of proof*. Whether or not this burden lies on the speaker is a feature that is determined by the context in a systematic way. I make room in the conversational score for a burden-of-proof parameter and give some first hints on which features of a conversation might have an effect on this parameter. However, there is no doubt that there is much further study needed to vindicate this approach.

11.2 What is Constructivity?

I would like to end this work with some general philosophical musings about the material I presented. In particular, I would like to leave you with two questions. The first is this: What makes a logic constructive? We have seen a host of logics, all introduced with a claim to be constructive. Are they? What is essential for making the decision?

The second question: What exactly is the connection between the constructivist program and the phenomenon of paraconsistency? We have seen many paraconsistent logics in the course of the last chapters. Is there a specific constructive or anti-realistic motivation for paraconsistency (or even dialetheism) that is underlying all these occurrences, or is the fact that these logics do not validate Explosion merely accidental? I believe that this question is closely related to the first one.

So let us start there and ask again: What is a constructive logic?

For many, “constructive logic” and “intuitionistic logic” are simply synonymous. However, H. Wansing writes:

In a situation where there are no clear, agreed-upon, individually necessary and jointly sufficient conditions for the constructiveness of a logical system, it seems quite difficult or next to pointless to designate one particular logic as the correct constructive logic. Nevertheless,

for some reasons certain logics may still be regarded as constructive logics (Wansing 2008, p. 342).

What are those reasons? Is there a syntactic feature that divides constructive logics from non-constructive ones? I do not mean meta-logical properties of proof systems; there might well be definitive characteristics that divide constructive proof systems from non-constructive ones. This is a deep and important issue, but I have bracketed it all throughout the book, and I will not start to go into it now.

What I mean is this: Can we tell just from knowing which inferences are valid in a logic whether the logic is constructive or not? More specifically, is there a specific form of inference that clearly marks the divide? Which inference would that be? We have seen all Double Negation Laws and De Morgan's laws validated by N_3 , surely a constructive logic. A natural assumption that is often made is that the most central and essential syntactic marker of constructivity is that failure of the Law of Excluded Middle, $\models A \vee \neg A$. But we have seen logics such as N_{3f} that even validate LEM. Is this a reason to deny that it is a constructive logic?

In fact, between them, the logics I discussed in Chaps. 8 and 9 validate each and every inference of classical logic. So, if they indeed deserve to be called constructive (and I think they do), then no single inference can claim to divide constructive from non-constructive logics.

However, it is hard not to have noticed that all the constructive logics that validate LEM are paraconsistent, which brings us to my second concern.

Priest takes up the question whether or not paraconsistency has a special relation to the anti-realistic/constructivist project and comes up with a negative answer: The question whether or not a logic is paraconsistent is completely independent of whether or not it is suitable to the constructivist.

[T]here are many (...) paraconsistent logics, of widely different kinds. To determine on which side of the realism/anti-realism fence each sits requires its own investigation. Sometimes this will be obvious. For example, if the logic verifies the LEM, it is not going to sit on the anti-realist side (Priest 2012, p. 190).

As I just suggested, I think that no inference on its own, be it LEM or another one, can decide whether a logic is constructive or not. Thus a paraconsistent logic is not ruled out for constructive purposes, as Priest suggests, just because it validates LEM.

However, I am of course not claiming that the question which inferences a logic validates and fails to validate is irrelevant when we try to decide on its constructivity. The failure of LEM *for the right kind of reason* is indeed essential to the constructivity of intuitionistic logic and N_3 .

What is this reason? It is the nature of the semantic values. At the very outset of this book, I have defined constructivism as the view that semantic values must be epistemically accessible. But even if such an explicit explication of the notion of constructivity in terms of semantical values is not assumed, I believe that without a semantical account, the question whether or not a logic is constructive seems quite pointless. But given the semantics, we have seen for intuitionistic logic and N_3 , the failure of LEM quite distinctly signals the constructivity of the logic.

In exactly the same way, the paraconsistency of the falsificationistic logics we have seen is quite essential. And the interesting fact here is this: The reason for which LEM fails for N_3 is exactly the same reason for which Explosion fails for N_{3f} , and this reason is the existence of *gaps* between the constructive semantic values in the semantics. After reading this book, the surprise inherent in this result may well have worn off, but it is worth taking a step back and appreciating it anew: Paraconsistency is induced, not by *gluts*, but by gaps, palpable manifestations of our epistemic humility.

If it were not for those gaps, our models would have no right to call themselves constructive models. That is, if we were to restrict our attention to models in which every statement is either verifiable or falsifiable at each world, we can no longer maintain that these senses of “verified” and “falsified” are actually constrained by our epistemic achievements and potentials any more.¹

Now, given that the root of falsificationistic paraconsistency lies in the gaps, it is clear that *dialetheism* has no hope of gaining motivation here. If anything, what we are dealing with is a form of *analetheism*. As I presented analetheism, it is a doctrine about truth and falsity and the empty space between the two. So, for falsificationism to entail a view of this sort, it seems we need to be willing to make the identification of verifiability and truth on the one hand, and falsifiability and falsity on the other. I was not showing much enthusiasm for this identification in the last chapter. But even if we do not speak of truth and falsity, the structural similarity seems so close and clear that it would seem to me to be apt to call the falsificationistic theories cases of *constructive analetheisms*.

In sum, Priest is partly right: The question whether a logic is constructive or not is in a sense independent of the question whether it is paraconsistent or not. However, against Priest I want to contend that the sense of this independence is the same in which constructivity and the LEM are independent. Given a certain semantic setup, either LEM or paraconsistency becomes essential features of a constructive logic of one type or another.

¹ This is not meant to rule out the intuitionistic models as non-constructive. But while it is true that there are no gaps between the values 1 and 0 in these models, this is because the values are to be read as “provable/verifiable” and “not provable/verifiable,” not as “verifiable” and “falsifiable.”