Chapter 1
Introduction

The central themes of this book are certain non-classical logical systems, their philosophical motivation, and the meanings of their constants. My point of departure is the general constructivist line of argument that Michael Dummett has offered over the last decades. This argument expands to touch on a dazzlingly large number of important philosophical topics, but its root lies in Dummett's philosophy of language.

Even though most of his project is a modern version of verificationism, there is a clear strand of falsificationistic thinking in his writing that merits closer scrutiny. Indeed, Dummett himself has recently bemoaned the fact that this strand has not received much attention by his commentators.[1] This book explores where this largely untrodden path might lead.

Very succinctly put, the difference between the usual verificationistic picture and the alternative falsificationistic one is this: Understanding language consists in understanding under what circumstances an assertion would be correct. The verificationist story is that this consists in knowing under what circumstances an assertion would be *verified*. The falsificationistic story, in contrast, has it that one needs to know what would *falsify* the assertion, because an assertion will have to count as correct unless it is falsified.

One of the most interesting aspects of Dummett's new constructivism is that it seems to entail a revision of classical logic. For Dummett, the logic that should be adopted by a verificationist is intuitionistic logic, a logic that was developed in response to the philosophic views of L. Brouwer. Intuitionistic logic shows many characteristic differences compared to classical logic.

As will become clear in my discussion, I think that the central aspect of intuitionistic logic that makes it suitable for Dummett's verificationism is its rejection of the Law of Excluded Middle (henceforth LEM), while other peculiarities such as the rejection of Double Negation Elimination are quite accidental and dispensable features. The LEM says that, as a matter of logical validity, "*A* or not *A*" will always be assertible, no matter what sentence *A* stands for. For the verificationist, this is

[1] Auxier and Hahn (2007), p. 694.

A. Kapsner, *Logics and Falsifications*,
Trends in Logic 40, DOI: 10.1007/978-3-319-05206-9_1,

only true if either "A" or "not A" can be verified, but this cannot be assumed to hold for every A, so the LEM must be rejected.

Now, one of the main claims I wish to make is this: If the LEM is what has to be given up in verificationism, then the move to falsificationism will have a different casualty, the principle of *Explosion*. This classically and intuitionistically valid principle, also known as *ex contradictione quodlibet*, tells us that we can infer whatever we wish from a contradiction: $A \wedge \neg A \vDash B$. Any logic that does not support such inferences is called a paraconsistent logic.

The normal way of supporting the principle of Explosion is this: A contradiction such as "A and not A" will never be true, no matter what sentence A we choose to plug in. An inference is valid iff (if and only if) the conclusion is true whenever the premises are true; therefore, the inference from something that can never be true to an arbitrary statement is always valid.

However, I claim that this kind of argument will not hold if we take the statement "A and not A" to be correctly assertible if it is not falsifiable, for the most natural way of giving falsification conditions for a conjunction will turn out to be the requirement that one of the two conjuncts is constructively falsified. But just as there was no guarantee that we can always verify an arbitrary statement or its negation, there is no guarantee that we can falsify either of them. Thus, given the falsificationistic account of what a correct assertion is, there is no guarantee that a contradiction will never come out assertible.

Falsificationism represents quite a radical departure from the usual verificationistic picture. I will spend quite some time in this book trying to get to grips with it. However, there are also more subtle ways in which falsifications can enter into a constructivist semantics. I will display the full spectrum of options and discuss the logical systems most suitable to each one of them. There are many forks on this path, and only one of them leads to intuitionistic logic.

The book is divided into three large parts. In the first part, important background information about Dummett's program, intuitionism, and logics with gaps and gluts is supplied. The second part is devoted to the introduction of falsifications into the constructive account. It turns out that there is more than one way in which one can do this. In the third part of the book, I detail the logical effects of these various moves. Below, I give a chapter by chapter overview of what is to come.

1.1 Analytical Table of Contents

Chapter 1 Introduction

This book examines the effects on logic that introducing the concept of *falsification* as a central notion of semantic theories will have. This introduction gives a first idea of what this might mean, and an overview of the following chapters.

Introduction to Part I: Background

The first part of the book consists of three chapters that will provide the foundation for the later discussion.

Chapter 2 Constructivism

This chapter, like the next two, is an introductory one that presents the critique of classical logic that Dummett put forward. It is grounded in his account of how meaning theories should be constructed, namely not in terms of mind-independent truth conditions, but rather in terms of proof or verification conditions. A semantic theory that gives the proof (verification) conditions of logically complex statements in terms of the proof (verification) conditions of their constituent statements is the basis of such an explication of meaning. The logical inferences that are licensed by this semantic theory are, arguably, those of intuitionistic logic.

Chapter 3 Intuitionism

The original ideas behind intuitionistic logic, its axiomatics, and two semantic theories are presented. Intuitionistic mathematics was the brainchild of L. Brouwer, who took mathematics to be about mental constructions, not abstract objects. These ideas were captured in a logical system by his student A. Heyting. I present the Brouwer–Heyting–Kolmogorov interpretation and the Kripke semantics for intuitionistic logic.

Chapter 4 Gaps, Gluts, and Paraconsistency

In this chapter, some semantical theories that allow for gaps and/or gluts are introduced, and the logics that can be based on them. In particular, I will present First Degree Entailment (FDE), strong Kleene (K3), and the Logic of Paradox (LP). An important idea that is introduced here is that logical consequence need not be defined as truth preservation, but might also be defined as non-falsity preservation. This makes no difference in some cases, in others it does. The notions of paraconsistency, dialetheism, and analetheism are introduced. Again, all this is important background material for the later chapters.

Introduction to Part II: Falsifications

This second part contains the most exegetical work. I try to analyze and systematize what Dummett has to say about the role of falsifications in semantic theories.

Chapter 5 From Proofs to Verifications, and on to Falsifications

I claim that the move from mathematical discourse to the empirical realm will have an influence on the logic that is motivated by the constructivistic semantic theory. This is because the intuitionistic explanation of negation is highly problematic in this setting. Dummett acknowledged that falsifications are necessary to fix the verification conditions of logically complex expressions. He then went even further and suggested that falsification might even be regarded as the central concept in a semantic theory and that logical consequence should transmit non-falsifiability.

To get some order into what Dummett offers us, I first give a clear account of the broad distinction I make between *verificationism* and *falsificationism*. The central tenet of verificationism is that an assertion is correct iff it is verifiable. The central idea of falsificationism, on the other hand, is that an assertion is correct iff it merely is not falsifiable.

Making more fine-grained distinctions, I then discern five stages of possible involvement of falsifications in a semantic theory. They are, in ascending order of falsificationistic predominance:

(I) pure verificationism, the view presented in the first two chapters leading to intuitionistic logic; (II) expanded verificationism, a verificationism (in the sense above) that uses falsifications to fix the meaning of complex statements; (III) hybrid strategies that rely both on verifications and falsifications in equal parts; (IV) expanded falsificationism, a falsificationism that relies on verifications to explain complex statements; and (V) pure falsificationism, the complete expulsion of all verifications.

These five stages, I will argue, all come with their own distinctive logics, and the last part of the book will go through these stages and their logics one by one.

Chapter 6 Falsificationism

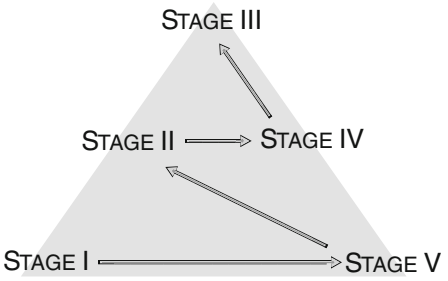
In this chapter, the idea of a falsificationistic theory of meaning is examined, without regard to the exact kind of falsificationism (pure or expanded) at issue. I further analyze Dummett's arguments for the idea that assertibility is nothing more than non-falsifiability, and I strive to give arguments of my own that make this position as plausible as possible. To this end, I give examples of areas of discourse that might be governed by a falsificationistic norm of assertion.

One of these examples concerns the assertions a defendant makes at a criminal trial. As most legal systems are working under the presumption of innocence, the defendant will be able to correctly make any assertion, as long as this assertion is not falsified. In other words, the prosecution has to bear the *burden of proof*, a concept that will feature prominently in the last chapter of Part III.

The second example I focus on is taste talk. If I say "Sushi is tasty", then it is up to my audience to prove me wrong. If they cannot, my assertion will be correct, even if not all members of the audience are bound to like sushi. I will make a proposal what it would mean to prove me wrong in this case.

Introduction to Part III: Logics

In the last part of the book, I will present the logics corresponding to the different stages I mentioned above. As I will make plain, it will make sense to go through them in the following zig-zag pattern:



Chapter 7 Stage Five: Pure Falsificationism and Dual Intuitionistic Logic

As Stage I (intuitionism) has already been dealt with in Part I, I start this last part by presenting Dummett's own proposal for a logic for Stage V: A logic that is only based on falsifications, the paraconsistent logic known as *dual intuitionistic logic*. This logic preserves not verifiability (as the concept of a verification is not utilized), but unfalsifiability. Choosing this property ensures that a speaker who asserts the premises will not incur further liabilities by asserting the conclusion. I present dual intuitionistic logic in a different semantical guise than Dummett did, which will make it more accessible. It will turn out, however, that this logic suffers from problems with complex statements, similar to those that intuitionistic logic had.

Chapter 8 Stage Two: Expanded Verificationism and the Logic N_3

In the first account that combines verifications and falsifications, I start with a preliminary discussion of how verifications and falsifications should be related. The upshot is that there are gaps (statements that are neither verified nor falsified), but no gluts (statements that are both verified and falsified). Based on this assessment, I will present the logic that most naturally arises. It is a species of the so-called Nelson logics. When it comes to conditionals and negations, there are some options to be explored, and I will go through the most important ones.

Chapter 9 Stage Four: Expanded Falsificationism and the Logic N_{3f}

Turning then to an account that pays tribute to Dummett's idea of falsificationism, I will show how to modify the logic of the previous chapter so that it transmits non-falsifiability. There are some worrying features of this new logic, as it seems to allow assertions that are quite incoherent. On the one hand, the logic is a paraconsistent one, which I claim is a good thing. This feature ensures the possibility for two persons to be correct, even though they contradict each other. However, it seems to allow the assertion of outright contradictions, and it does not satisfy modus ponens. I will then present a strategy to overcome these problems.

Chapter 10 Stage Three: Hybrid Strategies

I show in this chapter how verificationism and falsificationism can be combined. I give three main strategies (which might be used alongside each other): First, to simply differentiate areas of discourse in which assertions are correct iff verifiable, and others in which assertions are correct iff unfalsifiable. The second strategy is to open up a space for assertions that are neither correct (verifiable) nor incorrect (falsifiable). The last strategy is to make the norm of assertion dependent on the burden of proof. Paradigmatic here is legal discourse, where we might see the assertions of the defendant to be correct iff they are not falsifiable, and those of the prosecution as correct iff they are verifiable.

Chapter 11 Summary

I review the findings of the book and end by drawing some further philosophical conclusions.

1.2 Symbols and Abbreviations

A note on the logical symbols that will appear in this work: I will use $\neg$ to denote either classical negation or a generic unspecified negation (context will disambiguate). $\sim$ will stand for intuitionistic negation, $-\circ$ for dual intuitionistic negation, and $-$ for Nelson negation. I will only use two different symbols for conditionals, namely $\rightarrow$ for the material conditional and $\supset$ to denote a number of conditionals that are constructive in one sense or another (again, context will most of the time disambiguate sufficiently; if not, I will use subscripts as in $\supset_{\mathrm{TOL}}$).

I will employ the following abbreviations (for logical principles and often cited works of Dummett's):

DNE	Double Negation Elimination
DNI	Double Negation Introduction
EOI	Elements of Intuitionism (Dummett 2000)
LBM	The Logical Basis of Metaphysics (Dummett 1991)
LEM	Law of Excluded Middle
TOE	Truth and Other Enigmas (Dummett 1978)
TRUTH	Truth (Dummett 1978, pp. 1–25)
WTM	What is a Theory of Meaning (II) (Dummett 1993, pp. 34–94).