New Economic Windows

Marisa Faggini
Anna Parziale *Editors*

Complexity in Economics: Cutting Edge Research

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Preface

The crisis happened in the world in the last few years, describing a whole of interdependencies and interactions, highlighted fundamental flaws of neoclassical economic theory: its unedifying focus on prediction and, above all, its inability to explain how the economy really works. The reductionist approach, applied by this theory, too often overlooks the dependencies or interconnections among elements and their influence upon macroeconomic behaviour. Its focus is not to study the unfolding of the patterns its agents create, but rather to simplify its questions in order to make it easy and seek closed. These principles, imposed by the Cartesian paradigm of simplification, have created a separation between reality and its formal representation.

Economic scientists relying on seeing the social system as a static system-with linear relationships, equilibrium, and connections that fit relatively simple equations have to turn to new economic theories in order to understand how the economy really works and how governments might manage the economic system more effectively.

One of fundamental assumptions of many economic models is that the system is in equilibrium or at least, if disturbed, has a tendency to move back. Now, if we assume equilibrium, as essential status of an economy, we place a very strong filter on what we can see in it. Under equilibrium there is no improvement or further adjustment, no exploration, no creation, so anything in the economy that takes adjustment—adaptation, innovation, structural change, history itself—must be bypassed or dropped (Arthur 2013). The result may be a beautiful structure, but it is one that lacks authenticity, aliveness, and creation. Economics is viewed as a discipline that is mainly concerned with "simplistic" theorizing, centred upon constrained optimization. As such, it is ahistorical and it does not deal with economic processes.

A more natural picture of our economic system rather seems to be that of a complex dynamical system with many nonlinearly interacting components. So it is time to explore new ways of managing our economy, oriented at evolution and change rather than only at pursuit of competition, efficiency, and growth.

Economies are complex systems composed of a large number of interacting components and of the relationships between them. The goal of complex systems research is to explain, in a multidisciplinary way, how complex and adaptive behaviour can arise in systems composed of large numbers of relatively simple components, with no central control and with complicated interactions. Not more aggregate reduced to the analysis of a single, representative, individual agent ignoring by construction any form of heterogeneity and interaction, but the aggregate emerging from the local interactions of heterogeneity agents.

The Complexity Theory challenges these fundamental orthodox assumptions and seeks to move beyond market transactions, static equilibrium analysis and homo oeconomicus, emphasising the power of networks, feedback mechanisms, and the heterogeneity of individual. Not more by simplifying, linearizing and dividing, but observing the relevance of interrelationships among the components of systems—as well as their relationships with the environment and vice versa—in determining collective behaviours.

This approach is not just an extension of standard economics, nor does it consist of adding agent-based behaviour to standard models. It is a different way of seeing the economy. It gives a different view, one where actions and strategies constantly evolve, where time becomes important, where structures constantly form and reform, itself where phenomena appear not visible to standard equilibrium analysis, and where a meso-layer between the micro and the macro becomes important.

Complexity theory describes how people and organizations respond to the chaos around them. Within complexity theory, chaos does not mean disarray or being out of order; rather, it means order emerging. In fact, chaos means order with no predictability. Events appear to be random; hence, confusion is erroneously inferred.

Other than chaos its main concepts include emergence, adaptation, self-organization, patterns, agents, networks, wholeness, and interdependent interactions among divergent yet connected parts. Other concepts include learning and memory, change and evolution, surrounding environments, relationships between entities, internal system substructures, and holism and synergy (Manson 2001).

This book starts from the premise that there is a lot wrong with conventional economics and that insights from new economic thinking need to be taken seriously.

It focusing the attention on innovative components of Complexity Theory in economics, results from the idea to investigate economic phenomena not as derived from deterministic, predictable, and mechanistic dynamics, but as history-dependent, organic, and always evolving processes.

The volume by exploring such perspectives, with an emphasis on the more recent view of what Economic Complexity is, contains papers regarding methodological, theoretical and applied aspects of various research fields such as Non-linear Dynamics, Chaos Theory, Network Theory, Fractal Analysis, Neural Network.

By casting light on a variety of topics in the field, it will provide an ideal platform for researchers wishing to deepen their understanding and identify areas for further investigation.

The contributions offer a set of tools and techniques of a cutting-edge research that is highlighting the potential of complexity approach in economic fields by overcoming the limitations of last researches, offering new insights, to bring together into a coherent picture of the different light that are lighted in science so any researcher can observe them and deepen them.

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Contents

Chapter 1
Applications of Methods and Algorithms of Nonlinear Dynamics in Economics and Finance

Abdol S. Soofi, Andreas Galka, Zhe Li, Yuqin Zhang and Xiaofeng Hui

Abstract The traditional financial econometric studies presume the underlying data generating processes (DGP) of the time series observations to be linear and stochastic. These assumptions were taken face value for a long time; however, recent advances in dynamical systems theory and algorithms have enabled researchers to observe complicated dynamics of time series data, and test for validity of these assumptions. These developments include theory of time delay embedding and state space reconstruction of the dynamical system from a scalar time series, methods in detecting chaotic dynamics by computation of invariants such as Lyapunov exponents and correlation dimension, surrogate data analysis as well as the other methods of testing for nonlinearity, and mutual prediction as a method of testing for synchronization of oscillating systems. In this chapter, we will discuss the methods, and review the empirical results of the studies the authors of this chapter have undertaken over the last decade and half. Given the methodological and computational advances

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of the recent decades, the authors of this chapter have explored the possibility of detecting nonlinear, deterministic dynamics in the data generating processes of the financial time series that were examined. We have conjectured that the presence of nonlinear deterministic dynamics may have been blurred by strong noise in the time series, which could give the appearance of the randomness of the series. Accordingly, by using methods of nonlinear dynamics, we have aimed to tackle a set of lingering problems that the traditional linear, stochastic time series approaches to financial econometrics were unable to address successfully. We believe our methods have successfully addressed some, if not all, such lingering issues. We present our methods and empirical results of many of our studies in this chapter.

Keywords Nonlinear deterministic dynamics · Financial integration · Nonlinear prediction · Synchronization of stock markets · Correlation dimension · Time-delay embedding

Introduction

The traditional empirical financial and economic studies presume the underlying data generating processes (DGP) of the time series observations to be linear and stochastic. However, recent advances in statistical physics, probability theory, and ergodic theory, which are summarized under the rubric of dynamical systems theory and algorithms have enabled researchers to observe complicated dynamics of time series data, and test for validity of these assumptions. These developments include theory of time delay embedding and state space reconstruction of the dynamical system from a scalar time series (Takens 1981; Sauer et al. 1991), methods in detecting chaotic dynamics by computation of invariants such as Lyapunov exponents (Pesin 1977; Wolf et al. 1985) and correlation dimension (Grassberger and Procaccia 1983), surrogate data analysis (Schreiber and Schmitz 1996) and the other methods of testing for nonlinearity (McLeod and Li 1983; Tsay 1986; Brock et al. 1996), and mutual prediction as a method of testing for synchronization of oscillating systems (Fujisaka and Yamada 1983; Afraimovich et al. 1986; Pecora and Carroll 1990).

Traditionally, the numerical algorithms of nonlinear dynamical systems are mostly used in analyses of experimental data of physics and other physical and natural sciences; however, over the last two decades, these methods and algorithms have found extensive use in finance and economics also (Scheinkman and LeBaron 1989; Soofi and Cao 2002a; Soofi and Galka 2003; Das and Das 2007; Zhang et al. 2011; Soofi et al. 2012).

These advances have opened up possibilities of gaining further insights into the dynamics of financial/economic data. Even though from a theoretical point of view these methods are as applicable to economic data as they are to financial data, in practice one observes more frequent applications of these methods to financial data compared to economic data. The reason for this mismatch in applications is low frequency nature of most economic time series data (most economic time series observations are monthly, quarterly, or annual), which leads to limited observations. The algorithms of nonlinear dynamical systems require very large set of time series observations. The financial time series with adequate number of observations for use in nonlinear dynamical analysis could be obtained from the financial markets.

At the outset, we should point out that applicability of these methods and algorithms and the validity of the empirical results hinge on nonlinearity of time series observations. The name nonlinear deterministic dynamics, which is known chaos theory also, should make this requirement absolutely clear. Accordingly, tests for nonlinearity of the series under investigation assume a paramount importance in nonlinear data analyses, and are an absolute requirement before applying any of the above mentioned methods to the data. Nonlinearity is a necessary condition for nonlinear deterministic (chaotic) as well as nonlinear stochastic dynamics.

In this chapter, we will discuss the methods, and review the empirical results of the studies the authors of this chapter have undertaken over the last decade and half. Given the methodological and computational advances of the recent decades, the authors of this chapter have explored the possibility of detecting nonlinear, deterministic dynamics in the data generating processes of the financial time series that were examined. We have conjectured that the presence of nonlinear deterministic dynamics may have been blurred by strong noise in the time series, which could give the appearance of the randomness of the series. Accordingly, by using methods of nonlinear dynamics, we have aimed to tackle a set of lingering problems that the traditional linear, stochastic time series approaches to financial econometrics were unable to address successfully. We believe our methods have successfully addressed some, if not all, such lingering issues. We present our methods and empirical results of many of our studies in this chapter and leave the judgment of how successful we have been in resolving the lingering issues in the financial econometrics to reader.

Specifically, section "Defining Chaotic or Nonlinear Deterministic Dynamics" gives an overview of concepts and definitions of nonlinear dynamical systems. In section "Surrogate Data Analysis and Testing for Nonlinearity", we discuss surrogate data analysis as a test for nonlinearity. Section "Determining Time Delay and Embedding Dimension" reviews time-delay and embedding dimension methods that are used in phase space reconstruction of nonlinear dynamical systems from a single set of observations of the dynamics. In section "Nonlinear Prediction", we discuss the use of nonlinear deterministic method in predictions of the financial time series. Section "Discriminate Statistics for Hypothesis Testing in Surrogate Data Analysis" discusses discriminate statistics that are often used in surrogate data analysis and in tests for detection of chaotic systems. Section "Nonlinear Predictions of Financial Time Series: The Empirical Results" reviews the empirical results of nonlinear prediction of financial time series. In section "Noise Reduction and Increased Prediction Accuracy" the effect of noise reduction on prediction accuracy is examined. Section "Mutual Prediction as a Test for Integration of the Financial Markets" reviews method of mutual prediction as a test for integration of financial markets. Finally, section "Summary and Conclusion" concludes the chapter.

Defining Chaotic or Nonlinear Deterministic Dynamics

It is useful for our subsequent analyses to start with concise definitions of some of the terminologies of nonlinear dynamical systems theory. However, before giving formal definitions of these terms, we give a general description of nonlinear dynamical systems.

Economies (and financial markets), like population biology and statistical physics, consist of large numbers of agents (elements), which are organized into dynamic, volatile, complex, and adaptive systems. These systems are sensitive to the environmental constraints and evolve according to their internal structures that are generated by the relationships among the individual members of the systems. Of course, each of these disciplines has its own peculiarities, the knowledge of which necessitates development of expertise in the respective discipline. However, synthetic microanalytic approach to study the systems is their common characteristic. This implies that one could aim to understand the behavior of the system as a whole by relating the system's behavior to the conducts of its constituent parts on one hand, and by considering interactions among the parts on the other.

For example, in finance one might be interested in learning how trading by thousands of investors in the stock market determines the daily fluctuations in the stock indexes; or in physics, one might be interested to explain how interactions among countless number of atoms result in transformation of a liquid into solid.

Given the evolutionary nature of economic (financial) systems, dynamical systems theory is the method of choice in studying these complex, adaptive systems. A dynamical system is a system whose state evolves over time according to some dynamical laws. The evolution of the system is in accord with working of a *deterministic evolution operator*. The *evolution operator*, which can assume a differential or a difference equation form, a matrix form, or a graph form provides a correspondence between the initial state of the system and a unique state at each subsequent period. In real dynamical systems random events are present, however, in modeling these real systems the random events are neglected.

Let the state of the dynamical system be described by a set of d state variables, such that each state of the system corresponds to a point $\boldsymbol{\xi} \in \mathbf{M}$, where $\mathbf{M}$ is a compact, differentiable d-dimensional manifold. $\mathbf{M}$ is called the *true state space* and d is called the *true state space dimension*.

The states of dynamical systems change over time, hence the state is a function of time, i.e., $\boldsymbol{\xi}(t)$.

In continuous cases a curve or a trajectory depicts the evolutionary path of $\boldsymbol{\xi}(t)$. If the current state of system $\boldsymbol{\xi}(0)$, where one arbitrarily defines the current time $t = 0$, uniquely determines the future states $\boldsymbol{\xi}(t)$, $t > 0$, the system is a *deterministic dynamical system*. If such unique correspondence between the current state and the future states does not exist, the system is called a *stochastic dynamical system*. The completely uncorrelated states are called *white noise*.

In practice, it is not feasible to observe $\boldsymbol{\xi}(t)$, the true states of the dynamical systems. However, measurement of one or several components of the system might be possible. Therefore, using a measurement function $h : \boldsymbol{R}^d \rightarrow \boldsymbol{R}^{d'}$ on the true state $\boldsymbol{\xi}$, we measure a time series $x(t) = h(\boldsymbol{\xi}(t)) + \eta(t)$, where $\eta(t)$ is measurement error (noise) and $d' < d$.

The properties of the evolution operator define the characteristics of the system. A dynamical system is *linear* if its evolution operator is linear; otherwise the system is *nonlinear*.

We need to define *attractor* of a dynamical system before further discussions of the possible forms of behavior of the dynamical systems. To do so, we start with a formal re-statement of *deterministic dynamical systems*.

Start with a system in the initial state of $\boldsymbol{\xi}(0)$. If the system is deterministic, a unique function f^t maps the state at time 0 to state at time t: $\boldsymbol{\xi}(t) = f^t(\boldsymbol{\xi}(0))$. We assume the f^t to be differentiable function, which has a smooth inverse. Such a function is a *diffeomorphism*.

Depending on the structure of f^t, the behavior of $\boldsymbol{\xi}(t)$ for $t \rightarrow \infty$ (after the transient states) varies. In a dissipative dynamical system, where energy of the system is not conserved, all volumes in the state space shrink over time and evolve into a reduced set $\boldsymbol{A}$ called *attractor*. Accordingly, we define an attractor as a set of points in the state space which are invariant to flows of f^t. The transient state is the state in which the process of convergence of the neighboring trajectories to a set of points A of attractor is taking place.

Four types of attractors are observed, which are defined below.

- Fixed points
  The initial state converges into a single point. The time series of such system is given by $x(t) = x(0)$, implying a constant set of observations.
- Limit cycles
  The initial state converges to a set of states, which are visited periodically. The time series corresponding to limit cycles is defined by $x(t) = x(t + T)$, where T is the period of periodicity.
- Limit tori
  A limit torus is the limit cycle with more than one incommensurable frequency in the periodic trajectory.
- Strange attractors
  Strange attractors are characterized by the property of attracting initial states within a certain basin of attraction, while at the same time neighboring initial states on the attractor itself are propagated on the attractor in a way such that their distance will, initially, grow exponentially. When the distance approaches the size of the attractor, this growth will stop due to back-folding effects.
  The time series representing the dynamical systems with strange attractors appear to be stochastic, even though they are completely deterministic. These dynamical systems are called chaotic or nonlinear deterministic dynamics.

We defined nonlinear systems in the context of evolution operators above. However, an intuitive way to gain an understanding of the difference between linear and nonlinear systems is described below.

Perturb the system by x_1 and record its response y_1. Next perturb the system by x_2 and record its response y_2. Then perturb the system by $(x_1 + x_2)$ and record its response y_3. Finally compare $(y_1 + y_2)$ and y_3. If they are equal for any x_1 and x_2 then the system is linear. Otherwise it is nonlinear (Balanov et al. 2009).

Many models depicting chaotic behavior have been developed. Among these models we name the most widely used ones such as the Lorenz attractor (Lorenz 1963), Henon map (Henon 1976), tent map (Devany 1989), and logistic map (May 1976).

Surrogate Data Analysis and Testing for Nonlinearity

As stated above, an extensive literature dealing with different methods for testing for nonlinearity in time series observations has evolved over the last two decades. These methods were used in a number of studies that point to possible nonlinearity in certain financial and economic time series[1] (e.g. Scheinkman and LeBaron 1989; Hsieh 1991; Yang and Brorsen 1993; Kohzadi and Boyd 1995; Soofi and Galka 2003; Zhang et al. 2011; Soofi et al. 2012).

The dynamics of short, noisy financial and economic time series could be the outcome of working of nonlinear determinism in its varieties (periodic, limit tori, and chaotic), stochastic linearity and nonlinearity, and random noise emerging from either or both the dynamics itself and from measurement. Accordingly, in applications of methods and algorithms of nonlinear dynamical systems the first task is to delineate and disentangle all these influences on the observed data set. Given the daunting task of accounting for above listed influences, in practice most analysts focus on determining the role nonlinearity plays in the observed series.

One of the most popular methods of testing for nonlinearity of time series is the surrogate data technique (Theiler et al. 1992). In the surrogate data method of testing for nonlinearity of the series one postulates the null hypothesis that the data are linearly correlated in the temporal domain, but are random otherwise. Among the most popular test statistics for hypothesis testing we mention correlation dimension and some measures of prediction accuracy. We have used both correlation dimension as well as root mean square errors as test statistics for hypothesis testing within the framework of surrogate data analysis on a number of exchange rates and stock market time series studies. We will discuss these quantities below in section "Discriminate Statistics for Hypothesis Testing in Surrogate Data Analysis" after introduction of the method of phase space reconstruction by time-delay embedding.

Presence of noise in the data and insufficient number of observations may point to nonlinearity of a stochastic time series even though the series might be linear (see for example, Osborne and Provencale 1989). To exclude the possibility of receiving such

[1] As it will become clear in the discussion of surrogate analysis below, *nonlinearity* is not a property of a series; it is the absence of the property of linearity that is often detected. However, it is more straightforward, even though less accurate, to speak of presence of nonlinearity in a series throughout this chapter.

misleading signals, surrogate data analysis is often used for testing for nonlinearity of a series. One of the methods used in surrogate data analysis generates a number of surrogates for the original series by preserving all the linear correlations within the original data while destroying any nonlinear structure by randomizing the phases of the Fourier transform of the data. Alternatively one might wish to describe the linear correlations within the original data by generating the linear surrogates from an autoregressive model of order p model, $\mathrm{AR}(p)$, and then using the surrogates for estimation of the autocorrelation function (see Galka 2000).

In many practical cases of data analysis, one is faced with a single set of short, noisy, and often non-stationary observations. In such cases, the application of the nonlinear dynamical methods leads to point estimates leaving the analyst without measures of statistical certainty regarding the estimated statistics. One approach to overcome this problem is artificial generation of many time series which by design contain the relevant properties of the original time series, which are obtained through the estimated statistics.

The strategy in surrogate data analysis is to take a contrarian view. The analyst should choose a null hypothesis that contradicts his/her intuition about the nature of the time series under investigation. For example, if one is testing for presence of nonlinear deterministic dynamics in the series, one should select a model that directly contradicts these properties and use a linear, stochastic model to generate the surrogate data, which are different realizations of the hypothesized linear model. Using the surrogate, the quantity of interest, for example, correlation dimension as a discriminating statistics, is estimated for each realization. The next step in this strategy is formation of a distribution using the estimates of the discriminating statistics from the surrogates. The resulting distribution is then used in a statistical test, which might show that the observed data are highly unlikely to have been generated by a linear process.

By estimating the test statistics for both the original series and the surrogates, the null hypothesis that the original time series was linear is tested. If the null is true, then procedure for generating the surrogates will not affect measures of *suspected* nonlinearity. However, if the measure of nonlinearity is significantly changed by the procedure, then the null of linearity of the original series is rejected.

An alternative approach in determining the unknown probability distribution of measures of nonlinearity is the parametric bootstrap method (Efron 1982), which aims to extract explicit parametric models from the data. The validity of this approach hinges on successful extraction of the models from the data. The main shortcoming of parametric bootstrap methods is that one cannot be sure about the true processes underlying the data. The surrogate data method, which can been characterized as a *constrained realization* method, overcomes the weakness of parametric bootstrap method, which can be characterized as a *typical realization* method, by directly imposing the desired structure onto the randomized time series.

To avoid spurious results it is essential that the correct structure (according to the null hypothesis) is imposed on the original series. One approach in ensuring validity of statistical test is determining the most likely linear model that might have generated the data, fitting the model, and then testing for the null hypothesis that the data have been generated by the specified model (Screiber 1999, pp. 42–43).

The number of surrogates to be generated depends on the rate of false rejections of the null hypothesis one is willing to accept (i.e., on the *size* of the test). In most practical applications generating 35 surrogate data series should suffice. A set of values of the discriminating statistics $q^1, q^2, \ldots q^{35}$, is then computed from the surrogates.

Rejection of the null hypothesis may be based either on rank ordering or significance testing. Rank ordering involves deciding whether q^0 of the original series appears as the first or last item in the sorted list of all values of the discriminating statistics $q^0, q^1, q^2, \ldots q^{35}$.

If the qs are fairly normally distributed we may use significance testing. Under this method rejection of the null requires a t value of about 2, at the 95 % confidence level, where t is defined as:

$$t = \frac{|q^0 - \langle q \rangle|}{\sigma_q} \tag{1.1}$$

where $\langle q \rangle$ and σ_q are the mean and standard deviation, respectively, of the series $q^1, q^2, \ldots q^{35}$ (for an in-depth discussion of surrogate data analysis see Kugiumtzis 2002 and Theiler et al. 1992).

Note that a software for generating phase-randomization surrogate data, *fftsurr* (fast Fourier transform surrogates) has been made available by Kaplan (2004); it is written in MATLAB. Phase-randomized surrogate data generated by *fftsurr* have the same spectral density function as the original time series. A further improvement of phase-randomization surrogates can be achieved by creating *improved amplitude-adjusted phase-randomization (IAAPR)* surrogates (sometimes also known as *polished surrogates*). These surrogates have a distribution of amplitudes which is identical to that of the original data, in addition to the preservation of the spectral density function. This is achieved by reordering the original series in a way such that the power spectrum of the surrogates and the original series are (almost) identical.

For data with non-Gaussian distribution, phase-randomized surrogates without amplitude adjustment may result in spurious rejection of the null hypothesis. This result is due to difference between the distributions of the surrogates and the original series. To remedy this problem one should distort the original data so that it is transformed to a series with Gaussian distribution. Then from the distorted original series, now a Gaussian series, a set of surrogates is created by phase-randomization. Finally, the surrogates are transformed back to the same non-Gaussian distribution as the original data (for further details see Galka 2000, Chap. 11).

Soofi and Galka (2003) employed the algorithm of Schreiber and Schmitz (1996) for the generation of IAAPR surrogates in the context of the estimation of the correlation dimension of the dollar/pound and dollar/yen exchange rates. They found evidence of presence of nonlinear structure in the dollar/pound rate, however, no such evidence was found for dollar/yen exchange rate.

Zhang et al. (2011) using the IAAPR algorithm generated 30 surrogate series for 4 daily dollar exchange rates data including Japanese yen, Malaysian ringgit, Thai baht, and British pound for testing for presence of nonlinear structure in the exchange rate series. They found evidence of nonlinear structure in dollar/pound rate. However, it was observed that all the exchange rate series go through periods of linearity and nonlinearity intermittently, a characteristic that was not observed for the simulated data generated from the chaotic Lorenz system.

Testing for nonlinearity of the Chinese stock markets data (Soofi et al. 2012) used algorithms that generate phase-randomization surrogates and amplitude-adjusted surrogates (Kaplan 2004), and found evidence of nonlinearity in all three stock market indices in China: Hong Kong stock Index (HSI), Shanghai Stock Index (SSI), and Shenzhen Stock Index (SZI).

Determining Time Delay and Embedding Dimension

Advances in mathematical theory of time-delay embedding by Takens (1981) and later by Sauer et al. (1991) allow understanding of the dynamics of the nonlinear system through observed time series. These algorithms have had a large number of applications in detecting nonlinear determinism from observed time series, e.g., economic and financial time series (Soofi and Galka 2003; Soofi and Cao 2002a; Cao and Soofi 1999; Bajo-Rubio et al. 1992; Larsen and Lam 1992) .

Given the significance of methods of time-delay embedding and phase space reconstruction in nonlinear dynamical time series analyses, we will discuss these techniques in detail below.

Choosing Optimal Model Dimension

Before a discussion of method of determining the optimal embedding dimension, let us define the *dimension* of a set of points. Geometrically speaking a point has no dimension, a line or a smooth curve has a single dimension, planes and smooth surfaces have two dimensions, and solids are three-dimensional. However, a concise, institutive definition is given by Strogatz (1994, p. 404) who stated that "...the dimension is the minimum number of coordinates needed to describe every point in the set."

Given a scalar time series, $x_1, x_2, \ldots, x_N$, one can make a time-delay reconstruction of the phase-space with the reconstructed vectors:

$$\mathbf{V}_n = (x_n, x_{n-\tau}, \ldots, x_{n-(d-1)\tau}), \tag{1.2}$$

where τ is time-delay, d is embedding dimension, and $n = (d-1)\tau + 1, \ldots, N$.

d represents the dimension of the state space in which to view the dynamics of the underlying system. The time-delay (time lag), τ, represents the time interval between the successively sampled observations used in constructing the d-dimensional embedding vectors.[2]

According to the embedding theorems (Takens 1981; Sauer et al. 1991) if the time series is generated by a deterministic system, then there generically exists a function (a map) $\mathbf{F}: R^d \mapsto R^d$ such that

$$\mathbf{V}_{n+1} = \mathbf{F}(\mathbf{V}_n), \tag{1.3}$$

if the observation function of the time series is smooth, has a differentiable inverse, and d is sufficiently large. The mapping has the same dynamic behavior as that of the original unknown system in the sense of topological equivalence.

In practical applications, we usually use a scalar mapping rather than the mapping in (1.3), that is,

$$x_{n+1} = f(\mathbf{V}_n), \tag{1.4}$$

which is equivalent to (1.3).

In reconstructing the phase space, the remaining problem is how to select the τ and d, i.e., time-delay and embedding dimension, in a way that guarantees existence of the above mapping. But in practice, because we have only a finite number of observations with finite measurement precision, a good choice of τ is important in phase space reconstructions. Moreover, determining a good embedding dimension d depends on a judicious choice of τ. The importance of choosing a good time-delay is that it could make minimal embedding dimension possible. This implies that optimal determination of embedding-dimension and time-delay are mutually interdependent.

There are several methods to choose a time delay τ from a scalar time series, such as mutual information (Fraser and Swinney 1986) and autocorrelation function methods.

The more interesting issue is the choice of the embedding dimension from a time series. Generally there are three basic methods used in the literature, which include *computing some invariant (e.g., correlation dimension, Lyapunov exponents) on the attractor* (e.g., Grassberger and Procaccia 1983), *singular value decomposition* (Broomhead and King 1986; Vautard and Ghil 1989), and *the method of false neighbors* (Kennel et al. 1992). However, all these methods contain some subjective parameters or need subjective judgment to choose the embedding dimension.

Dealing with the problem of subjective choice of embedding dimension Cao (1997) modified the method of false neighbors and developed a method of the *averaged false neighbors*, which does not contain any subjective parameter provided the time-delay has been chosen. A more general method based on zero-order approximations has been developed by Cao and Mees (1998), which can be used to determine the embedding dimension from any dimensional time series including scalar and multivariate time series.

[2] For details, see an excellent introductory book by Hilborn (1994).

For an unfolding of a time series into a representative state space of a dynamical system, optimal embedding dimension d and time delay τ are required. The methods of computing embedding dimension and time delay, however, presuppose prior knowledge of one parameter before estimation of the other. Accordingly, calculating one parameter requires exogenous determination of the other.

Soofi et al. (2012) adopted the method of simultaneous estimation of embedding dimensions and time delays.[3] They selected that combination of the embedding dimension and time delay in generation of the dynamics that would lead to the minimum prediction error using nonlinear prediction method.

Specifically, let $\zeta_i = f(d_j, \tau_k, \eta_i)$, $[i = 1, \ldots, N; j = k = 1, \ldots, M]$, where ζ_i, d_j, τ_k, and η_i are the ith prediction error, the jth embedding dimension, the kth time delay, and the ith nearest neighbors, respectively. Then one would search for that combination of d_j, τ_k, and η_i that minimizes ζ_i.

Below we briefly describe the Cao method. Note that the method takes τ as given, however, the method estimates an embedding dimension that minimizes the prediction error.

For a given dimension d, we can get a series of delay vectors $\mathbf{V}_n$ defined in (1.2). For each $\mathbf{V}_n$ we find its nearest neighbor $\mathbf{V}_{\eta(n)}$, i.e.,

$$\mathbf{V}_{\eta(n)} = \text{argmin}\{||\mathbf{V}_n - \mathbf{V}_j|| : \quad j = (d-1)\tau + 1, \ldots, N, j \neq n\} \tag{1.5}$$

Note, $\eta(n)$ is an integer such that

$$||\mathbf{V}_{\eta(n)} - \mathbf{V}_n|| = \min\{||\mathbf{V}_n - \mathbf{V}_j|| : \quad j = (d-1)\tau + 1, \ldots, N, j \neq n\}$$

where the norm

$$\begin{aligned}||\mathbf{V}_n - \mathbf{V}_j|| &= ||(x_n, x_{n-\tau}, \ldots, x_{n-(d-1)\tau}) - (x_j, x_{j-\tau}, \ldots, x_{j-(d-1)\tau})|| \\ &= [\sum_{i=0}^{d-1}(x_{n-i\tau} - x_{j-i\tau})^2]^{1/2}.\end{aligned}$$

Then we define:

$$E(d) = \frac{1}{N - J_0}\sum_{n=J_0}^{N-1} |x_{n+1} - x_{\eta(n)+1}|, \; J_0 = (d-1)\tau + 1. \tag{1.6}$$

where $E(d)$ is the average absolute prediction error of a *zero-order approximation* predictor for a given d. Note that a zero order predictor f is $\hat{x}_{n+1} = f(\mathbf{V}_n)$ and $\hat{x}_{n+1} = x_{\eta(n)+1}$, where $\eta(n)$ is an integer such that $\mathbf{V}_{\eta(n)}$ is the nearest neighbor of $\mathbf{V}_n$. Furthermore, note that the N in (1.6) represents only the number of available

[3] The method was suggested by Liangyue Cao.

data points for fitting, which does not include the data points for out-of-sample forecasting.

To choose the embedding dimension d_e, we simply minimize the E, i.e.,

$$d_e = \text{argmin}\{E(d) : d \in \text{Z} \text{ and } d \geq 1\}. \tag{1.7}$$

The embedding dimension d_e we choose gives the minimum prediction error if we use a zero-order approximation predictor. It is reasonable to infer that this d_e will also give good predictions if we use a high-order (e.g., local-linear) approximation predictor, since a high-order predictor is more efficient than a zero-order predictor when making out-of-sample predictions.

In practical computations, it is certainly impossible to minimize the E over all positive integers. So in real calculations we replace (1.7) with

$$d_e = \text{argmin}\{E(d) : 1 \leq d \leq D_{\text{max}}\}, \tag{1.8}$$

where D_{max} is the maximum dimension with which one would like to search the minimum value of $E(d)$.

In summary, the above method is to find the embedding dimension by minimizing the 1-step prediction errors using a zero-order approximation predictive model. For details about this method, see Cao et al. (1998a).

Nonlinear Prediction

Reconstruction of phase space from a scalar time series allows prediction of the series. The reconstructed phase space allows approximation of a function representing the dynamics that could be used for prediction. Below we discuss the local-linear prediction method as one of the methods used in function approximation.

Local-Linear Prediction

Having solved the problem of choosing embedding dimension and time-delay for the vectors $\mathbf{V}_n$ defined in (1.2) we now use model (1.4) for prediction.

The next problem is how to approximate function f. Several approximation techniques, such as local-linear approximation, polynomial approximation, neural networks, radial basis function, and wavelet decomposition are available. One of the more straight forward method is local-linear approximation, because it requires a lower computational time.

Suppose we have N_f samples of time series data available for fitting the function, i.e., we have $x_1, x_2, \ldots, x_{N_f}$.

Therefore we have time-delay vectors $\mathbf{V}_n,\ n = J_0, J_0 + 1, \ldots, N_f$

and $J_0 = (d-1)\tau + 1$. We want to predict x_{N_f+1}.

Steps in the local-linear approximation method are listed below:

1. Impose a metric on the delay-vector space, denoted by $|| \ ||$. An example is the root-square norm, i.e, $||\mathbf{a}|| = ||(a_1, a_2, \ldots, a_d)|| = (\sum_{i=1}^{d} a_i^2)^{1/2}$.
2. Find the l nearest neighbors of $\mathbf{V}_{N_f}$, denote them by $\mathbf{V}_{j_1}, \mathbf{V}_{j_2}, \ldots, \mathbf{V}_{j_l}$, $J_0 \leq j_k < N_f$, $(k = 1, 2, \ldots, l)$, then for any $k = 1, 2, \ldots, l$, $||\mathbf{V}_{j_k} - \mathbf{V}_{N_f}|| \leq ||\mathbf{V}_n - \mathbf{V}_{N_f}||$ ($J_0 \leq n < N_f$ and $n \neq j_k$ for any $k = 1, \ldots, l$).
3. Construct a local-linear predictor, regarding each neighbor $\mathbf{V}_{j_k}$ as a point in the domain and x_{j_k+1} as the corresponding point in the range. That is, fitting a linear function to the l pairs $(\mathbf{V}_{j_k},\ x_{j_k+1})$ $(k = 1, 2, \ldots, l)$.

   We use the least-squares method to fit this linear function. Denote it by $\hat{F}$, then we have $\sum_k |x_{j_k+1} - \hat{F}(\mathbf{V}_{j_k})|$ minimized.
4. The predicted value of x_{N_f+1} is $\hat{F}(\mathbf{V}_{N_f})$, i.e.,

$$\hat{x}_{N_f+1} = \hat{F}(\mathbf{V}_{N_f}).$$

Discriminate Statistics for Hypothesis Testing in Surrogate Data Analysis

In this section we will discuss two quantities that we have used in various empirical studies as a discriminating statistics in hypothesis testing for nonlinearity of the financial data that were under consideration.

Correlation Sum and Correlation Dimension

One could select from a set of measures as the test statistics in surrogate data analysis as the first step in determining the behavior of the time series. One of the more popular discriminating statistics in nonlinear dynamical system analysis is correlation dimension. Moreover, in addition to being used as a discriminating statistics in hypothesis testing for presence of nonlinearity in the data, correlation dimension may point to the chaotic nature of the nonlinear dynamical system. This is due to the observation that stochastic processes always use all available dimensions of the state space, while deterministic processes may evolve on a manifold of much lower dimension. This results in the observation that the fractal dimensions are substantially smaller than d-degree of freedom of the dynamical system leading to the evidence of determinism. Below, we give a formal definition of correlation dimension.

Starting with a scalar time series, $x_1, x_2, \ldots, x_N$, which might describe the states of a system or may be the result of a time delay embedding of a univariate time series described by

$$\boldsymbol{x}_i = (x_1, x_{i-\tau}, x_{i-2\tau}, \ldots, x_{i-(d-1)\tau}), \tag{1.9}$$

where τ and d are the time delay and the embedding dimension, respectively.

From these vectors the correlation sum[4] is defined by:

$$C(r) = \binom{n}{2}^{-1} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} I(r - \|\mathbf{x}_i - \mathbf{x}_j\|), \tag{1.10}$$

where $I(.)$ is an indicator function, such that $I(x) = 1$ for positive x and $I(x) = 0$ otherwise. $\|.\|$ denotes maximum norm, though other norms could also be employed. $C(r)$ estimates the probability of finding two vectors in the set which are separated by a distance not larger than a radius r (in d-dimensional state space).

To avoid spurious results due to unwanted dynamic correlations in the set of vectors $\mathbf{x}_i$ it is advisable to omit all those distances $\|\mathbf{x}_i - \mathbf{x}_j\|$ from the correlation sum for which $\mathbf{x}_i$ and $\mathbf{x}_j$ are too close together in time, i. e. for which $i - j < W$ with a fixed integer parameter W (Theiler 1986). The absence of this correction corresponds to $W = 1$. The choice of W is not critical, provided a sufficiently large value is chosen.

For sufficiently small radius r the correlation sum is expected to display a scaling

$$C(r) = a\, r^{d_c} \tag{1.11}$$

a is a constant. Hence the correlation dimension d_c can be obtained by

$$d_c = \lim_{r \to 0} d_c(r) = \lim_{r \to 0} \frac{\partial \log C(r)}{\partial \log r}. \tag{1.12}$$

The derivative is carried out numerically and yields a dimension estimate $d_c(r, m)$, which still depends on radius r and embedding dimension m.

An Information Theoretic Approach in Estimating a Test Statistics

A number of existing methods for direct testing of nonlinearity such as highly popular residual-based methods, and bispectrum (Hinich 1982) exists. However, none of these methods provide an efficient test statistics that is based on a discrete parametric model. The discrete parametric modeling or information theoretic method of testing for nonlinearity provides such an efficient test statistics (Galka and Ozaki 2001).

Given a time series x_i, $i = 1, \ldots, N$, with zero mean and unit variance (this can be realized by simple linear transformation), we can get an autoregressive model

$$x_i = f(x_{i-1}, \ldots, x_{i-p}) + \eta_i, \tag{1.13}$$

[4] We use the term *correlation sum* because we are dealing with discrete time series. In cases that deal with continuous time series, the term *correlation integral* is used.

where p is the model order and η_i is the dynamical noise. Take $f(\cdot)$ to be a linear function, we get an AR(p) model

$$x_i = \sum_{j=1}^{p} a_j x_{i-j} + \eta_i =: \hat{x}_i + \epsilon_i, \tag{1.14}$$

where $\hat{x}_i$ is the prediction value or conditional mean of x_i.

An exponential autoregressive (ExpAR) model is defined as follows:

$$x_i = \sum_{j=1}^{q} (a_j + b_j \exp(\frac{-x_{i-1}^2}{h})) x_{i-j} + \eta_i =: \hat{x}_i + \eta_i, \tag{1.15}$$

where the bandwidth h for each time series can be estimated by

$$h = -\frac{\max x_{i-1}^2}{\log c}, \tag{1.16}$$

and c is a small number selected in advance. The choice is based on the idea of selection of a bandwidth, h, such that the exponential term becomes essentially zero for large amplitude. Since the exponential function is always positive it never becomes exactly zero. Therefore, we must assign a very small positive number for the exponential term and we call this small constant c. If we choose $\log(c) = -30$, this corresponds to $c = 9.3576e - 014$. This is a number close to the machine precision of computers.

Based on the two models represented in Eqs. (1.2) and (1.3), the test statistic is constructed as follows:

$$\delta(p) := \frac{1}{N}(AIC(\text{AR}(p)) - AIC(\text{ExpAR}(q))) \tag{1.17}$$

where AIC is the Akaike Information Criterion (Akaike 1974) and is defined as

$$AIC = N \log[\frac{1}{N-p} \sum_{i=p+1}^{N} (x_i - \hat{x}_i)^2] + 2(P+1) \tag{1.18}$$

where p is model order, and P denotes the number of the parameters a_i and b_j in the model.

Model (1.17) uses AIC as a measure of the quality of fitted model. Note that the smaller the value of AIC, the better the selected model for the data that is being modeled. Accordingly, $\delta > 0$ implies that the nonlinear *ExpAR(p)* model is a better model compared to the linear *AR(p)* in fitting the data. For $\delta < 0$, the models reverse role.

The Empirical Results Based on the Information Theoretic Model Using the Exchange Rates

Zhang et al. (2011) tested for nonlinearity of the daily dollar exchange rates time series. They used the discrete parametric modeling approach (Galka and Ozaki 2001) to compute an efficient test statistic for nonlinearity of daily dollar exchange rates for 3 Asian currencies and British pound series.

To explore whether the underlying dynamics of Asian financial systems went through changes during the Asian Crisis of 1997–1998, they examined the nonlinear properties of currencies of Thailand and Malaysia before, during, and after Asian financial crises, and obtain highly interesting results. They performed the same analysis using yen and pound rates also. They used yen as the currency of an Asian industrialized country that was immune to the Asian Contagion. They used the pound rate because of observed nonlinear structure in the series by other researchers (Soofi and Galka 2003) as a non-Asian currency for the purpose of a control time series in the study.

According to the results of nonlinearity test, Thai baht shows a nonlinear structure for pre-crisis period. However, the nonlinearity is totally absent for crisis and post crisis periods. For Malaysian ringgit, they observe a mild nonlinearity which corresponds to a period of time close to the early July of 1997, when the monetary authorities in Thailand abandoned the pegging of baht to the dollar this may imply cross-country contagion effect. Again, the data support no nonlinearity of the currency during and after the crisis periods. For Japanese yen, they find no evidence of nonlinearity for any period under study here. Finally, for British pound, they observe a very mild nonlinearity in pre-crisis period, observe no evidence of non-linearity during the crisis period, and detect evidence of a very weak nonlinearity immediately after the first post-crisis period. For the remaining post-crisis periods no evidence of nonlinearity is present. Based on these observations one may conclude that a period of high nonlinearity of the exchange rate may be a prelude to a major financial crisis. Constant monitoring of the behavior of an exchange rate using the present method may be a highly effective early warning system for financial crisis and collapse of currency value.

Nonlinear Predictions of Financial Time Series: The Empirical Results

Soofi and Cao in several works (1999, 2002a) used the nonlinear prediction (local linear approximation) method discussed in section "Nonlinear Prediction" for out-of-sample forecasting of several foreign exchange rates. In all of these prediction exercises the nonlinear prediction method out-performed the competing predictors.

Specifically Cao and Soofi (1999) predicted five daily dollar exchange rates time series: Canadian dollar (Ca$), British pound, German mark, Japanese yen, and French franc, from October 1, 1993 to October 3, 1997. In that study they found evidence that the exchange rate data tested have some deterministic dynamics. In fact, from the theoretical patterns of embedding dimensions for different systems showed that it is very unlikely that the above exchange rate return data are generated by purely random processes. They may be generated by high dimensional systems contaminated by (measurement) noise or nonlinear deterministic systems with stochastic driving forces, i.e., dynamic noise and measurement noise.

Furthermore they tested out-of-sample prediction of the above five exchange rate return time series using the local linear method. They evaluated the prediction by local-linear method with mean value predictor, and calculated the root-mean-square error. The results showed their predictions outperform the mean value predictor for the pound/dollar and the yen/dollar rate returns, but not for the three remaining exchange rate returns.

Soofi and Cao (2002a) used the same prediction method in prediction of monthly black market renminbi/dollar (Feb. 1955–June-1989), monthly black market rial/dollar (Jan. 1957–May 1988), and daily fixed renminbi/dollar (4 Jan. 1993–29 Dec. 2000) exchange rates. They found that in all cases the nonlinear prediction method out-performed the benchmark mean predictor.

Finally, Soofi and Cao (1999) performed out-of-sample predictions on daily peseta/dollar spot exchange rates using a simple nonlinear deterministic technique of local linear predictor. They compared the predictions based on local-linear method with those by two simple benchmark predictors: random walk model and mean-value predictor. The results on the differenced time series indicate that their predictions are better than those by the random walk model, and marginally better than the results from the mean-value predictor.

Noise Reduction and Increased Prediction Accuracy

It is well known that noise can seriously limit the performance of prediction techniques on time series. Effective methods are currently still lacking on noisy time series forecasting. The main difficulty is the absence of prior knowledge on what is noise and what is determinism in real time series, especially when the noise takes part in dynamical evolution of the systems, that is, so-called dynamic noise.

There are obviously two possible approaches to predict noisy time series. One is, ignoring the presence of noise, to fit a predictive model directly from noisy data with the faith on possibility to extract the underlying deterministic dynamics from the noisy data. It seems that the technique of neural networks is helpful in doing such kind of fitting (e.g., Albano et al. (1992)). The other is, filtering the noise beforehand, to fit a predictive model from the filtered or noise-reduced data of the noisy time series with the hope that the noise level in the noisy time series has been reduced. We may need to mention that the latter approach should be more effective than the former one at least in the case of short-term predictions (e.g., see Cao et al. (1998b)).

Suppose a noisy time series $\{x_n\}$ is generated in the following way:

$$\begin{aligned} x_n &= h(\mathbf{y}_n) + \eta_n, \\ \mathbf{y}_n &= \mathbf{f}(\mathbf{y}_{n-1}), \end{aligned} \tag{1.19}$$

where h is a measurement function (observable); η_n is additive noise; $\mathbf{y}_n$ and the iterative equation(s) defined by the function $\mathbf{f}$ are the unknown underlying dynamic variable(s) and dynamic equation(s), respectively.

In the former approach, one should fit a predictive model,

$$x_{n+1} = G(x_n, x_{n-\tau}, \ldots, x_{n-(d-1)\tau}) \tag{1.20}$$

based on the observed time series data using some techniques such as neural networks, where d and τ are the so-called embedding dimension and time-delay, respectively.

Obviously the function G in (1.20) changes the additive noise contained in $x_n, x_{n-\tau}, \ldots, x_{n-(d-1)\tau}$ to dynamic noise. In this sense the predictions must be inaccurate if the noise level is relatively high, as the dynamic noise destroys the determinism of the future dynamic behavior completely.

In the latter approach, on the other hand, one should first obtain the noise-reduced data from the observed noisy time series. Assume the noise-reduced time series having been obtained by some noise reduction method, e.g., local projective and singular value decomposition methods (Grassberger et al. 1993), and denote it by $z_1, z_2, \ldots, z_n, \ldots$. So,

$$x_n = z_n + \varphi_n, \quad n = 1, 2, \ldots, \tag{1.21}$$

where the term φ_n is the noise which was removed by the noise reduction method. The ideal result of noise reduction is $z_n = h(\mathbf{y}_n)$ or $\eta_n = \varphi_n$ for each n, see (1.19) for how the $\{x_n\}$ was generated.

Then a predictive model is fitted based on the noise-reduced time series, that is,

$$z_{n+1} = H(z_n, z_{n-\tau}, \ldots, z_{n-(d-1)\tau}). \tag{1.22}$$

Using this predictive model, the future z_{n+1} can be predicted, and *the value predicted can then be regarded as the predicted value of the future x_{n+1}, i.e, the actual data to be observed*. In fact, the predicted value $\hat{z}_{n+1}$ at the time $n+1$ should be the optimal predicted value of x_{n+1} because the noise term φ_{n+1} can never be predicted, see (1.21). If the noise has been significantly reduced in the noisy time series, then the latter approach is expected to give much better predictions than the former one.

Given that most financial time series contain noise: measurement noise, dynamic noise or both of them together, prediction of financial time series is certainly very challenging. It has attracted much attention on development of methods to improve the predictions. Besides traditional linear methods such as autoregression method, some nonlinear methods have also been applied to forecast financial time series (e.g., Cao et al. 1996; Lisi and Medio 1997; Cao and Soofi 1999). These studies are based on Takens' embedding theorem (Takens 1981). In these applications of nonlinear methods or linear methods, however, the predictive models were generally fitted directly from the original noisy data, i.e., the first approach on prediction of noisy time series mentioned above, see the Eq. (1.20). Not much work has been done using the second approach (see the Eq. (1.22) in prediction of financial time series, although it is expected that the second approach should provide better prediction than the first approach in forecasting of noisy time series as explained earlier.

In Soofi and Cao (2002a) both approaches are applied and compared on predicting two real financial time series- daily mark/dollar exchange rate and monthly U.S. Consumer Price Index(CPI), to see how the noise reduction could improve the predictions.

Nonlinear Noise Reduction

Power spectrum is traditionally used in separating noise with a flat or broad band spectrum from the periodic or quasi-periodic signals with sharp spectral lines. This method, however, has been shown inapplicable in dealing with noise in nonlinear time series, particularly chaotic time series, because the method is unable to differentiate between broad-band spectra from signals of chaotic systems and from signals of purely random noise (Grassberger et al. 1993). Therefore, some newly nonlinear noise reduction methods should be used when dealing with noisy nonlinear time series or noisy chaotic time series; for a review of nonlinear noise reduction methods, see e.g., Kantz and Schreiber (1997) and Ott et al. (1994).

The methods of local projective (LP), singular value decomposition (SVD) (Grassberger et al. 1993), and 'simple' nonlinear noise reduction (SNL) (Schreiber 1993) were adopted by Soofi and Cao (Soofi and Cao 2002a) to reduce the noise in the time series tested in the study.

The LP method rests on the hypothesis that the deterministic part of a noisy time series lies on a low-dimensional manifold in a high-dimensional state space reconstructed by the time-delay embedding, while the effect of noise is to distribute the data in the immediate surroundings of the manifold. The method is designed to identify the low-dimensional manifold and project the time series data onto it. Interested readers are referred to Schreiber (1998) for a detailed description of the method and relevant discussions.

Applying SVD to a time series tends to optimize the signal to noise ratio. In filtering data with SVD, the singular vectors of the covariance matrix of the time series are first computed; then the reconstructed m dimensional vectors are projected to a q dimensional space, where q $(< m)$ is the number of singular values computed (see Grassberger et al. (1993) for details).

The idea of the 'simple' nonlinear noise reduction method is to locally approximate the dynamics of the underlying system. Unlike the LP and the SVD methods, this method does not require to project the system to a lower dimensional system.

Mark-Dollar Exchange Rates

Soofi and Cao (2002a) used daily mark-dollar exchange rate time series, for sample observations for the period from October 1, 1993 to October 3, 1997.

Non-filtered data

Prediction test is first conducted on the time series without filtering. That is, no noise reduction is made on the differenced-log time series of mark/dollar exchange rates. This test was done in our earlier work (Cao and Soofi 1999). The RMSE between the out-of-sample predicted and the actual data was 1.08.

The finding that RMSE in the prediction was greater than 1 implies that the prediction by the local linear method is not better than the prediction by a mean value predictor. This negative result was actually expected because the behavior of exchange rates is so complicated that any deterministic predictions may not lead to better performance than the prediction by a simple mean value predictor. High level of noise in the exchange rate time series is also commonly regarded as one of the reasons for the failure of nonlinear deterministic prediction.

Filtered data

Given that one noise reduction method may work well in some cases, while it may not in the others , three sets of filtered data were generated using the simple nonlinear noise reduction (SNL), local projective (LP), and the SVD methods. The last two methods require a prior projection dimension (q). This q is generally not known for real time series (interested readers may consult the literature in noise reduction, e.g., Grassberger et al. (1993), for the selection of q).

The RMSE for the case with LP method for mark/dollar exchange rate was less than 1, which implies that the prediction by the local linear method was better than that by a mean value predictor. This means that noise reduction improves prediction of the exchange rate time series provided that an appropriate noise reduction method as well as suitable parameter values for the method is used. At this stage, the improvement is not statistically significant based on the statistic provided by Harvey et al. (1997) at a 10 % nominal level; however, the improvement is statistically significant at a 20 % nominal level.

U.S. Consumer Price Index

Monthly US consumer price index (CPI) time series was also used by Soofi and Cao (2002b) for out-of-sample prediction exercises. The reason they chose the CPI time series was that it is believed deterministic dynamics should be stronger in the CPI time series than that in the exchange rate time series. Therefore, nonlinear deterministic techniques should have a better chance to provide good prediction on the CPI time series than on the exchange rate time series.

Following the same procedures as for the exchange rate time series, the results for the CPI time series showed that the RMSE ($=0.87$) for the non-filtered data was less than 1, which means that the local linear deterministic prediction is better than the mean value prediction. Comparing with the corresponding results of the exchange rate time series, the much smaller RMSE for the CPI time series indicates that the deterministic dynamics in the CPI time series should be stronger than that in the exchange rate time series as we mentioned earlier.

For all other cases, the predictions with noise reduction are even worse than the prediction without noise reduction. This means that noise reduction may have distorted the deterministic dynamics in the CPI time series, therefore, the prediction on the filtered data becomes even more difficult. This could be often the case given it is not known what is the noise and what is the determinism in a real time series. However, this should not be taken as discouragement to use noise reduction in prediction of real time series. It implies that one should carefully select which noise reduction method as well as its related parameter values should be used for a particular time series, because a noise reduction method may work better in some cases, while it may not in other cases.

Mutual Prediction as a Test for integration of the Financial Markets

Another application of the methods of nonlinear dynamics is in testing for integration of economies and financial markets. There exists a vast literature on the subject of financial integration, which uses terms such as integration, globalization, and interdependence interchangeably. However, none of these terms is given a concise, quantitative definition. Soofi et al. (2012), however, using methods from science of nonlinear dynamical systems provided an exact quantitative definition of financial integration and treated terms such as financial integration and interdependence of financial markets synonymously.

The basis for the quantitative definition is the notion that interdependence of two or more financial markets implies that the observed time series of these systems originate from the *different parts* of the *same* dynamical system. The rational for this argument is that the equity markets are the subsystems of the global economic or financial system. Specifically, presence of dynamical interdependence among the subsystems (the individual equity markets) implies that:

1. The subsystems communicate, that is, they are coupled together and information flows between them (news arrival in the financial markets), and/or
2. They are coupled to a common driver, where in the case of the stock markets the driving force is profit motive.

It should be noted that even for coupled, but otherwise independent dynamical systems, it is possible that their temporal evolutions might become “synchronized" as one adjusts the coupling strength between them, even though their temporal evolution might not be identical.

The study of dynamical interdependence of nonlinear systems, commonly known as synchronization in physics literature, has its origin in the works of Fujisaka and

Yamada (1983), Afraimovich et al. (1986) and Pecora and Carroll (1990). A variety of approaches to synchronization studies, including system-subsystem synchronization, synchronization in unidirectional and bidirectional coupled systems, anti-phase synchronization, partial synchronization, pulse-coupled synchronization, and generalized synchronization have been developed.

Oscillating systems evolve along their attractors. In certain situations where the oscillators are asymmetrically coupled, there may exist a one-to-one mapping between each attractor. In presence of such mapping, it is possible to predict behavior of one system given the attractor of another one.

Dynamical interdependence, as described in Rulkov et al. (1995), which adopts a generalized synchronization approach, implies predictability of the *response* system's behavior by the *driving* system. This is the starting point for testing for interdependence of two systems which assumes existence of function ϕ that projects values from the trajectories of the driving system D space into the trajectories in the response system R space. In practice, however, when the degrees and directions of the coupling between the systems are unknown, one aims to reconstruct the dynamics of the two systems by time-delay embedding method, and then estimates statistics for testing for dynamical interdependence between the reconstructed systems. This is the basis for the mutual prediction method for testing for interdependence of two dynamical systems (Pecora and Carroll 1990; Schiff et al. 1996; Breakspear and Terry 2002), a method used by Soofi et al. (2012).

Mutual prediction is a method for testing for synchronization of completely independent, but coupled oscillating systems. Examples of synchronization of completely independent, yet coupled, oscillating systems from biological and physical realms include synchronized intermittent emissions of light by tens of thousand fireflies to random openings of ion channels in cell membranes, to organ pipes, just to name a few. In short, synchronization is interaction among different systems or subsystems, which at the times before or after synchronization, operate independently from each other. This means that these coupled, different, and independent systems or subsystems adjust the time scales of their oscillations due to the interaction (Balanov et al. 2009).

We search for evidence of coupling between these markets by considering their dynamics that are represented by the following differential equations:

$$\frac{d\mathbf{X}}{dt} = f(\mathbf{X}(\mathbf{t}) + \bar{f}(\mathbf{X})\,\boldsymbol{\xi}_1(t)) \tag{1.23}$$

$$\frac{d\mathbf{Y}}{dt} = g[(\mathbf{Y}(\mathbf{t}) + \bar{g}(\mathbf{Y})\boldsymbol{\xi}_2), h_c(\mathbf{X}(\mathbf{t}) + \bar{f}(\mathbf{X})\boldsymbol{\xi}_1(t), \mathbf{Y}(\mathbf{t}) + \bar{g}(\mathbf{Y})\boldsymbol{\xi}_2)] \tag{1.24}$$

where functions f and g generate local dynamics, function h transmits the influence of $\mathbf{X}(t)$ to $\mathbf{Y}(t)$, and constant c measures the strength of coupling. Moreover, $\boldsymbol{\xi}_1(t)$ and $\boldsymbol{\xi}_2(t)$ are random dynamical noise reflecting random decisions of traders in the two markets. These random terms, not to be confused by the measurement noise of equations (1.25) and (1.26) below, may induce oscillatory dynamics in the model, opening up the possibility that the markets meet the self-sustaining oscillations requirement for synchronization.

Let $\mathbf{X}$ and $\mathbf{Y}$ be two potentially coupled dynamical systems with the time series observations of x_i and y_i $(i = 1, \ldots, N)$, respectively. Often, in practice, the state variables are not directly observable, and one has no *a priori* knowledge of their individual dynamics or their dynamical interdependence. Instead, their evolutions are measured by the scalar variables

$$x_i(t) = h(\mathbf{X}(t)) + \eta_1(t) \tag{1.25}$$

$$y_i(t) = k(\mathbf{Y}(t)) + \eta_2(t) \tag{1.26}$$

where h and k are the measurement functions (possibly nonlinear), and η_1 and η_2 are the error terms representing noise in the data.

On many occasions one might have to analyze time series data that have values in a wide range. In such cases one should standardize the series by the following transformations:

$$\hat{x}_i = \frac{x_i - \bar{x}}{\sigma_x} \tag{1.27}$$

$$\hat{y}_i = \frac{y_i - \bar{y}}{\sigma_y} \tag{1.28}$$

where $\bar{x}$, $\bar{y}$, σ_x, and σ_y are the mean and standard deviation of the x_i and y_i series, respectively.

Next using the time-delay embedding of section "Determining Time Delay and Embedding Dimension" we would reconstruct the phase spaces for both $\mathbf{X}$ and $\mathbf{Y}$.

Surrogate data analysis is the method of choice in physics and nonlinear dynamical systems analysis. Hence, the mutual prediction method of test for nonlinear interdependence uses this approach also. See section "Surrogate Data Analysis and Testing for Nonlinearity" above for a discussion of this method.

The algorithm of computing the time delay τ with mutual information technique in Soofi et al. (2012) is Shannon's entropy method, and consists of first constructing a histogram for the probability distribution of the data. For details see Soofi et al. (2012).

Algorithm for Mutual Prediction Method

One starts with a possible functional relationship between $\mathbf{X}$ and $\mathbf{Y}$ as

$$\mathbf{Y} \stackrel{?}{=} \phi(\mathbf{X}) \tag{1.29}$$

and aims at empirically verifying existence of the functional relationship ϕ between the two reconstructed systems $\mathbf{X}$ and $\mathbf{Y}$. If such a relationship exists, then two close states in the phase space of the **X** system correspond to two close states in the phase space of the **Y** system.

It does not matter which state variable we choose as autonomous or response variable. For measuring nonlinear interdependence what counts is h_c function, and coupling strength coefficient c. Existence of a continuous, differentiable map ϕ, where in presence of synchronization creates a one-to-one correspondence between the orbits of **X** onto the orbits of **Y** in case of $\mathbf{Y} = \phi(\mathbf{X})$, and maps **Y** onto **X** in case of $\mathbf{X} = \phi(\mathbf{Y})$ is the important consideration.

Select an arbitrary point x_0 in the **X** space. Suppose the nearest neighbor of x_0 has a time index of n_{nnd}. Then if function ϕ exists, that is, if the two systems are coupled, then point y_0 in the **Y** space will have point $y_{n_{nnd}}$ as a close neighbor also. This means that the nearest neighbors of both points x_0 and y_0 share the same time indexes.[5] For example, if the nearest neighbor of point x_0 is a three-dimensional vector with time indexes $(1, 5, 8)$, then the vector that is the nearest neighbor of point y_0 has the same time indexes $(1, 5, 8)$.

In implementing the mutual prediction method of testing for nonlinear interdependence of Chinese stock markets, Soofi et al. (2012) followed the method discussed by Breakspear and Terry (2002) which is a modified, improved version of Schiff et al. (1996) as discussed below.

- Construct in **X** a simplex around an arbitrary selected point $x(t_i)$ in time $t = t_i$ with $2d_1^x$ vertices each consisting of another vector in **X**. d_1^x is the embedding dimension of **X**.
- Choose these embedding vectors (vertices) such that the size of the simplex is minimized.
- Denote the points satisfying the criteria of being a vertex in the minimized simplex as $x_j(t_{ij})$, $j = 1, \ldots, 2d_1^x$. Also denote the time indices of the vertices as t_{ij}, $j = 1, \ldots, 2d_1^x$.
- Use the time indices t_{ij} of $x_j(t_{ij})$ to construct a simplex in the state space **Y** with vertices $y(t_{ij})$, $j = 1, \ldots, 2d_1^x$.
- Take the weighted average of the vertices in $y(t_{ij})$ to locate the vector $y(t_{ij})$ that was predicted by the vector $x(t_i)$

$$y_{pred.}(t_i) = \frac{\sum_{k=1}^{2d_1^x} \omega_{ik} y(t_{ik})}{\sum_{k=1}^{2d_1^x} \omega_{ik}} \tag{1.30}$$

  where the weighting factors ω_{ik}, are determined by the distances of the vertices in **X** from $x(t_i)$, giving

$$\omega_{ik} = (|x(t_{ik}) - x(t_i)|)^{-1}. \tag{1.31}$$

- To calculate the mutual prediction error, take the difference of the predicted vector and the actual vector

[5] Note that we have unfolded the time series into d-dimensional space.

$$\epsilon_{y(x)} = |y_{pred}(t_i) - y(t_i)|. \tag{1.32}$$

- To compare the prediction error $\epsilon_{y(x)}$ with a prediction error based on a randomly selected element of the time series observations calculate

$$\epsilon_{rand} = |y_{rand} - y(t_i)|, \tag{1.33}$$

  where y_{rand} is calculated using the same procedure used in prediction of $y_{pred.}(t_i)$, except that the simplex in **X** is a random combination of points on the orbit **X** weighted with respect to another randomly selected point. This corresponds to the null hypothesis of no interdependence between the markets.
- The normalized predicted y, $\nabla_{y(x)}$, as predicted by x, is calculated by

$$\nabla_{y(x)} = \frac{\langle \epsilon_{y(x)} \rangle_{rms}}{\langle \epsilon_{rand} \rangle_{rms}} \tag{1.34}$$

  where $\langle\rangle_{rms}$ is the root mean square.
  $\nabla_{y(x)} = 1$ implies no interdependence (no synchronization). $\nabla_{y(x)} = 0$ implies complete synchronization.
- Calculate the vertices of simplex in **Y** as above and then iterate them H-step ahead on their respective orbits to obtain the vertices $y(t_{ij} + H)$, $j = 1, \ldots, 2d_i^y$
- Compare the weighted predicted vector $y_{pred.}(t_i + H)$, $j = 1, \ldots, 2_i^y$ to the actual forward iterate $y(t_i + H)$ to obtain future prediction errors.
- Normalize the H-step ahead prediction errors by a vector generated from random vertices in **X** to yield the normalized future prediction error:

$$\nabla_{y(x)}^H = \frac{\langle \epsilon_{y(x)}^H \rangle_{rms}}{\langle \epsilon_{rand} \rangle_{rms}} \tag{1.35}$$

  $\nabla_{y(x)}^H = 1$ implies no interdependence between the systems at H-step prediction.

Note that in presence of generalized synchronization the error grows at a rate determined by the Lyapunov exponents, and is less than one for some time steps into the future.

After generating a number of surrogates,which share the spectral density functions with the original time series use one-step ahead mutual prediction method described above, and conduct H forecasts of the original time series and the surrogate time series separately. If the one-step ahead nonlinear prediction errors of the original series are smaller than those for any of the surrogates, predictions are significant.

A plot of H prediction errors as well as prediction interval for the original and surrogate series based on the above mentioned algorithm would aid in determining nonlinear interdependence of the markets.

The deterministic interdependence is detected if the graph of the cross-prediction errors of the original series is below the graphs of cross-prediction errors for the surrogate sets, but above the lower bound of the 95 % confidence interval.

Synchronization of Chinese Stock Markets

Soofi et al. (2012) considered three Chinese stock markets: Shanghai (SSI), Shenzhen (SZI) and Hong Kong (HSI), as nonlinear dynamical oscillating systems. They further considered two indexes at a time for testing and took $\mathbf{X}(\mathbf{t})$ as the driver system and $\mathbf{Y}(\mathbf{t})$ as the response system. Furthermore, they reconstructed the phase space of each stock market as a dynamical system using time series observations of the daily average stock prices.

We note that synchronization can be bi-directional or unidirectional. In a *forced synchronization* one system influences the second one without being influenced by it. One has bidirectional synchronization where both systems are mutually interacting and influencing each other. Hence, in the forced synchronization case, if $\mathbf{X}$ is not influencing $\mathbf{Y}$, it does not necessarily mean that $\mathbf{Y}$ is not influencing $\mathbf{X}$. (for excellent discussions of synchronization, see Balanov et al. (2009)).

They constructed 19 bivariate surrogate data with the same amplitude distribution, auto correlation function, and cross-spectral density function as the original data. However, non-linear structure contained within and between the surrogate series are destroyed. Thus the surrogate algorithm allows testing of the null hypothesis that the time series are produced by a cross-correlated stochastic system.

The results of Soofi et al. (2012) show that there is nonlinear mutual (bidirectional) predictability between SSI and SZI. Moreover, there exists unidirectional predictability from SSI to HSI and from SZI to HSI. However, the results don't provide statistically significant evidence that Hong Kong market predicts the stock markets in mainland China.

In sum, the study concluded that Shanghai, Shenzhen, and Hong Kong stock market data are nonlinear, and are nonlinearly dependent on each other. This implies that the stock index observations of the three stock markets are originated from different parts of the same dynamical system, and hence the markets are well integrated.

Comparing the Results with the Results Based on a Traditional Linear Method

Comparing the results for synchronization of the Chinese stock markets based on mutual prediction method with the results based on a linear method of testing for integration of financial markets, Soofi et al. (2012) used results from Zhu et al. (2003). Zhu et al. (2003) have used cointegration, fractional cointegration, and Granger causality methods in testing for integration of Chinese stock markets. The tests in Zhu et al. (2003) show no evidence of cointegration (either integrated or fractionally integrated) among the stock markets. They could not find any evidence for presence of causality among the markets either. Hence, the mutual prediction method of testing for interdependence of Chinese stock markets data shows completely different results from those obtained by the traditional linear stochastic methods used in Zhu et al. (2003) study. The evidence pointing to nonlinearity of the stock markets as dynamical systems, should support the conclusion that the linear models have failed to detect interdependence, while the mutual prediction method succeeded in finding the evidence of dynamical interdependence between the markets.

Summary and Conclusion

Advances in nonlinear dynamical system theories and methods have opened up new possibilities for applying them in finance and economics. The authors of the present chapter have applied a number of these methods in testing for nonlinearity, predictions, and calculation of invariants such as correlation dimension of the some exchange rate data. They have used these methods in testing for synchronization (interdependence) of the stock markets also.

Even though tests uniformly show presence of nonlinearity in many financial data that were analyzed, determination of whether the data generating processes are deterministic is inconclusive because of the short sample observations and presence of noise in the observed data. Further advances in theory of nonlinear stochastic dynamical systems in the last decades promises to be useful in further applications on the financial data. Applications of these methods, specially mutual prediction method as warning system for imminent emergence of financial contagion is very promising also. Hitherto, the methods of nonlinear dynamic systems unravel the dynamics in many financial time series observations that could not be detected by the tradition linear stochastic methods.

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Chapter 2
Kaldorian Assumptions and Endogenous Fluctuations in the Dynamic Fixed-Price IS-LM Model

Giovanni Bella, Paolo Mattana and Beatrice Venturi

Abstract With the aim of better understanding the conditions which determine endogenous fluctuations at business cycle frequencies, recent literature has revived interest in the Schinasi's variant of the dynamic, intermediate-run, IS-LM model (Schinasi 1981, 1982). Results, however, remain confined to Kaldorian-type economies, namely to those economies which present a greater-than-unity marginal propensity to spend out of income. This paper contributes to the debate by showing that, in the case of a negative interest rate sensitivity of savings, stable endogenous cycles can actually emerge as equilibrium solutions of the model also in the case of non Kaldorian-type economies. To this end, we combine the instruments of the global analysis, specifically the homoclinic bifurcation Theorem of Kopell and Howard (1975), with numerical methods.

Keywords Multiple steady states · Homoclinic bifurcation · Oscillating solutions

JEL classification C61 · C62 · E32

Introduction

Fixed-price, dynamic IS-LM models of Schinasi's type (1981, 1982) have recently re-gained centrality in the literature regarding deterministic fluctuations at business cycle frequencies. A strand of contributions is using the imposition of time-delayed

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feedbacks in the tax collection function to obtain or suppress endogenous fluctuations (cf *inter al*., Cai 2005; de Cesare and Sportelli 2005; Fanti and Mandredi 2007; Neantu, Opris and Chilarescu 2007). Other scholars (cf, *inter al*., Gandolfo 1997; Makovinyiova 2011; Neri and Venturi 2007, Sasakura 1994; Zimka 1999) remain more in line with the original structure of the model, and look for the existence of oscillating solutions in specific regions of the parameters space mainly by the use of the Hopf bifurcation Theorem. In general terms, it is important to point out that the interest for the model is largely undermined by the severe functional restrictions needed to generate the required oscillating behavior of the variables. In particular, a Kaldorian *S*-shaped investment function, in turn implying the possibility of an upward sloping IS curve, is appealed by this literature to show that the model can originate endogenous fluctuations.

This paper contributes to the debate by showing that, provided that the interest elasticity of savings is negative, stable endogenous cycles can emerge as equilibrium solutions of the model also in the case of non Kaldorian-type economies. To prove this, we combine the instruments of the global analysis, specifically the homoclinic bifurcation Theorem of Kopell and Howard, 1975,[1] with numerical methods, to show that a trajectory, starting in the vicinity of a saddle steady state (where the economy is of non Kaldorian-type) can approach from outside a limit cycle enclosing a non saddle steady state (where the economy is of Kaldorian-type).

The paper develops as follows. The second section introduces the model and studies the long-run equilibrium. The third section establishes some stability properties of the long-run equilibrium from the perspective of the local analysis. In particular, we provide here confirmation that stable limit cycles can only emerge if the economy is of a Kaldorian type. The fourth section shows that the more powerful instruments of the global analysis allow us to prove the possibility of stable limit cycles also for a non-Kaldorian economy. The fifth section discusses an example. A brief conclusive section reassesses the main findings of the paper. All necessary proofs are provided in a specific Appendix.

The Model

Schinasi, in a series of different papers, revises "classical" Kaldor's (1940) business cycle model by replacing the capital stock with the interest rate and taking into account financial markets and a Government budget constraint in which both money and bond financing are alternative means of financing budget gaps. A non linearity in the income variable is assumed in the investment function. The shape of the investment function, crucial for the derivation of feasible oscillating solutions, is

[1] The Theorem is largely used in mathematics, physics and biology, but has found a surprisingly limited application in economics: to the best of our knowledge, the only applications in $\mathbb{R}^2$ planar systems is in Benhabib et al. (2001) for a Taylor-rule monetary model, and in Benhabib et al. (2008) for a growth model. An application in the $\mathbb{R}^3$ dimension is in Mattana et al. (2009).

originally postulated by Kaldor (1940) as a way to model a non-linear reaction of investors to changes in market conditions such as excess demand and excess supply. More formally, Kaldor's view assumes that

$$I_Y > 0; \qquad I_{YY} \begin{Bmatrix} > 0, & \text{for } Y < Y^* \\ < 0, & \text{for } Y > Y^* \end{Bmatrix}$$

where I is investment and Y income. I_Y and I_{YY} represents first and second-order partial derivatives of the investment function with regard to income, respectively. Finally, Y^* is the "normal" level of output.[2] What is crucial for us is that:

Remark 1 Since the investment function is not constrained to be linear, the IS curve needs not be linear too.

In a formal perspective, let $S(R, Y^D)$ represent savings as a function of the interest rate and disposable income, with $Y^D = Y - T(Y)$ where $T(Y)$ is the tax collection function. Given the non-linearity in the investment function, as output increases above expected levels, firms will increase investment but *less* than they would have in a linear model, since they expect Government to be "active" in stabilizing economic activity. Therefore, it could be the case that the crucial quantity $I_Y - \left[S'(1 - T') - T'\right]$ be negative. Notice that, as it will become clearer below, the quantity above presented connected with the slope of the IS curve in a (R, Y) space, with regard to which presents an opposite sign (cf. Schinasi 1981, for a detailed discussion).

After this preparatory discussion, we are ready to present the system of differential equations originated by Schinasi's variant of the IS-LM model. Referring to Sasakura (1994) for a detailed derivation, in the case of an instantaneous adjustment in the money market, the following planar system of first order differential equations is implied

$$\begin{aligned} \dot{R} &= \tfrac{G-T(Y)}{L_R(R,Y)} - \alpha \tfrac{L_Y(R,Y)}{L_R(R,Y)} \left[I(R, Y) - S(R, Y^D) + G - T(Y)\right] \\ \dot{Y} &= \alpha \left[I(R, Y) - S(R, Y^D) + G - T(Y)\right] \end{aligned} \qquad (\mathcal{M})$$

where $\dot{R} = dR/dt$ and $\dot{Y} = dY/dt$. It is assumed that all functions are continuously differentiable at a suitable order. $L(\cdot)$ is the liquidity preference function, relating the demand for money to R, the (real) interest rate, and Y, the income level. It follows that $L_Y(R, Y)$ and $L_R(R, Y)$ are partial derivatives of the liquidity preference function with respect to income and the interest rate, respectively. $I(R, Y)$ is the investment function which is assumed to depend on income and on the interest rate. Finally $G > 0$ is the (constant) government expenditure and α is a scale parameter. Notice that system $\mathcal{M}$ is a crucially extended variant of standard Schinasi's model

[2] The idea has been derived from a dynamic theory of the firm in which agents expect aggregate demand to fluctuate around a trend and believe Government attempts to stabilize output around the trend.

(1981) and (1982) where the $S(\cdot)$ function takes into account the interest rate as a further argument (cf., *inter al.*, Cai 2005; and Makovinyiova 2011, for similarly augmented models).

For the sake of a simple representation, we shall assume that the tax raising function is linear in Y, so that we have $T(Y) = \tau Y$. Thus $Y^D = Y - T(Y) = (1-\tau)Y$ is disposable income. The signs of the derivatives are crucial for the scopes of the paper. Consider first the liquidity preference and the tax raising functions, for which the literature assumes

$$L_Y(\cdot) > 0;\, L_R(\cdot) < 0$$

Less clear is the case of the investment and savings functions; whereas there is no theoretical and empirical disagreement on the following

$$I_R(\cdot) < 0;\, I_Y(\cdot) > 0;\, S_Y(\cdot) > 0$$

the sign of $S_R(\cdot)$ remains ambiguous (cf. *inter al.* Abrar 1989). Basic economic courses show that the interest elasticity of saving can be decomposed into a (positive) "substitution" effect and an "income" effect, which works in opposite directions. Which effect prevails depends on the specific model and/or parameter configurations. We shall assume, in the rest of the paper, that it is possible for the savings interest rate sensitivity $S_R(\cdot)$ to be negative.

Steady State

We obtain now some information on the long-run properties of system $\mathcal{M}$. Let (R^*, Y^*) be values of (R, Y) such that $\dot{R} = \dot{Y} = 0$. To simplify notation, and considering $Y^D = (1-\tau)Y$, we define the following

$$H(R, Y) = I(R, Y) - S(R, Y)$$

Simple algebra shows that, at the steady-state, we have

$$Y^* = \frac{G}{\tau} \tag{2.1}$$

$$H(R^*, Y^*) = 0 \tag{2.2}$$

We now study conditions for existence and uniqueness of the steady state. Let $\phi \to \mathbb{R}$ be defined as:

$$\phi(R) = H(R, Y^*) \tag{2.3}$$

with ϕ conveniently smooth in all its arguments. Let also $\phi'(R)$ and $\phi''(R)$ be the first and second-order derivative of $\phi(R)$ with respect to R. If $\phi'(R)$ is negative (positive) in the domain $D = \{(R) : R > 0\}$, the function $\phi(R)$ monotonically decreases (increases) with the interest rate and we can only have one intersection with the R-axis (one steady state). Conversely, if it changes sign, for specific values of the interest rate, it can have multiple intersections with the R-axis (multiple steady states). To simplify the analysis, without loosing generality, consider the following regularity condition

Assumption 1 $\phi''(R)$ does not change sign in the domain $D = \{(R) : R > 0\}$.
Assumption 1 implies that, if $\phi'(R)$ changes sign in D, $\phi(R)$ is unimodal, and the maximum number of possible intersections with the R-axis is limited to two. Let now $\omega \in \Omega$ represent the set of all parameters. Let also $\bar{\Omega} \equiv \{\omega \in \Omega := R^* \in D\}$. Then

Lemma 1 *Recall Assumption* 1. *Let* $\hat{\Omega} \equiv \{\omega \in \bar{\Omega} := \phi'(R)$ *is positive or negative*$\}$ *and, complementarily,* $\check{\Omega} \equiv \{\omega \in \bar{\Omega} := \phi'(R)$ *changes sign at* $R = \hat{R}\} = \bar{\Omega} - \hat{\Omega}$. *Then, if* $\omega \in \hat{\Omega}$, *the steady state is always unique. Consider now* $\omega \in \check{\Omega}$ *and assume* $\phi''(R) > 0$. *Then, if*

$\phi(\hat{R}) < 0$ *there are two steady states, one with a low interest rate* $\left(R^*_-, Y^*\right)$ *and one with a high interest rate* $\left(R^*_+, Y^*\right)$;
$\phi(\hat{R}) = 0$ *there is one steady state;*
$\phi(\hat{R}) > 0$ *there are no steady states.*

The statements are inverted if $\phi''(R) < 0$.

Proof Let $\omega \in \hat{\Omega}$. Since, by assumption, the first derivative does not vanish in D, the function $\phi(R)$ is always monotonically decreasing/increasing in D and only one steady state is possible. Let now $\omega \in \check{\Omega}$. Then $\phi(R)$ is unimodal. Assume first $\phi''(R) > 0$. Assume, furthermore, there is a (generic) parameter with the properties in Proposition. Then, the three intersection possibilities with the $\phi(R) = 0$ axis must apply. Inverse statements apply in the case of $\phi''(R) < 0$.

Some numerical applications will be provided in Section "Some Numerical Simulations", for specific functional forms.[3]

Local Stability Analysis

Consider trajectories in which R and Y remain bounded in a small neighborhood of the steady state. In Appendix A, we show that the linearization matrix associated with system $\mathcal{M}$ evaluated at the steady state, is

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[3] Notice that Lemma 1 is also of notable interest for related fields. For instance, the possibility of conceputalizing via multiple steady states some paradoxical features of real world time series is of considerable importance in the monetary economics literature (cf., *inter al.*, Bullard and Russel 1999; Bullard 2009).

$$\mathbf{J}^*_{\mathcal{M}} = \begin{bmatrix} -\alpha \frac{L^*_Y}{L^*_R} H^*_R & -\alpha \frac{L^*_Y}{L^*_R} H^*_Y + \left(\alpha \frac{L^*_Y}{L^*_R} - \frac{1}{L^*_R}\right)\tau \\ \alpha H^*_R & \alpha(H^*_Y - \tau) \end{bmatrix} \tag{2.4}$$

where, for the sake of a simple notation, the arguments of the partial derivatives have been dropped. Consider the characteristic polynomial associated with $\mathbf{J}^*_{\mathcal{M}}$

$$\mathrm{Det}\left(\lambda \mathbf{I} - \mathbf{J}^*_{\mathcal{M}}\right) = \lambda^2 - \mathrm{Tr}(\mathbf{J}^*_{\mathcal{M}})\lambda + \mathrm{Det}(\mathbf{J}^*_{\mathcal{M}}) \tag{2.5}$$

where $\mathbf{I}$ is the identity matrix. $\mathrm{Tr}(\mathbf{J}^*_{\mathcal{M}})$ and $\mathrm{Det}(\mathbf{J}^*_{\mathcal{M}})$ are Trace and Determinant of $\mathbf{J}^*_{\mathcal{M}}$, respectively. In Appendix A, they are shown to be

$$\mathrm{Det}(\mathbf{J}^*_{\mathcal{M}}) = \alpha \frac{\tau}{L^*_R} H^*_R$$

$$\mathrm{Tr}(\mathbf{J}^*_{\mathcal{M}}) = \alpha \left(H^*_Y - \frac{L^*_Y}{L^*_R} H^*_R - \tau\right)$$

To study the stability properties in a planar system from the local analysis perspective, it is crucial to establish the signs of $\mathrm{Det}(\mathbf{J}^*_{\mathcal{M}})$ and $\mathrm{Tr}(\mathbf{J}^*_{\mathcal{M}})$. Simple algebra shows that necessary conditions for the birth of attracting orbits, from the perspective of the local analysis, are the following

$$H^*_R < 0 \tag{2.6}$$

$$H^*_Y - \frac{L^*_Y}{L^*_R} H^*_R - \tau > 0 \tag{2.7}$$

which guarantee that the steady state is an unstable node or focus. More precisely

Proposition 1 *Recall Lemma* 1. *Let* $\omega \in \hat{\Omega}$ *and first assume* $H^*_R < 0$. *Then, the (unique) steady state is an unstable node or focus if* (2.7) *is satisfied. Conversely, if* $H^*_R > 0$, *the steady state is a saddle.*

Let now $\omega \in \check{\Omega}$ *and assume first* $\phi''(R) > 0$. *As shown in Lemma* 1, *we can either have a dual steady state, one steady state or no steady states at all. In the former case, at* $\left(R^*_-, Y^*\right)$, $H^*_R < 0$, *so that the low interest rate equilibrium is an unstable node or focus if* (2.7) *is satisfied. The other steady state* $\left(R^*_+, Y^*\right)$ *has* $H^*_R > 0$ *and is therefore a saddle. The low interest rate steady state and the high interest rate steady state interchange their stability properties if* $\phi''(R) < 0$.

Proof To exclude a saddle we need $\mathrm{Det}(\mathbf{J}^*_{\mathcal{M}}) > 0$, which happens if (2.6) applies. Furthermore if (2.7) is satisfied, Tr $(\mathbf{J}^*_{\mathcal{M}}) > 0$ and the steady state is an unstable node or focus.

Recall that

Remark 2 Conditions in (2.6), imply an upwards sloping IS curve. As above discussed, this property can be justified in a Kaldorian perspective, namely with the assumption of an S-shaped investment function.

We find it useful, for a clear presentation of the main results of the paper, to give the following

Definition 1 *Let an economy (not) satisfying conditions in* (2.6) *be called a (non) Kaldorian-type economy.*

Thus

Corollary 1 *Recall Lemma* 1. *Then, from the perspective of the local analysis, only a Kaldorian-type economy can give rise to stable deterministic cycles.*

Proof From the local analysis point of view, only in the neighborhood of the non-saddle steady state we can have oscillating solutions. Therefore the case $H_R^* > 0$ must be discarded. We are only left therefore with the $H_R^* < 0$ possibility. In this case, since $L_R^* < 0$, Tr $(\mathbf{J}_{\mathcal{M}}^*) = H_Y^* - \frac{L_Y^*}{L_R^*} H_R^* - \tau$ is positive only if $H_Y^* - \tau > 0$. As above discussed this implies a positively sloping IS curve which, in turn, can be justified with an S-shaped Investment function.

Remarkably, it is interesting to point out here that a negative interest elasticity of savings is also crucial for the emergence of endogenous cycles in discrete-time overlapping generations models (cf. *inter al*. Azariadis and Guesnerie 1986; and Grandmond 1985).

Global Analysis

In contrast with the conclusion of Section. "Steady State", the global bifurcation analysis will allow us to prove that, when system $\mathcal{M}$ admits a dual steady state, a non Kaldorian-type economy can undergo endogenous fluctuations. What we actually do is to show that there exist trajectories originating in the neighborhood of the saddle steady state (the one at which the economy is of a non-Kaldorian type) which are bound to converge to a limit cycle around the non-saddle steady state (the one at which the economy is of a Kaldorian type). To obtain this result, we make use of the homoclinic bifurcation Theorem of Kopell-Howard, 1975, which allows to locate the regions in the parameter space implying the existence of a closed orbit or of a saddle connection. The application of the Theorem is not trivial and requires several steps to be accomplished (cf Appendix C).

In any case, before proceeding, we need first to assume some specific functional forms. Therefore, with regard to the original system $\mathcal{M}$:

1. the liquidity preference function $L(R, Y)$ is assumed linear in its two arguments (cf. Cai, 2005, for a similar approach). Therefore, $L(R, Y) = -\beta R + \gamma Y$ where $(\gamma, \beta) > 0$ measure the sensitivity of $L(\cdot)$ to the interest rate and income, respectively;

2. a convenient explicit form for the $H(\cdot) = I(\cdot) - S(\cdot)$ function is harder to propose. In Makovinyiova (2011) the investment and savings functions are assumed to have the following form

$$I = \varepsilon_1 \sqrt{Y^3} - \varepsilon_2 R$$
$$S = \varepsilon_3 \left(Y^D\right)^2 + \varepsilon_4 R + \varepsilon_5$$

to match the characteristics of the Slovakian economy, where $(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5) \in \mathbb{R}^{++}$ and R (the real interest rate) is expressed in percentage terms. However, to account for a dual steady state,[4] we need the $H(\cdot)$ function to be non-linear in R. Therefore, we propose the following generalization of the $H(\cdot)$ function

$$H(R, Y) = \varepsilon_1 \sqrt{Y^3} - \varepsilon_2 R - \varepsilon_3 \left(Y^D\right)^2 - \varepsilon_4 R - \varepsilon_5 - \delta \tfrac{Y}{R} \tag{2.8}$$

with $\delta > 0$. Notice that we can interpret the factor Y/R as a proxy for wealth. This makes it easier to introduce a negative savings sensitivity to the interest rate. It is useful to point out that parameters lie in the $\check{\Omega}$ set and that Assumption 1 is satisfied; consequently, recalling Lemma 1, system $\mathcal{S}$ can give rise to two steady states.

With Eq. (2.8), and recalling $Y^D = (1-\tau)Y$, system $\mathcal{M}$ becomes

$$\begin{aligned} \dot{R} &= -\tfrac{1-\gamma\alpha}{\beta}(G - \tau Y) + \tfrac{\gamma\alpha}{\beta}\left[\varepsilon_1\sqrt{Y^3} - \varepsilon_2 R - \varepsilon_3(1-\tau)^2 Y^2 - \varepsilon_4 R - \varepsilon_5 - \delta\tfrac{Y}{R}\right] \\ \dot{Y} &= \alpha\left[\varepsilon_1\sqrt{Y^3} - \varepsilon_2 R - \varepsilon_3(1-\tau)^2 Y^2 - \varepsilon_4 R - \varepsilon_5 - \tfrac{\delta Y}{R} + G - \tau Y\right] \end{aligned} \tag{$\mathcal{S}$}$$

As shown in Appendix B, the linearization matrix associated with system $\mathcal{S}$ is

$$\mathbf{J}^*_{\mathcal{S}} = \begin{bmatrix} \tfrac{\gamma\alpha}{\beta} H^*_R & \tfrac{\gamma\alpha}{\beta} H^*_Y - \tfrac{\gamma\alpha-1}{\beta}\tau \\ \alpha H^*_R & \alpha(H^*_Y - \tau) \end{bmatrix}$$

where

$$H^*_R = -(\varepsilon_2 + \varepsilon_4) + \delta \frac{Y^*}{R^{*2}} \tag{2.9}$$

$$H^*_Y = \frac{3}{2}\varepsilon_1 \sqrt{\frac{G}{\tau}} - 2\varepsilon_3 (1-\tau)^2 \frac{G}{\tau} - \frac{\delta}{R^*} \tag{2.10}$$

Simple algebra leads to the following

[4] Recall that we need $\phi''(R) \neq 0$ to account for multiple steady states.

$$\mathrm{Det}(\mathbf{J}^*_{\mathcal{S}}) = -\frac{\alpha\tau H^*_R}{\beta}$$
$$\mathrm{Tr}(\mathbf{J}^*_{\mathcal{S}}) = \alpha\left(H^*_Y + \frac{\gamma}{\beta}H^*_R - \tau\right)$$

A first requirement of the Kopell Howard Theorem is that there exist regions in the parameter space at which the linearization matrix $\mathbf{J}^*_{\mathcal{S}}$ has a double-zero eigenvalue. In the two-dimensional case, this happens if $\mathrm{Det}(\mathbf{J}^*_{\mathcal{S}})$ and $\mathrm{Tr}(\mathbf{J}^*_{\mathcal{S}})$ vanish simultaneously. Taking δ and τ as bifurcation parameters, we can state the following

Lemma 2 *Let $\left(\bar{R}^*, \bar{Y}^*\right)$ be the levels of the interest rate at the bifurcation point. Let furthermore $\bar{\delta}$ and $\bar{\tau}$ be the values of δ and τ which solve Det($\mathbf{J}^*_{\mathcal{S}}$) $= 0$ and Tr($\mathbf{J}^*_{\mathcal{S}}$) $= 0$, respectively. Then, if $\delta = \bar{\delta}$ and $\tau = \bar{\tau}$, $\mathbf{J}^*_{\mathcal{S}}$ has a zero eigenvalue of multiplicity* 2. *Considering* (2.9) *and* (2.10)*, simple algebra shows*

$$\bar{\delta} = \frac{(\varepsilon_2 + \varepsilon_4)\bar{R}^{*2}}{G}\bar{\tau}$$

and

$$\frac{3}{2}\varepsilon_1\sqrt{\frac{G}{\bar{\tau}}} - 2\varepsilon_3(1-\bar{\tau})^2\frac{G}{\bar{\tau}} - \frac{\bar{\delta}}{\bar{R}^*} - \bar{\tau} = 0.$$

where

$$\bar{R}^* = \frac{\varepsilon_1\sqrt{(\frac{G}{\tau})^3} - \varepsilon_3(1-\tau)^2(\frac{G}{\tau})^2 - \varepsilon_5}{2\bar{\delta}}$$

Proof To have a linearization matrix with a zero eigenvalue with multiplicity 2 at the bifurcation, we need to make sure that the determinant and the trace vanish simultaneously. Since $\mathrm{Det}(\mathbf{J}^*_{\mathcal{S}})$ and $\mathrm{Tr}(\mathbf{J}^*_{\mathcal{S}})$ vanish, respectively, when $\delta = \bar{\delta}$ and $\tau = \bar{\tau}$ the statement in Lemma 2 is implied.

We are now ready to prove the main proposition.

Proposition 2 *Recall Lemma* 2. *Assume $H(R, Y)$ be approximated by the expression in (2.8). Then, for δ and τ close to the bifurcation values ($\bar{\delta}$, $\bar{\tau}$), there exist trajectories originating in a close neighborhood of the saddle steady state which either spiral towards the non-saddle steady state or converge to a limit cycle around it. By Proposition* 1*, the saddle steady state is $\left(R^*_-, Y^*\right)$ whereas the non-saddle steady state is $\left(R^*_+, Y^*\right)$.*

Proof To prove the Proposition, we show in Appendix C that system $\mathcal{S}$ satisfies, for specific parameter values, the Kopell-Howard's homoclinic bifurcation Theorem. Since in our case $\phi''(R) = -2\delta\frac{Y^*}{R^{*3}} < 0$, by Proposition 1, $\left(R^*_-, Y^*\right)$ is a saddle while $\left(R^*_+, Y^*\right)$ is non-saddle. In the case of a saddle connection, spiralling towards the non-saddle steady state requires $\mathrm{Tr}(\mathbf{J}^*_{\mathcal{S}})|_{(R^*_+, Y^*)} < 0$. Convergence of trajectories starting in the neighborhood of the saddle steady state to a limit cycle requires the orbit to be attractive. We will show in the next section, by means of numerical simulations, that this ordinarily happens for system $\mathcal{S}$.

Table 2.1 Baseline parameter

α	β	γ	G	ε_1	ε_2	ε_3	ε_4	ε_5
0.1	0.8	0.1	0.78	$0.0\bar{6}$	0.04	0.00132	0.03	$0.381177\bar{3}$

Proposition 2 immediately implies the following

Corollary 2 *Recall Definition 1. Then, a non Kaldorian-type economy can give rise to stable deterministic fluctuations.*

Proof Since there exist trajectories originating in the neighborhood of the saddle steady state (where conditions in 2.6 and 2.7 do not apply) which approach a limit cycle, then non-Kaldorian type economies can exhibit deterministic fluctuations (also cf Definition 1).

Interestingly, a negative interest elasticity of savings is also crucial for the emergence of endogenous cycles in discrete-time overlapping generations models (cf. *inter al*. Azariadis and Guesnerie 1986; and Grandmond 1985). We conjecture that the result is also likely to arise in models with alternative specifications of the non linearity in the interest rate in both the investment and savings functions.

Although the existence of a limit cycle approached by trajectories originating in the neighborhood of the saddle steady state is our main result, it must not be underestimated the possibility of a saddle connection between the two steady states which, in the case of a low decay factor, is not inconsistent with the observation of a fluctuating behavior of real economies.

In Section "Some Numerical Simulations" we provide an extensive simulation study based on system $\mathcal{S}$.

Some Numerical Simulations

Let now $\check{\Omega}_{\mathbf{M}} \equiv (\alpha, \beta, \gamma, G, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5)$ be set as in Table 2.1.[5]

At the bifurcation, there is a unique steady state such that $\left(\bar{R}^*, \bar{Y}^*\right) \approx (1.36, 4.39)$.[6] The parameter values are essentially taken from Makovinyiova with some crucial differences. First of all, to obtain a reasonable amplitude of the cycle, in the case of an economy starting close to the saddle steady state (in the case of a large distance between the two steady state values of the interest rate), we found it necessary to lower the parameter α to 0.1. Furthermore, γ, the elasticity of the demand for money

[5] Notice that, for system $\mathcal{S}$, since $\phi'(R)$ can change sign in the domain D, the parameters lie in the $\check{\Omega}$ sub-sector.

[6] Notice that, with these parameter values, the saving sensitivity to the interest rate equals, at the bifurcation

$$S_R^* = \varepsilon_4 - \bar{\delta}\frac{Y^*}{\bar{R}^{*2}} = -0.040733821$$

which is consistent with the simulations in Abrar (1989).

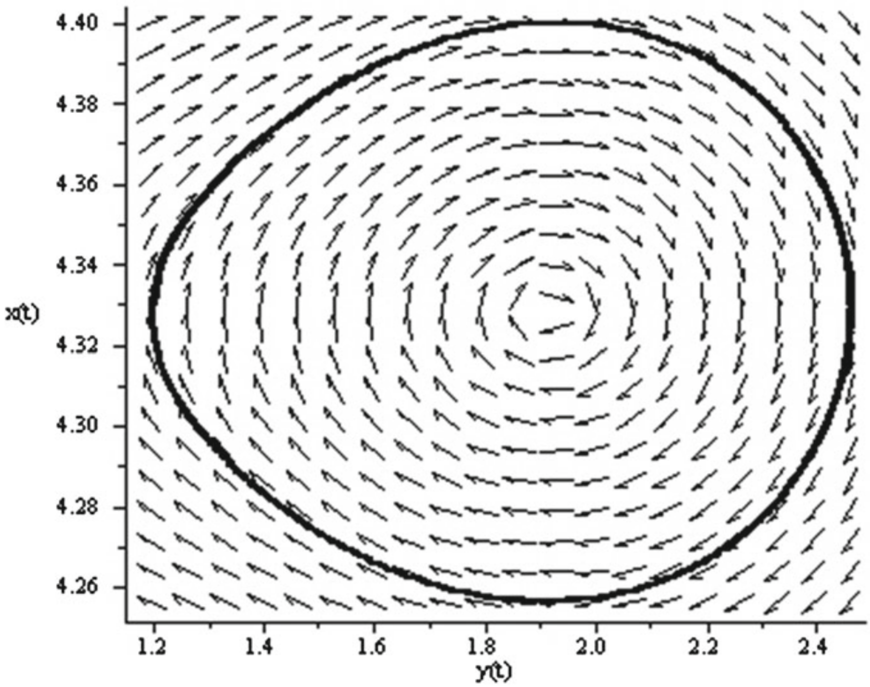

Fig. 2.1 A non-Kaldorian type economy (*slowly*) converging to a cycle

with respect to Y, is set to 0.1 instead of 0.2. Finally ε_3 is set to 0.00132 instead of 0.001. These further changes are necessary to allow for a positive $\mathrm{Tr}(\mathbf{J}^*_{\mathcal{S}})|_{(R^*_+,Y^*)}$, and therefore for the most interesting case of an attracting orbit to be obtained.

The implied critical values of the bifurcation parameters in the baseline simulation are respectively $\left(\bar{\delta}, \bar{\tau}\right) \approx (0.03787, 0.17732)$. Notice that the critical value of the tax rate is very close to the value of the tax rate reported in Makovinyiova for the Slovakian economy in 2007.

Consider now the following example. Set $\check{\Omega}_{\mathbf{M}}$ as in Table 2.1. Assume furthermore $\delta = 0.03 < \bar{\delta}$ and $\tau = 0.1801 > \bar{\tau}$. Then, a dual steady state emerges. We have $(R^*_-, Y^*) \approx (0.9613, 4.331)$ and $(R^*_+, Y^*) \approx (1.9309, 4.331)$. We also obtain $\mathrm{Det}(\mathbf{J}^*_{\mathcal{S}})|_{(R^*_-,Y^*)} > 0$, $\mathrm{Det}(\mathbf{J}^*_{\mathcal{S}}|)_{(R^*_+,Y^*)} < 0$, $\mathrm{Tr}(\mathbf{J}^*_{\mathcal{S}})|_{(R^*_-,Y^*)} < 0$ and $Tr(\mathbf{J}^*_{\mathcal{S}})|_{(R^*_+,Y^*)} > 0$. Therefore, the low interest rate steady state is a saddle and the high interest rate steady state is a source.

The following Fig. 2.1 shows, for the above reported parameter values, the convergence to a limit cycle of a non Kaldorian-type economy starting at (1.2,4.33). It is interesting to observe that for these parameter values, the high interest rate steady state is virtually a center, since the trajectory approaches the orbit from outside at a very slow speed.

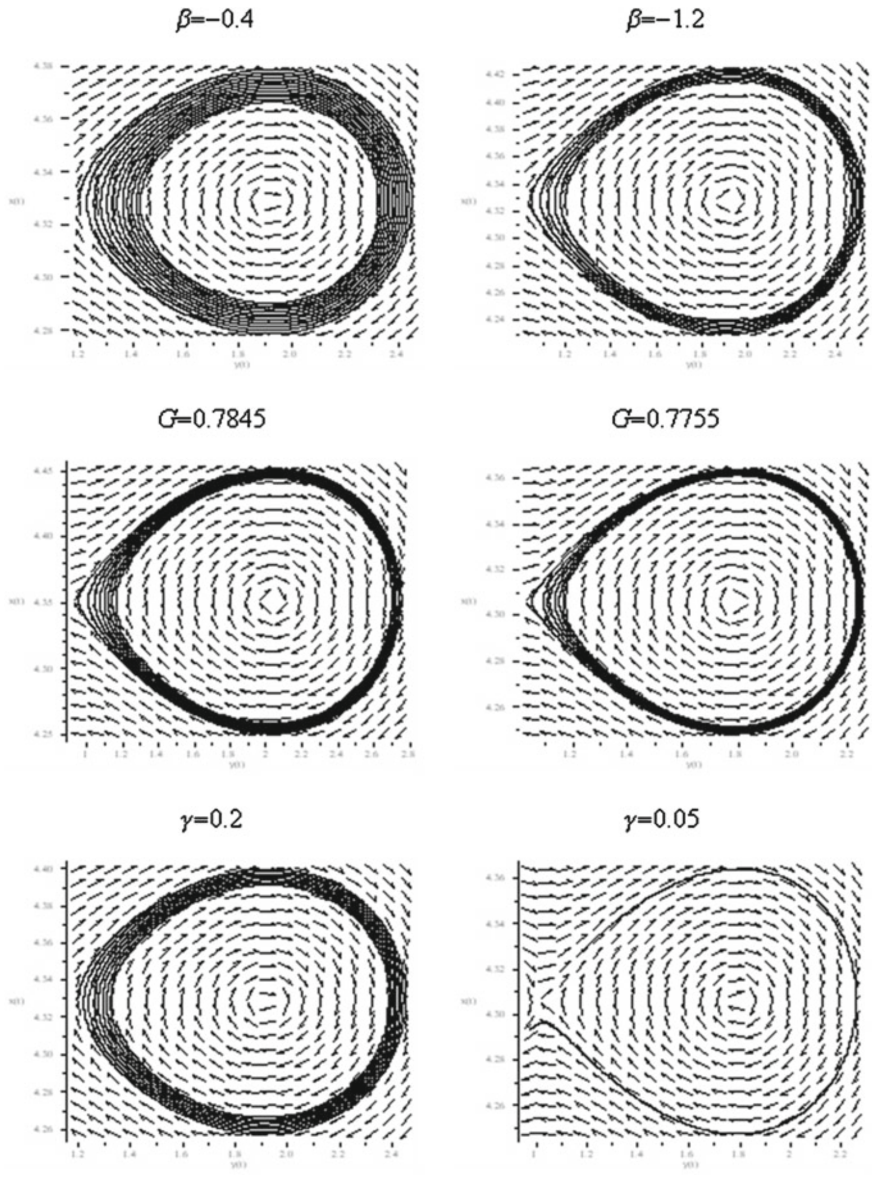

Fig. 2.2 Solution trajectories for varying parameters

We have also conducted a sensitivity analysis by changing some crucial parameters. As it appears clear in Fig. 2.2, what we find is that raising (decreasing) β and γ with respect to their baseline values stabilizes (destabilizes) the non saddle steady state. Moreover, we find that small variations of G, measuring the "size" of public expenditure, cause the double steady state to disappear. In Fig. 2.2, therefore, we report trajectories obtained for very small variations of G with respect to its baseline value.

Conclusions

This paper innovates the literature regarding dynamic IS-LM models of Schinasi's type (1981) and (1982). First of all, we find that, if the interest rate sensitivity of savings is negative, the model admits a dual steady state, characterized by the same long-run level of income but by different interest rates. One of these steady states is a saddle and the other is a non-saddle equilibrium. From the local analysis perspective, our results remain in line with the "Kaldorian tradition", namely endogenous fluctuation can only arise if the IS curve is upward sloping.

However, the global analysis provides a different perspective. By means of the homoclinic bifurcation Theorem of Kopell and Howard (1975) we are able to prove that (for specific functional forms and parameter configurations) there exist trajectories originating in the neighborhood of the non-Kaldorian steady state which spiral towards the other steady state, or to a limit cycle around it. This implies that an economy not satisfying the Kaldorian assumptions can start, at some point in time, to exhibit oscillating behavior.

We conclude the paper by proposing the results of an extensive sensitivity analysis.

Appendix A

Linearization matrix associated with system $\mathcal{M}$.

As shown in the text, Schinasi's model (1981) and (1982) gives rise to the following system of first-order differential equations

$$\begin{aligned}\dot{R} &= \tfrac{G-\tau Y}{L_R(R,Y)} - \alpha \tfrac{L_Y(R,Y)}{L_R(R,Y)}\left[H(R,Y)+G-\tau Y\right] \\ \dot{Y} &= \alpha\left[H(R,Y)+G-\tau Y\right]\end{aligned} \qquad (\mathcal{M})$$

Let $\mathbf{J}^*_{\mathcal{M}}$ be the Jacobian of the right hand side of system $\mathcal{M}$ evaluated at the steady state. The single elements of $\mathbf{J}^*_{\mathcal{M}}$ are

$$\begin{aligned}j^*_{11} &= \partial\dot{R}/\partial R|_{ss} = -\alpha\frac{L^*_Y}{L^*_R}H^*_R \\ j^*_{12} &= \partial\dot{R}/\partial Y|_{ss} = -\alpha\frac{L^*_Y}{L^*_R}H^*_Y + \tfrac{\alpha L^*_Y - 1}{L^*_R}\tau \\ j^*_{21} &= \partial\dot{Y}/\partial R|_{ss} = \alpha H^*_R\end{aligned}$$

$$j_{22}^* = \partial \dot{Y}/\partial Y|_{ss} = \alpha(H_Y^* - \tau)$$

where, for the sake of a simple representation, the arguments of the functions have been dropped. Therefore, we have

$$\mathbf{J}_{\mathcal{M}}^* = \begin{bmatrix} -\alpha \frac{L_Y^*}{L_R^*} H_R^* & -\alpha \frac{L_Y^*}{L_R^*} H_Y^* + \frac{\alpha L_Y^* - 1}{L_R^*} \tau \\ \alpha H_R^* & \alpha(H_Y^* - \tau) \end{bmatrix} \tag{A.1}$$

The eigenvalues of (A.1) are the solutions of the characteristic equation

$$\det\left(\lambda \mathbf{I} - \mathbf{J}_{\mathcal{M}}^*\right) = \lambda^2 - \mathrm{Tr}(\mathbf{J}_{\mathcal{M}}^*)\lambda + \mathrm{Det}(\mathbf{J}_{\mathcal{M}}^*)$$

where $\mathbf{I}$ is the identity matrix. $\mathrm{Tr}(\mathbf{J}_{\mathcal{M}}^*)$ and $\mathrm{Det}(\mathbf{J}_{\mathcal{M}}^*)$ are Trace and Determinant of $\mathbf{J}_{\mathcal{M}}^*$, respectively. We obtain

$$\mathrm{Tr}(\mathbf{J}_{\mathcal{M}}^*) = \alpha \left(H_Y^* - \tau - \frac{L_Y^*}{L_R^*} H_R^* \right)$$

$$\mathrm{Det}(\mathbf{J}_{\mathcal{M}}^*) = \alpha\tau \frac{H_R^*}{L_R^*}$$

Appendix B

Linearization matrix associated with system $\mathcal{S}$.

Consider now system $\mathcal{S}$ in the text

$$\dot{R} = -\tfrac{1-\gamma\alpha}{\beta}(G - \tau Y) + \frac{\gamma\alpha}{\beta}\left[\varepsilon_1\sqrt{Y^3} - \varepsilon_2 R - \varepsilon_3(1-\tau)^2 Y^2 - \varepsilon_4 R - \varepsilon_5 - \tfrac{\delta Y}{R}\right] \tag{$\mathcal{S}$}$$

$$\dot{Y} = \alpha\left[\varepsilon_1\sqrt{Y^3} - \varepsilon_2 R - \varepsilon_3(1-\tau)^2 Y^2 - \varepsilon_4 R - \varepsilon_5 - \tfrac{\delta Y}{R} + G - \tau Y\right]$$

Let $\mathbf{J}_{\mathcal{S}}^*$ be the Jacobian of the right hand side of system $\mathcal{S}$ evaluated at the steady state. The single elements of $\mathbf{J}_{\mathcal{S}}^*$ are the following

$$j_{11}^* = \partial \dot{R}/\partial R|_{ss} = -\tfrac{\gamma\alpha}{\beta}\left(-\varepsilon_2 - \varepsilon_4 + \tfrac{\delta}{R^{*2}}\right)$$
$$j_{12}^* = \partial \dot{R}/\partial Y|_{ss} = -\tfrac{\gamma\alpha}{\beta}\left(\frac{3}{2}\varepsilon_1\sqrt{\tfrac{G}{\tau}} - 2\varepsilon_3(1-\tau)^2\tfrac{G}{\tau}\right) - \tfrac{\gamma\alpha-1}{\beta}\tau$$
$$j_{21}^* = \partial \dot{Y}/\partial R|_{ss} = \alpha\left[-\varepsilon_2 - \varepsilon_4 + \tfrac{\delta}{R^{*2}}\right]$$

$$j_{22}^{*} = \partial \dot{Y}/\partial Y|_{ss} = \alpha \left(\frac{3}{2}\varepsilon_1 \sqrt{\frac{G}{\tau}} - 2\varepsilon_3 (1-\tau)^2 \frac{G}{\tau} - \tau \right)$$

where, for the sake of a simple representation, the arguments of the functions have been dropped. Therefore, we have

$$\mathbf{J}_{\mathcal{S}}^{*} = \begin{bmatrix} \frac{\gamma\alpha}{\beta} H_R^{*} & \frac{\gamma\alpha}{\beta} H_Y^{*} - \frac{\gamma\alpha-1}{\beta}\tau \\ \alpha H_R^{*} & \alpha(H_Y^{*} - \tau) \end{bmatrix} \tag{B.1}$$

where $H_R^{*} = -\varepsilon_2 - \varepsilon_4 + \frac{\delta Y^{*}}{R^{*2}}$ and $H_Y^{*} = \frac{3}{2}\varepsilon_1 \sqrt{\frac{G}{\tau}} - 2\varepsilon_3 (1-\tau)^2 \frac{G}{\tau} - \tau$. Therefore,

$$\mathrm{Tr}(\mathbf{J}_{\mathcal{S}}^{*}) = \alpha(H_Y^{*} - \tau) + \frac{\gamma\alpha}{\beta} H_R^{*}$$

$$\mathrm{Det}(\mathbf{J}_{\mathcal{S}}^{*}) = -\frac{\alpha\tau H_R^{*}}{\beta}$$

Appendix C

For the sake of a simple discussion, we shall refer to the original version of the two-parameter homoclinic bifurcation Theorem in Kopell and Howard (1975) (Theorem 7.1, p. 334).

Let (δ, τ) be our control parameters. Posit $\mu = \delta - \bar{\delta}$ and $\nu = \tau - \bar{\tau}$ where $\bar{\delta}$ and $\bar{\tau}$ be the critical values of our bifurcation parameters. Let also $\bar{R}^{*}$ and $\bar{Y}^{*}$ be the particular steady state values of the interest rate and income implied by $\mu = \nu = 0$.

Preliminarily, we translate our system of differential equation to the origin and provide a second-order Taylor expansion.

Let $\tilde{R} = R - \bar{R}^{*}$ and $\tilde{Y} = Y - \bar{Y}^{*}$. We have, from system $\mathcal{S}$

$$\begin{aligned} \dot{\tilde{R}} &= \tfrac{\gamma\alpha}{\beta} \tilde{H} \left((\bar{Y}^{*} + \tilde{Y}), (\bar{R}^{*} + \tilde{R}), (\bar{\delta} + v), (\bar{\tau} + \mu) \right) \\ &\quad + \tfrac{\gamma\alpha-1}{\beta} \left[G - (\bar{\tau} + \mu)(\bar{Y}^{*} + \tilde{Y}) \right] \\ \dot{\tilde{Y}} &= \alpha \tilde{H} \left((\bar{Y}^{*} + \tilde{Y}), (\bar{R}^{*} + \tilde{R}), (\bar{\delta} + v), (\bar{\tau} + \mu) \right) + \alpha [G - (\bar{\tau} + \mu)(\bar{Y}^{*} + \tilde{Y}))] \end{aligned} \tag{C.1}$$

where

$$\begin{aligned} \tilde{H} &= \varepsilon_1 \sqrt{(\bar{Y}^{*} + \tilde{Y})^3} - \varepsilon_2 (\bar{R}^{*} + \tilde{R}) - \varepsilon_3 (1 - \bar{\tau} - \mu)^2 \left(\bar{Y}^{*} + \tilde{Y} \right)^2 \\ &\quad - \varepsilon_4 (\bar{R}^{*} + \tilde{R}) - \varepsilon_5 - \frac{(\bar{\delta} + v)\left(\bar{Y}^{*} + \tilde{Y} \right)}{(\bar{R}^{*} + \tilde{R})} \end{aligned}$$

System C.1 corresponds to the generic two-parameter family of ordinary differential equations $\dot{X} = F_{\mu,v}(X)$ in Kopell-Howard's original Theorem. We present now, in sequence, the computation necessary to apply the homoclinic bifurcation Theorem 7.1 in Kopell and Howard to system C.1.

1. Computation of $dF_{\mu,\nu}(\mathbf{0})$. We obtain

$$dF_{\mu,\nu}(\mathbf{0}) = \begin{bmatrix} \frac{2\alpha\gamma}{\beta}\varepsilon_3(1-\bar{\tau}-\mu)\bar{Y}^{*2} - \frac{\alpha\gamma-1}{\beta}\bar{Y}^* & -\frac{\alpha\gamma}{\beta}\frac{\bar{Y}^*}{\bar{R}^*} \\ 2\alpha\varepsilon_3(1-\bar{\tau}-\mu)\bar{Y}^{*2} - \alpha\bar{Y}^* & -\frac{\alpha\bar{Y}^*}{\bar{R}^*} \end{bmatrix} \tag{C.2}$$

Simple algebra gives

$$Tr\ dF_{\mu,v}(0) = \frac{2\alpha\gamma}{\beta}\varepsilon_3(1-\bar{\tau}-\mu)\bar{Y}^{*2} - \frac{\alpha\gamma-1}{\beta}\bar{Y}^* - \frac{\alpha\bar{Y}^*}{\bar{R}^*}$$
$$\det dF_{\mu,v}(0) = -\frac{\alpha\bar{Y}^{*2}}{\beta\bar{R}^*}$$

At $(\mu, v) = (0, 0)$ (C.2) becomes

$$dF_{0,0}(0) = \begin{bmatrix} \frac{2\gamma\alpha}{\beta}\varepsilon_3(1-\bar{\tau})\bar{Y}^{*2} - \frac{\gamma\alpha-1}{\beta}\bar{Y}^* & -\frac{\gamma\alpha}{\beta}\frac{\bar{Y}^*}{\bar{R}^*} \\ 2\alpha\varepsilon_3(1-\bar{\tau})\bar{Y}^{*2} - \alpha\bar{Y}^* & -\frac{\alpha\bar{Y}^*}{\bar{R}^*} \end{bmatrix}$$

Since $dF_{0,0}(0)$ has a double zero eigenvalue, the first requirement of the Theorem is satisfied.

2. Computation of the mapping $(\mu, v) \to \big(\det dF_{\mu,v}(0),\ \mathrm{Tr}\ dF_{\mu,v}(0)\big)$.
We have

$$\begin{bmatrix} \frac{\partial}{\partial\mu}\det dF_{\mu v}(0) & \frac{\partial}{\partial v}\det dF_{\mu v}(0) \\ \frac{\partial}{\partial\mu}\mathrm{Tr} dF_{\mu v}(0) & \frac{\partial}{\partial v}\mathrm{Tr} dF_{\mu v}(0) \end{bmatrix}$$

which reduces to

$$\begin{bmatrix} 0 & 0 \\ -2\frac{\alpha\gamma}{\beta}\varepsilon_3\bar{Y}^{*2} & 0 \end{bmatrix} \neq 0$$

Therefore the second requirement of the Theorem is satisfied.

$$\tilde{H} = \varepsilon_1\sqrt{(\bar{Y}^* + \tilde{Y})^3} - \varepsilon_2(\bar{R}^* + \tilde{R}) - \varepsilon_3(1-\bar{\tau}-\mu)^2\left(\bar{Y}^* + \tilde{Y}\right)^2$$
$$- \varepsilon_4(\bar{R}^* + \tilde{R}) - \varepsilon_5 - \frac{(\bar{\delta}+v)\left(\bar{Y}^*+\tilde{Y}\right)}{\bar{R}^*+\tilde{R}}$$

3. Computation of the $Q(e, e)$ matrix.

Let $\mathbf{P}_i$, $i = 1, 2$ be the matrices of the second order derivatives of system C.1 evaluated at $(\mu, v) = (0, 0)$. We have

$$\mathbf{P}_1 = \frac{\alpha\gamma}{\beta}\begin{bmatrix} -\frac{2\bar{\delta}\bar{Y}^*}{\bar{R}^{*3}} & \frac{\bar{\delta}}{\bar{R}^{*2}} \\ \frac{\bar{\delta}}{\bar{R}^{*2}} & \varepsilon_1 \frac{3}{4}\frac{1}{\sqrt{\bar{Y}^*}} - 2\varepsilon_3(1-\bar{\tau})^2 \end{bmatrix}$$

$$\mathbf{P}_2 = \alpha\begin{bmatrix} -\frac{2\bar{\delta}\bar{Y}^*}{\bar{R}^{*3}} & \frac{\bar{\delta}}{\bar{R}^{*2}} \\ \frac{\bar{\delta}}{\bar{R}^{*2}} & \frac{3\varepsilon_1}{4}\frac{1}{\sqrt{\bar{Y}^*}} - 2\varepsilon_3(1-\bar{\tau})^2 \end{bmatrix}$$

Let us now compute the **right** eigenvector $\mathbf{e} = (e_1, e_2)^T$ of $\mathbf{J}^*_{\mathcal{S}}$. A possible candidate is

$$\mathbf{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} -\frac{\frac{\alpha\gamma}{\beta}\left(\frac{3}{2}\varepsilon_1\sqrt{\bar{Y}^*} - 2\varepsilon_3(1-\tau)^2\bar{Y}^* - \frac{\delta}{\bar{R}^*}\right) - \frac{\alpha\gamma-1}{\beta}\bar{\tau}}{\frac{\gamma\alpha}{\beta}\left(-(\varepsilon_2+\varepsilon_4)+\delta\frac{\bar{Y}^*}{\bar{R}^{*2}}\right)} \\ 1 \end{bmatrix}$$

Therefore

$$Q(e,e) = \frac{1}{2}\begin{pmatrix} e^T\mathbf{P}_1 e \\ e^T\mathbf{P}_2 e \end{pmatrix} = \frac{\alpha}{2}\begin{pmatrix} -\left(\frac{j^*_{12}}{j^*_{11}}\right)^2 \frac{\gamma}{\beta}\frac{2\bar{\delta}\bar{Y}^*}{\bar{R}^{*3}} + \frac{\gamma}{\beta}\left(\frac{3\varepsilon_1}{4}\frac{1}{\sqrt{\bar{Y}^*}} - 2\varepsilon_3(1-\bar{\tau})^2\right) \\ -\left(\frac{j^*_{12}}{j^*_{11}}\right)^2 \frac{2\bar{\delta}\bar{Y}^*}{\bar{R}^{*3}} + \frac{3\varepsilon_1}{4}\frac{1}{\sqrt{\bar{Y}^*}} - 2\varepsilon_3(1-\bar{\tau})^2 \end{pmatrix}$$

Finally

$$\left[dF_{0,0}(0), Q(e,e)\right] = \begin{bmatrix} \frac{2\alpha\gamma}{\beta}\varepsilon_3(1-\bar{\tau})\bar{Y}^{*2} - \frac{\alpha\gamma-1}{\beta}\bar{Y}^* - \left(\frac{j^*_{12}}{j^*_{11}}\right)^2 \frac{\gamma}{\beta}\frac{2\bar{\delta}\bar{Y}^*}{\bar{R}^{*3}} \\ +\frac{\gamma}{\beta}\left(\frac{3\varepsilon_1}{4}\frac{1}{\sqrt{\bar{Y}^*}} - 2\varepsilon_3(1-\bar{\tau})^2\right) \\ 2\alpha\varepsilon_3(1-\bar{\tau})\bar{Y}^{*2} - \alpha\bar{Y}^* - \left(\frac{j^*_{12}}{j^*_{11}}\right)^2 \frac{2\bar{\delta}\bar{Y}^*}{\bar{R}^{*3}} \\ +\varepsilon_1\frac{3}{4}\frac{1}{\sqrt{\bar{Y}^*}} - 2\varepsilon_3(1-\bar{\tau})^2 \end{bmatrix} \tag{C.3}$$

where $j^*_{11} = \frac{\gamma\alpha}{\beta}\left(-(\varepsilon_2+\varepsilon_4) + \delta\frac{\bar{Y}^*}{\bar{R}^{*2}}\right)$ and $j^*_{12} = \frac{\gamma\alpha}{\beta}\left(\frac{3}{2}\varepsilon_1\sqrt{\bar{Y}^*} - 2\varepsilon_3(1-\tau)^2\bar{Y}^* - \frac{\delta}{\bar{R}^*}\right) - \frac{\alpha\gamma-1}{\beta}\tau$. Since (C.3) has rank 2, the third requirement of the Theorem is satisfied.

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Chapter 3
Determining the Relationship Between Co-creation and Innovation by Neural Networks

Giacomo di Tollo, Stoyan Tanev, Kassis Mohamed Slim and Davide De March

Abstract The growing complexity of markets, business development and administration has fostered the application of more sophisticated quantitative methods aiming at the analysis of common features and differences amongst different businesses. Amongst those quantitative methods, Neural Networks are gaining support of both practitioners and scholars. This is due to their generalisation capabilities which make them apt to be used without any preliminary assumptions about the variables at hand or about the specific types of the corresponding models. To this extent, we are using them to classify firms w.r.t. the relationship between the perception of their innovativeness and the degree of their involvement in value co-creation activities—the extent to which they involve end users in the definition of their final products and services. We will show that businesses from specific sectors could have a higher degree of involvement in value co-creation. The mapping between the type of firms and the degree of their involvement in value co-creation is of particular

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interest since they describe attributes and activities and a completely different heuristic level. We have also studied businesses belonging to stock Exchange indexes, which are regarded as the specimen of the economic and financial situation of a Country. Our main contribution will be in translating the applicability of ANN in innovation research.

Keywords Innovation complexity · Value co-creation · Artificial Neural Networks (ANN)

Introduction

The concept of Value Co-creation (Chesbrough 2011; Ramaswamy and Gouillart 2010; Tanev et al. 2009; Mindgley 2009) is nowadays well established in the marketing and innovation communities (Prahalad and Krishnan 2008; Lusch and Vargo 2006; Prahalad and Ramaswamy 2004), describing how customers and end users could be involved as active participants in the process of value creation (Prahalad and Ramaswamy 2004; Etgar 2008; Payne et al. 2008). It refers to the situation in which businesses enable users to become an active part in the different stages of the production process, by letting them define the main components of the market offer and thus shape specific personalized products, services and experiences. The focus on the relationship between production and customer emphasizes the collaborative nature of the interaction between firms and consumers (Hoyer et al. 2010). There is a growing emergence of participatory platforms and co-creation practices focusing on further enhancing the opportunities for user innovation, in particular when they are enabled by a broader and more systematic positioning of customers and end users across the entire innovation lifecycle. Individual co-creation experiences are becoming essential for the emergence of innovation networks through the development, access and dynamic reconfiguration of appropriately designed technological, business process and human resource infrastructures (Prahalad and Krishnan 2008). In this sense, the co-creation paradigm can be described as a market-driven approach based on an open innovation business philosophy.

The participation of customers in co-creation activities should impact the innovation outcomes (Kristenson et al. 2008; Prahalad and Krishnan 2008; Nambisan and Baron 2009; Bowonder et al. 2010; Ramaswamy and Gouillart 2010). Literature about value co-creation is predominantly qualitative-oriented, and most of the works emphasize the role of the customer in both co-creating the innovation outcomes and contributing to the reduction of innovation cost and time-to-market as well as to better quality and capacity to address emerging market needs (Kristenson et al. 2008; Prahalad and Krishnan 2008; Nambisan and Baron 2009; Nambisan 2009; Midgley 2009; Romero and Molina 2009; Bowonder et al. 2010; Ramaswamy and Gouillart 2010). Furthermore, co-creation practices tend to be assessed from innovation and cooperation perspectives alone, neglecting collateral effects such as brand perception, customer satisfaction, or customer relationship quality (Nambisan and Baron 2009; Nambisan 2009).

In our work instead we want to carry a quantitative analysis of the relationship between co-creation and innovation, showing that there is a positive association between the intensity of the co-creation activities of a business and its innovation potential. A first hint in this direction has been given by Tanev et al. (2011), who use linear regression and neural networks in order to examine the relationship between the degree of firms' involvement in value co-creation activities and the frequency of their online comments about their new products, processes and services. This work has focused on a dataset composed of OSS oriented businesses. We want to improve this approach by applying the analysis provided therein to other datasets, which are not focused to OSS business only, and which represent more closely the reality of the economic and financial situation of a country, in order to see whether the conclusions about innovation and co-creation drawn by Tanev et al. (2011) hold without loss of generalisation. The aforementioned previous study has shown that firms with a higher degree of involvement in co-creation activities have a better opportunity to articulate the innovative features of their new products, processes and services. We want to see whether this assertion can be generalised to broader business categories.

Value Co-creation and Innovation

As stated in the Introduction, co-creation makes economic subjects to re-think their concept of innovation (Prahalad and Krishnan 2008; Kristenson et al. 2008; Tanev et al. 2009), by stressing two points: first, the customer-driven aspect of the value co-creation activities; then, the focus on a balance between cooperation and competition, called *co-opetition*.

As for the first point, we can say that value co-creation platforms can be seen as a natural extension of some key aspects of user-driven innovation initiatives (von Hippel 2006a,b) by focusing on the development of participation platforms (von Hippel 2001; Nambisan and Nambisan 2008; Nambisan and Baron 2009), which enables a broader and more systematic positioning of customers and end users across the entire innovation lifecycle. For this reson, the development of value co-creation platforms is increasingly recognised as a promising innovation strategy (Prahalad and Ramaswamy 2003; Nambisan and Baron 2009; Nambisan 2009; Romero and Molina 2009; Midgley 2009; Bowonder et al. 2010) leading firms to adopt adaptive leadership and adaptive management practices (Prahalad and Ramaswamy 2004; Desai 2010).

As for the second point, the balance between cooperation and competition have us think more dynamic scenarios of the innovation processes: the economic mechanisms which trigger innovation are based on complex relationships an transactions amongst customers, partners, and suppliers, at all points on the value network. In those scenarios, a key role is played by customers and end users, since in this way they are eventually capable to control the relationship between price and user experience (Etgar 2006; Prahalad and Ramaswamy 2004), to create specific value chain configurations, and to determine new ways of using existing products.

These two points are of the utmost relevance, expecially because the traditional innovation and marketing literature has been so far predominantly focused on the firms' activities, making customers unapt to become part of the value creation process (Vargo and Lusch 2004, 2008).

Instead, in a co-creation context, end user (which are referred to as *innovators* or *co-creators*) are rather active stakeholders who can define the type of interaction and the specific personal context of the encountering event (Prahalad and Ramaswamy 2003): this new paradigm could lead to a disruptive innovation business models (Christensen 2006), in which firms have to change their approaches to reach customers (Payne et al. 2008), by re-defining their understanding of the value chain into a dynamic value network, producing complex relationships between producers, suppliers, customers and end users.

Research Methodology

Our research is focused on analysing public data available on the Internet to understand the importance given by firms to the concepts and to the activities represented by some given regular expression.

This kind of analysis has already been used by (Hicks et al. 2006; Ferrier 2001), and applied to co-creation and innovation matters by (Allen et al. 2009; Tanev et al. 2011). The rationale behind the latter approaches is that such methodology could be used to classify value co-creation practices, and one need to formalise the key steps of the data gathering and analysis work flow, showing that the frequencies of a specific set of regular expressions can be used to extract the key components of value co-creation activities, and using those ideas to outline a detailed research process. This process can be summarised as follows:

- To construct a set of regular expressions in order to represent the different value co-creation constitutive dimensions and to measure the frequency of use of each of those regular expressions on companies' websites;
- To construct a set of regular expressions in order to represent the perception of firms' innovativeness and to measure the frequency of use of each of the regular expressions on companies' websites;
- To use Principal Component Analysis (PCA) to identify emerging groups of regular expressions that could be associated with specific self-consisting groups of activities (components);
- To apply correlation analysis and ANN approach to model the relationship between co-creation and innovation (if any).

Benchmarks

We will apply the procedure outlined in the section "Research Methodology" to the same research sample used by (Tanev et al. 2011) (which represents a set of innovation-oriented firms) and to firms that are included in the FTSE and DOWJONES indexes (which represent instead a global image of the economic and financial situation of two different countries). Our goal is to understand whether co-creation and innovation are nowadays considered as an important asset by large part of the firms, or just by a small subset of them, whose interest for these paradigms is justified by their main activity (i.e., the product and services they supply). The choice of Stock Exchange Indexes from the USA and from the UK is justified by the fact that regular expressions have been defined in English. Further work will be devoted to define regular expressions in several different languages in order to assess the specific situation of several countries.

The sample used by (Tanev et al. 2011) relies mainly on OSS firms, which are good representatives of firms mastering the main building blocks of value co-creation (Prahalad and Ramaswamy 2004) since they actively contribute to the development of OSS participation platforms and engage in dialogue with multiple external contributors who are most often the end users. They also provide access to their source code, to their internal resources and development processes. In addition, they share IP management and development risk with external contributors and end users as well as enable a high degree of transparency through development forums and newsgroups. In this category we have also selected, amongst firms belonging to Eclipse OS Foundation, Open Source Experts—www.opensourceexperts.com, and Canadian Companies Capabilities Directory of OS Companies—http://strategis.ic.gc.ca/epic/site/ict-tic.nsf/en/h_it07356e.html, 271 firms whose revenue relies on OSS.

FTSE and DOWJONES barely need explaination since they are representatives of the markets of two amongst the most industrialised country of the world. They include businesses from several sectors, carrying out (purportedly) different policies of innovation and co-creation (if any). Up to the authors knowledge, this is the first time that businesses belonging to those indexes are analysed w.r.t. their focus on innovation and co-creation. Both indexes contain 100 firms. In our dataset we have selected 98 firms from the DOWNJONES index and 95 from the FTSE one since some website was not working properly. In sections "Value Co-creation Components" and "Innovation Variable" we are going to describe the metrics used to model co-creation and innovation. These metrics will be used for the experimental analysis in what follows.

Value Co-creation Components

The first step of the research process consists on measuring the frequency of the specific regular expressions on the several businesses websites. This step was repeated for each of the three benchmarks. This has been made by mean of a tool developed by Giacomo di Tollo and available at http://lisibox.univ-littoral.fr/CoCreation.

The next step consisted in the application of Principal Component Analysis (PCA) Landau and Everitt 2004 to the three datasets collected in the previous step. The goal is to identify independent groups (PCA components) of regular expressions (and the co-creation activities associated with these regular expressions) that tend to appear together on firms websites.[1]

The number of components in each of the instances was determined by a classical of the Scree plot analysis (see (Field 2005, p. 633)). The application of this analysis resulted in 3 co-creation components for the OS related benchmark, 4 components for the NASDAQ firms and 2 components for the FTSE firms. The meaning of the components has been interpreted on the basis of the relevance (loading values) of each of the regular expression within a given component (Field 2005). The keywords with the highest loading values were given higher priority in the interpretation. Hence, for the OSS related benchmark, the 3 components have been interpreted as:

1. Using information about customer preferences and insights from tests and beta trials to shape design solutions;
2. Cooperation and partnerships with customers focusing on using their experiences as a basis for the evolution of existing products, processes and services;
3. Addressing IP issues with customers using companys integrated online services to disclose and reveal relevant information.

For the NASDAQ related benchmark, the 4 components have been interpreted as:

1. Providing integrated online services to reduce customers cost and exposure to risk;
2. Using customer partnerships and user forums to learn about customer experiences;
3. Using/providing internal resources to help customer reveal and disclose relevant information by taking care of all emerging IP issues;
4. Using product and process modularity to provide users with simulation and modeling toolkits, virtual world applications, and software development kits.

For the FTSE related benchmark, the 2 components have been interpreted as:

1. Using information revealed by customers to enable cost reduction by providing a variety of resources aiming at design and process flexibility;
2. Using customer forums and user networks to enable cooperation and partnerships focusing on learning from customer experiences and risk management.

The regular expressions' composition of each of the components has been used to determine the co-creation variables for each of the firms in the three samples by summing up the rating of each of the regular expressions weighted by their specific loadings.

[1] As for the Principal Component Analysis we have used the Vari-max rotation method. Parameters for accepting the PCA analysis were: $correlation\ table\ determinant > \frac{10}{-5}$, $Kaiser-Meyer-Olkinmeasure > 0.5$, $Bartletts\ significance\ level < 0.05$.

Table 3.1 Main statistics of the co-creation variables

Component	Mean	STD	Skewness	Kurtosis
OSS 1	0,398	0,359	2,202	9,794
OSS 2	0,357	1,642	10,072	104,752
OSS 3	0,403	1,831	6,711	49,357
NASDAQ1	0,111	0,159	3,434	16,724
NASDAQ2	0,508	0,410	1,117	1,259
NASDAQ3	0,069	0,121	3,667	15,496
NASDAQ4	0,142	0,169	2,529	7,761
FTSE1	0,419	0,299	1,619	4,075
FTSE2	0,442	0,356	1,816	5,290

Table 3.2 Main statistics of the innovation variable

Innovation Metric	Mean	STD	Skewness	Kurtosis
OSS	0,270	0,1819	1,130	1,734
NASDAQ	0,270	0,166	0,549	0.003
FTSE	0,342	0,276	3,064	12,301

Table 3.1 shows the main descriptive statistics all co-creation variables for all instances that were constructed by adding up the ratings of each keyword weighted by its loading (see (Tanev et al. 2011) for further details about this procedure).

Innovation Variable

As for the innovation metric, we have used the firm's own *perception* of innovativeness, defined by (Tanev et al. 2011) by measuring the frequency of firms' online comments about new products, services and processes by looking for the following regular expression: new $\wedge$(product $\vee$ service $\vee$ process $\vee$ application $\vee$ solution $\vee$ feature $\vee$ release $\vee$ version $\vee$ launch $\vee$ introduction $\vee$ introduce $\vee$ "new product" $\vee$ "new service" $\vee$ "new process" $\vee$ "new solution" $\vee$ "product launch"). The frequency of this regular expression has been searched for on the three instances at hand by mean of a tool developed by Giacomo di Tollo and available at http://lisibox.univ-littoral.fr/CoCreation. Please notice that, only one regular expression is used, so no Principal Components Analysis is needed. Table 3.2 shows the main descriptive statistics of the innovation variable for all instances.

Correlation Analysis

After having defined the co-creation and innovation variables, the next step of the procedure outlined in (Tanev et al. 2011) is to find the correlation between the value co-creation component variables and the degree of firms articulation of their

Table 3.3 Rank Based Correlation, OSS instance

	Innovation Metric	Co-creation 1	Co-creation 2	Co-creation 3
Innovation Metric	1,000	0,662	0,564	0,451
Co-creation 1	0,662	1,000	0,632	0,591
Co-creation 2	0,564	0,632	1,000	0,515
Co-creation 3	0,451	0,591	0,515	1,000

Table 3.4 Rank Based Correlation, NASDAQ instance

	Innovation Metric	Co-creation 1	Co-creation 2	Co-creation 3	Co-creation 4
Innovation Metric	1,000	0,622	0,649	0,575	0,300
Co-creation 1	0,622	1,000	0,584	0,695	0,505
Co-creation 2	0,649	0,584	1,000	0,644	0,533
Co-creation 3	0,575	0,695	0,644	1,000	0,464
Co-creation 4	0,300	0,505	0,533	0,464	1,000

Table 3.5 Rank Based Correlation, FTSE instance

	Innovation Metric	Co-creation 1	Co-creation 2
Innovation Metric	1,000	0,355	0,488
Co-creation 1	0,355	1,000	0,588
Co-creation 2	0,488	0,588	1,000

innovativeness. (Tanev et al. 2011) uses linear regression to this extent. We have considered the hypotesis of performing linear regression, but the analysis of the descriptive variables' statistics for each of the three instances indicated an high degree of skewness. Hence, instead of linear regression, we have used the Spearman ranked-based correlation coefficients, which are reported in Tables 3.3, 3.4 and 3.5.

The analysis of the data shows that, on average, the sample of firms dominated by the OSS firms manifests the highest degree of correlation between the value co-creation components and the online innovation metric. This result is not surprising since firms related to OSS development tend to use their web pages as participation platforms which allows for a better articulation of the innovativeness of their products and services. Please notice that all paiswise correlations but the *innovation* VS *NASDAQ components 4* are statistically significant accordingly to the Rank Based analysis. Combining these results with the ones shown in Table 3.2, we can already draw an interesting conclusion: the FTSE benchmark shows higher innovation values than NASDAQ and OSS business, even if with an higher standard deviation. This is quite an interesting consideration, since our hypotesis was that OSS business would have shown the highest values. It apparently shows a concrete feature of the British companies, for which the focus on innovation represents an asset.

A Neural Network Approach to Model Innovation Based Outcomes

In this section we will apply a neural network approach to determine whether a relationship between the innovation and co-creation variables exists, on the three instance sets we have defined in section "Research Methodology". A similar approach has been introduced by (Tanev et al. 2011), but just focused on the first instance set. Instead, we want to see whether such a relationship holds over more generical dataset, in order to understand if innovation and co-creation are important features of the actual economic and financial context or not.

Artificial Neural networks (ANN) (Hykin 1999; Angelini et al. 2008) are algorithms whose behavior mimics the human brain, which are composed of elementary units (neurons) that are connected to create a network capable of solve complex problems, especially when the problem model is unknown in advance and when the relationships amongst the different components are non-linear. Neurons are connected by synapses, which represent the parameters of the neural network: each synapses is associated a numerical value that (can) changes over time by means of a learning procedure, by which the network better fit with the observed output.

The most common ANN model is the feed-forward neural network, which is regarded to as a general function approximator and which is composed of neurons which are partitioned in *layers*, in which no neuron belonging to a layer can have connections with other neurons belonging to the same layer. Given an ordinal sequence of layers $1 \cdots n$, every neuron belonging to layer l $(0 \leq l \leq n)$ is allowed to have only unidirectional synapses to layer $l + 1$. In this framework, a layer h $(0 < h < n)$ is referred to as *hidden layers*. Generally, one hidden layer only can approximate any function with a finite number of discontinuities, arbitrarily well, given sufficient neurons in the hidden layer (Hagan et al. 1996). We have to remark that Neural Networks are robust with respect to noisy and missing data, which do not hinder the network operations (but of course trigger some degree of tolerable performance degradation). All those requirements make their use appropriate for our problem. We forward the interested reader to (Bishop 1996; Rumelhart et al. 1986) for more insight about Neural Networks.

In this research we consider the three value co-creation components, defined in section "Value Co-creation Components", as the input variables and the perception of innovation, as defined above.

Neural Networks to Determine the Correlation Between Co-creation and Innovation

The results obtained by ANN clearly indicate that there is a relationship between the actual and desired outputs, and this assertion is of the utmost importance, since it is observed over the test set. It suggests that, since the network has been trained using

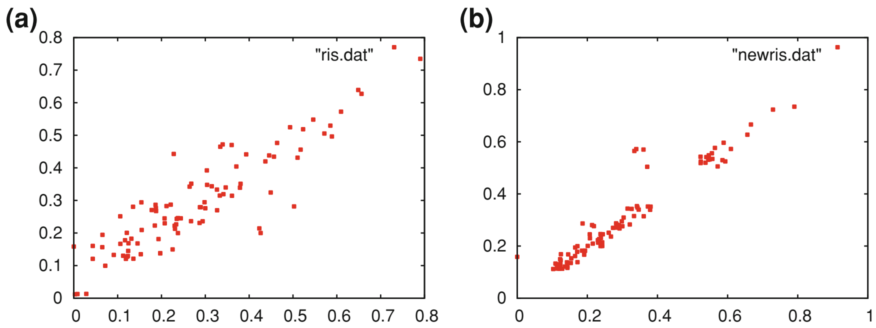

Fig. 3.1 Relationship between desired and actual output for two different train-test partitions. The x-axis corresponds to the expected output value ($perception$); the y-axis corresponds to the actual network value

the co-creation component values, the variation of the co-creation components is able to explain firms' perception of innovation. This could be seen on Fig. 3.1, where the expected output (innovation) for the test examples is shown along the x-axis, and the actual network output (innovation) for the same dataset is shown along the y-axis. Just results obtained by tackling two different partitions of data are reported. Other partitionings lead to comparable behaviors.

In order to verify if there is a generalised trend of correlation between the current network output and the desired output (innovation metric) found over the 30 different training-test partition, we plot, in Fig. 3.2, the cumulative empirical distribution of Spearman's rank based correlation (Spearman 1904) value between desired and current network's output values. We decided to introduce a correlation analysis instead of to define an error measure due to the lack of such an error measure in previous research. In order to assess an error measure, we should have introduced a subjective threshold, without no guarantee on the soundness of this threshold. A correlation analysis instead, just relies on data, without further manipulation and without taking into account the variable scale. Furthermore we decided to use the rank-based correlation rather than, i.e., Pearson correlation, in order to evidence non-linear features between variables. It is nonetheless worthwhile to notice that rank based correlation and Pearson indicator lead to comparable results.

We can see that there exists a positive rank-based correlation on the variables under examination, and even in the cases where this relationship appears to be weaker, it is never smaller than 0.5. The correlation measure is greater than 0.85 in 70 % of the cases, i.e. the positive relationship between variables appears to be robust. Hence, we can conclude that Neural Networks can be used to examine the relationship between the co-creation component and firms perception of innovation (see section "Neural Networks to Determine the Correlation Between Co-creation and Innovation"). This result is in agreement with the results from the linear regression analysis provided by (Tanev et al. 2011), which shows that there is a statistically significant positive association between the perception of innovation and the value co-creation components

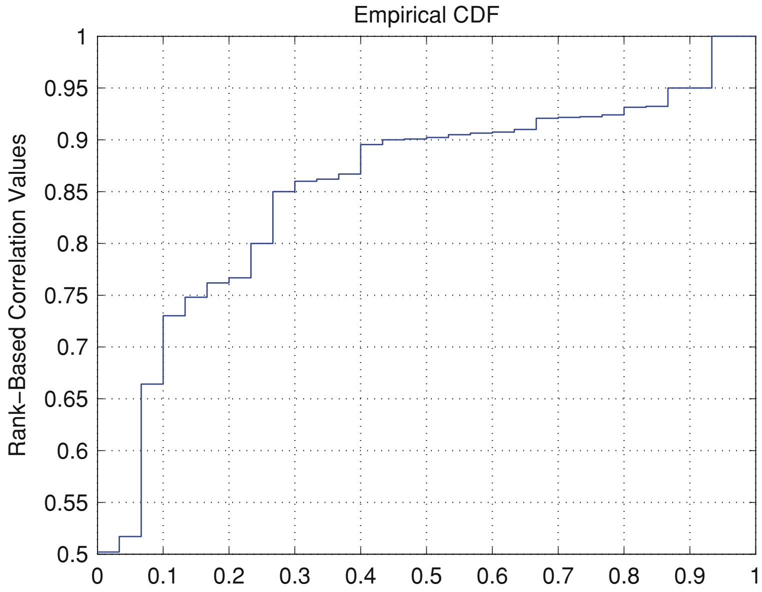

Fig. 3.2 Cumulative Distribution of the Spearman's Rank Based Correlation values between desired output (*perception*) and actual output over 30 train-test partitions

"Customer relationships enabled through partnerships and cooperation" and "Mutual learning mechanisms". The agreement and the high explanatory power of the linear regression model (49.0 %, assessed by the adjusted R square value) suggest that linear models are quite adequate in describing the relationship between value co-creation and the perception of innovation, also showing with the additional advantages of being less time consuming as well as being able to identify the dominant role of specific co-creation components. The combination of the results from the ANN and linear regression analysis provides evidence in support of one of initial hypothesys that more co-creative firms are in a better position to differentiate themselves by emphasising the innovative aspects of their new products, processes and services.

The good agreement between the ANN and linear regression does not address the question of how good the online innovation metric is in describing the innovative capacity of firms. The answer to this question requires the additional research focusing on the relationship between the three value co-creation components and some traditional innovation metrics based on the number of new products, processes and services.

Conclusions

The present study provided an ANN analysis to examine the relationship between the degree of value co-creation activities and firms' innovativeness. Although, it is impossible to claim the existence of a causal relationship, the results suggest that value co-creation practices could be considered good indicators of the firms innovation-related outcomes such as the degree of online articulation of the innovative aspects of their new products, processes and services. The advantage of such approach can be found in the opportunity to test the existence of this relationship without any preliminary assumption about its specific functional form. This opportunity appears to be highly relevant given the early stage of quantitative value co-creation research and the still limited knowledge about the relationship between co-creation and innovation.

The main contributions of this work should be seen in the specific methodological setting, since it could open the way for applications of ANN modeling to co-creative innovation research.

Up to the authors knowledge, this is the first application of these connectionists approaches to innovation and co-creation. These connectionist approaches have shown a high degree of flexibility and performance in adaptation and prediction.

We could however suggest as a subject of future research the development and the comparison of different neural networks in terms of topologies and connections in order to generate reliable and robust models to predict more complex innovation activities. One should also compare our approach with other unsupervised appraoches to determine, whether or not, a neural network model platform could be suited to simultaneously model and classify such kind of data sets. The potential value of such modeling could be found in their ability to take into account the inherent complexity and the emerging nature of value co-creation networks.

We stress out the fact that the results shown here were based on an online innovation metric that was introduced recently in the literature.

Such an approach will provide an opportunity for future research to focus on the development of specific online innovation metrics to overcome the limits of more traditional ones, such as the ones suggested in the OSLO manual: this could open new research areas focusing on the development of business intelligence and innovation research tools that would increase the utility of both managers and researchers.

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Chapter 4
On the Fractal Characterization of a System for Tradings on Eurozone Stocks

Marina Resta

Abstract It is a common habit among practitioners to maintain under strong control the behavior of a bunch of indexes that are known to capture the movements of Eurozone stocks. Baltic Dry Index (BDI), RJ/CRB Commodity Price Index (CRB), Chicago Board Options Exchange Volatility Index (VIX) and Deutsche Bank G10 Currency Future Harvest Index (DBHI), in fact, are supposed to exhibit a kind of anticipatory behavior with respect to that of Eurozone economy: understanding their dynamics should therefore imply to know in advance how the economic system will behave. The rationale of this chapter is to verify to what extent the use of tools relying on chaos theory and complexity studies (in our case: multiscaling analysis) can be of any help to capture such anticipatory movements. To do this, we performed two separate tasks: we evaluated the Hurst exponent of the aforementioned indexes using a set of techniques, to give robustness to the results; we then moved to compute for each of them the Hölderian function values. The results suggested us the track along which developing a trading system based on the fractal characterization of the Eurostoxx 50 index whose performance will be provided and discussed as well.

Keywords Hurst exponent · Hölderian function · Trading system

Introduction

Studying financial indexes is generally acknowledged as a method (a) to provide a robust explanation (in mathematical sense) of how does the market work, and (b) to assure proper room for profit. With this spirit, during the past decades a huge literature apparatus bloomed with focus on the challenging issue to develop quantum-based trading strategies: i.e. a set of rules (generally automated into a programming

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language code) that suggest the investor the position to assume into the market: long (buy), short (sell), or null (doing nothing). Contributions include neural networks-based strategies (Azoff 1994; Fernandez-Rodriguez et al. 2000; Resta 2009), systems based on genetic algorithms (Allen and Karjalainen 1999; Arifovic and Gencay 2001; Bauer 1994), and hybrid solutions (Dourra and Siy 2002), coping together soft computing and technical analysis (Murphy 1999). However, a common aspect of the existing literature relies in the poor compliance of the suggested techniques to the very features of observable markets: models are good enough in theory, but they are rarely applied by either operators or investors. On the other hand, tools employed by practitioners like candlesticks (Nison 1994), and technical analysis can seem satisfactory, but generally suffer for severe drawbacks due to poor statistical soundness.

The point of our contribution is to offer a different approach to the problem, trying to define a common ground where practice meets academia: instead of suggesting a new technique to operate in the market, we move from an existing technique, commonly applied by operators, and we suggest a way to improve its performances based on the idea of the Hurst exponent (Hurst 1951), and more generally on multi-scaling analysis (Mandelbrot et al. 1997).

Going deepest into the detail, we examined the so called Ghost Index[1] (GI), which is supposed to exhibit a kind of anticipatory behavior with respect to that of the Eurozone economy. Studying GI under both the practitioner and the econometrician's points of view, we indicate the path towards which improving its financial performances as well as its statistical robustness. This way implies the use of the Hurst exponent to characterize the GI index components and then the estimation of the correspondent Hölderian function values (Ayache and Lévy Véhel 2004).

The chapter structure is therefore as follows. Section "Lights and Shadows on the Ghost Index" after a brief description of the Ghost Index (GI) and its components, examines the financial performance of GI and its statistical coherence; this will lead us in search of a better characterization of GI. This is provided in section "H-Characterization of the Ghost Index", where we denote as H-characterization the analysis of both GI and its components by way of the Hurst exponent first, and the Hölderian function later. The task is performed by way of various techniques, in order to give more robustness to the results. Moving from this renewed version of GI, in section "Tradings with Time-Dependent H" we tested its anticipatory features on the Eurostoxx 50 index. Section "Conclusion" concludes.

Lights and Shadows on the Ghost Index

From the mathematical viewpoint, the Ghost Index (GI from now on) is defined as follows:

[1] *See* http://www.finanzaonline.com/forum/25986512-post23.html

$$Gl(t) = \frac{\log BDI(t)}{2} + \frac{30}{VIX(t)} + \log CRB(t) + \frac{\log DBHI(t)}{3}$$

where BDI is the Baltic Dry Index[2], VIX is the Chicago Board Options Exchange Volatility Index[3] (Brenner and Galai 1989), CRB stands for Thomson Reuters/ Jefferies Commodity Price Index[4], and DBHI is the Deutsche Bank Currency Future Harvest Index[5].

The capability of GI to provide operative signals is entirely contained in its components: whereas BDI gives an assessment of the price of moving the major raw materials by sea, VIX measures the implied volatility of S&P 500 index options, and as such, it is generally acknowledged to represent a measure of the market's expectation of stock market volatility over the next 30 day period. Finally, while CRB provides a dynamic representation of broad trends in overall commodity prices, on the other hand DBHI reflects the return from investing long in currency futures, for currencies with relatively high yielding interest rates, and going short for currencies with relatively low yielding interest rates. In practice, combining those components into a single index assures the investor with a broad range coverage of various market aspects. Moreover, GI is claimed to have a kind of anticipatory behavior with respect to the Eurostoxx 50 (EXX50), Europe's leading Blue-chip index for the Eurozone[6], so that it is often employed for active speculation on that market.

Figure 4.1 provides a view on the behavior of the logarithm of GI and EXX50, respectively, observed in the period 28 July 2009–31 January 2012, from which the reader may take an idea of how (apparently) the two indexes seem to co-move.

However, in order to be sure whether the goodness of results can be due either to the index, or to the trader's skill, we regressed the log returns of EXX50 on those of GI in the period of observation; early 70 % records (from 28 July 2009 to 28 April 2011) were employed as training set, the remaining 30 % (from 29 April 2011 to 31 January 2012) as test set:

$$r_{Exx50}(t) = \beta_0 + \sum_{k=1}^{l0} \beta_k r_{GI}(t-k) + \varepsilon(t)$$

where $r_{Exx50}(t)$ is the log-return[7] of EXX50 at time t, β_0 is the constant term, β_k (k $=$ 1, ..., 10) is the coefficient associated to the k-th regression term $r_{GI}(t\ k)$,

[2] http://www.balticexchange.com/

[3] http://www.cboe.com/

[4] http://www.jefferies.com/

[5] https://index.db.com/staticPages/DBCFH.html

[6] http://www.stoxx.com/indices/index_information.html?symbol=SX5E

[7] Remember that the log-return $r(t)$ between times t $-$ 1 and t can be derived from the corresponding prices levels $p(t-1)$ and $p(t)$ as:

$$r(t) = log^{p(t)}_{p(t-1)}$$

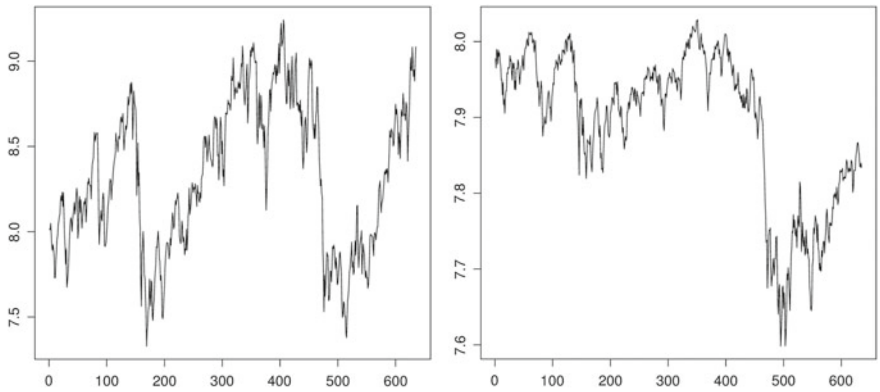

Fig. 4.1 From *left* to *right*: the behavior of the GI index in the period July 2009–31 January 2012 (*left*), compared to that of EXX50 (*right*). On *x axes* we reported the days, being 1 = 28 July 2009. On *y axis* we provided the log value of the examined indexes

Table 4.1 Results for the regression of *log* EXX50 on *log* GI

Coefficient	Value	SE	t-statistics	p-value
β_0	0.000271	0.000321	0.7837	0.4333
β_1	0.281472	0.011412	0.1741	0.0024
β_2	−0.02218	0.010001	−0.8871	0.1233
β_3	0.000072	0.012362	0.1989	0.1517
β_4	0.003259	0.017847	0.1798	0.1771
β_5	−0.02880	0.016845	−0.0845	0.0321
β_6	−0.00349	0.026787	−0.091	0.0329
β_7	0.003478	0.015325	0.9987	0.7058
β_8	0.005677	0.015587	0.5335	0.3554
β_9	0.007440	0.016530	0.9122	0.2597
β_{10}	0.000203	0.016974	0.9154	0.3480

Estimated coefficients values are in Column 2, corresponding Standard Error (SE), t-statistics and p-value are provided in Columns 3–5

and $\varepsilon(t)$ is the normally distributed error term. Table 4.1 shows the results of the regression up to the 10th lag.

Looking at the values in Columns 4–5, one might conclude that by combining the pure first and fifth terms of the regression, it is possible to get a proper forecast of the desired variable. More simply, those results suggest that a good forecast of $r_{Exx50}(t)$ may be given by combining $r_{GI}(t-1)$ with $r_{GI}(t-5)$:

$$\hat{r}_{Exx50}(t) = 0.281472\ r_{GI}(t-1) - 0.02880 r_{GI}(t-5)$$

It is a matter of fact that using this GI-based system to anticipate the movements on the EXX50, i.e. taking long (buying) positions when GI log returns started to

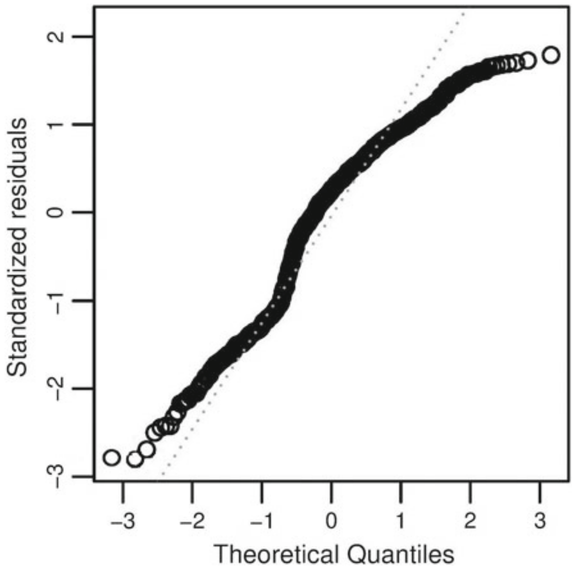

Fig. 4.2 Quantile–Quantile plot opposing theoretical quantiles to standardized residuals

Table 4.2 White's test for residual heteroscedasticity

Coefficient	Value	t-statistics	p-value
α_0	0.000341	7.959	4.17e-015
α_1	0.090574	5.054	5.12e-05
α_2	0.105976	7.590	1.19e-017
α_3	0.123914	9.435	3.47e-025
α_4	0.140179	8.534	2.86e-021
α_5	0.167898	12.921	3.13e-032

In order to reject the presence of ARCH effects, the p-value = Pr(Chi-square(5) > LM) is asked to stay beneath 1.811005e-186

become positive, and going short (selling) when they began to be negative, made possible to maintain a 59 % of successfully trades in the test set.

Nevertheless, the model residuals are far to be normally distributed, as shown by the quantile-quantile plot (Chambers et al. 1983) in Fig. 4.2.

This first glance impression is confirmed by the results of the White test (White 1980) for heteroscedasticity in residuals, as shown in Table 4.2.

This lead us to preliminary conclude that while it is possible to build an anticipatory model of the EXX50 based on the GI, the resulting system is of poor statistical soundness. The very key issue is therefore to verify if it is possible to improve the statistical robustness of the GI index: in next section we are going to provide some evidence in such sense.

H-Characterization of the Ghost Index

A Brief Guideline on the H Value

By the symbol H we conventionally denote a dimensionless statistical index, known either as Hurst exponent (in honor of the hydrologist Harold Edwin Hurst) or Hölder exponent (in honor of the mathematician Ludwig Otto Hölder).

The H value is generally employed to characterize the memory features of a process: roughly speaking, claiming a process to have long memory implies that its past realizations may have a significant effect on the present; this, in turn, if we refer to financial data, means to have a key to understand the current behavior of the market by way of past prices, and it would imply a strong predictability of the price return, and hence room for profits.

From a formal viewpoint, it is possible to define H in several ways. Assume, for instance, a stationary process with finite second order statistics: we say it has long-range correlations if its covariance function C(n) decays slowly as $n \to \infty$, i.e., for $0 < q < 1$:

$$\lim_{n \to +\infty} \frac{C(n)}{n^{-q}} = c$$

where c is a finite positive constant. The parameter q is related to the Hurst exponent via the equation: $q = 2 - 2H$. Alternatively, in the frequency domain, the weakly stationary time-series X_t is said to exhibit long range dependence if:

$$f(\lambda) \sim C_f |\lambda|^{-\eta},$$

as $\lambda \to 0$, for some $C_f > 0$, and some real parameter $\eta \in (0, 1)$, $f(\lambda)$ being the spectral density of the time-series. In this case (Clegg and Dodson 2005) the parameter η is related to the Hurst exponent by the relation: $H = (1 + \eta)/2$.

Note that H can also vary over time: depending from H being either a constant function, or a (continuous) function, the related process is said to be either monofractal or multifractal.

Starting from monofractals, it is possible to estimate the Hölder exponent in various ways: the oldest method is known as Rescaled Range (R/S) Analysis, and it was developed by Hurst himself (Hurst 1951, 1955). In order to recall Hurst's original procedure, we may consider the observed time-series X and turn it into the series of log-returns R, hence going on as follows (Peters 1994).

Step 1 Divide R into d sub-series of length n.
Step 2 For each sub-series $m = 1, \ldots, d$, evaluate the corresponding mean value (E_m) and standard deviation (S_m).
Step 3 Normalize the data by subtracting the sample mean:

$$Z_{i,m} = R_{i,m} - E_m, \text{ for } i = 1, \ldots, n.$$

Step 4 Create the cumulative time-series $Y_{i,m}$:

$$Y_{i,m} = \sum_{j-1}^{i} Z_{i,m}$$

for $i = 1, \ldots, n$.

Step 5 Find the range:

$$R_m = max\{Y_{1,m}, Y_{2,m}, \ldots, Y_{i,m}\} - min\{Y_{1,m}, Y_{2,m}, \ldots, Y_{i,m}\}$$

Step 6 Rescale the range dividing R_m by S_m:R_m/S_m. Averaging over the whole set of sub-samples d, the mean value of the rescaled range for sub-series of length n is then:

$$(R/S)_n = \frac{1}{d} \sum_{m-1}^{d} \frac{R_m}{S_m}$$

After the analysis is conducted for all possible divisors of N, one can plot the $(\mathrm{R/S})_n$ statistics against n on a double-logarithmic scale. If the returns process is white noise, then the plot will be roughly a straight line with slope 0.5. If the process is persistent (i.e. it exhibits long memory), the slope will be greater than 0.5; if it is anti-persistent (and hence it has short-term memory), then the slope will be lower than 0.5.

In general, since for small n values there is a significant deviation from the 0.5 slope, the theoretical values of the R/S statistics are approximated by (Weron and Przybylowicz 2000):

$$E(R/S)_n = \begin{cases} \frac{n-\frac{1}{2}}{n} \frac{\Gamma\left(\frac{n-1}{2}\right)}{\sqrt{\pi}\Gamma(\frac{n}{2})} \sum_{i=1}^{n-1} \sqrt{\frac{n-i}{i}}, & n \leq 340 \\ \frac{n-\frac{1}{2}}{n} \frac{1}{\sqrt{n\frac{\pi}{2}}} \sum_{i=1}^{n-1} \sqrt{\frac{n-i}{i}}, & n > 340 \end{cases}$$

where Γ is the Euler gamma function (Abramowitz and Stegun 1972).

In order to put more light on this concept, we are going to discuss an example on close prices of the VIX index, observed in the time range: 2 January 19907–31 January 2012, for an overall number of 5565 records. Let us consider latest 101 observations, we will denote them by: $x_1, x_2, \ldots, x_{101}$. Now, we turn them into the corresponding log-returns: $R = \{r_1, r_2, \ldots, r_{100}\}$, and we compute their mean value E: E=(1/100) $[r_1 + r_2 +, \cdots, +r_{100}]$. We move to calculate the deviations from the mean z_i, and the cumulative time-series y_i, $(i = 1, \ldots, 100)$. Now we find the maximum and the minimum Y values: by subtracting one to each other we get the range R. Finally, we calculate the standard deviation over the log-returns' time-series. Now we can repeat the above scheme for 110 points, then for 120 points, and so on: each time we will generate a point on our chart, as we have shown in Fig. 4.3.

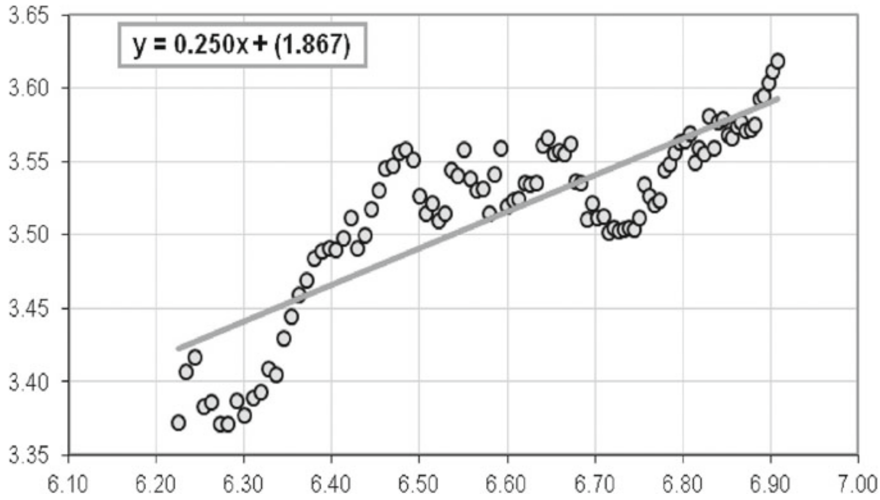

Fig. 4.3 R/S statistics for the VIX index

The slope of the interpolating line joining together the points $(\log(n), \ \log(R/S_n))$ provides an estimation of the *H* value. In particular, in the examined case we got $H = 0.250$, that means that VIX seems to exhibit short term memory.

The accuracy of the R/S statistics, however, has been widely questioned (Taqqu et al. 1995; Lo AW and MacKinlay 1999). The main objection moved to the technique is that it cannot properly work with smaller datasets, with the possibility of finding long memory in random series, or viceversa by rejecting the evidence of long memory where effectively it is present. For this reason, in order to give robustness to our analysis, we estimated the Hurst exponent for both GI, its components, and the EXX50 index by a battery of techniques, including: the Aggregate Variance method (Taqqu et al. 1995), the Aggregated Absolute Value Moments (Taqqu et al. 1995), Higuchi's method (Higuchi 1988), Peng's method (Peng et al. 1994), the periodogram (Liu et al. 2009) technique, and the wavelet estimation (Abry and Veitch 1998): a short description for each of them will be provided in Appendix A. The results in Table 4.3 refer to EXX50 index, whose case is going to be discussed in full details.

The estimations sensitively vary according to the technique in use: the Peng's value corresponds to the best estimation, in terms of standard error (SE); moreover the reader can easily note that six methods over nine agree on the (weak) anti-persistence of the index, whereas the R/S completely fails by reporting evidence of long-term memory.

In general, the method of Peng provided the best performance in the H characterization of all examined indexes: Table 4.4 shows the estimations computed by way of this technique for all the indexes under examination.

Looking at the results suggests a number of observations. The first one concerns CRB (the commodities index), which is the sole index whose H value is greater

Table 4.3 Results of H estimation for the EXX50 index by way of various techniques:

	H	a	Hurst exponent diagnostic		
			SE	t-value	Pr (> | t |)
AVM	0.534662	−0.93068	0.004492	119.0217	3.094809e-313
AbsMOM	0.576600	−0.42340	0.005916	97.46108	7.67e-280
Higuchi	0.532467	−1.46750	0.010028	53.09664	1.04e-182
Peng	0.461704	0.92340	0.002562	180.1803	2.203e-321
RS	0.603274	0.60327	0.012911	46.72461	1.71e-101
Pgram	0.482502	0.03499	0.025633	18.82343	2.21e-63
Wav	0.471145	−0.05771	0.018089	26.04521	2.11e-07

Aggregate Variance method (AVM), Aggregated Absolute Moment method (AbsMOM), Higuchi and Peng's methods, R/S analysis (RS), Periodogram (Pgram) and wavelet method (Wav)

Table 4.4 Estimated H values via the Peng's method for GI, BDI, VIX, CRB, DBHI and EXX50

	H	a	Hurst exponent diagnostic		
			SE	t-value	Pr (> | t |)
GI	0.406846	0.933693	0.024409	19.1257	4.08e-51
BDI	0.489342	0.978684	0.009063	53.9911	1.01e-148
VIX	0.357680	0.715360	0.004139	86.4155	6.05e-221
CRB	0.522505	1.045010	0.011943	43.7495	2.88e-120
DBHI	0.419948	0.839895	0.008118	51.7289	1.40e-146
EXX50	0.461704	0.923400	0.002562	180.1803	2.203e-321

than 0.5. However, in this case it would be exceeding to say it exhibits long memory, as well as in the case of BDI it could be inappropriate to report a short memory effect: in both cases, in fact, the evidence is rather that log-returns are really close to be truly independent and uncorrelated, and there is no trend effect in the index levels. A second remark concerns the indexes whose H value is lower than 0.5, evidencing a short memory (mean-reverting) effect. Unlike as it makes sound, this information is as precious as that of long memory: a mean-reverting process, in fact, can be used for active speculations, because, in general, when the current market price is lower than the average price, the stock is considered attractive for purchase, with the expectation that the price will rise; conversely, when the current market price is above the average price, the market price is expected to fall and hence attractive for selling.

A conclusive remark concerns the relations existing between the H values of EXX50 and GI. As already highlighted, the H value for EXX50 is lower than 0.5, and the same applies for GI, VIX and DBHI. The fact that the H value of GI is closer to those of VIX and DBHI raises doubts about its real effectiveness as information source: as an index mimicking the behavior of EXX50, one could straightforwardly expect the correspondent H values to be more similar one to each other than as appearing. The present difference is undoubtedly a consequence of the construction principle underlying the Ghost Index. However, a method to assure the significance of the estimated H value (and hence of previous conclusions) could be that of testing

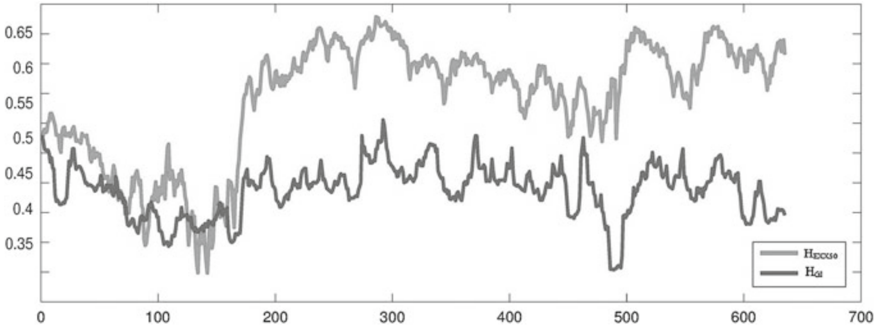

Fig. 4.4 Behavior of H(t) for the GI index (H_{GI}) and the EXX50 (H_{EXX50}): lighter gray line denotes H_{EXX50}, while H_{GI} is represented by black line

whether it fluctuates over time or not. This shifts the focus to the estimation of the H value in a dynamic fashion, i.e. the Hölderian function values H(t) that we estimated with the wavelet transform modulus maxima (WTMM) method (Mallat 1989; Yalamova 2006).

Figure 4.4 shows the behavior of H(t) estimated for both GI and EXX50 indexes, while Fig. 4.5 compares the behavior of H(t) of EXX50 to the log-behavior of the index itself.

From Fig. 4.4 it immediately sticks out to eyes that (a) H_{GI} has an anticipatory behavior with respect to H_{EXX50}, but (b) it maintains sensitively beneath H_{EXX50}. On the other hand, looking at Fig. 4.5, one can observe that rises in H_{EXX50} correspond to volatility bursts in EXX50.

Joining together the observations exploited by the comparison of H_{GI}versus H_{EXX50}, and by the analysis of the dynamics of EXX50 in the light of the behavior of H_{EXX50}suggests the possibility to interpret the variations of EXX50 by means of the bursts and blazes in the values of H(t). Moreover, if it were possible to assure a better tuning of H_{GI}, it could be possible to exploit its fluctuations to anticipate $H_{EXX50}$and hence EXX50, making room for profit.

Next section will focus on some models to improve the performance of the GI towards this direction.

Tradings with Time-Dependent H

We have already higlighted that speculating on the EXX50 by way of the Ghost Index can be effective from a financial point of view, but not from the statistical perspective. By examining the scaling features of both GI, its components and EXX50 (H characterization) we found out that the lower statistical robustness of GI might be imputed to the fact that by construction GI is made more similar to some of its components (namely the VIX and CRB indexes), enhancing its mean reverting features more than necessary. We turned then the attention to the multiscaling features

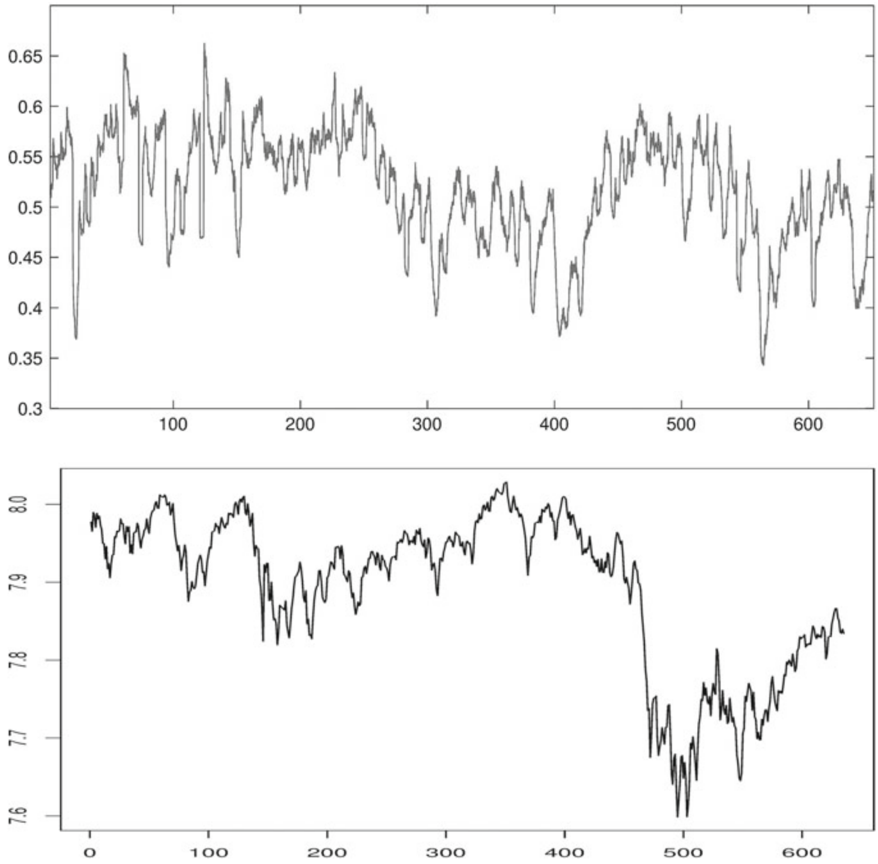

Fig. 4.5 From *top* to *bottom*: the behavior of $H_{EXX50}(t)$ and the logarithm of EXX50

of the above indexes, discovering that booms and burst on EXX50 seem to be anticipated in the behavior of the Hölderian function.

Now we are facing the challenging issue to exploit such information in order to make the Ghost Index more effective.

Let us assume to denote by $H_{BDI}, H_{VIX}, H_{CRB}, H_{DBHI}$, the time-series corresponding to the estimated Hölderian function for the indexes: BDI, VIX, CRB and DBHI, observed in the period: 28 July 2009–31 January 2012. As usual, earlier 70 % of records data will serve as learning set, while the remaining 30 % will be use for backtesting purposes.

The first model we studied is based on the regression of H_{GI} on H_{BDI}, H_{VIX}, H_{CRB}, and H_{DBHI}:

Table 4.5 Results for the regression of H_{GI} on H_{BDI}, H_{VIX}, H_{CRB}, and H_{DBHI}

Coefficient	Value	SE	t-statistics	p-value
β_0	0.0818	0.0250	3.2747	0.0011
β_1	0.2009	0.0451	4.4523	0.0000
β_2	0.1916	0.0538	3.5581	0.0004
β_3	−0.3066	0.0486	−6.3048	0.0000
β_4	0.3425	0.0528	6.4917	0.0000

Notational conventions are the same as discussed in Table 4.1

Table 4.6 White's test for residuals heteroscedasticity

Coefficient	Value	t-statistics	p-value
α_1	0.0726	4.2189	0.0841
α_2	0.0030	4.2415	0.0499
α_3	0.0018	4.2376	0.0378
α_4	0.0043	4.2525	0.0487
α_5	0.0042	4.2574	0. 0107

Notational conventions are the same as discussed in Table 4.2

$$H_{GI}(t) = [\beta_0 \quad \beta_1 \quad \beta_2 \quad \beta_3 \quad \beta_4] \begin{bmatrix} 1 \\ H_{BDI}(t-1) \\ H_{VIX}(t-1) \\ H_{CRD}(t-1) \\ H_{DBHI}(t-1) \end{bmatrix} + \varepsilon(t)$$

where β_0, *ldots*, β_4 are the regression coefficients, and $\varepsilon(t)$ is the normally distributed residual at time t. The results are provided in Table 4.5.

The second step was then to apply the values found by the regression to build the estimated H_{GI} index:

$$\widehat{H_{GI}}(t) = [0.0818 \quad 0.2009 \quad 0.1916 \quad -0.3066 \quad 0.3425] \begin{bmatrix} 1 \\ H_{BDI}(t-1) \\ H_{VIX}(t-1) \\ H_{CRD}(t-1) \\ H_{DBHI}(t-1) \end{bmatrix}$$

Table 4.6 shows the results for the White's test in search of residuals heteroscedasticity. The analysis, performed up to the lag 5, highlights only a violation, a sensitive enhancement, with respect to what observed in section "Lights and Shadows on the Ghost Index".

In this case we chose to directly employ $\widehat{H_{GI}}(t)$ to develop operative signals on EXX50. In particular, every time $\widehat{H_{GI}}(t)$ felt down the value 0.5, this was assumed as a reverting signal, i.e. either to sell, if previous action was buying, or to buy if we had gone short at previous time. On the contrary, every time $\widehat{H_{GI}}(t)$ went up 0.5, we hold the position (either selling or buying) that have been previously assumed.

Table 4.7 Results for the regression of H_{EXX50} on H_{GII}

Coefficient	Value	SE	t-statistics	p-value
β_0	0.5853	0.0089	5.8258	0.0001
β_1	−0.1139	0.0567	−2.0071	0.0452
β_2	0.0190	0.0889	0.2142	0.8305
β_3	0.0222	0.0890	0.2496	0.8030
β_4	−0.0263	0.0568	−0.4639	0.6429
β_5	0.0254	0.0312	0.7584	0.6121

Notational conventions are the same as discussed in Table 4.1

Table 4.8 White's test for residuals heteroskedasticity

Coefficient	Value	t-statistics	p-value
α_1	0.0267	5.1995	0.0384
α_2	0.0387	5.2118	0.0499
α_3	0.0289	5.2065	0.0078
α_4	0.0053	5.1938	0.0094
α_5	0.0088	5.1955	0.0110

Notational conventions are the same as discussed in Table 4.2

By applying this strategy to the test set we get a 58 % of successful trades: a percentage aligned to the one of practitioners' GI we discussed in section "Lights and shadows on the Ghost Index". However, the results of the new trading system are better than those examined in section "Lights and Shadows on the Ghost Index" from the statistical point of view, as the heteroscedasticity of residuals is now limited to the lag 1.

We then developed a second model, where the H_{EXX50} is regressed on H_{GI} and estimated values are used to create signals to anticipate the behavior of EXX50. Regressions coefficients are provided in Table 4.7, while Table 4.8 shows the results of White's test for ARCH effects.

Searching for ARCH effects on residuals gave affirmative feedbacks in one over five cases, but this time financial performances lowered to 54 %.

Conclusion

In this chapter we examined the fractal features of a number of indexes that practitioners maintain under strict control to trade on Eurozone stocks. In particular, we focused our attention to the so-called Ghost Index (GI) that by construction assures a broad range coverage of various market aspects, and it is therefore employed for active speculation on the Eurostoxx 50 (EXX50), and, more generally, on the Eurozone market.

The starting point for our analysis was the observation that while GI, as built by practitioners, can proficiently work from a financial point of view, nevertheless is statistically weaker.

We then moved in search of a way to give more robustness to the index, and we found out that analyzing what we have called the H-characterization of both GI, its components and EXX50 can be of help. In practice, this involved the estimation of the Hurst exponent H of the indexes; this task was performed by way of a bunch of techniques letting us to give robustness to our conclusions. We then moved to test whether such H values maintain constant (monofractal process) or vary (multifractal process) along time. In this way we were able to verify that (a) the Hölderian function values of GI (H_{GI}) tends to anticipate movements in H_{EXX50}, and (b) booms and burst in the EXX50 behavior are mirrored, generally at least one day in advance, by the Hölderian function values H_{EXX50}.

As final act we developed two models that implement such information to generate active trading signals. The results we obtained are aligned to those of the original Ghost Index from a financial point of view, but are statistically more robust in both examined models. In this way we provided evidence that multiscaling analysis is very important not only at theoretical level, but also in practice, where it can be of aid to practitioners to tune existing techniques to give them more financial results coped to statistical soundness. This, in turn, highlights that tools relying on chaos theory and complexity studies may be employed to develop practical applications achieving successful tradings on financial markets.

Appendix A: Estimation Techniques of the Hurst Exponent

Starting from the original technique suggested by Hurst, several methods have been developed to estimate the parameter *H*, which operates both in time and frequency domain.

The aggregate variance is a time domain method useful for non-stationary time series. Using the same notational conventions adopted in section "H-Characterization of the Ghost Index", assume that *R* is the series of returns derived from the original time-series *X*. Then:

Step 1 Aggregate *R* into *d* sub-series of length *n*, with $n = 2, \ldots, [N]/2$, where *N* is the original length of *R*, and [.] indicates the integer part.
Step 2 For each sub-series consider the related sample variance.
Step 3 Graph the variances of such aggregate time-series in a log–log plot versus the different levels of aggregation and provide a least square line to fit the data. The slope of such line gives the estimation for *H*.

The modulus of the aggregate series works in a similar fashion, but uses the modulus of the aggregate time series variance instead of its variance.

The periodogram method, on the other hand, is a technique working in the frequencies domain. The periodogram for the returns series *R* is defined as:

$$I(\lambda) = \frac{1}{2\pi N} \left| \sum_{j=1}^{N} R_{j e^{ij\lambda}} \right|^2$$

where λ is the frequency. For a series with finite variance, I(λ)(is an estimate of the time-series spectral density. Fitting a log-log plot of I(λ)(with a least square line, one should obtain a slope of $1 - 2H$ close to the origin.

Turning back to time-domain models, the Higuchi estimator is based on the fractal dimension D of a time series. Starting from the original time-series R, the procedure works as follows.

Step 1 Create sample sequences:

$$Z_k^m = \left\{ Y(m), Y(m+k), Y(m+2k), \ldots, Y\left(m + \left[\frac{(N-m)}{k} \right] k \right) \right\}$$

where $Y(m) = r_1 + r_2 + \cdots + r_m$ are partial sums, $k = 1, 2, \ldots, N$; $m = 1, 2, \ldots, k$, and the operator [.] stands, as already seen in previous rows, for the integer part.

Step 2 For each sequence Z_k^m the normalized curve length is computed as:

$$L_m(k) = \frac{N-1}{K^2 \left[\frac{N-m}{k} \right]} \sum_{i=1}^{\left[\frac{N-m}{k} \right]} |Y(m+ik) - Y(m+(i-1)k)|$$

and the curve length $L(k)$ for each lag k is:

$$L(k) = \frac{1}{k} \sum_{k}^{m=1} L_m(k)$$

Hence, $E(L(k)) \approx C_{2k}^{-D}$ for $k \to \infty$, where $D = 2 - H$. The H parameter is then estimated by classical regression techniques with $logL(k)$ opposed to $log(k)$.

The Peng's method, also known as Detrended Fluctuation Analysis (DFA) makes possible to estimate H in case of non-stationary time-series.

At first, one should divide the sequence R of length N into $[N]/s$ non-overlapping boxes, each containing s points. The linear local trend $z(t) = at + b$ in each box is the standard linear least-square fit of the data points belonging to that box. The detrended fluctuation function F is then defined by:

$$F_k^2(s) = \sum_{t=ks+1}^{(k+1)s} |r(t) - z(t)|^2$$

Averaging $F_k^2(s)$ over the $[N]/s$ intervals gives the fluctuation $E(F^2(s))$ as a function of s , being:

$$E(F^2(s)) = \frac{s}{N} \sum_{\mathrm{k}=0}^{\frac{[N]}{s}-1} F_k^2(s)$$

If the observable returns are realizations of random uncorrelated variables or short-range correlated variables, the behavior of $\mathrm{E}(\mathrm{F}^2(\mathrm{s}))$ is expected to be a power law:

$$\sqrt{E(F^2(s))} \sim S^H$$

The value H can be then easily derived.

To conclude, an interesting estimation method has been introduced at the eve of the new century based on the wavelet approach. The wavelet transform is a mathematical tool for representing signals as sum of “small waves”. It is a better substitute of the Fourier transform: while this latter is used to transform a signal from the time domain to the frequency domain, the wavelet transform, on the other hand, is capable of providing the time and frequency information of a signal simultaneously.

The input signal is represented in terms of dilated versions of a prototype of high-pass wavelet function ψ_{ij} and shifted version of a low-pass scaling function (ϕ_{ij}), based on the scaling function ϕ_0 and the mother wavelet basis function (ψ_0), being:

$$\begin{aligned} \phi_{ij}(t) &= 2^{-i/2}\phi_0(2^{-i}t - j), \qquad i \in \mathbb{Z} \\ \psi_{ij}(t) &= 2^{-j/2}\psi_0(2^{-j}t - j), \qquad j \in \mathbb{Z} \end{aligned}$$

The approximation information of sequence R is then given by:

$$approx_i(t) = \sum_j a_r(i,j)\phi_{ij}(t)$$

where the coefficient $a_r(i,j)$ is given by calculating the inner product:

$$a_r(i,j) = \langle R, \phi_{ij} \rangle$$

The detail information ($detail_i$) of sequence R is given by:

$$detail_i(t) = \sum_j d_r(i,j)\psi_{ij}(t)$$

where the coefficient $d_r(i,j)$ is given by calculating the inner product of R:

$$d_r(i,j) = \langle R, \psi_{ij} \rangle$$

Multi Resolution Analysis (MRA) represents the information about the sequence X as a collection of details and a low resolution approximation:

$$\begin{aligned} r(t) &= approx_N(t) + \sum_{i=1}^{N} detail_i(t) \\ &= \sum_j a_r(N, j)\phi_{Nj}(t) + \sum_{i=1}^{N}\sum_j d_r(i, j)\psi_{ij}(t) \end{aligned}$$

The function ϕ_0 produces an approximation of signal R and it must be a low-pass filter. The mother wavelet function ψ_0 must be high-pass filter, and it performs a differential operation on the input signal to produce the detail version.

The wavelet-based Hurst parameter estimator is based on a spectral estimator obtained by performing a time average of the wavelet *detail* coefficients $|d_r(i, j)|^2$ at a given scale:

$$S_r = \frac{1}{N_i}\sum_j |d_r(i, j)|^2$$

where N_i is the number of wavelet coefficients at scale i, i.e., $N_i = 2^{-i}N$, and N is number of data points. The estimator first performs Discrete Wavelet Transform (DWT) on the input signal, employing wavelets from the Daubechies family. After computing the DWT, the estimator calculates the estimates of $log_2 E[d(i, j)]^2$ and variance of these estimates and performs a linear regression hence finding the slope γ. H is then calculated as: $\mathrm{H} = 0.5(1 + \gamma), 0 < \gamma < 1$.

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Chapter 5
Managing Uncertainty in Complex Projects

Giancarlo Nota and Rossella Aiello

Abstract The need for complex systems has grown in recent years to react to the different requirements arisen in an increasingly interconnected and interdependent environment. Examples range from air traffic control systems, systems managing transactions on the stock exchange or environmental monitoring systems. The increasing complexity of such systems brings to the higher complexity of project management activities, which requires the review of commonly used methodologies to provide a better response to expectations of success for the planned projects. In this work we present a method for project monitoring and control based on the idea of a project view: given an analysis dimension such as time, cost and quality, each participant in the project has its own view on the project execution. The unavoidable differences between the views of various stakeholders have been formalized in the function *gap* that can be interpreted as an inverse measure of the degree of consonance between two participants with respect to the dimension of considered analysis. The method can leverage on the consonance seeking among stakeholders as a reduction factor of project risks.

Keywords Project views · Uncertainty management · Consonance · Complex projects

Introduction

In the last years, both dimension and complexity of projects managed by public or private organizations have greatly grown. Williams (2002) indicates that the causes can be attributed to the complexity growth of the products to be developed and to

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New Economic Windows, DOI: 10.1007/978-3-319-05185-7_5,

the containment of development times. Projects are increasingly managed with a very rapid pace, contributing to add difficulties and reduce the ability of achieving objectives such as on-time delivery, costs within budget, the expected quality.

Statistical evaluations made by Standish Group (2009), or Gartner Group in Bloch et al. (2012) on project execution show that projects fail at an alarming rate. Common causes are vague project objectives and specifications as well as inadequate solutions deriving from poor design or wrong implementations, but also the lack of involvement in the project of the various stakeholders, especially when their points of view are not consistent.

The traditional approach of project management assumes that the project context remains unchanged and the key factors for the success of a project are attributable to unambiguous elements for management and control. Many project plans are based on a static view of the world, set at the project start time; as a result, the plan remains valid until what has been planned as a model for the future continues to hold. Actually, the project environment rarely remains static, and the planning assumptions change over time.

Due to this increasing complexity, projects are now considered as complex systems and both researchers and practitioners, such as Crawford et al. (2011), Remington and Pollack (2008), Thomas and Mengel (2008), have started to apply complexity theory also in the field of project management. A huge number of studies and scientific works like Holland (1998), Waldrop (1993), Miller and Page (2007), Senge (1996) can be found in literature about complexity theory in many different areas, such as physics, economics, biology, ecology and so on. Complex adaptive systems are characterized by a large number of entities with a high level of nonlinear interactivity; they exhibit some different characteristics and multiple kinds of systemicity such as hierarchy, interconnectedness, control, communication, emergence, adaptiveness. Complex Adaptive Systems learn and evolve by adapting and thereby surviving by processing information and building schemas based on experience. According to Barile and Saviano (2011) "*..complexity manifests itself as the incapacity to orient and act using criteria and rules that were previously deemed useful; consequently, the indications to bring about change in lieu of recovery of stability cannot be directly and immediately inferred from past method*"; moreover different observers perceive a different level of complexity but also the "same" observer, at different moments, perceives different levels of complexity.

In these cases, implementing a strategic action becomes a serious challenge to managers of organizations. The process becomes more and more difficult to handle because of characteristics like unpredictability, uncertainty, and the wide variety of interactions among multiple autonomous agents. Strategies become the results of a collaboration process in which thinking and acting go together. The relation between creation and implementation of strategies is a continuous process in which the strategist gives form to the strategy by personal touch as discussed by Remington and Pollack (2008). Barile and Saviano (2011) states that in a complex context, human beings take decision strongly based on psychological factors: decisions are made on the basis of value categories. In Holmdahl (2005), the author points out that culture and the view of the world determine the strategies to perceive and, consequently, the plans and techniques to apply.

No one approach to project management is appropriate for every situation. According to Pich et al. (2002) two fundamental strategies become necessary when the project information is inadequate: learning and selectionism that is pursuing multiple approaches and choosing the best one ex post. In Hurtado (2006) a conceptual framework for the strategy management process is proposed: "*when complexity is irreducible, the traditional planning has to be complemented by additional techniques, shifting the focus from the full-fledged intended strategies towards interactive organizational learning, scenario planning and organizational mindfulness: this means to operate considering the evolution not only of the organization-environment ensemble but also of interrelated groups of organizations*".

A highly topical issue in the management of complex projects is the handling of uncertainty, as shown in Atkinson et al. (2006), Cleden (2009), Curlee and Gordon (2010), Meyer et al. (2002), Perminova et al. (2008), Ward and Chapman (2003). A common management of uncertainty is a necessary condition for an effective project management, but this requires more attention together with the use of more sophisticated methods and techniques than those of current common practice. In order to cope effectively with complex projects managers must adopt a pluralistic approach to practice. They must be able to draw from a wide range of tools and ways of thinking to develop their own methods, their own patterns of practice, freely, according to the exigencies of the particular project.

If the projects become more complex for a number of reasons ranging from the size of the technical complexity, or environmental conflicts or political constraints, then the usual project management methodologies need to be adapted as well as managers must be able to analyze situations from different perspectives and work using a larger number of models. Therefore, planning uncertainty implies a willingness to review the project plan whenever the situation demands it. Atkinson et al. (2006) says that uncertaint may also arise from gaps in different areas of knowledge, concerning,e.g., contextual information on the project, or the degree of understanding of basic processes, or an underestimation of specific past events. When the world is complex, the agile adaption to circumstances and latest gained knowledge allow to obtain successful actions in handling uncertainty Holmdahl (2005). Some other works, such as Montoya-Weiss and Calantone (1994) and Verganti (2001) focus on the principle of anticipation or based on the ability to anticipate the knowledge generation to the early stages of project planning, and thus to reduce uncertainty on contextual factors as quickly as possible. However, there are certain types of uncertainty that are difficult to assess by an analytical approach: events or unknown causalities can play a role or the evaluation of the effects of actions could not be possible due to a large number of interacting variables. In other cases, it may happen that a series of random events in combination with each other cause an unexpected result that could be a risk or an opportunity as Hillson (2002), Olsson (2007). Meyer et al. (2002) show the example of pharmaceutical companies that invest extensive resources in the drugs testing; on one hand there always exists the risk that some

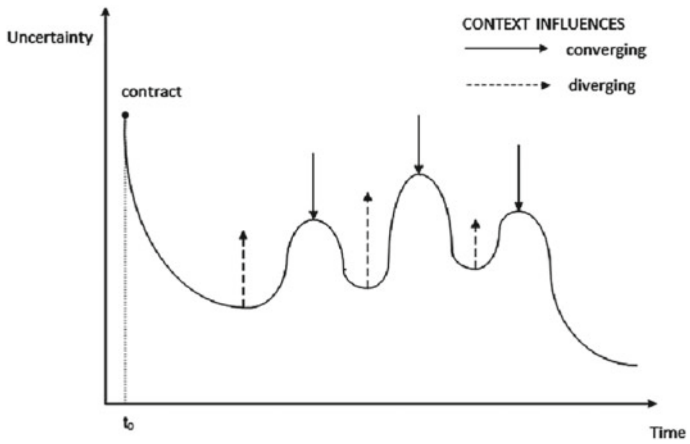

Fig. 5.1 Project Uncertainty in dependence of the environmental influences

unforeseen combination of environmental events (such as the intake of other drugs by the patient) may create dangerous side effects. On the other hand, for Pfizer a great opportunity was obtained with the Viagra: it was created as a heart medication to improve blood flow by relaxing the arteries, but when clinical studies found that it also increased sexual performance, the company with huge success changed the marketing approach midway through the original project.

Figure 5.1 illustrates how the environment can affect the curve of project uncertainty positively or negatively, by applying forces that either make tend uncertainty to zero (*influences of convergence*) or make diverge it (*influence of divergence*) causing sudden increases in the trend. As an example, a force that acts as influence of convergence at time t_0, when a project starts, could be the act of defining and signing of a contract by stakeholders. Conversely, the delay with which a public authority grants an authorization to build a structure represents an influence of divergence which increases uncertainty concerning the prediction of execution times estimated in the project planning.

In this work, the focus will be on management of complex projects and, specifically, on the monitoring and controlling tools for the foreseen execution of activities necessary for achieving a set of project goals. In particular, the chapter introduces a method for the management of complex projects, where contract management together with dispute management act as a regulatory instruments to maintain adequate levels of consonance and resonance among vital systems cooperating for the achievement of a common goal. According to Viable System Approach considered by Barile (2008) and Golinelli (2010), *consonance* is considered as a potential compatibility between systems that facilitates their connection. The result of a virtuous interaction (harmonizing) between the two drivers is the *resonance*. While consonance concerns structural concepts, resonance is a systemic concept. The consonance represents a situation aiming at the reaching of a harmony or agreement among two or more systems, but they become resonant when an effective harmonic interaction between components exists. Resonance can be also considered as the "variation in time of the consonance as a result of the activation frequency of the relations and quality of the information exchanges among systemic entities". To achieve resonance, decision makers must have visions of future scenarios, not as linearly determined outcomes of past facts (causality), but as emotionally anticipated desired future events. Then, by acting upon common feeling and desiderata, they create conditions of consonance, so being able to involve all relevant components and stakeholders into the achievement of a shared goal. The chapter structure is therefore as follows. Section "The Searching of the Consonance as Reduction Factor of Project Risks" introduces the concept of consonance in a project management environment. In Sect. "The Formal Method" some basic definitions about the concepts of view and gap are provided; Sect. "Monitoring and Control Loop of Stakeholder Views" discusses a process to reduce the dispute management will be discussed. Finally, Sect. "The case study" closes the chapter.

The Searching of the Consonance as Reduction Factor of Project Risks

Contract is one of the most valuable tools for planning and managing the interactions among the project stakeholders. It defines a certain number of clauses in the form of obligations, permissions and prohibitions, which regulate the implementation of project activities. The contract between two parties involves the identification of operational guidelines aimed at achieving a particular goal: it is desirable that, in this case, decision makers analyze some possible situations verifiable in the future and they take them into account in the operational planning of activities bound by the contract.

However, even if a contract imposes constraints and delineates the guidelines towards the achievement of the project objectives, usually it is not enough as a control tool. Uncertainty of an unforeseeable event or the lack of information can produce negative effects on the smooth running of the project; violations of contractual obligations, e.g. delays in delivery or quality level not compliant with expected deliverables quality also contribute to the project failure and to the loss of trust among stakeholders that affects the future business relations. In community and international legal order, the award for the provision of works, goods and services is governed by specific rules. What is interesting, for the purposes of this chapter, is to emphasize how the agreement on a contractual basis for a project realization creates a situation of consonance among the parties that appears to be the minimum necessary to enable the resonance. The stakeholders will to contract indicates that exists a relationship of trust and harmony that comes from sharing the same design goals. When strategy, mutual commitments, values and goals are shared, from the system composed of stakeholders emerges a cooperative behaviour Dietrich et al. (2010). Franco et al.

(2008) stated that if some conflict of interest arise, each stakeholder (and, in general, each system) must identify the best possible way to cooperate, possibly changing its behavior and to reach the convergence of individual perspectives. However, precisely because of the complexity and external factors related to the influences of other systems in the operative context, it is not always possible to predict in advance and manage the risks of adverse events to the project realization; in addition, due to the conflicting and opportunistic roles that stakeholders hold, different views on one or more project activities often emerge. In these cases, the balance achieved in the degree of initial system consonance is disturbed and some corrective actions need to be provided, by one or both parties, to try to restore consonance to initial levels or at levels considered necessary in the project continuation.

The method that will be introduced below acts as a tool trying to restore stakeholders harmony to a sufficient value for entering into resonance again and triggering the processes necessary to reach the shared objectives. To limit the impact of the forces of divergence operating in the context of which a project is proposed, the result of the work is a method that integrates aspects of the disputes handling with the project and contract management.

The Formal Method

In this section we give some basic definitions related to the project plan and to the different stakeholders perceptions on the state of project realization. The unifying aspect of these definitions is the concept of activity, understood as the unit of work that can be allocated and analyzed in order to monitor and control project progress. Let be defined a project P at time t_0 as a set of quadruple:

$$P = \{[a_1, c_1, s_1, q_1], [a_2, c_2, s_2, q_2], \ldots, [a_m, c_m, s_m, q_m]\} \tag{5.1}$$

where the set represents the project activities $a_1, \ldots, a_m$ in which

- $c_1, \ldots, c_m$ are the planned costs for such activities,
- $s_1, \ldots, s_m$ are the scheduled times with s_i=[t_{start}, t_{end}, $state$] $i = 1, \ldots, m$ where $state \in \{$`started,not started,in progress,completed`$\}$ and
- $q_1, \ldots, q_m$ define output specifications constrained by quality requirements.

We assume that two contractors A and B have signed a contract at time t_0. At a generic time instant $t > t_0$, each contractor $x \in \{A, B\}$ has a point of view with respect to the execution of P:

$$View_x(P, t) = \{[a_i, c_{it}, s_{it}, q_{it}]\}, i = 1, \ldots, m \tag{5.2}$$

where

- c_{it} represents costs assigned by x at time t with respect to a_i;
- s_{it} are the observed times for the realization of a_i until t;
- q_{it} indicates the quality degree according to x at observation time.

Note that, at time t_0, $View_A = View_B$ because all values in the project are intended as *planned values*, that is, expected values on which the parties have already reached a contractual agreement. Consonance, at time t_0, is the minimum necessary for the entry into resonance of stakeholders; with a contract the foundation for the project realization has started and from then on, the necessary conditions for the activation of the processes are created. As time goes by, a view contains the "*observed values*" deriving from the perception that a contractor has on the execution of the project activities. Since the point of view is a subjective value, it frequently happens that the stakeholder perceptions tend to vary over time. The function defined below is intended to formalize this idea of divergence obtained from the values of the project variables.

Suppose that project milestone M is observed at the time t_k; the function Δ represents the *gap* between the views perceived at time t_k by two observers A and B:

$$\Delta(View_A, View_B, t_k) = \{[a_i, \Delta_i^c, \Delta_i^s, \Delta_i^q]\}\ i = 1, \ldots, m \tag{5.3}$$

where Δ_i^c, Δ_i^s, Δ_i^q can be considered as the projections of the distance between $View_A$ and $View_B$ to the dimension of cost, time and quality respectively. A more detailed description of these measures is given in the following.

The total gap on the project P at time t_k according a dimension d (such as cost, time or quality) can be computed as the sum of the gaps on all the activities:

$$T\Delta_d(t_k) = \sum_{i=1}^{m} \Delta_i^d \tag{5.4}$$

The analysis of the $T\Delta_d$ trend also gives us an idea of how the project consonance between A and B changes, with respect to the considered dimension (Fig. 5.2). If the views diverge at the observation time, the harmony among the stakeholders tends to decrease. In case of irreconcilable points of view a dispute can arise that slows down or possibly stopping the project. The launch of a series of informal negotiations among stakeholders can be necessary to re-establish an acceptable level of consonance or, the initiation of a formal process of dispute resolution governed by a mediator.

By setting the time as required dimension, the analysis of time schedule and their difference, allow to answer questions like:

- Is the project taking longer than scheduled?
- Could the project be completed in time?
- Is the supplier able to deliver the products before the deadline?

Resuming the project definition, the duration dur at time t of a schedule s_i for an activity a_i can be described as:

$$dur(s_{i,t}) = \begin{cases} t_{end} - t_{start} & \text{if state} = \text{completed} \\ t - t_{start} & \text{if state} = \text{started} \\ \mathit{undefined} & \text{if state} = \text{not\_started} \end{cases} \tag{5.5}$$

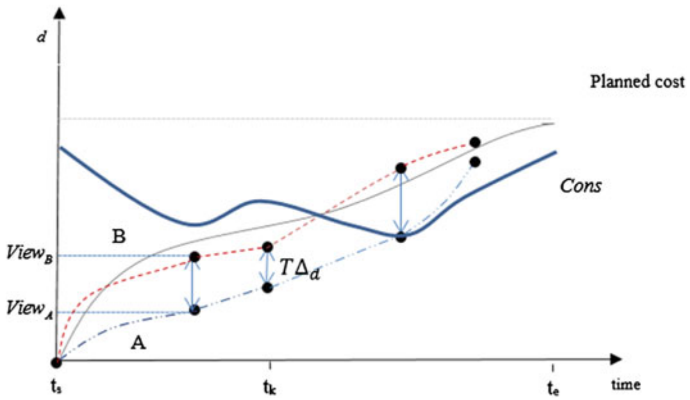

Fig. 5.2 Trend of the total gap between two project views with respect to a dimension *d*

Now, considering the time dimension, a possible gap between View$_A$ and View$_B$ on activity *ai* can be selected among the following measures:

$$\Delta_{i,t}^{start} = |t_{start}^{A} - t_{start}^{B}| \tag{5.6}$$

$$\Delta_{i,t}^{end} = |t_{end}^{A} - t_{end}^{B}| \tag{5.7}$$

$$\Delta_{i,t}^{dur} = \begin{cases} undefined & \text{if } \exists x \in \{A, B\} \ni' dur(s_{i,t}^{x}) = \text{undefined} \\ |dur(s_{i,t}^{A}) - dur(s_{i,t}^{B})| & \text{otherwise} \end{cases} \tag{5.8}$$

If the result is equal to *undefined*, the stakeholder behavior is driven by the following cases:

1. both parties agree that the activity has not yet started and the measures are postponed to a future time;
2. only one stakeholder considers the activity as already started; the consonance degree related to the activity decreases.

For other methods of gap computation relatively to the other dimensions of cost and quality, we refer to what is defined in Nota et al. (2011a).
Assuming for example, that the beginning of the project occurs at time $t = 0$, and considering two stakeholders *A* and *B* that express their views on the project at time $t = 50$, Table 5.1 lists some examples of gap calculation of three activities. As can be seen from the table, it is possible to calculate three distinct gaps: one that is obtained by comparing the view of *A* with that of *B*, while the other two gaps arise by comparing respectively, views of *A* and *B* with planned values for the project. For example, the value $\Delta_{2,50}^{dur}(A, B)$ for the activity a_2 is computed as follows:

$$\Delta_{2,50}^{dur}(A, B) = |dur(s_{2,50}^{A}) - dur(s_{2,50}^{B})| = |(48 - 12) - (50 - 12)| = |36 - 38| = 2.$$

Table 5.1 Examples of gap computation on time dimension

	Attribute	Plan	View$_A$	View$_B$	$\Delta^{dur}_{i,50}(A,B)$	$\Delta^{dur}_{i,50}(Plan,A)$	$\Delta^{dur}_{i,50}(Plan,B)$
a_1	Start	5	10	5			
	End	30	45	35	5	10	5
	State	Completed	Completed	Completed			
a_2	Start	10	12	12			
	End	40	48	–	2	6	8
	State	Completed	Completed	Running			
a_3	Start	45	–	45			
	End	60	–	–	Undefined	Undefined	0
	State	Running	Not started	Running			

The implementation of projects involving different stakeholders is generally governed by contracts. A contract on the basis of an agreement among the parties, establishes the purpose of the provision, the role of the stakeholder, the period of validity. In the written contracts, the agreement is defined as a set of clauses formalizing the full coincidence between the declarations of intent of each stakeholder (consonance). For the purposes of the monitoring and control method described in the following paragraph, a clause is defined as:

$$clause = (id, type, role, description, result, validity) \tag{5.9}$$

where:

- *id* is the unique identifier of the clause within a contract;
- *type* is one of the deontic concepts of obligation, permission or prohibition. For more details on deontic logic see von Wright (1967), Governatori and Milosevic (2006), Governatori (2010);
- *role* indicates who is the recipient of the clause or who is responsible for its satisfaction;
- *description* indicates what the role should (or should not) do, depending on the clause type;
- *result* expresses the satisfaction or violation of the clause. At time t_0 the value is undefined;
- *validity* is the time interval in which the clause should be applied.

An example of obligation between two contractors A and B is when the Gantt attached to the project/offer is considered as integral part of the contract:

$c_1 = $ (3,obligation,[A, B], "Gantt is part of the contract"-, [date$_{signedContract}, -$]);

A similar formulation has the following clauses: A requires that the contractor B does not disclose confidential information that have been learned during the contract execution, while the second allows A the right to request further valid documentation for the project, at any time before the end of the contract:

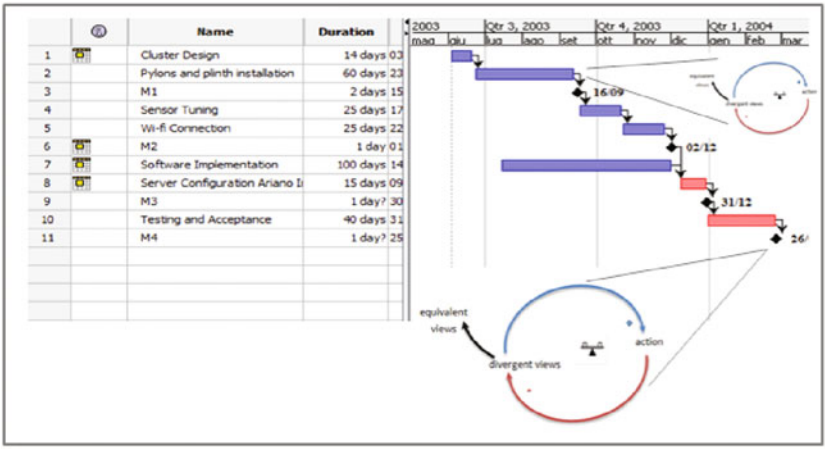

Fig. 5.3 Examples of application of the monitoring cycle on a single task or a milestone

$c_2 =$ (5,prohibition,B, "do not disclose confidential information"$-$, [$date_{signContract}, -$]);
$c_3 =$ (9,right,A, "require additional documentation"$-$, [$date_{signContract}, date_{endContract}$]);

Monitoring and Control Loop of Stakeholder Views

The model of monitoring and control of project views borrows by Senge (1996) the concept of balancing loop. This is a balancing process aiming at the reduction of a gap between a current state and a desired state. Starting from the concepts shown in Sect. "The Formal Method", in Nota et al. (2011a) was defined a cycle model for the dispute management in which:

- the current state of the system represents the contractors points of view;
- in the desired state divergent points of view converge towards a shared vision;
- the gap is the distance from a point of view to another, and its presence is symptomatic of a potential dispute;
- an action can be taken by one or more stakeholders to try to reduce the divergent views. A negotiation between the parties may be required to seek for a reconciliation and to promote the natural project continuation; see also Nota et al. (2011b) for more details.

Figure 5.3 shows the application of the balancing loop in different points of the project:

- on a *single task* if the project manager chooses, for example, to focus the attention on the monitoring of activities that have a great impact on the whole project. This approach tries to avoid a high gap to manage near the project deadline;

- *at the milestone*: the gap evaluation may be periodically performed, e.g. at the project milestones when the release of deliverables happens and there is a greater exchange of information among stakeholders.

So far, the discussed method was focused on project management and, in particular, on an activity or set of activities relevant to the entire project. Since, in general, the project becomes an integral part of a contract, its definition in terms of activities (each with time, cost and quality planned) is a subset of the clauses specified in the contract.

However, a contract contains other clauses (in the form of obligations, permissions or prohibitions) that also need to be regularly observed and evaluated in order to detect possible violations. The method here discussed can be generalized by defining a metalevel for the monitoring and controlling of contractual clauses in which each stakeholder expresses an opinion with respect to the fulfillment or violation of the observed clause. In this case, it may be helpful to plan "proactive" control loops that, at predefined time interval before a deadline, check that a clause has been fulfilled or not. This also contributes to avoid incurring additional requirements (such as. applicable penalties) in the event of violations (Contrary-to-Duty) Prakken and Sergot (1996).

The Case Study

The case study here presented considers issues related to the project named "e-Territory" for the realization of an integrated system whose main purpose is the ground surveillance and automatic fire detection in specific areas of "UFITA Mountain Community" (UCM or UFITA, in the following) located in a region of South of Italy, Campania.

The goal of "e-Territory" is the creation of a distributed system, composed of some stations for detection of environmental variables installed on a regional territory (in the geographical areas of Casalbore, Greci, Trevico, Frigento, Savignano and S. Sossio Baronia) and a centralized structure for data collection located at the data processing center (DPC) in Ariano Irpino.

Stations on the territory continuously measure the environmental variables and transmit them to the DPC with the main purpose to carry out the environmental monitoring, both for the automatic detection of forest fires and also of other variables concerning the health status of the territory (quality of air and water, electromagnetic pollution and seismic monitoring). In addition, a mobile station was also accomplished with the possibility to use the vehicle not only for the fire control but also to perform withdrawals of physical quantities of territory areas not covered by the fixed stations. Data collected from fixed and mobile stations converge in the databases of the collection centre located in Ariano Irpino where they are processed and displayed on the information portal. Finally, e-Territory project also implemented a Geographical Information System (GIS) for the management of cartographic databases.

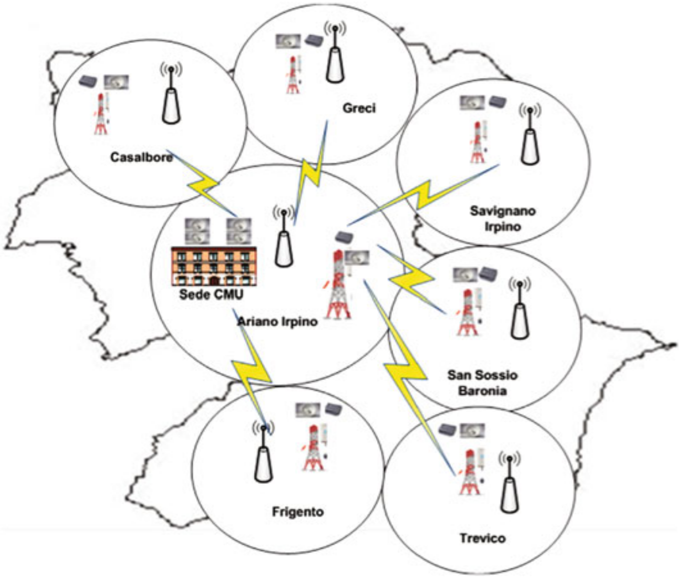

Fig. 5.4 The seven clusters of the e-Territory project

To simplify the project implementation, UMC was decomposed into seven areas (Fig. 5.4), each with the task of monitoring a specific part of the territory. A *cluster* is a focusing unit with the same territory characteristics for the purposes of monitoring. Each cluster was equipped with a peripheral station for fire monitoring, completed with the necessary sensors and a wireless communication network for the information transmission to the DPC.

A typical installation for a cluster comprises the weather station made by a set of sensors for the detection of environmental variables, such as temperature, humidity, wind direction; similarly, the supervisory and fire detection system includes cameras for detections both in the visible and infrared.

Having specified, albeit briefly, the object of the realization of e-Territory project, we are in presence of a complex system consisting of a relatively large number of elements with multiple relationships between them, a variety of elements and relationships, and a variability of the structure over time. However, project management does not cover only aspects related to the complexity of the "technological" system, but it is necessary to take into account other factors that increase the complexity degree of systems, such as the needs and expectations of stakeholders and the influences that suprasystems thrust upon the system under observation.[1]

[1] In this context, "UMC territorial system" stands for "the set of tangible and intangible values such as such people, culture, the historical legacy, heritage and urban art, infrastructure, location and any other kind of situation that will maximize the total value of the various elements". Kotler et al. (1993).

According to Barile and Golinelli (2008) definition,[2] a possible approach to territorial analysis identifies the following logical levels of local government:

- the administration of Campania represented the *authorizing officer subject* that, first with the publication of Operational Programme Campania 2000–2006 Measure 6.2 "Information Society", has established guidelines to promote the development of the information society in Campania, and support the dissemination of Information and Communication Technology (ICT) in public administrations and in production systems; then, with Rural Development Programme (RDP) Campania 2007–2013 Size 226 Action C, has prepared a series of economic aids to comply with preventive actions and reconstitution of forestry potentially damaged by natural disasters;
- funding operations by Campania represent for UMC government an opportunity to pursue the goal of improving the management of the territory. On such premise, UMC assumed the role of *coordinator subject* that adopted the guidelines and the time limit dictated by the officer for completion of e-Territory project;
- finally, companies have responded by offering a commitment to implement the project, as defined in the tender notice, played the role of the *proponent subjects*.

These logical levels can be analyzed from different perspectives such as the decision-making territorial process, the administrative procedure aimed at the project realization, the monitoring process and ex-post analysis of the benefits arising from the investment, and so on. Figure 5.5 shows the BPMN (Business Process Modeling Notation)[3] high-level administrative procedure aimed at the realization of e-Territory. BPMN provides visual tools for:

a. the *focusing*, through the clarification of activities and sub-processes that make up the process to define;
b. the *abstraction* concerned instead the relationships that occur between organizations in terms of exchange of data and information without the need to explain the internal processes.

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[2] "VSA provides interpretive schemes congenial to the representation and analysis of the dynamics of the territorial governments formation, through the identification of three logical levels of government:

- the officer subject (OS), responsible for identifying action lines derived from a subjective interpretation of the environment that, through the identification of vocations, leads to the extraction of one or more contexts to be submitted to any subject coordinators;
- one or more coordinators (CS), able to develop proposals within the context identified by the OS;
- one or more proponents (PS), involved in implementation of projects connected with the proposals made by the CS."

[3] BPMN was developed by Business Process Management Initiative (BPMI) and is currently managed by the Object Management Group (OMG). Further information on BPMN can be found on the organization's website at: http://www.bpmn.org/.

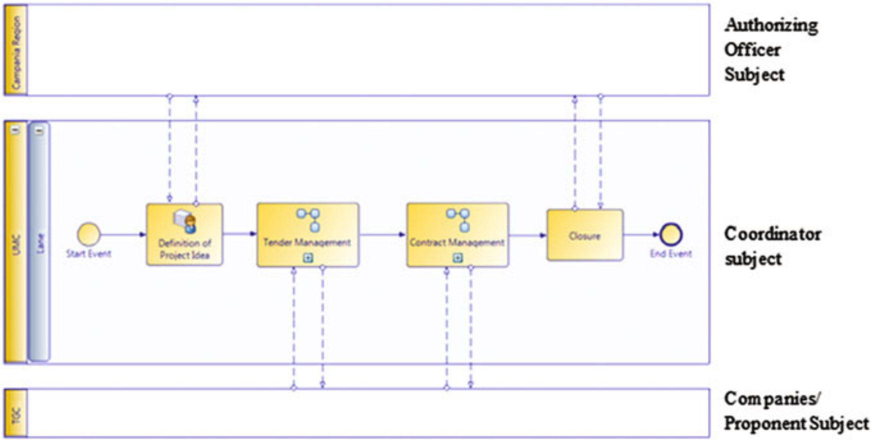

Fig. 5.5 High level process and interaction among subjects

In the representation of Fig. 5.5 the interest of the observer[4] is assumed to be focused on the process inside the CS and the main relationships with the OS and PS.

Dashed arrows represent the relationships among subjects on which the interactions are defined mainly through the exchange of messages or data (e.g., documents, emails, phone calls). Solid arrows represent the control flow governing the execution order of activities and sub-processes.

In the following, the focus will be on observation of the emergent system behaviour obtained by the union of UMC and a set of proponents.

The high-level process shows the different macroactivities: the call for funding of Campania represents the input to the UMC for promoting the creation of a new project (*Definition of project idea*). By identifying the objectives to achieve and the definition of the call, the subprocess *Tender Management* can start; at this stage, interested companies interact with UMC by submitting the required documentation in the manner and terms specified in the notice. After the choice of the winner proposal, the real project implementation phase begins. The contract, signed by UMC and the winning company, represents the tool used to protect the parties and regulate the execution phases of the project. The activity *Contract Management* defines the subprocess in charge of implementation and monitoring of project activities and, eventually, of the management of contractual disputes arising among the parties. Finally, last activity (*Closure*) defines the obligations to be performed to end the project; in this phase, UMC interacts with Campania to report the final project documentation and the result of system inspection.

[4] In Golinelli (2010) (Introduction to Chap. 3): "A system as such does not exist in reality. It is the result of a cognitive operation that an observer performs distinguishing a particular entity and assigning it a meaning of its own."

Table 5.2 Some summary data about e-Territory project

e-Territory project	Core project	Extension
Start date	June 2003	September 2010
End date	July 2010	October 2012
Expected duration	12 months	10 months
Project cost	€1.5M	€1M
no. of management meetings	$\geq$ 100	14
no. of informal disputes	12	3
no. of formal disputes	2	0
no. of competitive tenders	3	1

Table 5.2 shows some data summary of e-Territory, comparing the two project realizations: the core project started in June 2003 and extension started in September 2010.

Some kinds of significant events acted on the project negatively resulting in a greater uncertainty as well as in a longer execution time and project failure risk:

- the insufficient skill to realize some activities critical to the success of the project;
- delays in authorizations or building permits from the local authorities;
- delays in payments by the authorizing officer subject;
- differences and conflicts within the stakeholders;
- the change of the governance of the coordinator during the period of project development;
- the failure of one proponent company.

The increase of uncertainty is inevitably reverberated on the consonance level between authorizing officer and proponents, but also among the own partners components causing, on several occasions, informal and formal disputes designed to unlock a stalemate. The use of monitoring and control loop was only introduced in the second version of the project: whenever an influence of divergence (Fig. 5.1) occurred, a monitoring loop was executed to restore acceptable levels of consonance among stakeholders. This approach helped to provide practical support to both project manager and the principal contractors allowing them to identify some problems more quickly and encouraging the quick resolution of some disputes in an informal manner. The coordinator has been able to apply more control about work in progress and the proponent was able to demonstrate the goodness of what has been achieved (for some activities in dispute) making use of the potential offered by an open source document management system (KnowledgeTree) shared with the UMC.

Conclusion

The proposed method addresses to the uncertainty management in complex projects and it can make a contribution in reduction of risks about project failure or uncontrolled increase in project times and costs, maintaining a watchful attention on planned quality.

The set of measurements made on a project activity allows us to estimate the degree of consonance between the participants in all the activities. When the total gap continuously raises, the consonance among stakeholders decreases and the potential of disputes grows. The proposed method provides the continuous monitoring of the project and identifies the guidelines for the resolution of disputes. Divergent views concern both the implementation of project activities and, in general, the fulfillment of contractual clauses governing the conduct of the parties in the project. The realignment of views allows to restore the level of consonance to acceptable levels and to reactivate the resonance for achieving a common goal.

The case study presented has allowed a first validation of the proposed method. The results in terms of reducing the time of project completion for the e-Territory extension, compared to the first project to implement, have benefited from the method of continuous monitoring and dispute resolution, especially when influences of divergence due to the context caused uncertainty that was unforeseen at the project beginning.

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Chapter 6
On the Concept of Endogenous Volatility

Orlando Gomes

Abstract Most financial and economic time-series display a strong volatility around their trends. The difficulty in explaining this volatility has led economists to interpret it as exogenous, i.e., as the result of forces that lie outside the scope of the assumed economic relations. Consequently, it becomes hard or impossible to formulate short-run forecasts on asset prices or on values of macroeconomic variables. However, many random looking economic and financial series may, in fact, be subject to deterministic irregular behavior, which can be measured and modelled. We address the notion of endogenous volatility and exemplify the concept with a simple business-cycles model.

Keywords Endogenous volatility · Volatility clustering · Nonlinear dynamics · Chartists and fundamentalists · Periodicity and chaos · Business cycles

Introduction

The forces that shape the evolution of a price or the motion in time of a given real variable are so many that economists are often faced with a sentiment of frustration. No matter how much we know about the way markets are organized, or about the relative weight of each market participant, or even about the intrinsic complexity governing the relations between agents, we will never be able to accurately understand how economic and financial variables will evolve in the near future. This implies that there is a random component underlying this evolution. The future always brings unexpected events, introducing uncertainty into the economic environment and an

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impossibility of exactly knowing what the next movement of, e.g., an asset price or an interest rate, will be.

A fundamental question is whether the observed volatility is entirely associated to unpredictable events or if there is a more or less significant part of this volatility that is endogenous, emerging from the type of relation that is established between the relevant economic or financial variables. The answer to this question has huge practical implications—endogenous volatility will correspond to the predictable component of the observed fluctuations; if we can discern this, we will be able to isolate a smaller component of true unpredictability and, in this way, mitigate the uncertainty associated with the time trajectory of the variable(s) under consideration.

The observation of the behavior over time of some economic aggregates provides an indication that, effectively, some of the displayed volatility is endogenous. For instance, Mandelbrot (1963) has identified the presence of 'volatility clustering' in the evolution of prices. Under volatility clustering, periods of large volatility alternate with periods of small changes in prices of commodities and assets. The existence of this type of phenomena was pervasively recognized in the academic profession and it begun to be modelled resorting to statistical models of the ARCH type (Engle 1982; Bollerslev 1986).[1] From the statistical analysis one infers that the volatility displayed by financial and economic time series must be associated with some kind of endogeneity, in the sense that if the volatility was purely random, it would not display any type of regularity, as the one described.

As Adrangi et al. (2010) refer, many random looking time series may contain deterministic fluctuations and the state of the scientific knowledge allows, at the present, to conduct some analyses in order to distinguish what is endogenous from what is purely random or 'noise'. Unfortunately, many of the undertaken studies so far are not completely conclusive; we will get back to this idea in the beginning of section "Making Sense of Theoretical Nonlinearities".

We will be concerned with endogenous volatility essentially at two levels: policy implications and possible theoretical approaches. If we accept the intuition that observed fluctuations are, at least partially, endogenous, we can open the door to short-run predictability. A stochastic / random time series is completely unpredictable; if the fluctuations are deterministic, on the other hand, even when they are irregular the possibility to forecast future values with accuracy exists if one is able to fully understand the law of motion governing the relation between endogenous variables. Deterministic cycles may be periodic, of any possible periodicity, or completely a-periodic, i.e., we may have endogenous irregular fluctuations which can be associated with the notion of chaos. Chaotic time series, that we will address with some detail in section "The Theory of Nonlinear Dynamics", are characterized

[1] These statistical models became sufficiently sophisticated in order to be possible to search for the reason of volatility clustering, namely when the ARCH analysis is combined with models for the conditional mean, giving place to TAR-ARCH models (see Tong 1990). Other variants of the ARCH class also provide relevant insights on what determines the type of observed volatility (see Bollerslev et al. 1992; Engle 2002 and Bollerslev 2010). I thank the referee for suggesting these references on the subject.

by being deterministic (and, thus, full predictability exists) but also by being subject to sensitive dependence on initial conditions (SDIC), what means that even if we are in the possession of the actual law of motion governing the economic or financial relation, we can radically fail in providing good forecasts on future values of the series, namely if an error occurs in understanding the initial state of the system; even the slightest difference in initial conditions leads to a complete divergence of the considered orbits.

On the theoretical perspective, we will be concerned with emphasizing the idea that a simple nonlinear dynamic relation is capable of generating endogenous cycles. The only pre-requisite for these cycles to emerge is, in fact, the lack of linearity. Noticing that the reality is complex and that most of the relations in the financial and economic realms are necessarily nonlinear, we infer that endogenous volatility is not difficult to explain from a theoretical point of view. This is the strong idea that this manuscript explores, first by surveying theory and applications in this field of study and, in a second stage, by illustrating the presence of endogenous cycles in a macroeconomic business cycles model.

The remainder of the text is organized as follows. Section "The Theory of Nonlinear Dynamics" reviews the most meaningful notions on nonlinear dynamics. Section "Making Sense of Theoretical Nonlinearities" describes some of the recent applications on economics and finance. In section "An Illustration: Inflation Dynamics", an illustration is explored; this illustration relates to a business cycles macro model, relatively to which we can address the dynamics of the inflation rate. Section "Conclusion" concludes.

The Theory of Nonlinear Dynamics

Economic relations are typically addressed in a multi-period framework. What has happened in the past has an impact on today's economic activity and current events shape future time paths. Moreover, past expectations on today's outcomes and current expectations about future outcomes are, most of the times, present in this dynamic interpretation of the observed reality. The dynamics can be formally addressed through models involving differential equations (in continuous time) or difference equations (in discrete time). These equations reflect the kind of relations one expects to exist between economic variables; most of the times these are not just ad-hoc relations but the outcome of the optimizing behavior of rational agents.

If the referred relations take a nonlinear form, the dynamic process characterizing the evolution in time of the assumed variables may depart from the trivial results of pure convergence to a fixed-point steady-state (stability) or pure divergence from such point (instability). Cycles of any periodicity or even completely a-periodic motion might arise; in this case, we will be in the presence of bounded instability or endogenous volatility: there will be a perpetual fluctuation around the steady-state point without ever occurring a complete convergence or a complete divergence relatively to that point.

As referred in the introduction, the discovery of fluctuations determined by the type of connection between variables has significant impact over the way we understand the evolution of financial and economic time series—irregular behavior is not necessarily synonymous of stochastic behavior and we can find some type of predictability in a series with apparent erratic behavior.

In this section, we address the most significant notions and results on deterministic nonlinear dynamics. Our intention is not to be thorough in the presentation of the theory but rather to highlight the most important tools and intuition at this level. A more detailed presentation of concepts and mechanisms can be found in Medio and Lines (2001), Lines (2005), Barnett et al. (2006), Gomes (2006) and Grandmont (2008).

Our starting point is a difference equation defined in some m dimension (we shall proceed the presentation assuming that time is discrete):

$$X_{t+1} = G(X_t), \;\; X_0 \text{ given.}$$

Function G is a map from an open subset U of $\mathbb{R}^m$ into $\mathbb{R}^m$, i.e., $G : U \subset \mathbb{R}^m \to \mathbb{R}^m$, and X_t is a vector with m variables x^i, $i = 1, 2, ..., m$. The evolution of the set of variables x^i in time will depend on the particular shape of function G.

If G is linear, only two outcomes are possible: the elements of X_t converge from X_0 towards a fixed-point X^* or, alternatively, they will (at least some of them) diverge from X_0 relatively to X^*. Let $G(X_t) = A + BX_t$, with A a vector of parameters of length m and B a $m \times m$ square matrix also of parameters. Defining the steady-state, balanced growth path or long-term equilibrium as the point in which the system remains at $X_{t+1} = X_t := X^*$, the system's steady-state will be $X^* = (I - B)^{-1}A$ with I a $m \times m$ identity matrix. Transitional dynamics (the behavior of the system from $t = 0$ to $t \to \infty$) will be determined by the properties of matrix B; more accurately, the dynamics will depend on the values assumed by the m eigenvalues of the matrix. The number of eigenvalues for which $|\lambda_i| < 1$, i.e. the number of eigenvalues inside the unit circle, determine the dimension of the stable arm; the number of eigenvalues such that $|\lambda_i| > 1$, i.e. the number of eigenvalues that fall outside the unit circle, correspond to the unstable dimension of the system. For instance, a three dimensional system with two eigenvalues lower than 1 in absolute value has a stability dimension of order 2.

In contrast, when nonlinearities are present, we will possibly encounter long-term outcomes that differ from the simple convergence or divergence behavior. A nonlinear system can be linearized in the vicinity of the steady-state and addressed as explained above. Such a procedure may be helpful in order to understand what occurs in a local perspective, i.e. when the initial locus of the system is located nearby the long-term fixed-point result. However, such an approach may hide global dynamics involving much more sophisticated intertemporal behavior. Typically, the local analysis of a nonlinear system is able to separate a region of stability, in the space of the system's parameters, from a region of divergence relatively to the fixed-point. The frontier between these two regions is a bifurcation line. The consequences of passing through this bifurcation line become clear once we look at global dynamics: as we abandon the stability area, the bifurcation may trigger the formation of low-periodicity cycles, that may degenerate in more complex long-run outcomes before plain instability sets in. This eventual process of formation of irregular cyclical behavior is the outcome of varying the values of the key parameters of the model.[2]

In a systematic form, we present quick and straightforward definitions of the several types of long-term outcomes we can find when analyzing nonlinear maps (i.e., nonlinear systems in discrete time):

1. Fixed-point: X^* is a fixed-point of X_t if $X^* = G(X^*)$;
2. Cycle of n-order periodicity: X^* is a periodic-point of order n of X_t if there is a constant $n = 2, 3, \ldots$ such that $X^* = G^{(n)}(X^*)$, where $G^{(n)}(X_t)$ represents the nth iterate of X_t;
3. A-periodicity: long-term dynamic outcome of a system for which it is not possible to identify the existence of a fixed-point or of a cycle of a defined periodicity;
4. Chaos: particular form of a-periodicity, which can be defined through the Li and Yorke (1975) theorem—a continuous system $X_{t+1} = G(X_t)$ exhibits chaos if it is possible to identify a periodic point of a period that is not a power of 2.

Some examples are trivial in this literature. The most frequently addressed are the logistic map and the tent map. They are both defined in one dimension and are good examples of how one evolves from stability to chaos by varying some parameter's value. Their analytical representations are as follows:

- Logistic map:
$$x_{t+1} = ax_t(1 - x_t), \quad x_0 \text{ given}, a > 1$$
- Tent map:
$$x_{t+1} = \begin{cases} (1/b)x_t, 0 \leq x_t \leq b \\ (1/b)(1 - x_t), b < x_t \leq 1 \end{cases}, \quad x_0 \text{ given}, b \in (0, 1)$$

Figures 6.1 and 6.2 present, for each one of the maps, the corresponding long-term trajectories for values of parameters in which the chaotic zone is reached; Fig. 6.1 also includes a panel with the bifurcation diagram of the logistic map. The trajectories show that the variables evolve around the corresponding fixed-points, but they will never stabilize in order to rest forever in that specific point. Such behavior is just the result of the shape of the considered nonlinear relation. In the second case, the tent map, we realize that a discontinuity is a possible cause of bounded instability in deterministic dynamics.

There are several ways in which we can define nonlinear results and particularly chaos. The developments in nonlinear dynamic theory along the past few decades have allowed to develop important tools to measure chaos in theoretical and empirical

[2] Furthermore, we should note that the local stability analysis may not be sufficient to take a defnitive conclusion about stability, since in certain circumstances a locally unstable system may be globally stable.

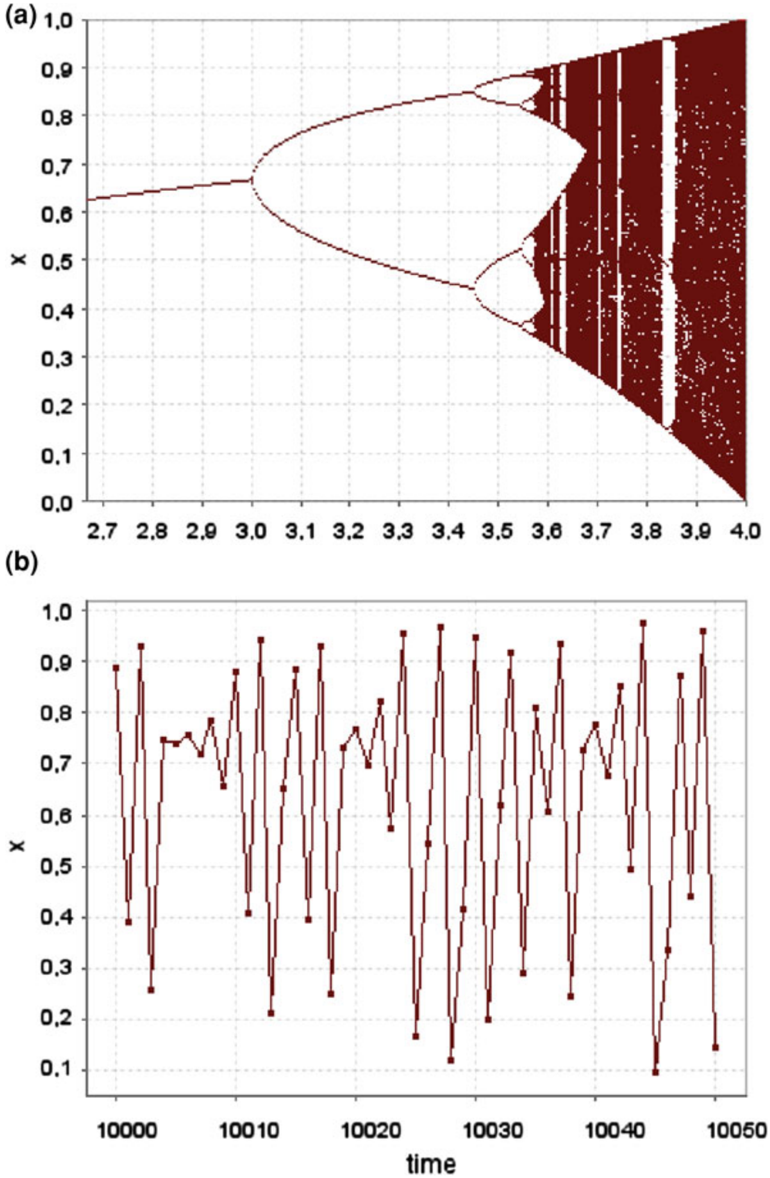

Fig. 6.1 Logistic map: bifurcation diagram and long-term time series

terms. It is not the purpose here to deal with these in further detail, but we should stress again the relevance of encountering nonlinear deterministic processes that look random. The property of SDIC implies that, although deterministic, a chaotic system positioned at slightly different states will rapidly evolve towards dramatically

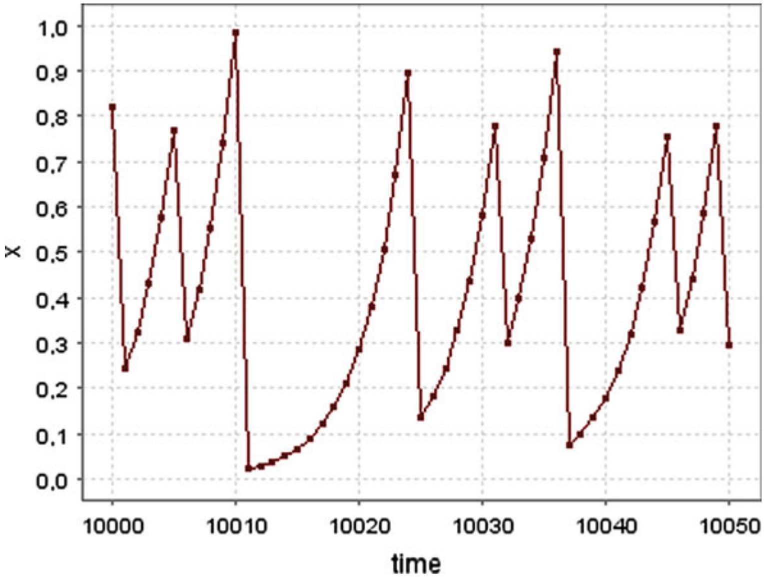

Fig. 6.2 Tent map: long-term time series

different trajectories, which turn forecasting difficult but not impossible. Knowing with accuracy the underlying dynamic process associated with some financial or economic relation and accurately perceiving, as well, the state the system is in, at a precise moment, may allow, at least in the short-run, to predict part of the observed time series volatility.

Making Sense of Theoretical Nonlinearities

The usefulness of nonlinear dynamic models in addressing financial and economic phenomena was made clear in the previous sections. Interpreting all observed volatility as 'noise' or 'unexplainable shocks' is a recognition of the incapacity of the researcher in explaining one of the two components of any time series: although the forces shaping long-run trends are relatively easy to address, fluctuations around such trends are typically associated with random events that no theory should ever dare to approach if one wants to keep scientific knowledge as an objective and non-speculative entity.

Although tests exist to measure the possible presence of chaos in observable time series, the results so far are far from conclusive. Tests on chaos for stock prices, interest rates or exchange rates (Barnett and Chen 1988; Serletis and Gogas 1997) point to a possible affirmative answer, but even if we agree that observed fluctuations are chaotic more than they are random, we will have to face a second challenge: to discern where can we find the source of deterministic fluctuations, i.e., what kind of nonlinear relation effectively exists in order to generate such type of volatility. This is the main question we place in this section—if chaos explains at least partially observed fluctuations, how should we look at economic relations in order to explain the existence of this phenomenon?

The raised question should be approached taking into account the strength of the evidence on chaos, but also under the more philosophical notion, that Ruelle (1994) emphasizes, that 'noise' is only 'noise' as long as we are unable to find some relation that explains it and, thus, a full understanding of how the world works could hypothetically give us a model where all volatility could be endogenously explained.

One of the most influential models in economics and finance involving an interpretation for deterministic randomness is the heterogeneous beliefs model of Brock and Hommes (1997, 1998). In this model, two types of agents are considered: fundamentalists, who are well informed agents that formulate rational expectations, and chartists or trend followers, who rely on past information in order to predict future values. These two types of agents will exist both in financial markets and in commodity markets, and the relevant expectations are typically associated to asset prices or prices of goods and services. By combining agents' heterogeneity with a mechanism of discrete choice and evolutionary selection, this kind of framework is able to generate endogenous fluctuations and, therefore, to provide an endogenous explanation for part of the observed volatility.

The mechanism of discrete choice, developed by McFadden (1973), is based on a concept of bounded rationality. Under rational expectations, agents' heterogeneity would not persist; agents with expectations other than the ones involving the fundamental outcome would be expelled from the market because they would be systematically wrong. This notion, when applied to financial markets, is known as the efficient market hypothesis (EMH): only the fundamental outcome matters, because any expectations other than rational ones imply incurring in systematic mistakes and, thus, lead to an irrational behavior. In this case, markets should be efficient and past prices must not be used to predict future prices. In other words, there is no place in an efficient market for chartists or technical traders; an efficient market is a market of homogeneous and rational traders. A corollary of this reasoning is that an efficient market is also a market where all observed volatility is necessarily exogenous.

Empirical results pointing to phenomena of excess volatility (Shiller 1981) or to the notion of volatility clustering, already referred in the introduction, indicate that markets are not efficient, rationality may be bounded and technical traders that extrapolate future outcomes from past performance are able to remain on the market without incurring in systematic losses. While fundamentalists believe that prices return to their fundamental value (the discounted sum of future dividends, in the case of asset prices), technical traders exploit particular episodes of more or less strong market activity. This second type of traders works as a destabilizing force, while fundamentalists have the role of stabilizing the market. It is the interplay between these two types of agents that gives rise, at least partially, to the kind of bounded instability price dynamics one observes in practice.

The popularity of the Brock-Hommes framework relates to its capacity of offering a simple but convincing explanation for observed market volatility. This interpretation is associated to the idea that price movements are driven by endogenous laws of motion which can be discovered only if we relax the notion of market efficiency. Heterogeneity of beliefs and bounded rationality (supported on a mechanism of gradual evolutionary selection) seem to be the required ingredients to build a mechanism that is able to support the stylized facts of financial series: excess volatility, volatility clustering, speculative bubbles, crashes and fat tails for the distribution of returns.

In the last few years, the Brock-Hommes framework has been extended in several directions. Some are relatively straightforward, as the work developed by Brock et al. (2005), who generalize the original framework to include many trader types; in this case, the authors introduce the notion of large type limit in order to show that independently of the degree of heterogeneity, an adaptive evolutionary system is capable of generating endogenous volatility. Boswijk, Hommes and Manzan (2007) resort to the same kind of setting of heterogeneous agents that are boundedly rational; again, the evolutionary selection mechanism is considered and the emphasis is placed on the incentives to change strategy—relative past profits determine investment decisions. Gaunersdorfer et al. (2008) address, as well, the fundamentalist-chartist setup in financial markets; the technical analysis of bifurcations in this paper allows adding some important insights concerning the issue of volatility clustering. Other studies on complex evolutionary systems involving competing boundedly rational trading strategies in financial markets include Manzan and Westerhoff (2007) and Dieci and Westerhoff (2010), who extend the benchmark framework to the foreign exchange market or, more precisely, to the interaction of stock markets of different countries through the exchange rate market. Many other studies follow the referred approach to address price fluctuations.

The presence of chaos in the mentioned type of model is particularly relevant, as highlighted by Wieland and Westerhoff (2005), because if fluctuations are, even partially, chaotic, then chaos control methods can be applied by central authorities in order to reduce observed volatility. Chaos control may be a fundamental tool in order to solve problems of excess volatility in the markets.

Hommes et al. (2005, 2008) conduct laboratory experiments to test the empirical plausibility of the fundamentalist-chartist framework. Markets are simulated from a pre-defined adaptive evolutionary system and results seem to concur with what theory predicts and with what reality shows: bubbles emerge endogenously and, thus, the advanced explanation can be accepted as successful in replicating the stylized facts of financial environments.

The fundamentalist-chartist approach to endogenous fluctuations has been applied also outside the realm of financial markets. Branch and Evans (2007), Branch and McGough (2009) and Lines and Westerhoff (2010) apply the adaptive evolutionary setup to economic relations and, in particular, to macro relations involving the time paths of output and inflation. Volatility clustering is also found in macroeconomic time series (namely, in what concerns the 'Great Moderation' of the 1980s, period in which inflation and output volatility has fallen dramatically). To explain this phenomenon, the cited authors build frameworks where agents either use a sophisticated costly predictor or a simple cheap predictor. Evolutionary competition concerning the performance of these two predictors will, also in this case, determine a scenario of deterministic long-term cycles, in which there will be a systematic change on the shares of agents that choose to remain with one rule or switch to the other rule. Economic time series are also exposed to changes in the intensity of volatility and the same setup used for the financial analysis can have here a decisive role in order to encounter a reasonable explanation for observed behavior.

A brief inspection of the economic literature allows to find many other sources of endogenous volatility that help explaining relevant stylized facts. Some recent examples include: Chen et al. (2008), who apply different types of expectations (perfect foresight, myopic expectations and adaptive expectations) to an overlapping generations model; they conclude that dynamics are simple under perfect foresight, but myopic and adaptive expectations may induce cycles and chaotic motion. In Fanti and Manfredi (2007), a standard neoclassical labor market is studied; in this case, cycles and chaos are the result of a setting where consumption and leisure are considered sufficiently low substitutes.

In Hallegatte et al. (2008), a growth model under the absence of market clearing is explored; the authors call it non-equilibrium dynamic model (NEDyM). The NEDyM might generate endogenous business cycles under peculiar conditions. These conditions involve two types of inertia: delays in the mutual adjustment between production and demand and a delayed dependence of investment on past profits. Thus, part of the observed volatility associated to business cycles can, in this perspective, be associated with these two inertia effects. Chaotic motion is also found and explored by Sushko et al. (2010) in a growth setting where investment decisions are central: investment will be delimited from above and from below given capacity limits and capital depreciation, respectively. The ceiling and the floor are ingredients that are likely to generate cycles and chaos for reasonable combinations of parameter values. Many other studies, involving different types of explanations explore endogenous fluctuations in macroeconomic environments. Just to give two additional references, we cite the work of Yokoo and Ishida (2008), who find an explanation for endogenous business cycles on deficiencies on the access to and interpretation of relevant information (what they call misperceptions), and Kikuchi and Stachurski (2009), who study international growth and attribute fluctuations to the interaction through credit markets when countries have asymmetric economic conditions.

An Illustration: Inflation Dynamics

In this section, we present our own illustration on how endogenous volatility might arise once we take some acceptable changes over a benchmark macroeconomic model.

We follow Mankiw and Reis (2002) and consider a monopolistically competitive market environment, where firms want to set an optimal or desired price, that is obtained by taking a trivial profit maximization problem. We define the desired price as p_t^*; the aggregate price level is given by p_t and y_t represents the output gap (the relation between effective and potential output). All these variables are expressed in logarithms of the corresponding original values. The firms' optimization problem leads to the desired price

$$p_t^* = p_t + \alpha y_t \tag{6.1}$$

with $\alpha \in (0, 1)$ a measure of real rigidities (this parameter translates the degree of substitutability between different varieties of the assumed good; $\alpha = 0$ brings us back to perfect competition). The equation states that the price firms intend to set at some period t will be larger than the observed price level in periods of expansion ($y_t > 0$) and smaller than the observed price level in periods of recession ($y_t < 0$).

We will assume that a share λ of firms collects information on the state of the economy at the current period, while the remaining share $1-\lambda$ resorts to old information, obtained one period in the past. Thus, the observed price level will be

$$p_t = \lambda p_t^0 + (1 - \lambda) p_t^1 \tag{6.2}$$

with $p_t^0 = p_t^*$ and $p_t^1 = E_{t-1}(p_t^*)$. We also assume that if expectations are formed at $t - 1$, firms will perceive the current period as the long-run steady-state and the expectation will correspond to the observed desired price at $t - 1$ plus the rate of growth to t; as it will become clear at a later stage, the growth rate of the output gap will be zero in the steady-state and inflation will grow at some rate π^* that we will be able to present in explicit form. The expectation is, then, $E_{t-1}(p_t^*) = p_{t-1} + \pi^* + \alpha y_{t-1}$.

We define the inflation rate as $\pi_t := p_t - p_{t-1}$. Putting together the previous information, we arrive to a supply side relation or Phillips curve that establishes a link between the output gap and the inflation rate:

$$\pi_t = \frac{\alpha\lambda}{1-\lambda} y_t + \alpha y_{t-1} + \pi^* \tag{6.3}$$

On the demand side of the economy, we will have to take a trivial utility maximization problem. We also consider information stickiness on the part of households. As for firms, stickiness will translate on a share λ of consumers that set their consumption plans at t and a share $1 - \lambda$ that has updated their information set at $t - 1$. Thus,

$$c_t = \lambda c_{t,0} + (1 - \lambda) c_{t,1} \tag{6.4}$$

Variable c_t represents the logarithm of aggregate consumption and $c_{t,0}$, $c_{t,1}$ are, respectively, the consumption levels (in logs) of each one of the two types of assumed households. According to Mankiw and Reis (2006), $c_{t,j} = -\theta E_{t-j}(R_t)$, $j = 0, 1$, with $\theta > 0$ the intertemporal elasticity of substitution of consumption and $R_t = E_t\left(\sum_{i=0}^{\infty} r_{t+i}\right)$ the long real interest rate. If the real interest rate, r_t, is expected to converge to its long-run value, which is zero, at a rate $a \in (0, 1)$, then, we can simplify the expression: $R_t = \sum_{i=0}^{\infty}(1-a)^i r_t = \frac{1}{a} r_t$.

Assuming market clearing, i.e., $y_t = c_t$, expression (6.4) will be equivalent to:

$$y_t = -\frac{\theta}{a}\left[\lambda r_t + (1-\lambda) r_{t-1}\right] \tag{6.5}$$

Equation (6.5) can be further rearranged by taking the Fisher equation, $i_t = r_t + E_t(\pi_{t+1})$, where i_t stands for the nominal interest rate. We must consider as well a monetary policy rule; here, the assumption is that the central bank reacts to deviations of the expected inflation rate relatively to a target value $\overline{\pi}$, when setting the nominal interest rate,

$$i_t = \phi\left[E_t(\pi_{t+1}) - \overline{\pi}\right] \tag{6.6}$$

The monetary policy is considered active, in the sense that the monetary authority responds aggressively to changes on the inflation rate, that is, $\phi > 1$.

Assuming households exhibit perfect foresight when predicting future inflation, the demand side equation takes the form,

$$y_t = -\frac{\theta(\phi-1)}{a}\left[\lambda \pi_{t+1} + (1-\lambda)\pi_t - \pi^*\right] \tag{6.7}$$

The behavior of this economy is fully described by the supply and demand relations (6.3) and (6.7). Combining the two, we can suppress the output-gap from the analysis and arrive to a system that explains the evolution of the inflation rate in time. This system is:

$$\begin{cases} \pi_{t+1} = -\left[\frac{a(1-\lambda)}{\alpha\lambda^2\theta(\phi-1)} + \frac{2(1-\lambda)}{\lambda}\right](\pi_t - \pi^*) - \left(\frac{1-\lambda}{\lambda}\right)^2 (z_t - \pi^*) + \pi^* \\ z_{t+1} = \pi_t \end{cases} \tag{6.8}$$

with $\pi^* = \frac{\phi}{\phi-1}\overline{\pi}$ the steady-state inflation level. Note that the steady-state inflation rate is larger than the target that is set by the central bank; this is the result of considering a non-optimal monetary policy rule.

The dynamics of system (6.8) can be addressed under a local perspective, i.e., in the vicinity of the steady-state and under a global analysis. The first typically allows for separating regions of stability and instability; the global analysis confirms the location of the stability area and allows to perceive if the region of instability involves some kind of cyclical motion.

To address the model's dynamics, we consider an additional assumption: we assume that λ is not constant; this degree of attentiveness will respond to the inflation observed in the previous period, i.e., $\lambda = \lambda(z_t)$. The idea is that agents will be more attentive if they have previously observed a larger inflation level; low levels of inflation would not require such an attentive behavior. Thus, λ will be an increasing function of z_t. If we define a floor $\underline{\lambda} \in (0, 1)$ that corresponds to the lowest possible

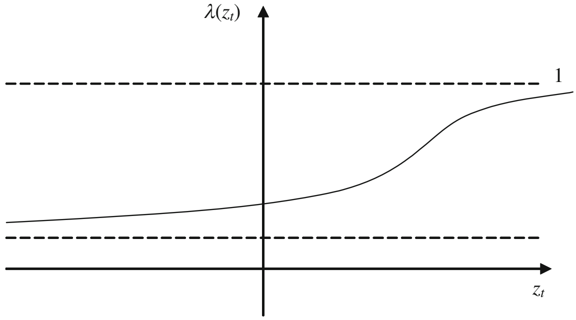

Fig. 6.3 $\lambda(z_t)$ function

level of attention of firms and households and an attentiveness share $\lambda_0 \in (\underline{\lambda}, 1)$ such that $\lambda_0 = \lambda(0)$, the function that translates the behavior of the degree of attentiveness will be something as presented in Fig. 6.3, where large levels of past inflation imply full attentiveness (i.e., all agents will be attentive to relevant news); in other words, as $z_t \to +\infty$, λ converges asymptotically to 1.

The function in Fig. 6.3 can be analytically represented in the following form[3]:

$$\lambda(z_t) = \frac{1+\underline{\lambda}}{2} - \frac{1-\underline{\lambda}}{\pi} \arctan\left[\tan\left(\frac{\pi}{2}\frac{1+\underline{\lambda}-2\lambda_0}{1-\underline{\lambda}}\right) - z_t\right] \tag{6.9}$$

Local dynamics are similar with a constant and with an increasing degree of attentiveness. The linearized system is:

$$\begin{bmatrix} \pi_{t+1} - \pi^* \\ z_{t+1} - \pi^* \end{bmatrix} = \begin{bmatrix} -\left[\frac{a(1-\lambda^*)}{\alpha\lambda^{*2}\theta(\phi-1)} + \frac{2(1-\lambda^*)}{\lambda^*}\right] & -\left(\frac{1-\lambda^*}{\lambda^*}\right)^2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \pi_t - \pi^* \\ z_t - \pi^* \end{bmatrix} \tag{6.10}$$

with λ^* the steady-state level of $\lambda(z_t)$. Applying stability conditions $1-Det(J) > 0$, $1-Tr(J)+Det(J) > 0$ and $1+Tr(J)+Det(J) > 0$, with J the Jacobian matrix of the above system, we conclude that only the second condition is universally satisfied. The first and the third conditions imply the following inequalities, respectively,

$$\lambda^* > 1/2; \;\; \phi > 1 + \frac{a(1-\lambda^*)}{\alpha\theta\left[1-4\lambda^*(1-\lambda^*)\right]}$$

That is, stability requires a relatively high degree of attentiveness and, simultaneously, a relatively aggressive monetary policy.

Through a global dynamic analysis we can confirm that the region of stability is bounded according to the found inequalities and that endogenous volatility exists for specific combinations of parameter values. Take the following set of reasonable values: $a = 0.01$, $\alpha = 0.1$, $\theta = 1$, $\underline{\lambda} = 0.1$, $\overline{\pi} = 0.02$ and $\phi = 1.5$. Figure 6.4 presents a bifurcation diagram that allows to perceive that different values

[3] Note that the π in the expression is not the inflation rate but the value 3.14159...

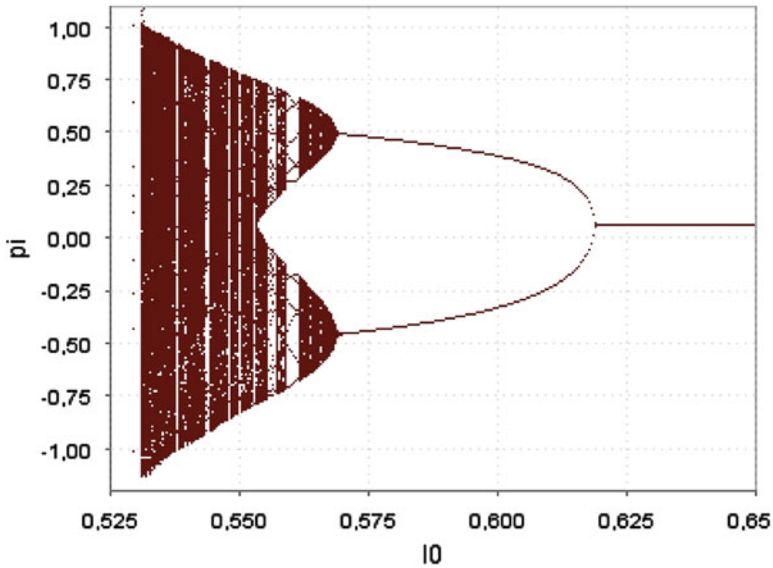

Fig. 6.4 Bifurcation diagram (π_t; λ_0)

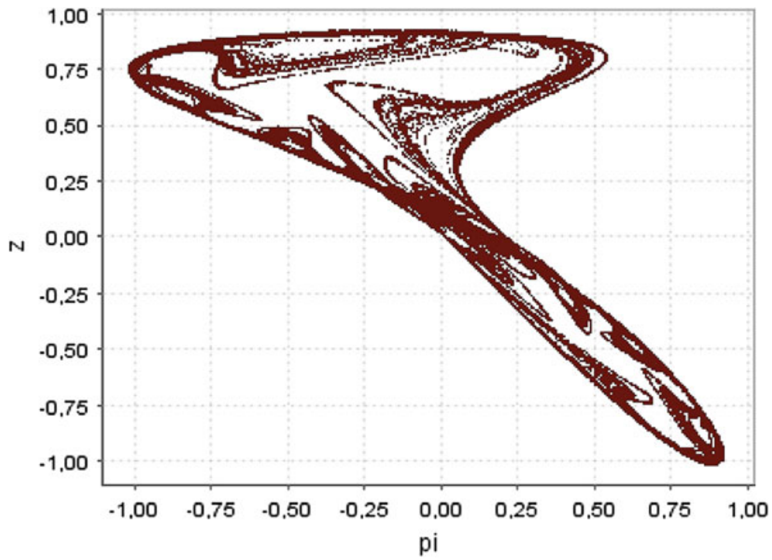

Fig. 6.5 Chaotic attractor (π_t, z_t); $\lambda_0 = 0.538$

of λ_0 generate qualitatively different outcomes in terms of dynamics. Chaotic motion exists for specific values of the referred parameter; for instance, for $\lambda_0 = 0.538$ there is chaos, as one observes by looking at the strange attractor in Fig. 6.5 and to the long-run time series of inflation in Fig. 6.6.

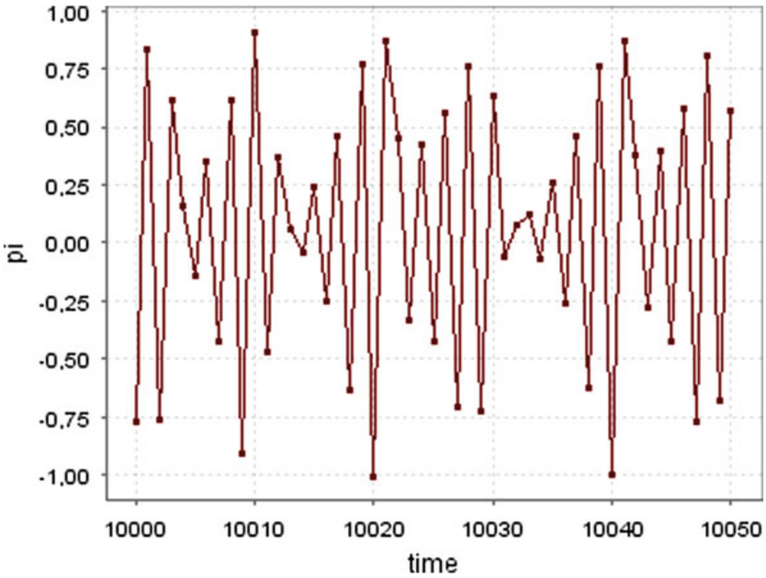

Fig. 6.6 Long-term time trajectory of π_t; $\lambda_0 = 0.538$

The above illustration shows how a conventional benchmark macroeconomic model, that takes as structural elements a Phillips curve and an aggregate demand equation, can be slightly modified in order to provide for the existence of endogenous volatility: in this case, the volatility is simply the result of how one approaches attentiveness relatively to the relevant information.

Conclusion

Time series of economic and financial variables display volatility and this volatility presents features that indicate that fluctuations do not correspond to pure noise. Being able to identify the reasons why volatility assumes some specific patterns may be helpful in understanding aggregate phenomena. A popular interpretation, namely in finance, considers a setting of heterogeneous boundedly rational agents. The interaction between fundamentalists and chartists in an evolutionary setting tends to generate endogenous fluctuations. Many other candidates to justify deterministic volatility exist, as discussed along the text.

We have presented our own illustration, where varying degrees of attentiveness trigger the formation of endogenous cycles for the inflation rate (and also for the output gap, because these two variables are correlated). The main lesson we withdraw is that endogenous volatility is a frequently obtainable result in any dynamic system involving nonlinearities. If we have the possibility of translating complex observable phenomena into nonlinear mathematical relations, we might be able to find convincing explanations on why observed fluctuations are not completely stochastic.

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Chapter 7
Chaotic Order: A Consequence of Economic Relativity

Rongqing Dai

Abstract A philosophical analysis on economic relativity is provided by philosophizing the dynamic reasons behind the coexistence of global disorder and local order in any sizeable economic system. The examples of economic development in China and Eurozone are discussed to demonstrate how economic relativity would affect economies in apparently very different situations. The vital and subtle role of fairness in economy is also discussed in this writing.

Keywords Economic relativity · Fairness · Communism · Capitalism · China · Eurozone

Introduction

For the past decades there have been many writings on the topic of chaos and order, and thus it is necessary for me to clarify the meaning of *chaotic order* in this context before any further discussion. Around 30 years ago when I took the "Chaos and Nonlinear Dynamics" course, the first class was about the Baker's transformation. Although much mathematical complexity has been attached to this issue as the name itself might attest, the so called Baker's transformation has a simple and clear practical background from our daily life. When a Baker works on a dough in a traditional handy way, after a few times of repeated rolling (stretching) and folding, the flour of the dough would be irreversibly very well self-mixed (Tél and Gruiz 2006). A good indication of this is that if he add a teaspoon of salt at one spot of the dough at the very beginning, after quite a limited number of above mentioned operations, the whole dough would become equally salty everywhere. Here what is relevant to the sense of *chaos* in everyday life is the *disorder* or *randomness* in the sense that after

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a few handy operations the distribution of salt completely loses its initially recognizable spatial orderliness. But what draws special interests of scientists about this *chaotic* issue is the fact that it only takes very limited (instead of unlimited as might be previously conceived a few centuries ago) number of well defined operations to make an ordered system into a completely disordered system, which they call as *deterministic chaos*. To add a little more scientifically salty flavor to this deterministic chaos, we might want to point out that as we could not predict the exact position of each salt particulate in a dough during the baker's operation, in general, deterministic chaos is referring to processes that are the deterministically generated but with unpredictable details. Furthermore, although the final salty equilibrium of the dough is obviously foreseeable, in general, the end state of a nonlinear dynamic system that exhibits some chaotic behavior could be very sensitive to the initial condition and thus unpredictable.

Almost thirty years passed, the writing of this article brought me back to the subject of deterministic chaos again. While it might sound much closer to everyday life (especially to nominally random and unpredictable economic events all over the world) than some other mind changing scientific discoveries fashioned in last century, such as relativity and quantum physics, chaos theory seems still very much remain as a fancy ivory decoration in scientist possessions. Even though chaos theory does unarguably provide valuable insights into the dynamics of Nature, after countless number of new literatures have been added to those familiar jargons such as strange attractor, bifurcation, fractal and so on (Grebogi and Yorke 1997), it is somehow a disappointment that not much practical fruits have been reaped from this golden tree (Bau and Shachrnurove 2003; Bayar 2005).

This practical futility of chaos theory brings up an interesting philosophical issue. As Wittgenstein stated (Philosophical Investigations 1953), "Philosophy is a battle against the bewitchment of our intelligence by means of our language." The philosophical issue behind this practical futility of chaos theory is indeed a language issue in the first place; and thus we might first look into this issue from its language perspective.

Regardless its sophisticated academic meaning, the strict scientific term *chaos* obviously has its origin from the casual usage of chaos in our daily life. The fundamental reason that people did not choose *harmony* or *order* or anything else but choose *chaos* instead as it is used in chaos theory is clearly related to the ordinary meaning of the word. More specifically, the reason that makes scientists believe that they could help to solve social issues including financial or more general economic problems clearly has its linguistic cause: one would describe social issues as chaotic for its highly disorderliness and unpredictability, without the need to know anything about chaos theory. But on the other hand, in their scientific contexts, professional scientists have been constructing the so called chaos theory by using the word *chaos* in a much narrower sense with some quite strict restrictions which might involve some mathematical measurement.

Now we see the clash: the temptation of applying chaos theory to solve complicated real life problems and the limited applicability of the theory itself. The temptation is largely due to the linguistic implication that this (ordinary) *chaos* is that (scientific) *chaos*; but whenever people wants to apply the theory to real life issues, they would clash with the incompatibility between the real life chaos and their theoretical chaos with some strict mathematical specification. Nonetheless, the temptation is extremely high, not only because so many great names of scientists have been attached to the theory, but also because of the astonishing mathematical beauty that has been displayed from the works that could be titled as chaos theory. As a result of this great temptation, we have witnessed a very interesting phenomenon that people have spent decades not for applying the chaos theory to solve problems but for finding problems that could be a good fit for the chaos theory. However, the intrinsic linguistic mismatch between the real life chaos and the scientifically canonized chaos has led to a huge disappointment so far for people who have been struggling to bring up some practical fruits of theory.

If we have enough faith in the unity of truth, then we might need to believe that the beauty we have seen so far from chaos theory should be a reflection of something deep in the general nonlinear world, which warrants some continuous efforts on bridging the theoretical beauty with everyday life reality. But on the other hand, we should also take the hitherto practical futility of chaos theory and the knowledge about the related linguistic mismatch discussed above as an alert that we might not have a good enough understanding about what is behind the word *chaos* for its most fundamental meaning as perceived by human beings throughout the history.

As a matter of fact, since the pioneer work of Edward Lorenz (Lorenz 1963) on butterfly effect, the philosophical impact of the chaos theory upon human civilization has much exceeded its mathematical advantages for practical problems. The general nonlinear nature of this world and its long term unpredictability due to its sensitivity upon initial conditions has become a common knowledge even for high school students. This common knowledge, like relativity and quantum physics, has fundamentally altered the world view of average people, and is an important reason for many to stop choosing the reductionism, which was once the norm in western philosophy, as the only way of thinking. Similarly, people would no longer consider the stochastic process as the only reason for the disorderliness in nature.

Nevertheless, philosophy and specific sciences are functioning in quite different ways. Philosophy changes the way of thinking, no matter for grand ontological questions or for detailed personal life management concerns. Therefore, even though throughout history people have witnessed countless cases when profound philosophical thinking could bring up direct solutions to practical problems, in general, philosophy aims to directing the thinking in a more efficient and profound way instead of laying out the specific protocols to get things done. But on the contrary, verifiable protocols for verifiable outcome are generally the basic requirement of any specific science, including the science of applying the mathematical utility of chaos theory to solve social or natural problems.

Moreover, even though philosophy could also be characterized for the preciseness and subtlety in its conceptual expressions, compared to the utilitarian goals of specific sciences, philosophy is more interested in acquiring profundity and extensiveness about the nature of being(s). This is indeed where the true power of philosophy lies and this is why and how the development of specific sciences could get help from philosophy. In this sense philosophy is more realistic than sciences since it could always reach certain type of goals while sciences, as we have witnessed in the case of chaos theory, might be over idealistic when constructing the theory but lack of applicability in reality.

The difficulty of making practical use of chaos theory due to the linguistic clash between ordinary meaning of chaos and the professional scientific term of deterministic chaos could tell that we might need deeper and wider understanding about the real life chaotic issues before we could more efficiently bring chaos theory to reality, which further suggests more philosophical thinking on the side of real life problems instead of the side of chaos theory.

On the other hand, even though persistence on theoretical endeavor might bring up great achievement in the future, we should never confuse ourselves by losing our vision on the end value of any theoretical endeavor: the real world issues. While the knowledge of scientifically identified *deterministic chaos* might lead to some potential solutions for economic problems, we should never forget that the real world common sense *chaos* is what makes all those theoretical endeavor meaningful.

For all the reasons discussed hitherto, the goal of this writing would not be another scientific discourse on how to apply chaos theory to economic events; instead this writing would be a writing to demonstrate some dynamic nature behind the real life economic phenomena by philosophizing through some logic involved in economic systems. In other words, this writing is aiming at helping solving real life economic issues instead of providing more thoughts of how to develop chaos theory. While I do believe in the long run a better understanding of the general chaotic phenomenon would definitely benefit the effort of applying the chaos theory to real problems, this writing itself is not coupled with any chaos theory conclusion.

Correspondently, hereafter in this writing, the term ***chaos*** or its adjective version ***chaotic*** would be used in its most common sense, or we may say in its phenomenological sense, without regard to its scientific specification or dynamic reasons according to the chaos theory. For further clarity, when it comes to ***economy***, the word ***chaos*** would be referring to the status (phenomenon) that the intentions, products or consequences of economic activities of different parties, sectors, or areas are not in synch with each other. In this sense, we might say the economy is in chaos when the general market demand could not be satisfied by the general supply, or the general supply could not be consumed by the general demand, and when there are enormous waste of human and material resources as a result of daily activities in the economic system, or when many people could not have a chance to work for living. As a synonym to chaos, economic ***disorder*** might also be used in the text for the same purpose, and the opposite to the chaos or disorder might be described as ***order*** in the text.

One of the key parameters behind the degree of economic disorder and order is what we called as *interest*, collective interest or personal interest. The pursuit of interest could set the local economy into order and the conflict of interests might drive the global economy into chaos. This is the ***economic relativity*** discussed in this writing and the adjectives *local* and *global* in this context are of relative sense as well. This economic relativity determines that the meaning of a good economy would be very relative when judged from different social positions and domains. For this very reason, a general solution to the question like "When the economy would be good?" would not be intended to answer by this writing, not only because the idea of getting a literal answer for such question is too much idealistic for any academic writing for today (since for otherwise we would not have to face the global economic crises that we are still facing to this moment given that so many Nobel prize winning economists as well as professional bankers and politicians are trying to resolve the issue), but also because that would be against the central theme of this writing: economic relativity.

Although the main interest of this writing is to philosophize the dynamics behind the real world economy in a way similar to the *methodological nominalism* postulated by Karl Popper (Popper 1945), it is understood that a relatively clear definition could be helpful for readers to understand the discourse in the text. Therefore, herein I would lay out the meaning of a good economy to an individual as *the capacity to access and dispose resources* (including material and human resources). Accordingly if everyone in a society could enjoy a good capacity to access and dispose resources then the people in that society are enjoy a good societal economy.

The chapter structure is therefore as follows.

Section "Chaotic Order of Economy" introduces and discusses the fundamental dynamics behind the economic relativity. It is further divided into four subsections: "Interest Based Economic Relativity" in which economic relativity is introduced and discussed in terms of the idea of conflicted interests; "Economic Wellbeing and Fairness" in which the relationship between economic wellbeing and fairness is discussed; "Economic Relativity Examples" in which the example of Chinese economic development and Eurozone crisis issue are discussed to demonstrate the universality of economic relativity; and "The Challenge" in which the ethic and economic challenges brought up by economic relativity is discussed. This is followed by Section "A Myth" in which a common linear way thinking that might mislead people to ignore the economic relativity is criticized. And finally the Section "Conclusion" concludes the whole writing by reiterating the importance of philosophical thinking in dealing with the economic challenges we are facing today globally.

Chaotic Order of Economy

Economic system of any sizeable region, no matter what type of economy, is virtually a chaotic system and the only difference from one system to another is the chaotic degree. This has been proved by historical practices around the world as we will see from the discussion in this text. There are some common factors behind the chaotic nature of economy and the most essential one is the conflicted interests, which is the key point for understanding all economically disorder phenomena. In everyday life, the expression *conflicted interests* might sound very personal to people; however, it is indeed more logical or mathematical than personal in the sense that given the limited natural and social resources available to a social system it is mathematically impossible for anyone to be always mutually equal-benefiting with everyone else. In general, people would care more and act accordingly for their own interests, which might not necessarily always be a positive contribution to the interests of others or the interest of the common community. Based on the empirical knowledge we all know that the increase of the order of the societal economy could imply an increase of mutual interest and thus a decrease of conflict of interests. Accordingly a better understanding of the interest-based economic relativity could help a better grasp of the art of reducing conflict of interests for the sake of a better economy.

Interest Based Economic Relativity

The conflicted interests factor has a dual-sided effect upon the chaotic order of an economic system: each economic unit (e.g. a private corporation) would manage to set forth its agenda and resources in good order within its own by absorbing energy from outside, while the competitions or irresponsible attitudes and actions between different economic units would increase the degree of disorder (chaos) at the societal level. Similarly, within each economic unit, there could be various subunits, and each subunit would tend to manage around the central interests of its own in competition with other subunits. Down to the smallest subunit of any economic unit—a single person, he might effort to best arrange his own agenda and resources to serve his own best interests. As a result, an economic system might behave similar to the water system when a river flow passing an obstacle, which is a turbulent mass of vortices of different sizes, and within a big vortex, there might be even smaller vortices. This would create a process in which the energy of the main stream flow is dissipated to the energy of vortices from the big ones down to the smallest ones. Each vortex is an ordered dynamic structure, but the whole mass of flow is a disordered turbulence.

This chaotic assembly of ordered units is a general picture of any sizeable economic system. Any serious study of economic dynamics should take this general picture into consideration; otherwise it might be misleading when attempting to define or understand the health or the wellbeing of an economic system since from different stand points of view we might get very different ideas about what is a good economy or what is a bad economy, which has been the central topic of political and economical debates.

Human beings are socialized creatures. Based on archaeological data and historical literatures presented to us in public domain, we could easily see that the very reason for human beings to have survived the natural selections to prosper was because of the advance of social collaboration during prehistorical and early historical ages. However, because of the conflicted interests between different social units, the existences of others are not always viewed as beneficial to everyone in this world. As a matter of fact, human beings have always lived a complicated interrelationship of mutual reliance and mutual competition or even mutual threat with each other. The fear of mutual competition is not necessarily always about the threat from any nominal enemy who might physically hurt someone, but rather is profoundly rooted in the scare of the lack of basic supplies of living. In ancient Sparta, people would consider those physically weak as their burden to discard. Even though the moral system of any modern society would prohibit the same practices of ancient Spartans, modern people are still living in the shadow of the fear of mutual competition from our own species all the time.

Today *team work* has become a popular political jargon because more and more people are realizing that their work could not be accomplished without the functional support of others; however, when it comes to job security, promotion vacancies, bonus shares, and business opportunities, people would very naturally view others who might potentially reduce their chances to benefit as rivals for survival. This dual attitudes of human beings towards other people in their endeavor to survive is indeed a psychological reflection of the logic possibility regards the values of others. Metaphysically speaking, for anyone in this world every single other co-living person has dual values: the value as a person whom can be count on for survival and the value as a person who could take away good things for survival. We might conceptually express the dual values as a duplet of positive and negative values, where the positive value refers to a beneficial contribution to one's survival and the negative value refers to a competition or threat to one's survival. Then we might say that in terms of the impact upon one's survival, every other co-living person is logically a duplet of positive and negative values, and the magnitudes of these values would be determined by many complicated factors such as mutual relationship and relative social status. Obviously, for a given person not everyone is of the same values. The negative value of a loving person might be close to zero, and the positive value of a savage enemy might be close to zero. When a friend turns to an enemy, his positive value would decrease drastically and his negative value would jump up greatly.

Very often what people consider to be good for them might not be truly good for them, and what they consider to be bad for them might be potentially very good for them; and thus when we talk about personal interest we might need to be aware that there is a difference between true interest and nominal interest, and the latter is what people consider to be good for themselves. There are many factors that would affect the nominal interest such as knowledge, emotional feeling, loyalty, empathy, altruism, cultural influence, political and religious faith, jealousy, greed, and more. However, although the difference between nominal interest and true interest might be significant for particular events, it will not change the general relative nature of economy as discussed in this context. This is because that even though true interest would make real difference in life, people would normally only be aware of the nominal interest before, during and after the presence of the interest; they might adjust their views about the nominal interest from time to time but those views would normally be always different from the true interest. Therefore, people would think and act according to what they consider as their own interest, not what hypothetically the true interests are. Even if there is no conflict at all between the true interests of two persons they would still view each other as a competitor as long as they subjectively think their interests are in conflict. Only if every person would view the interests of others as his own, the nominal interests of different people would no longer be in conflict and the conflicted interests would no longer be a psychological issue.

That would be the case of a fully altruist society which is impossible to exist due to the *paradox of altruism* that if everyone only cares about others then all people are counting on others to care about themselves.

Conflict of interests is a universal issue among individuals all over the world. However, the interests of different people might be closely related to each other through various social relations. A simple example is that people in the same family would often share many common interests even though conflict of interests within any family is not a rare thing. As another example, personal interests are closely related to personal social status and wealth, and personal social status and wealth is grouped into classes in any society around the world; consequently people in like social status or same economic classes might share many common patterns of personal interests, which means that the so called personal interest could be a social attribute labeling the social position of each person. Accordingly conflict of interests could also exhibit various social marks in the sense that the interests of different socially related groups could be in conflict with each other.

Since conflict of interests basically means that people are competing with each other for acquiring benefits or avoiding detriments, *fairness* becomes a fundamental issue for dealing with any social problems impacted by conflict of interests. At the social macroscopic level, the issue of fairness is closely related to the issue of social distribution of wealth. Production (various types of services could be viewed as production in a broad sense) and distribution of wealth in a society as a whole have always been two essential factors to determine the general quality of life in a society. If we could have an ideal distribution that is perfectly fair (if this kind of distribution could be defined) then undoubtedly the increase of production would benefit each individual as well as the society as a whole. But in real life with unfair distribution, productivity alone could not be used to determine the wellbeing of people in general. The reason is mathematically simple if we could ignore the mutual influences between the social effects of production and distribution and also ignore the relationship between production and natural resources plus environmental quality. With this linear assumption, we could have a very simple reasoning: in order to have a good life for everyone, we need to have a good supply for everyone; but in order to have a good supply for everyone, we first need to have a good total supply since if there is none in total there would not be any for anyone; however, even if we have a good total supply it does not mean that we could have a good supply for everyone since a good total supply might be taken by a very few people without sharing with others. If the distribution is not perfectly fair, especially when the distribution is extremely unfair, which means that very few people could be getting most of the products but the majority would be only getting very little, then even the increase of total production would not necessarily result in the improvement of life for many disfavored people.

In the above simple analysis we ignored the mutual influences between the social effects of production and distribution, as well as environmental resources. In real life, the social effect of distribution might be impacted by production and vice versa. A good production itself could become the weapon for certain group of people to take advantages of others in the game of distribution, and severely unfair distribution could also potentially ruin the general production in the society. Besides, since overproduction could hurt environment and natural resource reservation, the meaning of fair distribution would also infer the fair consumption of environmental and natural resources. Because of the imperfect distribution, not only the meaning of economic wellbeing is ultimately relative, but the moral meaning of a good production also becomes very relative. Therefore, a good understanding of the impact of demand of fairness upon the distribution rule in an economic system would be critical for a good understanding of the economy itself.

Economic Wellbeing and Fairness

I was growing up in a non-market economic system. Like all other traditional non-market economic systems, during the time of my childhood, good production was almost of the same meaning as good life in the society because the distribution rule was simply determined by non-economic concerns (e.g. ideological and political concerns). In market economic system, production is no longer the most important concern for the wellbeing of economy; not only the demand and supply takes the place of traditional production in an economy because of the importance of interpersonal transaction, but also many other factors such as the employment rate become important indexes of the economy.

The so called market is some physical or virtual places where people could sell what they have and buy what they need. The term *market* is almost equivalent to the term *trading* because without trading there would not be any meaningful market. Forceful deprivation, looting, systematically enforced submission of wealth by some groups of people to some other groups of people are examples of interest transferring ways that are very different from trading. What is special about trading is its implication of voluntariness and accordingly the fairness behind the trading, even though in reality trading is frequently not fair and not truly voluntary. Compared to other forms of interest transfer, trading establishes at least a nominal requirement for fair exchange of interests between different parties involved. Any violation of this fairness requirement for trading could provoke explicit or disguised protest and resistance. Therefore, the idea of fair trading would promote fairness in an economy and accordingly set up an ideal goal of improving general fairness in trading.

However, like many other moral concepts, the meaning of fairness itself is very relative. People might claim that an apparent unfairness in a particular short term issue might bring more fairness in the long run or at a large scale, which is virtually true no matter we like it or not. This uncertainty in fairness is often exploited for denying the fairness to some people by the excuse of some other more meaningful fairness. Owing to the ultimate connection between the idea of market economy and the demand of fairness, the uncertainty in the judgment of fairness would be inevitably reflected in the systematical practices of market economy. The judgment about social needs by makers of governmental economic policies or makers of real markets would in general be far short of the ideal fairness. This, in addition to the fairness paradox which will be discussed later, would lead to practical unfairness in any real life economy. Accordingly, even though fairness is the fundamental idea behind the market economy and thus the selling point for anyone who would promote market economy on this globe, the deviation from fairness in economy is a key point for a good appreciation of the relativity of economy, which is an important source to many problems that people have been facing to in any economy including market economy.

The demand of fairness from the public has always been such a social force that could not be completely ignored and severe unfairness would hurt general market. Consequently there have been different theories and correspondent practices around the world in history to solve the problem of unfair distribution. Communism and capitalism provide two extreme examples for this type of efforts. Communists attempted to reduce the chaotic dissipation by suppressing social competitions through centrally controlling and planning the economy; but they ended up with global economic bankruptcy among communist countries, basically because while centralizing the control over the system throughout the social hierarchy, they undervalued or even ignored the values and wills of the majority of individuals within the system. As a result, all communist governments not only failed to reasonably foresee many potential needs for running a good economy, but even failed to mobilize social resources for positive economic construction among the majority of their people from the very grass root to the top educated elite class.

On the other hand, capitalists attempted to build up the global economic order by fully mobilizing social resources through so called free market competition. However, they could not prevent the economy from going chaotic either. The reason is simple: the benefit of any individual person or company in an economic system is not necessarily in line with the benefit of the whole system by and large, and thus the best interest of any individual person or company is to pursue the benefit of his/its own, instead of the benefit of the whole system. Besides, we would encounter the *fairness paradox* whenever the so called fair competition is going on. Herein the paradox is a consequence of the conflict between the desired precondition of the competition and the goal of the competition. While it is always desired or demanded that any competition should be performed under fair condition, ironically, the end goal of the competition would normally be the unequal positions among the original competitors. As a result, the so called (pure) free market competition would not only generate huge amount of waste of natural and social resources but also lead to a polarization of wealth distribution among the people in the economic system.

Economic Relativity Examples: China and Eurozone

The relativity of economic wellbeing that I have discussed so far is by no means limited to theoretical speculation, but a very realistic issue when it comes to governmental spending and political decisions. No matter it is a democratic government or a non-democratic government, to ordinary people, the most important common nature of any government is its supreme power to access, collect, control, use, and distribute material and social resources. Therefore, governmental policies on how to collect and spend or distribute wealth among the people are of essential importance for the wellbeing or even survival of many people in a society. But by any means, governmental collection and spending could never be perfectly fair; as a matter of fact, there is even no any formula to tell the government what a perfectly fair collection and spending would be. Consequently there are always certain groups of people who receive the most benefits from the governmental operations and certain groups of people who are not much taken care or even sacrificed by the governmental operations. Similarly, the invisible hand (Smith 1776) of self-interests oriented free market could not change the relative nature of economic wellbeing either, but in fact would be very much under the influence of the economic relativity.

Because of the economic relativity, what people see as a good economy from outside an economic system might be quite different from what many people or sometimes even a majority of the people inside the system could personally sense day to day. What most impress people from outside might be the magnificence of infrastructure, the supply of goods and services available in the market, as well as the large amount of cash owned by the state or by individual citizens which could be spent inside and outside the country. However, even if the economy has been developed to such a stage that the purchasing power of the state and individual citizens could be on the top of the list in the world, it does not necessarily mean that the everyday life of average people has already been on the top of the list in the world.

Example of China. The rise of Chinese economy since the end of last millennium provides a very good example of how economic relativity would affect the wellbeing of the economy of a country. The Chinese political system turned into a hybrid of communism and capitalism in 1990s. Since late 1990s we have heard many predictions by many western economic experts that Chinese economy would crash very soon. However, over the past quarter a century, while western economy has gone into deep trouble as we are still experiencing now, Chinese economy has grown at a quite steady pace and become one of the top economic bodies in the world. Obviously, the wrong predictions tell us that the Chinese economic growth does not fit into the existing western economic models, just like what people have recently found that even the Eurozone crisis does not fit into the classic western economic models. It is obvious that Chinese economic success could not be explained with a flat economic view of equal opportunities in the market or the fully rational mindset of buyers and sellers etc. Actually since its taking off in 1990s, Chinese economy might be characterized by its disequilibrium or even polarization among its people.

For people outside China, the most important reason of Chinese economic success is its cheap labor plus low exchange rate of currency. With this advantage, China did have accumulated huge amount cash during the past quarter a century, which enables them to transfer China from the world factory into a mixture of world factory plus world market. This co-status of world factory and world market itself implies the coexistence of two opposite variables: a relatively low labor prices and a relatively high collective purchase power. This peculiar phenomenon is a great manifestation of economic relativity. In fact, even though the cheap labor plus low exchange rate of currency was one of the most important driving forces for the booming of Chinese economy back to 1990s, if we look from within China, neither the cheap labor nor the low exchange rate could completely represent the whole picture of economic status of its people since not everyone was cheap and not everyone was doing exportation. Inside China, the distribution of income and costs has never been flat among its people during the development. Not everyone could enjoy the sense of a good economy at the same time. As a matter of fact, at each stage of the development the interests of certain group of people could be the cost of the interests of some other benefited people. This is typical of economic relativity in any economic system in the world and the only difference is the degree of unevenness in the system; in some system the wealth distribution could be much more uneven than some others at certain time period. Therefore, from within China, we might find that a major driving force of the Chinese economic booming during the past quarter a century is indeed its economic unevenness, which is of course a manifestation of economic relativity.

When some western economic experts predicted a quick crash of Chinese economy years ago, obviously they did not examine the size of the pool of relatively low income or relatively disfavored in the country, whose interests were not equally served as those at economically upper levels or favored in other aspects. Since economic relativity as I am discussing here basically means that the judgment of economic wellbeing is different by different groups of people in a society, the interests of certain group of people could very much sacrificed by the economically powerful or governing group of people in order to sustain a good economy for the main stream or for specially favored groups. As a result, the degree of contrast between poor and rich or disfavored and favored in an economic system could play a significant role in determining various economic risks as well as the rate of economic development. This is because the existence of the poor or relatively disfavored, especially educated and trained ones provides a large financial buffer for the economic system, which, while also contribute greatly to the production of the economy, would be forced to absorb much negative impact upon the economy and thus virtually reduce the economic risk of the system. Therefore, any effort of predicting the development trend of an economic system like current Chinese economy must take into the consideration of this manifestation of economic relativity.

The fact that the existence of big contrast between poor and rich or relatively disfavored and favored could benefit the economic development implies that without that big contrast we might face bigger challenge to sustain a healthy development of economy. To better appreciate this challenge from the example of Chinese economic booming, we need to pay attention to two more facts. First of all, the hybrid of communism and capitalism in China could be characterized as the centrally controlled capitalist market. While it has to face the same challenge of fair trading as people in any other market economy have to, it is much easier for a centrally controlled system to avoid much risk for state-wise economy at the cost of the economic wellbeing of certain group of people. (Even so, as in any economic system, the principle of fairness would always act to battle any practice by the powerful or rich to sacrifice the interests of disfavored for the wellbeing of others or for the main stream economy). Secondly, compared to 20 years ago, even though the Chinese economy might be more polarized, but the main stream general economy is greatly improved and the living standard of ordinary people have been increased tremendously. This tells that even though at certain stage of economic development the interests of some group of people could be sacrificed because of the economic relativity, the accumulation of wealth due to the main stream economic development could also benefit people whose interests were not respected equally as others at earlier stages. Of course the improvement of the socio-economic status of any group due to the general economic development does not necessarily entail equality in its socio-economic status with other groups.

Furthermore, economic relativity is not limited within the border of a single country, but is a global issue. For the Chinese economic booming, in addition to the cheap labor cost and the low exchange rate as I mentioned earlier, the global economic relativity is also manifested in the great market potential created by its great population, especially a population that is getting ever richer than before since the beginning of this millennium.

Example of Eurozone. The Eurozone crisis provides another example of the significance of economic relativity. Creating a greater market was the ultimate concern to establish the Eurozone since the idea of capitalist free competition tells that market is the key for economic growth. However, the cause for the Eurozone crisis is also mainly a result from the pursuit of a greater market, or more precisely, from the ignorance of the negative side of the so called free competition in market economy. This ignorance is indeed one version of ignorance of the economic relativity by assuming that the so called free market competition is fair and mutually beneficial to everyone in the economic system.

If we could take a high level contemplation seeing through the dazzling financial figures presented to us by financial institutions around the world concerning the Eurozone crisis, we could see some very obvious and simple philosophical reason that is rooted in the capitalist market mechanism itself. Let's reiterate two important attributes of a capitalist market economy to facilitate the discussion here: (1) it promotes (nominally) fair trading with a respect of private interests of the public; (2) it encourages market based competitions. These two attributes are commonly acknowledged by economists and many ordinary people as the fundamental strengths that differentiate capitalist economy from other economic systems. The first of these two attributes is responsible for producing more to this world and the second is the key factor of rewarding the winner through distributing the products within the capitalist market system. Because of the free trading, capitalist economy could enjoy the greatest productivity over human history; however, because of the competition, capitalist economy would ultimately promote social polarization of wealth among people.

In a single country of capitalist market economy without any exchange with other countries, the social wealth would flow from some people to some other within that country constantly due to the capitalist competition. As a result, without special social assistance to counteracting this polarizing consequence, the general trend that could be expected with common sense would be that the rich would get richer and the poor would get poorer. During the past few centuries, one great achievement in the so called capitalist world is the establishment of social security systems and relatively reasonable domestic taxation systems within a democratic political framework, which greatly helped maintaining a happy middle class so that they could avoid getting poorer and poorer in capitalist competitions.

Now if there are two that kind countries having international trading with each other, and one of which is more competitive than the other in almost all economic sectors, then based upon the previous discussion we could expect a one-directional overall wealth flow from the less competitive country to the more competitive country. To prevent this from happening, there are many conventional measures taken by countries around the world, including tariffs at border, foreign exchange surcharges, or some special taxes toward cross border international businesses, and more. These measures function like cash dams to prevent surge of cash flow out of the country while people could enjoy the prosperity brought by international businesses within the country.

Now if those two countries decide to remove any trading barrier between them and also use the same currency in their daily life, the less competitive country would no doubt lose their protection on the border to prevent a severe cash flow out to the more competitive country. Then one question arises: why should we even worry about this since all the countries have social security systems to prevent the middle class people from getting poorer and poorer? The difference here is that the social benefit system of every country only serves its own people while the legal system of each country within a constitutional multi-country free market zone (e.g. Eurozone) is demanded to protect the free market business activities by people from all countries in that open market zone. Therefore, the two countries of free market business in the example here would face such an awkward situation that the competition mechanism of capitalist economy would drive the cash flow in the grand territory of all countries in the zone while the mechanism to counteract the wealth polarization of each country only function within its own territory. The poor in the less competitive country could only request help from their own government since the government of another country is not elected by them and not responsible for their welfare, and the politicians of their own government could not make use of the wealth accumulated in any other country for the rescue within their own country.

Under certain economical condition, the imbalance between the polarization resulting from capitalist competition and the counteracting social assistance power caused by the scenario discussed in last paragraph could potentially drag the less competitive country into financial austerity whilst the total wealth continues to accumulate within the grand market formed in those two countries due to the free market businesses in that market. Then when the general market is ruined by this imbalance to certain degree, the grand economy of the zone of those two countries would severely suffer as well. Even though the background assumption made in this analysis is much simplified compared to real situation in Eurozone, it does reveal the dynamic consequence that would result from the main conditions that characterize the Eurozone economy.

The Challenge

We could see that the economic relativity has played an important role in both Chinese economic booming and Eurozone crisis although they are apparently two completely different cases. In the example of Chinese economic booming, like the booming of many other economic systems in history, the economic relativity provides a buffer to absorb the negative impact of any detrimental happening and adverse condition for the economic development, in addition to provide a great working power at low cost; while in the example of Eurozone crisis, the economic relativity causes the economic austerity due to the breakdown of the social safety net to balance the negative effect of market economy. One common thing in these two cases is that the interests of certain group of people become the cost of the benefits of some other group of people.

The main difference between the Chinese case and the Eurozone crisis case is the starting level of the average living standard and the trend of the change of the living standard. The Chinese economic booming started with a low general living standard. Therefore, even though during the process the interests of certain group of people was not respected the same as some other group of people or sometimes even be sacrificed for the main stream economy, it would not cause much social tension because most people don't have the feeling of loss compared to their previous living standard. On the other hand, the living standard of Eurozone countries was at a relatively high level when the Eurozone was formed. Therefore, unless their living quality could be improved, people of any group within the Eurozone would not accept the fact that their living quality would be worsen as the cost of the benefits of others. Furthermore, closely related to the different starting levels of living standard, even though the Chinese society might be financially more polarized than before, the absolute living standard of the society as a whole is trending higher during the booming, while economic austerity has brought the economy of many Eurozone countries into a downward rail. This would create very different psychological consequences; the Chinese people might see some hope of a better future regardless the present unsatisfactory living condition, while people in Eurozone crisis might feel pessimistic about the future, which would obviously have very different impacts upon the social stability and very different impacts upon the economies.

Here comes the challenge, since sometimes the sacrifice of the interests of certain group of people could be benefiting the so called main stream economy, should people exploit this advantage and attempt to maintain a big pool of poor or disfavored for the sake of the wellbeing of the grand economy?

As a matter of fact, even though people might not be well aware of the philosophical reason behind the economic relativity, to use a large pool of disfavored or poor to benefit the wellbeing of favored or rich or the mainstream is a human practice with thousands years of history. Theories of maintaining low living standard of poor so that they would not demand much and thus would not cost much is nothing new but familiar to the rich around the world thousands of years ago. Even in the so called market economy, shrewd bosses know how to and eagerly do their best to get the most from their employees with lowest compensations. This means that people have been practically exploiting the apparent advantage of maintaining a pool of poor for what they think would be beneficial to their own wellbeing or to some imaginary societal wellbeing. There have even been the depopulation conspiracy theories that if only a small percentage of the population left on this globe then they would be much more prosperous with all the existing natural resources and human knowledge, which is some renewed global version of ancient Spartan way of thinking.

The cruel fact is that no matter how terrible the economy is at certain stage, there is always at least one chance for it to get better, not through more efficient operations for the existing market, but at the cost of the interests or even the survival rights of some people in the society. Even though this might sound foreign to many people, but there have always been some people who not only know it but really count on it. This means that, opposite to what many people assumed in their daily thinking, the goal of good economy is not *virtually* in line with the principle of fairness, which is the cold meaning of the *economic relativity*. This shares some similarity with the up and down of stock market. Over the history there have been quite a few times when some major stock markets crashed but the global market has always found its way to get back. However, what people usually tend to forget quickly after the stock market recovers is how many people could never get back to their original position due to the previous crash, especially those who has ended their life as a result of the crash. The full scale game of real life plays in a similar but sometimes much more cruel way. So much often the recovery of an economy signifies not only the end of a period of bad economy, but also the end of previously better life or previous prosperity or even rights of living of some people who have been either the victim of the bad economy or the cost of the recovery to the new economy.

Based upon our understanding of economic relativity, if we come back to look at the example of Eurozone crisis again, among many other potential options, we might see two logically viable possibilities for the Eurozone to get out of current crisis: (1) establishing centrally administered taxation across the zone, which would be used to help the less competitive nations in the zone so that the imbalance caused by the relative differences in competitiveness across the borders would be reduced; (2) to make the less competitive countries more competitive by restructuring their economy to be the exactly same as those more competitive countries, which would also reduce the above mentioned imbalance.

If either of the above 2 efforts could succeed, it could possibly solve the Eurozone crisis. However, since among the essential elements of an economic structure are the wealth distribution and social relationship, and also since every existing economic structure in this world is bound to the existing life style related culture, a recovery by a quick restructuring of economy might need to come at the cost of the interests of certain group of people, which indeed echoes what happened in China when their economic booming started a quarter century ago.

Now the question is that even though there is utilitarian benefit for the economy by maintaining or even creating a pool of disfavored or poor and it has been a practice for thousands of years to exploit this advantage, should the majority of this world endorse this type of practice for the sake of the main stream economy? To answer this question, each person should ask himself another question whether he would like to be the cost for the wellbeing of others or for the main stream economy or not. If his answer to that second question is “No” and he believes in the fairness principle, then he might also need to answer “No” to the question whether the majority of this world should endorse the practice of taking the advantage of the unfortunate for the sake of the main stream economy.

As a matter of fact, a “Yes” answer to the above question violates the fundamental idea of fair competition and fair market behind the idea of so called market economy, and thus in the long run would ruin the advantage provided by the trading-based market economy and hurt the economy itself as we have seen from the example of Eurozone crisis. Besides, it would be very hard to maintain a disfavored group while ensuring that their living standard would continue to improve when the economy develops as what happened during the past quarter a century in China. Therefore, it would be very risky for the economy in the long run to have a “Yes” answer to the above question.

However, even though it might sound simple to answer “No” here, practically it is not simple at all and most probably people would actually take a “Yes” position for various pressing concerns no matter what they might think with their conscience. This is because so much often the answer of “No” here might practically entail a bad economy to everyone or at least to the so called mainstream of the society, and thus to protect themselves, people might have to violate their own believes in fairness and to choose a “Yes” position to agree to sacrifice the interests of some people for the sake of the interests of their own or for the wellbeing of the main stream economy.

On the other hand, the danger for answering a “Yes” to above question is also very clear to most people that once the majority of people could tolerate social unfairness collectively for any reason, anyone of them could potentially be the next future victim of what they endorse currently.

Therefore, the economic relativity brings up two different but related goals for human civilization: (1) having a good economy in the society which everyone could potentially enjoy; (2) ensuring that each one of the society would not become the cost for the benefits of others or the mainstream society. The real challenge is to make these two ethically supposed to be consistent but practically often inconsistent goals to be consistent or at least as much consistent as possible. This is equivalent to making a globally chaotic system of locally ordered units more synchronized or less chaotic for the society as a whole.

However, due to the economic relativity, even though it would be nice and beautiful that we could still believe in the theory that a really good economy should always be an economy that maximizes the fair treatment of people in the society, unfortunately, people might not have the luxury to prove this theory with real life data when they need to bring a bad economy back to track if they don’t really know how to maximize the fairness with a really good economy. In other words, when facing to the economic relativity, a good will of being fair is not enough for sustaining social fairness; it needs some better knowledge including better knowledge about how economic relativity and fairness impact the market economy to help people to prepare for the fair and good economy in advance.

A Myth

The ignorance of economic relativity might lead to some popular myth in public life. One example is the myth that the more the employers earn the more they would spend for their hiring and their payment to employees. This myth is often cited by some politicians to support the argument that in order to boost economy the first and utmost governmental economic task should be to help the employers to acquire more capitals so that they could hire more and pay more to their employees. It would be nice if life is so simple. Unfortunately, this is quite a wrong assumption. The ultimate concern of the self-interest driven employers in general care most about their own earning instead of the living status of either their current employees or some potential employees in the market. Even though there might be some generous employers (at least under certain circumstances), but in general, employers would hire more only if that would be good for the business, and pay more to employees also only if there is some specific reason for them to benefit from the higher payment. Higher profit or higher earning does not necessarily entails the need for the employers to hire or to pay. There are plenty of examples in the history that big or small companies share the fruit of higher earnings only among owners or the high management team but not with ordinary employees, which is indeed a manifestation of economic relativity.

Conclusions

When people in the field of natural science are claiming that "Philosophy is dead" philosophy is and would continue to be reviving in the realm of social and economic sciences. The reason is simple that social and economic systems are open systems with virtually unlimited dimensions and highly nonlinear constitutive relations. Although the advance of technology and the avant-garde methodology might provide some beautiful curves with electronically collected or calculated data for social and economic issues, we might find that the narrowest bottle neck for any data-based analysis is the lack of knowledge about the dynamic nature of any social and economic system. Even though I would not comment much here on the saying that philosophy is dead even in natural science, I would like to emphasize the above mentioned difference between natural science and social-economic sciences because this difference determines that, instead of working with well defined parameters and well established formula as well as data collected from field or calculated using those formula for those parameters as in natural science, in the area of social and economic sciences, abstraction of proper parameters and discovery of right formula is still a critical task for people to have a better understanding of the subject at current stage.

This indeed warrants an important role for philosophizing in social and economic study even at this electronic and information age with advanced mathematical tools. Compared to application of any new technology or avant-garde methodology to reveal the inner pattern of social and economic dynamics from collected data, it would be much trickier and more challenging to construct an abstract framework with limited dimensions so that we could collect the data in a more meaningful way for social and economic study, which means we need to have a better understanding of the abstract dynamic relationship involved in social and economic processes. This is the job of philosophical analysis and it is what this writing is aiming to contribute to.

The ultimate concern of all economy related studies and practices is to help build a good economy or to avoid a bad economy. Therefore, the meaning of a good economy is not only of theoretical importance but also is essential for decision making and action taking in any economic practice. However, as we might see from the discussion of this writing that the judgment of a good economy would be ultimately affected by the economic relativity, and accordingly fairness should be treated with a great care to maintain a healthy and balanced market. Of course, economic relativity and the relevant social fairness is a big subject about very complicated issues, and thus the current discussion could only cover the very rudimentary aspects of the subject.

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Chapter 8
Restricted Coalition Formation

Giovanna Bimonte

Abstract Very often in social life individuals take decisions within groups (households, friendships, trade unions, local jurisdictions, networks, etc.). The formation of coalition may imply some theoretical difficulties, such as costs arising from forming a coalition or sharing information among agents. Coalition formation has the explicit purpose to represent the process of formation of coalitions of agents and hence modelling a number of relevant economic and social phenomena. Moreover, following this theoretical and applied literature on coalitions, the seminal chapter by Jackson and Wolinsky (1996) opened the way to a new stream of contributions using *networks* (graphs) to model the formation of links among individuals. In this chapter we will assume that only a subset $\mathscr{S}$ of the set of all possible coalitions in an economy is the set of admissible coalitions. We define the $\mathscr{S}$-core concept, as in Hervès-Moreno. We will extend to a model with both uncertainty and asymmetric informations the results showed in Okuda and Shitovitz.

Keywords Asymmetric information economies · Coalitions · Core allocations

Introduction

Very often in social life individuals take decisions within groups (households, friendships, trade unions, local jurisdictions, etc.). In a differential information economy the free coalition formation may imply some theoretical difficulties. It does not suffice to say that a coalition can be formed by several agents.The restriction of coalition formation is inflated by incomplete information. In a finite economy with N as the set of agents, it may happen that an agent will only know the preferences and endowments of a subset $K \subseteq N$ of people and can decide to form coalitions joint with

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agents from this group. Consequently, there is an upper maximum to the size of possible coalitions in the economy. Moreover, the formation of coalition may imply some theoretical difficulties, such as costs arising from forming a coalition or sharing information among agents. Indeed, incompatibilities among different agents may arise and a big amount of information and communication might be needed to form a coalition.

There are some consequences of placing an upper limit on the set of possible coalitions. Intuitively the core will be larger. We call a core with an upper maximum a *restricted core*. The first study on this direction were made by Schmeidler (1972), Vind (1972) and Grodal (1972).

We must take into account all limits imposed by the society to the aggregation in coalition. It is very simple to thing that agents are not free to form any coalition, especially in our framework. Indeed, it is usually argued that the costs, which arise from forming a coalition, are not all negligible. Moreover, traders will form a coalition only if they know one to each other. Incompatibilities among different agents may arise and a big amount of information and communication might be needed to form a coalition. Thus, it will be not enough to say merely that several agents form a coalition.

We define a set of all possible coalitions as the set of those coalitions that can be formed and joint by any agent. There exists, in this way, a rule imposed over coalition formation. We assume that only a subset S of Σ is allowed to be formed. In such way, we fix over the set of agents a rule of aggregation for which the coalitions can be formed only if they belong to this subset. We have restricted the set of coalitions that can be joined by traders.

Let T be the set of all traders. A coalition S is a measurable subset of T, such that $\mu(S) > 0$ which represents the size of coalition S. In the case of atomless economy, the size of a coalition S can be interpreted, following Schmeidler (1972), as the amount of information and communication, or costs, needed in order to form the coalition S. Then, it may be meaningful to consider those coalitions whose size converges to zero or, symmetrically, to one; that is, the coalitions that do not involve high costs to be formed.

The starting question is: suppose that in differential information economies a private allocation can be blocked, then "can it also be blocked by a coalition that is of a given structure"? Let $\mathscr{P} = (R_1, ..., R_k)$ be a partition of the grand coalition, with k large enough. We will prove that an optimal private allocation x belongs to the core if and only if it cannot be improved upon by any coalition that includes at least one of the element of the partition $\mathscr{P}$. Under differentiability the dimension of the cone of the efficiency price vector is one, then the condition k large enough becomes $k \geq 2$. Our statements becomes, for any coalition R, a private allocation x belongs to the private core of a market if and only if it cannot be blocked by any coalition that contains R. Then, we can classifying core allocations with respect to the family of all coalitions that include one of the members of partition.

The Model

We consider a Radner-type exchange economy $\mathscr{E}$ with differential information, with a finite number of types. The exogenous uncertainty is modelled by a measurable space $(\Omega, \mathscr{F})$, where Ω denotes a finite set of states of nature and the field $\mathscr{F}$ represents the set of all events. The space of traders is a measure space (T, Σ, μ), where T is the set of all traders, Σ is a σ-field of all coalitions, and μ is the Lebesgue measure. There is a finite number of goods, l, in each state. The information of traders $t \in T$ is described by a measurable partition Π_t of Ω. We denote by $\mathscr{F}_t$ the field generated by Π_t. If ω_0 is the true state of nature, trader t observes the member of Π_t which contains ω_0. Every traders $t \in T$ has a probability measure q_t on $\mathscr{F}$ which represents his *prior beliefs*: i.e. probability conditioned by their information set. The preferences of a trader $t \in T$ are represented by a state dependent utility function, $u_t : \Omega \times \Re^l_+ \to \Re$ such that $u_t(., \omega)$ is continuous, concave and strictly monotone a.e. in T. Moreover, each trader $t \in T$ has a fixed initial endowment $e : T \times \Omega \to \Re^l_+$, such that, $e(., \omega)$ is assumed to be μ-integrable in each state $\omega \in \Omega$ while $e(t, .)$ is $\mathscr{F}_t$-measurable, i.e. constant on each element of Π_t. The interpretation of this condition is that traders do not acquire any new information from their initial endowment. Let, for each $t \in T$, $M_t = \left\{x_t : \Omega \to \Re^l_+ \mid x_t \text{ is } \mathscr{F}_t - \text{measurable}\right\}$ be the set of all $\mathscr{F}_t$-measurable selections from the random consumption set of agent t. Throughout the chapter, we shall assume that $e(t, \omega) \gg 0$, and, for any function $x_t : \Omega \to \Re^l_+$, we will denote by $h_t(x) = \sum_{\omega \in \Omega} q_t(\omega) u_t(\omega, x(\omega))$ the *ex-ante expected utility* from x of trader t.

Definition 1 Let R be a fixed coalition. An allocation x is said to belong to the R-*inclusive core* if it cannot be improved upon by any coalition S that includes R; i.e. if there is no coalition S and an assignment y $\mathscr{F}_t$-measurable, $y : S \times \Omega \to \Re^l_+$ such that $R \subseteq S$, $\mu(S) > 0$, $\int_S y(t, .)d\mu \leq \int_S e(t, .)d\mu$ and $h_t(y(t, .)) > h_t(x(t, .))$ for almost every t in S.

Definition 2 A non-zero vector $p\colon \Omega \to \Re^l_+$ is an efficient price vector for the allocation x if μ a.e. in T, $x(t, \omega)$ is the maximal element of h_t over the efficiency set

$$B^*_t(p) = \left\{ z \in M_t \mid \sum_{\omega \in \Omega} p(\omega) \cdot z(\omega) \leq \sum_{\omega \in \Omega} p(\omega) \cdot x(t, \omega) \right\}.$$

We denote the cone of all efficiency price vectors for an allocation x by

$$P(x, \succ_t) = \left\{ p \in \Re^{l \times n}_+ : x \succ_t y \Rightarrow \sum_{\omega \in \Omega} p(\omega) \cdot x(t, \omega) \geq \sum_{\omega \in \Omega} p(\omega) \cdot y(t, \omega) \right\}$$

and its linear dimension by $r = \dim P$.

Definition 3 Let $\mathscr{S} \in \Sigma$ be the subset of all admissible coalitions, with $\mu(S) > 0$ for every $S \in \mathscr{S}$. A feasible allocation x belongs to the $\mathscr{S}$-private core of $\mathscr{E}$ if it is not privately blocked by any coalition $S \in \mathscr{S}$.

We denote this core as $\mathscr{S}$-$\mathbf{C}_p(\mathscr{E})$.

In each coalition S belonging to the subset $\mathscr{S}$ agents do not share their information, accordingly with the private blocking mechanism. Traders joint a coalition which belongs to $\mathscr{S}$, and they choose a private allocation over $\mathscr{S}$ which improves upon the allocation x.

From the definition of $\mathscr{S}$-core given $\mathscr{S}_1, \mathscr{S}_2 \subseteq \Sigma$ we can easily infer the following properties:

(i) if $\mathscr{S}_1 \subseteq \mathscr{S}_2$ then $\mathscr{S}_2$-$\mathbf{C}_p(\mathscr{E}) \subseteq \mathscr{S}_1$-$\mathbf{C}_p(\mathscr{E})$;
(ii) $\mathscr{S}_1$-$\mathbf{C}_p(\mathscr{E}) \cap \mathscr{S}_2$-$\mathbf{C}_p(\mathscr{E}) = (\mathscr{S}_1 \cup \mathscr{S}_2)$-$\mathbf{C}_p(\mathscr{E})$

From the property (i) it is deduced that if the private core is non-empty, then so is the $\mathscr{S}$-private core. The property (ii) implies that if $\Sigma = \bigcup_i \mathscr{S}_i$, then $\bigcap_i (\mathscr{S}_i - \mathbf{C}_p(\mathscr{E})) = \mathbf{C}_p(\mathscr{E})$. That is, for any partition $\mathscr{P}$ of the whole coalition set Σ the allocations belonging to the private core are exactly those allocations that belong to every $\mathscr{S}$-private core, with $S \in \mathscr{P}$, and the intersection of the S-private cores of a partition P does not depend on $\mathscr{P}$.

Preliminary Results

Given a fixed coalition $R \in \Sigma$, let

$$\mathscr{Q}_R = \{S \in \Sigma : R \subseteq S\}$$

be the set of all coalitions which contain R. This structure define the only coalitions that can be formed as those containing R.

Define with $T \backslash \mathscr{Q}_R$=$\{S \in \Sigma : R \cap S = \phi\}$.

Given this information structure, we turn to define the private core concept in a R-inclusive way.

Definition 4 Let R be a fixed coalition. An allocation x is said to belong to the R-inclusive private core if it cannot be privately improved upon by any coalition $S \in \mathscr{S}$, with $\mathscr{S} = \mathscr{Q}_R$; i.e. if there is no coalition S, with $\mu(S) > 0$, and a feasible assignment $y : S \times \Omega \to \Re^l_+$, $\mathscr{F}_t$-measurable, such that

(i) $R \subseteq S$,
(ii) $h_t(y(t, .)) > h_t(x(t, .))$ for almost every t in S.

Definition 5 A feasible allocation x is individually rational if $h_t(x) \geq h_t(e)$ for almost every t in T.

Definition 6 A non-zero vector $p : \Omega \to \Re^l_+$ is an efficient price vector for the allocation x if μ a.e. in T , $x(t, \omega)$ is the maximal element of h_t over the efficiency set

$$B^*_t(p) = \left\{ z \in M_t \mid \sum_{\omega \in \Omega} p(\omega) \cdot z(\omega) \leq \sum_{\omega \in \Omega} p(\omega) \cdot x(t, \omega) \right\}.$$

We denote the cone of all efficiency price vectors for an allocation $x(t, \omega)$ by $P(x)$ and its linear dimension by $r = \dim P$.[1]

We consider a finite and measurable partition $\mathscr{P} = (R_1, ..., R_k)$ of the grand coalition, with k large enough.[2] We prove that an optimal allocation x belongs to the core if and only if it cannot be improved upon by any coalition belonging to $\mathscr{Q}_{R_i}$ for all $i = 1, ...k$.

Lemma 1 *Let $x(t, \omega)$ be an allocation and let p be a non negative price, $p \in I\!B'^{\Omega}_+$. Then p is an efficient price vector for x if and only if $p \cdot G^*(t) \geq 0$ for almost all traders t.*

Proof The first implication is trivial.

Conversely, suppose that there exists a price p supporting the set $G^*(t)$ for almost all t in T. We want to show that $x(t, \omega)$ is the maximal element of the efficiency budget set $B^*_t(p)$ for almost all $t \in T$.

Suppose that $z \in B^*_t(p)$ and $h_t(z) > h_t(x)$. Then $\sum_{\omega \in \Omega} p(\omega) \cdot z(\omega) \leq \sum_{\omega \in \Omega} p(\omega) \cdot x(t, \omega)$. By continuity, there exists $\alpha < 1$ such that $h_t(\alpha z) > h_t(x)$. Therefore, $\sum_{\omega \in \Omega} p(\omega) \cdot \alpha z(\omega) \geq \sum_{\omega \in \Omega} p(\omega) \cdot x(t, \omega) \geq \sum_{\omega \in \Omega} p(\omega) \cdot z(\omega)$. If $\sum_{\omega \in \Omega} p(\omega) \cdot z(\omega) > 0$ the contradiction $\sum_{\omega \in \Omega} p(\omega) \cdot z(\omega) > \sum_{\omega \in \Omega} p(\omega) \cdot \alpha z(\omega)$ follows. If $\sum_{\omega \in \Omega} p(\omega) \cdot z(\omega) = 0$ then $\sum_{\omega \in \Omega} p(\omega) x(t, \omega) = 0$. Since $x(t, \omega) \gg 0$ for almost all agents, $p(\omega) = 0$ for all $\omega \in \Omega$. Then, x is the maximal element of the efficient budget set.

Lemma 2 *For a given allocation x, let F be a set-valued function such that $G^*(t) \subseteq F(t)$ for almost all traders t. If p is a non negative price such that $p \cdot \int F \geq 0$, then*

(i) (p, x) is an efficiency equilibrium,
(ii) $p \cdot f(t) \geq 0$ for all integrable selections f of F and almost all $t \in T$.

Proof For each $z \in \Re^l_+$, let $G^{*-1}(z) = \{t \in T : z \in G^*(t)\}$ be the set of all agents t for which the allocation z belongs to the preferred set $G^*(t) = \{z \in M_t : h_t(z) > h_t(x)\} - x(t, .)$.
Then from $G^{*-1}(z) = \{t : h_t(z(.) + x(t, .)) > h_t(x(t, .))\}$ we infer that this set is measurable for each z. Let N be the set of all rational points $r \in Q^{\Omega}$, where Q is a dense and denumerable set of $I\!B$, for which $G^{*-1}(r)$ is null. Obviously, N is

[1] As it is shown in Grodal (1972), it is always true that the linear dimension of the cone P of the efficiency price vectors is less than or equal to the number of commodities in the market, $l \cdot |\Omega|$, and that under classical assumption of differentiability and interiority $r = 1$.

[2] We refer to Okuda and Shitovitz (1985).

denumerable. Define with $S = \bigcup_{r \in N} G^{*-1}(r)$. Then S is a null coalition. Suppose that for some $t \notin S$, there is a bundle $z(.) \in G^*(t)$ with $\sum_{\omega\in\Omega} p(\omega)\cdot[z(t,\omega) - x(t,\omega)] < 0$. By continuity, we may find a rational point $r \in G^*(t)$ sufficiently close to z, so that we still have $\sum_{\omega\in\Omega} p(\omega)\cdot r < 0$.
Hence, for $t \notin S$ if $A = G^{*-1}(r)$ then $\mu(A) > 0$.
By desirability, for each $\varepsilon > 0$, we have an integrable selection $f = r\chi_A + \varepsilon q(t,.)\chi_{T\setminus A}$ from $G^*(t)$, where $q \in G^*(t)$. Hence, $f \in F(t)$. Therefore $0 \le \sum_{\omega\in\Omega} p(\omega)\cdot \int f = \sum_{\omega\in\Omega} p(\omega)\cdot r\mu(A) + \varepsilon \sum_{\omega\in\Omega} p(\omega)\cdot \int_{T\setminus A} q(t,\omega) \longrightarrow_{\varepsilon\to 0} \sum_{\omega\in\Omega} p(\omega)\cdot r\mu(A) < 0$ a contradiction.
Therefore, $\sum_{\omega\in\Omega} p(\omega)\cdot G^*(t) \ge 0$ for almost all traders t, and by Lemma 1, (p, x) is an efficiency equilibrium.
Let f be an integrable selection from $F(t)$.
Define with $A = \left\{t : \sum_{\omega\in\Omega} p(\omega)\cdot f(t,\omega) > 0\right\}$, then, for each $\varepsilon > 0$, the integrable function $f = r\chi_A + \varepsilon q(t,.)\chi_{T\setminus A}$ belongs to $F(t)$. Therefore $0 \le \sum_{\omega\in\Omega} p(\omega)\cdot \int f = \sum_{\omega\in\Omega} p(\omega)\cdot \int_A f + \epsilon \sum_{\omega\in\Omega} p(\omega)\cdot \int_{T\setminus A} q(t,\omega) \longrightarrow_{\epsilon\to 0} \sum_{\omega\in\Omega} p(\omega)\cdot \int_A f$.
Therefore, $\sum_{\omega\in\Omega} p(\omega)\cdot \int_A f \ge 0$, which implies by the definition of A that $\mu(A) > 0$.
This completes the proof of the Lemma.

The Equivalence $\mathscr{C}_p(\mathscr{E}) = \mathscr{S} - \mathscr{C}_p(\mathscr{E})$

The purpose of this section is to prove the equivalence between two private core concept: the classical one for a differential information economy, and the private core restricted defined in the previous section.

Proposition 1 *Let $x(t,\omega)$ be an allocation. Then x is Pareto optimal if and only if there exists an efficient price vector $p \in \mathbb{B}'^{\Omega}$ $(p \neq 0)$ such that $\sum_{\omega\in\Omega} p(\omega)\cdot \int_T x(t,\omega) = \sum_{\omega\in\Omega} p(\omega)\cdot \int_T e(t,\omega)$.*

Proof By contrary, suppose that x is not a Pareto optimal allocation. Then there exists an allocation $y : T\times\Omega \to \Re^l_+$, with $y(t,.) \in M_t$ such that $\int_T y(t,.) \le \int_T e(t,.)$ and $h_t(y) > h_t(x)$ for almost all $t \in T$. By assumption, there exists a supporting price $p : \Omega \to \Re^l_+$ such that $\sum_{\omega\in\Omega} p(\omega)\cdot y(t,\omega) > \sum_{\omega\in\Omega} p(\omega)\cdot x(t,\omega)$. By integrating over T, we get $\int_T p(.)\cdot y(t,.) > \int_T p(.)\cdot x(t,.)$. Since y is feasible, a contradiction follows.

For the converse, let us consider the correspondence G defined by $G(t) = \{z \in M_t : h_t(z(\cdot)) > h_t(x(t,\cdot))\}$.

We denote by $Z^*(t)$ the correspondence defined by $Z^*(t) = G(t) - e(t, \cdot)\ \forall t \in T$. By Pareto optimal assumption, we know that $0 \notin \int_T Z^*(t)$. Therefore, by Separation hyperplane Theorem, there exists a price $p \neq 0$ such that $p \cdot \int_T Z^* \geq 0$, i.e. (p, x) is an efficient equilibrium.
Since $\int_T x(t, .)$ belongs to the closure of $\int_T G(t)$ for almost all $t \in T$, then $\int_T x(t, .) - \int_T e(t, .) \in \overline{\int_T Z^*}$ and do to feasibility the conclusion follows.

Theorem 1 *Let $x(t, \omega)$ be a Pareto optimal allocation satisfying the smoothness assumption. Let $\mathscr{P} = (R_1, ..., R_k)$ be a measurable partition of T. If $k \geq 2$, then x belongs to the private core if and only if x belongs to the R_i-inclusive private core for all i, $i = 1, ..., k$.*

The proof of our results needs the following result:

Theorem 2 *Let $x(t, \omega)$ be an allocation and let R be a fixed coalition. Then x belongs to the R-inclusive core if and only if there exists an efficiency price vector $p : \Omega \to \mathbb{B}_+^{'}$ such that $\sum_{\omega \in \Omega} p(\omega) \cdot x(t, \omega) \leq \sum_{\omega \in \Omega} p(\omega) \cdot e(t, \omega)$ for almost each t in $T \backslash R$.*

Proof First assume that there exists an efficient price vector such that $\sum_{\omega \in \Omega} p(\omega) \cdot x(t, \omega) \leq \sum_{\omega \in \Omega} p(\omega) \cdot e(t, \omega)$ for almost each t in $T \backslash R$. Suppose by contrary that x does not belong to the R-inclusive private core, than there exist a coalition $S \supseteq R$ and a private allocation $y : T \times \Omega \to \Re^l_+$, with $y(t, \omega) \in M_t$ such that $\int_S y(t, .) \leq \int_S e(t, .)$ and $h_t(y) > h_t(x)$ for almost all $t \in S$. Let define with z a private measurable allocation in this way

$$z = y_{\chi_S} + e_{\chi_{T \backslash S}}$$

then for almost every $t \in S$

$$\sum_{\omega \in \Omega} p(\omega) \cdot z(t, \omega) = \sum_{\omega \in \Omega} p(\omega) \cdot y(t, \omega) > \sum_{\omega \in \Omega} p(\omega) \cdot x(t, \omega)$$

and for almost every $t \in T \backslash S$

$$\sum_{\omega \in \Omega} p(\omega) \cdot z(t, \omega) = \sum_{\omega \in \Omega} p(\omega) \cdot e(t, \omega) \geq \sum_{\omega \in \Omega} p(\omega) \cdot x(t, \omega).$$

Then for almost all $t \in T$

$$\sum_{\omega \in \Omega} p(\omega) \cdot \int_T z(t, \omega) > \sum_{\omega \in \Omega} p(\omega) \cdot \int_T x(t, \omega)$$

and
$\sum\limits_{\omega\in\Omega} p(\omega)\cdot\int\limits_T z(\omega) = \sum\limits_{\omega\in\Omega} p(\omega)\cdot\int\limits_S y(t,\omega) + \sum\limits_{\omega\in\Omega} p(\omega)\cdot\int\limits_{T\setminus S} e(t,\omega) \leq \sum\limits_{\omega\in\Omega} p(\omega)\cdot \int\limits_T e(t,\omega)$, and the contradiction.

Let us look at the "only if" part. Assume that x belongs to the R-inclusive private core. Then x s Pareto optimal.
Define with $F(t)$ the correspondence:

$$F(t) = \begin{cases} G^*(t) & \text{for } t \in R \\ G^*(t) \cup [e(t,\omega) - x(t,\omega)] & \text{otherwise} \end{cases}$$

where $G^*(t) = \{z(.) - x(t,.) | z(.) \in M_t \text{ and } h_t(z(.)) > h_t(x(t,\omega))\}$, $\forall\, t \in T$. By Pareto optimality $0 \notin \int_T F(t)$.

From supporting hyperplane Theorem there exists a price $p : \Omega \to \Re^l_+$ such that $\sum\limits_{\omega\in\Omega} p(\omega)\cdot\int F(t) \geq 0$. By Lemma 2 p is an efficient price vector for x. By monotonicity, there exists a measurable and integrable selection $f(t,.) = (e(t,.) - x(t,.))_{\chi_{T\setminus R}} + z(.)_{\chi_R}$, with $f(t,.) \in F(t)$ for almost all $t \in T$. Therefore, by lemma 2 $0 \leq p\cdot f(t,.) = p\cdot e(t,.) - p\cdot x(t,.)$ for almost all $t \in T\setminus R$

Let us try to give an interpretation. If we consider a partition of T into two sets, namely R and its complement we will say that a strictly positive allocation belongs to the R-inclusive core if and only if it is possible for individuals belonging to $T\setminus R$ to choose the efficiency price vector $p(\omega)$, in each state of nature, so that the value of their bundle is less than or equal to the value of initial bundle. So that, despite of the measure of the fixed coalition R, agents in R are not willing to leave this coalition to join its complement and to gain.

Now we can show the proof of the main theorem:

Proof (Theorem 1) Suppose that x belongs to each R_i-inclusive core. By theorem 2 there are efficient price vectors $p_i \geq 0$ for x, one for each R_i such that:

$$\sum_{\omega\in\Omega} p_i(\omega)\cdot x(t,\omega) \leq \sum_{\omega\in\Omega} p_i(\omega)\cdot e(t,\omega)$$

$\forall\ i = 1, ...k$ and for almost all $t \in T\setminus R_i$. Such $p_i(\omega)$ are linearly dependent for all $\omega \in \Omega$, i.e., there exist $\alpha_1(\omega), ...\alpha_k(\omega)$ not all vanishing, with $\sum\limits_{i=1}^k \alpha_i(\omega)p_i(\omega) = 0$ for all $\omega \in \Omega$. Let $I^+ = \{j : \alpha_j(\omega) > 0\}$ and $I^- = \{j : \alpha_j(\omega) < 0\}$. Since $p_i \geq 0$ for all $i = 1, ..., k$, I^+ and $I^-$are both nonempty. Let us define P by

$$P(.) = \sum_{i\in I^+} \alpha_i(.)p_i(.) = \sum_{i\in I^-} (-\alpha_i)(.)p_i(.)$$

P is the competitive price vector for x. Indeed,

(i) P is an efficient price vector for x since by definition P is a convex cone.

(ii) $\sum_{\omega\in\Omega} P(\omega)\cdot x(t,\omega) \leq \sum_{\omega\in\Omega} P(\omega)\cdot e(t,\omega)$ for almost each $t \in T$. Indeed, let t be in T. Since $(R_1, ..., R_k)$ is a partition of T, there exists i_0 such that $t \in R_{i_0}$. Assume, w.l.o.g., that $i_0 \notin I^+$. Therefore, for every $j \in I^+$, we have $j \neq i_0$, in particular $t \notin R_j$ and therefore, by definition of the $p_j(.)$, we have $\sum_{\omega\in\Omega} p_j(\omega)\cdot x(t,\omega) \leq \sum_{\omega\in\Omega} p_j(\omega)\cdot e(t,\omega)$. Since $\alpha_j(\omega) > 0$ for $j \in I^+$, we have $\sum_{\omega\in\Omega} \alpha_j(\omega)p_j(\omega)\cdot x(t,\omega) \leq \sum_{\omega\in\Omega} \alpha_j(\omega)p_j(\omega)\cdot e(t,\omega)$. Summing over I^+, we obtain the inequality

$$\sum_{\omega\in\Omega} P(\omega)\cdot x(t,\omega) = \sum_{\omega\in\Omega}\sum_{j\in I^+} \alpha_j(\omega)p_j(\omega)\cdot x(t,\omega) \leq$$
$$\sum_{\omega\in\Omega}\sum_{j\in I^+} \alpha_j(\omega)p_j(\omega)\cdot e(t,\omega) = \sum_{\omega\in\Omega} P(\omega)\cdot e(t,\omega).$$

for almost each $t \in T$.

Now, by Theorem 2, x is a core allocation.

Conclusion

In this chapter it is proved, for any coalition R, that a private allocation x belongs to the private core of a market if and only if it cannot be blocked by any coalition that contains R. Then, we have classified core allocations with respect to the family of all coalitions that include one of the members of partition. If whatever can be done by a coalition, can be done by any arbitrarily small coalition, then one only needs a few well informed people to take us to Walrasian equilibrium. In such way, these few well informed people can be considered as arbitrageurs. If the rest of people in the economy remaining passive, it is enough for this small group to do their duty and take to equilibrium.

Cooperation is modelled as a two stage process: first players form coalitions, while at the second stage formed coalitions interact in strategic setting. This process is a coalition formation game, in which a given rule of coalition formation maps players announcements of coalitions into a coalition structure, which determines the equilibrium strategies chosen by players at the second stage. Only in recent years, a widespread literature on endogenous coalition formation has the explicit purpose to represent the process of formation of coalitions of agents and hence modelling a number of relevant economic and social phenomena. Moreover, following this theoretical and applied literature on coalitions, the seminal chapter by Jackson and Wolinsky (1996) opened the way to a new stream of contributions using *networks* (graphs) to model the formation of links among individuals. Some natural extension of our model will be to involves a sequential formation of links among players and bilateral negotiations take place in some predetermined order. The exogenous rule determines the sequential order in which pairs of players negotiate to form a link. A link is formed if and only if both players agree and, once formed, cannot be broken.

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Chapter 9
The Strange Attractor of the Firm

Safieddine Bouali

Abstract In this chapter, we assume that corporate arrangement between owners, debt holders and managers builds modern firm. The governance of their interests and conflicts determines the crucial financial rules of the firm especially in the contemporary context of persistent financial crisis, accounting scandals, frauds and wasted earnings by the executive team. The lack of trust in their relationship with stockholders or bondholders pushes ownership to impose a strong discipline of payout mechanism extracting the free cash flows (FCF) from the manager hands. Accumulation of FCF and postponing payments should also imply a strong extra-dividend as a punishment to executives when they don't respect the discipline of payout policy. To identify the dynamical outcome of such financial governance, we define a 3D system of differential equations modeling a firm under the best standard of management principles and rules but embedding a payout mechanism. The computations reveal that state variables of this firm follow a wide spectrum of dynamics amongst them singular strange attractors. The main results show that chaotic oscillation is an intrinsic and endogenous characteristic of the modern firm, not derived from (exogenous) market failure. Paradoxically, automated disciplining payout policy injects dynamical risks in a deterministic model of firm without any stochastic leverage.

Keywords Modern firm · Financial governance · Nonlinear system · Payout policy · Managers · Equityholders · Debtholders

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Introduction

From a theoretical viewpoint, after that level of capitalized earnings was determined ex ante as a strategic target by the corporation, the free cash flow (FCF) is the remainder from liquidity expelled out of the firm in dividends (or stock repurchases or loan repayments).

However, Bates et al. (2009) confirm that excess cash dilemma is a consistent, persistent and an expanding phenomenon for a wide sample of US industrial firms.

Described in 1961 as "irrelevant" by Miller and Modigliani, the debate focused on the dividend policy and its dual face, the optimal rate of capitalization, appears not yet closed.

Indeed, Jensen (1986) identifies the deep corporate conflict between ownership and management about the destination of FCF and delays indirectly the confirmation of its "irrelevancy".

Optimal reinvestment level should emerge as an explicit or implicit agreement from corporate financial governance (Fama 1974). This is an item of the nexus of contracts building the firm.

Payout stream concerns residual earnings while permanent returns of capital are capitalized as self-financing assets even if Olson and McCann (1994) detects a volatility of the linkage between dividends and earnings. In this regard, to avoid underinvestment problem (Mayers and Smith 1987), and the symmetric risk of over-investment (Richardson 2006), the management should apply the prevailed and agreed dynamical path of reinvestment. Indirectly, frauds temptation (Agrawal and Chadha 2006; Agrawal et al. 1999) and wasted earnings dispatched in inefficient investments can be drastically reduced.

Indeed, to insure these disbursement stream (Grullon and Michaely 2002; Oswald and Young 2008) the (extra) dividend and/or repurchase programs are the two main ways to extract excess of cash. This additional corporate governance goal stimulated by multiple accounting scandals is allowed by faster disbursement stream of FCF. A strong speed of payout is reached by selecting nonlinear mechanism of adjustment. This rule as an item of the agency relationship triggering payout beyond a particular threshold of earning by corporate arrangement between owners and managers non only to improve the creation of wealth by the firm, to enhance its share value in the stock market but, simultaneously, allowing incomes to the shareholders.

This chapter focuses the dynamical outcomes of the FCF when the power of decision (Rajan and Zingales 1998) jumps from shareholders to debt-holders to define the preferred reinvestment rate. Nowadays, profits sharing between dividends and self-financing investment constitutes the main focus of the firm revealing the lack of trust between owners and executives (CEO and senior managers).

In order to study the dynamic implications of the (self) imposed discipline of capitalization via payout of the nonrecurring earnings, we propose a heuristic model of a representative (and hypothetical) firm with cash cows committed to pursue a sustainable reinvestment trend. The autonomous model of three ordinary differential equations selects Profit, Reinvestment, and Financial inflow (debt) having the status of the state variables. The system is defined to investigate the outcome of a particular nonlinear mechanism to pay free cash flows as dividends or stock repurchases (section "Theoretical Framework").

We simulate the dynamics of a firm with a targeted strategy of reinvestment where a nonlinear mechanism releases convergence by disbursements. We investigate the outcome of the management alignment to shareholders' interests then to debtholders' interests when the power of the firm is in their hands. The improved mechanism of payout will be revealed when the firm experiences a different calibration of the expected "normal" profit which is detected by its bifurcation diagram to prevent financial hazards. We explore by several numerical computations, the dynamics of the firm and their periodicity in the phase portraits of the state variables ("Computational Results"). We argue that nonlinear payout policy is, itself, the mechanism of fluctuation. The concluding remarks report some implications of our heuristic research and highlights on how automated payout procedures modify the firm behavior (section "Alignment of Payout Policy to Debtholders' Interests").

Theoretical Framework

We intend to emphasize stylized facts of corporate finance when it self-imposes a mechanism of convergence to a targeted level of capitalized earnings where the disbursements policy is a derived consequence. Obviously, our model describes elementary patterns of the firm and focuses only heuristic simulations of nonlinear adjustments of the convergence. Results are qualitative to a large extend.

We set up a modified version of a nonlinear system model proposed by Bouali (1999, 2002, 2011) in which Profits, Reinvestments and the (external) Financing of the firm's activity, are simultaneously determined. Written in three first-order differential equations, the model represents the first principles and rules of the disciplining finance of the modern firms.

Theoretically, in the first equation, the basic premise of the earnings' determination is allowed by the financial investment.

Indeed, the capital allows the creation of profits **P** which is made up of **R**einvestments and financed also by a capital inflow, i.e. the debts **F**.

$$\mathrm{dP/dt = v(R + F)} \tag{9.1}$$

v, rate of profits.

On the other hand, when the incentives to shareholder's underinvestment are reduced, the managers encourage reinvestment which expands the production capacity of the firm and enhances the shares value and avoids its dispersion. Reinvestment constitutes an important item of the global reliance of the corporate governance. We notice that additional investments have identical profitability (the scalar **v**) of the previous projects.

On the other hands, in a recent survey by Brav et al. (2005), operational and investment choices are made before dividend payout or stock repurchases.

In concordance with the investment prevalence, firstly the cash spent on capital acquisitions or mergers can be determined ex-ante by a corporate agreement as follows:

$$\mathrm{dR/dt = m\,P}$$

Meanwhile, interrelated to this targeted reinvestment in positive Net Present Values (NPV) projects, the payout starts to reduce potential overinvestment if extra cash is reinvested. In this direction, when the profit reaches the anticipated "normal" value (by the corporate management) **P* = 1** monetary unit (m.u.), m becomes the amounts of reinvestments and no dividends or stock repurchases are featured. "Normal" value of the expected Profits is necessarily a "normative" amount determined by an explicit or implicit evaluation from the corporate governance. Facing to multiple hypothesis of growth, "normal" value of profit chosen by corporate arrangement is a composition from pessimistic and optimistic viewpoints. This is the key of both capitalization and payout processes.

On the contrary, if $\mathbf{P \neq 1}$, it triggers off a mechanism of convergence to the selected level of capitalized earnings and the disbursement of the nonrecurring surplus cash (Bagwell and Shoven 1989; Lie 2000) or symmetrically holding more earnings.

Eventually, the complete equation becomes:

$$\mathrm{dR/dt = m\,P + (P^* - P^2)n\,R} \tag{9.2}$$

P*, amount of "normal" earnings which do not release any adjustment process.

Indeed for $\mathbf{P} = \mathbf{P^*} = 1$, the reinvestments trend takes the targeted **m** value and no procedures of payout are launched. However, in case of losses reaching value $\mathrm{P} = -1$, the firm initiates a divestitures at the trend: $-\mathrm{m}$.

Moreover, the nonlinearity allows a strong payout when Profits exceed **P*** and a decrease of the capitalization of earnings valued at the rate **n**. Beyond what a firm could invest, extra funds are strongly reduced to reach the m ratio of self-investment by the intensification of disbursements, or the more flexible stock repurchases, according to the gap between 1 and P^2.

Similar specification reveals lack of trust on corporate governance since this payout procedure implies an immediate extraction of the FCF from the manager's hand with accelerated speed.

Symmetrically, when the mass of Profits is lower to **P***, earnings are capitalized with a fast increase.

Nonlinear item arises strongly and pushes management to reduce payout to compensate the lack of profits. To prevent financial distress and underinvestment threat, Payout is slowed since cash flow shortage is a critical phenomenon (Uhrig-Homburg 2005). Capitalization must grow at a strong rate to converge to the m value and the stock buybacks, or the dividend payout, is decelerated.

When $\mathbf{P} < \mathbf{P^*}$, the Reinvestment Privilege or the automatic reinvestment of shareholders' dividends in more shares by a Dividend Reinvestment Plan (or scrip dividend) is used. The convergence pattern will keep funds to self-finance the capital assets. Besides, for the weak amounts of profits, the firm must resort to financing reinvestments by self-tender offers of new equities or shares' issuances into the open market. However, the firm divests its capital assets when accumulates losses.

The regulation's mechanism and its specification violate neither the "orthodox" behavior of the managers nor the principles and rules of the disciplining practice of finance governance. In fact, the aim of the mechanism is the driving of Reinvestment to desired level **m** and hinders the retention of excess liquid assets since that FCF, or its shortcoming, constitutes the hidden parameter of the second equation. P* is a threshold which separates between marginal reinvestments and triggering off payout.

It is worth noticing that equations encompass the payout stream with a simple pattern: only the profit drives the investment process.

In the actual international economic context of financial crisis and fraud scandals, a strong level of a monitoring activity is chosen introducing managerial inertia as a new agency cost. Indeed, in our model, even if identical profitable NPV investments are available, the firm pursues the prevailed governance arrangement of reinvestment rate (and its stabilization automaton) until the next managers', shareholders', bondholders' deliberations.

Eventually, the third equation is the account of the net capital inflow of the firm:

$$\mathrm{dF/dt} = -\mathrm{r\,P} + \mathrm{s\,R} \tag{9.3}$$

After deducting the capital outflow (**r** the debt service ratio), the corporate borrowing is obtained according to the debt/equity ratio **s.** In fact, the debt service is linked to the volume of loans but for ease of the simulations, our basic formulation simplifies the model and does not modify fundamentally the core of the corporate governance.

Simulations of the model serve to check the implications of the imposed (or self-imposed) discipline of capitalization policy when nonlinear mechanism of convergence is made.

Computational Results

The basic study begins with the detection of the solutions of the system:

$$\begin{aligned}
\mathrm{dP/dt} &= \mathrm{v(R + F)}\\
\mathrm{dR/dt} &= \mathrm{m\,P} + (\mathrm{P}^* - \mathrm{P}^2)\mathrm{n\,R}\\
\mathrm{dF/dt} &= -\mathrm{r\,P} + \mathrm{s\,R}
\end{aligned}$$

All variables are endogenous and the steady-state equilibria are obtained for $\mathrm{dP/dt} = \mathrm{dR/dt} = \mathrm{dF/dt} = 0$.

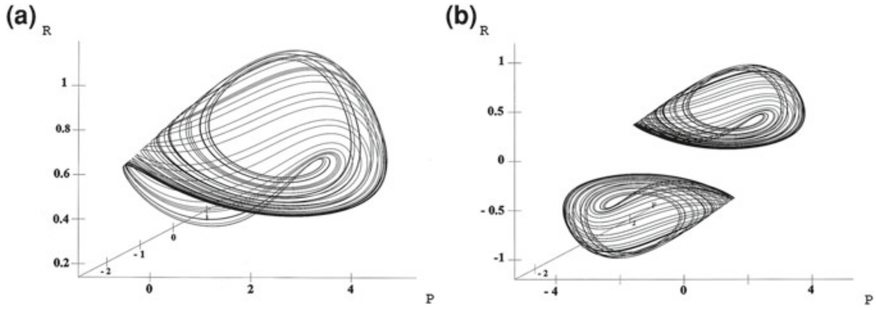

Fig. 9.1 Phase portrait of the chaotic attractors for $P^* = 1$. **a** A chaotic attractor centered on E1 appears in the *upper* sub-basin when initial conditions IC (P_0, R_0, F_0) are positive. The dynamic of the firm fluctuates chaotically around the equilibrium E1 between weak negative values and high levels of profits P. **b** With an additional and negative coordinates IC ($-P_0, -R_0, -F_0$) set in the *lower* sub-basin, the simulation displays another chaotic attractor anti-symmetrically shaped and centered around E_2. This dynamical bi-stability founds the Sensitive Dependency on Initial Conditions (SDIC) of the model

We get $F = -R$ from (9.1), $n(P^2 - P^*)R = mP$ from (9.2) and $P = s\,R/r$ from (9.3). The last two relations yielded the following equality: $[(P^2 - P^*)\, n\, r\, P/s] - mP = 0$.

The three roots of P are: $P_1 = 0$, $P_2 = [(ms/nr) + P^*]^{1/2}$ and $P_3 = -P_2$. Let $[(ms/nr) + P^*]^{1/2} = k$, the three equilibria become: $E_1(P, R, F) = (0, 0, 0)$, $E_2(P, R, F) = (k, rk/s, -rk/s)$ and the third solution $E_3(P, R, F) = -E_2$.

Jacobian matrix of the 3D system gives$|J| = v[nr(P^* - P^2) + ms - 2nPRs]$.

Numerical computations are carried out with the fifth-order Runge-Kutta integration method and 10^{-6} accuracy and the initial conditions are IC (P_0, R_0, F_0) $= (0.01, 0.01, 0.01)$.

We select a set of parameters as the financial statements of the firm C(v, m, n, r, s) = (0.25, 0.04, 0.02, 0.1, 0.3) and the expected "normal" profits: $P^* = 1$.

The trajectory of the system (Fig. 9.1a) follows an infinite orbit centered on the equilibrium: $E_1(P, R, F) = (2.64, 0.88, -0.88)$. The firm as a dynamical system oscillates without any periodicity in a restricted domain of the phase portrait of the state variables, profits, reinvestments and capital inflow.

In fact, the model studied reports a generalized bistability since the phase space displays simultaneously two attractors. In the present case, the space domain (R, P, F) is split in two independent sub-basins of attraction embedding each one its own strange attractor. According the initial conditions related to the positive values of R, and according the negative values of R, we obtain two independent chaotic trajectories confined respectively in the two sub-domains (Fig. 9.1b). This property of bistability attraction demonstrates the Sensitive Dependency on Initial Conditions (SDIC) of the system.

We notice the other equilibria: $E_0(P, R, F) = (0, 0, 0)$ and $E_2 = -E_1$. The model is conservative for the trajectories that are close to E_0 and dissipative particularly at the neighborhood of E_1 and E_2.

Theoretically, if and only if the initial conditions are E_1, the steady-state is obtained. In practice, the E_2 values cannot be attained with an infinite accuracy since a very weak lag pushes the trajectory farther from the equilibrium. Even if recorded financial data to built projections, analyst's forecasts or extrapolation of historical data have 10^{-9} accuracy, they do not allow reconstruction of the "real" model since the missed ten-thousandth fractions of the variables hinders the perfect estimation of the current financial statement. Paradoxically, the chaotic attractor as a dynamical object confined in a limited set of the phase portrait prevents predictions of the variable values and reflects an infinite number of dynamical periodicities. At the most fundamental level, the deterministic nonlinear mechanism in Eq. 9.2 can imply chaotic motion of the state variables of the firm.

Triggering off sequences of reinvestment-divestiture procedures marks persistent and non-transitory chaotic oscillations and a failed management practice leading to other governance arrangements. In fact, the management of public firms can be a subtle balance (and neutralization) of the stockholders and bondholders interests allowing to wide autonomous managerial actions with a minimum level of monitoring interference. Inefficient financial policy or unexpected and unpredictable instability yields to the lost of the management autonomy leading sometimes to a "big bath" (introducing a new agency cost!) requested by both groups of interests. Meanwhile, unbalanced interests could put *decision management rights* and *decision control rights* introduced by Fama and Jensen (1983), in the same hands of this or that group of holders which invades the operational field (Fluck 1999).

The instability leads to the adoption of an alternative governance arrangement substituting this failed management policy (Bhagat and Bolton 2008).

Alignment of Payout Policy to Debtholders Interests

If bondholders monitor the firm, they enforce their directives into the management. The power of decision (Rajan and Zingales, op. cit.) is now in their hands and they could postpone payout focusing on the extinction of instability.

In our application, the managers might decide to experience a new speed of adjustment and, *simultaneously*, define other "normal" earnings. The management can choose a different calibration of P* following objectives and beliefs of debtholders to avoid hazards of chaotic motion incurred by the previous convergence procedure. For example, the new payout strategy delays earning's distribution with $\mathbf{P^* = 1.8}$. Therefore, triggering the disbursement decision beyond a high threshold of cash flow thereby allows a resource which will boost the reinvestment rate and guides, in first principle, to a sustainable trend of growth. Indeed, zero-payout is yielded for the new strategic reinvestments rising from $dR/dt = 0.04$ up to 0.05.

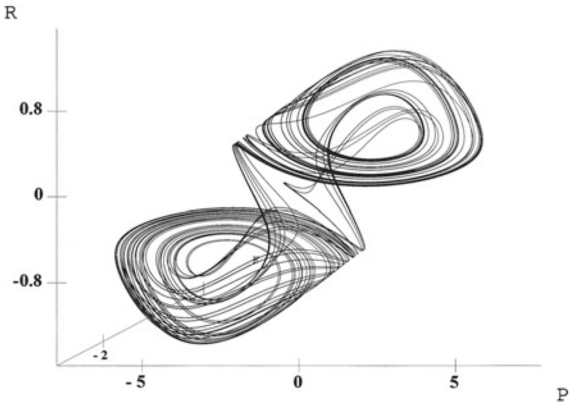

Fig. 9.2 Chaotic attractor for $P^* = 1.8$. The deterministic trajectory drives the firm to profitability and loss without any prediction. The basin of the chaotic attractor expanded to its anti-symmetric set leads to a wide array of losses. The unstable equilibria are $E_0(R, P, F) = (0, 0, 0)$, $E_1(R, P, F) = (2.79, 0.93, -0.93)$, and $E_2 = -E_1$. Changing P^* from 1 to 1.8, the system displays the Sensitive Dependency on Parameters (SDP) of the chaotic system. Associated with the SDIC, the chaos of the dynamical system becomes "strange"

The approach which governs the new strategy is focused not on the *present* earning's distribution but on the *future* creation of the profits. Moreover, postponing realization of capital gains through dividends allows the investors a preference to the "timing tax option" (Constantinides 1984), whose taxation of several annual earnings is less than that of the quarterly frequency tax.

Surprisingly, and contrarily to the expectations, the behavior of the state variables is projected to an unified basin of attraction covering the whole phase space where the profit P takes a wide range of gains but also a wide set of losses (Fig. 9.2). In other words, keeping cumulative surplus of liquid assets triggers disbursements of their *squared amounts* which are financed by earnings, equity issues, and also by debts.

The new attractor appears topologically different from the attractors displayed in the Fig. 9.1 by the variation of a unique parameter of the model, keeping the same Initial Conditions IC. Thus, the system expresses the property of the Sensitive Dependency on Parameters (SDP).

Furthermore, theoretically, the simultaneous presence of the SDIC -in the previous numerical computations- and currently the SDP, demonstrates the "strangeness" of the chaos detected in the model of the firm. Thereby, chaotic attractors can be called "strange attractors".

The key question appears: what is the optimal P* leading to the minimum of profit's instability? In fact, simulating the 3D system for a set of P* allows the detection of the dynamics of P and plots the diagram of bifurcation for the plane F = 0 (Fig. 9.3).

Against the orthodox principles of management, incremental P* guide the profits to an expanded chaotic bubble. Risk of negative profitability rises sharply beyond

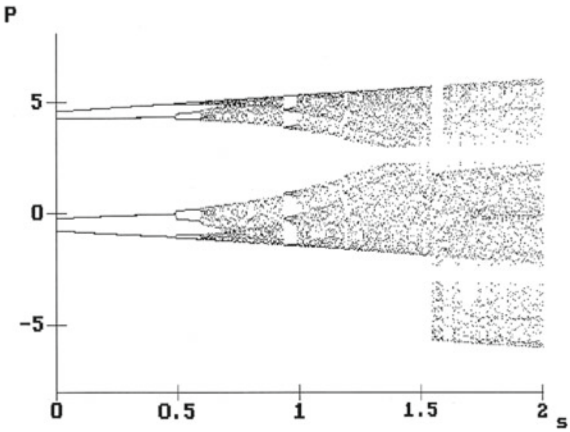

Fig. 9.3 Bifurcation diagram of the profit for $P^* \in]0, 2]$. Initial conditions $P_0^* = 0$, step-size $dP^*/dt = 10^{-5}$, and the C parameters. Postponing disbursement streams imply the amplification of the chaos bubble by the squared gap between the earnings and their expected level P*. We notice the period-doubling cascade. P begins with a period-4 dynamic until $P^* = 0.5$, then the system bifurcates to a period-8 solution. A chaotic bubble with intermittent stability windows appears signaling an infinite periodicity of the Profit

$P^* \approx 1.5$ since the possible worst performance of P moves from -2 to -6 despite postponing earning's distribution. The *deterministic chaos* vanishes when P* is selected in the stability windows of the bifurcation diagram.

Alignment of Payout Policy to Equityholders Interests

If insider ownership and the block of common shareholders inspire and monitor management (Schleifer and Vishny 1986; Blair 1995), they obviously evaluate "normal" earnings P* at a lower value. Triggering early payout stream even in case of weak profits reflects the management's alignment to shareholders interests in the earning's distribution since they constitute the powerful group of the firm.

If $\mathbf{P^* = 0.4}$, targeted reinvestments fall and zero-payout is yielded to a low motion: $dR/dt = 0.02$. The dynamical trajectory of the corporation follows period-4 orbit (Fig. 9.4). Efficient cash management is not only putting cash to applications immediately to produce earnings but, also exiting excess cash flows to reduce the periodicity of capital assets.

The result is consistent with high legal protection of shareholders which allows the increase of dividends even in case of low profitability (Laporta et al. 2000). Whether the payout mechanism injects oscillations for all values of P*, a low periodicity of the firm's variables is obtained when disbursements are released from very weak amounts of cash but with a relative underinvestment. We notice that modification

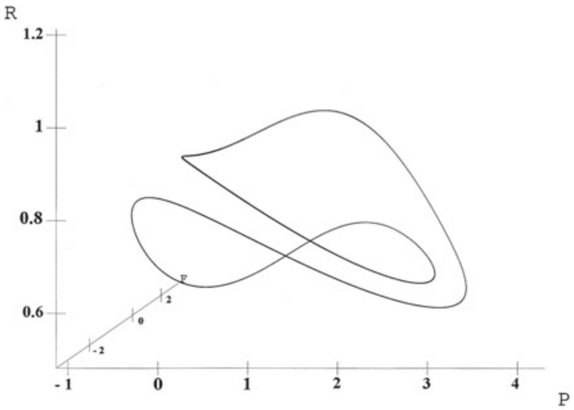

Fig. 9.4 Period-4 solution for $P^* = 0.4$. The state variables oscillate around the unstable equilibrium $E_1(R, P, F) = (2.53, 0.84, -0.84)$ in the simplest orbit reachable with a perfect and predictable recurrence if the equity holders impose a reduction of the reinvestment stream and an early payout triggering-off. The other equilibria are also unstable $E_0(R, P, F) = (0, 0, 0)$ and $E_2 = -E_1$

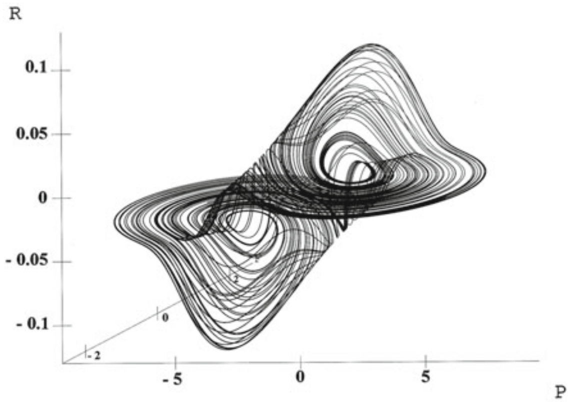

Fig. 9.5 Strange attractor for $P^* = 1$ and $C(v, m, n, r, s) = (1.02, 0.02, 0.3, 0.1, 10)$

of P* displays the morphological plasticity of the attractors and demonstrates the Sensitive Dependency on Parameters (SDP) of the 3D system. The wide range of patterns is obtained with only one nonlinear equation leading also to a singular anti-symmetric strange attractor (Fig. 9.5.).

We could argue that payout automaton generates an endogenous, singular and deterministic financial distress which is not driven by incomplete, imperfect market considerations or industry arguments.

Our 3D system is consistent with the objectives of bondholders if they enforce their interests to the management since delaying payout derives an overinvestment. Likewise, when stockholders inspire early disbursements, the system leads to underinvestment. In fact, an imposed payout requires a somewhat fine-tuning application to control oscillations: a small disbursement chain (low value of P*) breaking the chaotic expansion of the profit variable. It is a new justification of the "reluctance" to cut dividends (Kalay 1980; Frankfurter and Wood 2003).

Concluding Remarks

Corporate governance has now reached a level of sophistication far beyond our idealized numerical experiments. Yet, our three dimensional model of the firm where the disbursement policy is an implicit variable serves only as a heuristic tool to detect the implications of a mechanism of convergence to a targeted capitalization rate. It reflects well and rational choices of the corporate management art.

To our knowledge, this model is the first attempt to study dynamical findings of self disciplining profit's capitalization in the context of deep lack of trust between ownership and control. Numerical computations are carried out also when the power is transferred from shareholders to debt-holders. In our application, the nonlinear reinvestment regulation is investigated with the tools of the theory of deterministic chaos (Baumol and Benhabib 1989; Day 1994) which can complete the recently established framework of econophysics (Mantegna and Stanley 1999).

Singular results are made in opposition to rules and principles of finance governance built in static and *linear* framework since lead to a chaotic dynamics and strange attractors. Firm can lose its dynamic stability when targets fixed reinvestment stream. The gap between actual earnings and their expected amount releases the mechanics of payout but automated financial governance procedures imply costs.

Indeed, the main findings of our nonlinear and *ab initio* heuristic model show the negative implications of a self-imposed discipline of disbursements when the outflow earnings are erroneously triggered (amounts and frequency).

Committing to pay out excess of earnings resolves an agency conflict, but, it can also inject fluctuation leading to a chaotic hazard and bankruptcy threat added to the wider array of identified financial risks. For example, postponing disbursements of excess cash is not harmless and can introduce a critical dynamic motion. The normal earning's threshold P* which drives capitalization should be technically determined, for example, by a bifurcation diagram. P* is a key parameter of the corporation derived from the size and the profitability gathered in the financial statements parameters C (v, m, n, r, s).

Meanwhile, if the payout device maintains the interests of equityholders or bondholders, its automated mechanism plays against their interests themselves! The particular specification of the linkage between reinvestment target and payout policy inserts a chaotic dimension, or almost a periodicity into the profit.

Assuming the simplest formulation of the model, the self-financing procedure to focus level *m* is, itself, the turbulent process and not a transitory phenomenon. In a few words, the dispatching of the profit (to reinvestment and payout flows) modifies the dynamic stability of the profit itself.

The first insight from our application can be seen as disciplining profit capitalization policy is itself a mechanism of fluctuation. Oscillations are not an artifact yielded by the simulations but the outcome of the nonlinear disbursements behavior. Payout mechanism has an oscillatory nature since our heuristic system without the second item of Eq. (9.2) leads to an exponential growth of P. Our outlook is consistent with Baker and Smith (2006) conclusions. They indicate that some firms "…may follow a "modified" instead of "pure" residual dividend policy to avoid highly volatile dividend payments."

Intuitively, managers "disconnect" the payout's automaton and drive "manually" the earnings' disbursement to pull backward the system far from the chaotic bubble.

In our application, payout is not the residual of reinvestments but is triggered out by the nonlinear adjustment of capital reflecting high sensitivity of ownership to the cash emergence. The strong elasticity of payout to earnings can be a consequence of severe agency conflicts, and derived from scarce level of trust between stockholders and debtholders and managers (Farber 2005). The payout's simulation approach can be further applied to several sets of parameters particularly m, which is derived from the expected earnings stream. A more promising path for investigating the consistency of the present conclusions is carrying out simulations with other kinds of reinvestment convergence.

Deep insights in the shape of these regulators and their dynamical implications could enhance the practice of corporate finance and its reliability.

However, our basic model of the financial statements masks a loss of generality and deserves a sophisticated formulation. For example, managers perceive a substantial asymmetry between dividend cut's decision and its increase (Brav, op. cit.) and, also, between financial distress and profitability. The nonlinearity in Eq. (9.2) is adapted only to a perfect symmetry of the flows.

Meanwhile, fixing *ex ante* a level of m denying *per se* investments in unexpected NPV projects and hindering fund conversion in profitable assets is a credible agency cost of corporate finance.

Moreover, in case of loss, the opposite motion emerges since the reinvestments are converted into divestitures in Eq. (9.1) and the borrowing inflow to an outflow in Eq. (9.3).

Eventually, this study shows that chaotic oscillation can be an intrinsic and endogenous characteristic of the modern firm when an automated disciplining payout policy is implemented.

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Chapter 10
Interaction-Based Approach to Economics and Finance

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Abstract The chapter examines the characteristics of interaction-based models in economics and demonstrates that these models can be an important part of the future research in economics and finance. The economy is considered a complex system which consists of a large number of interacting units who are represented as software bits of data and act upon the specified rules of conduct. The multidisciplinary nature of the agent-based approach makes it highly applicable to examine heterogeneity, interaction, evolution, uncertainty and the agents' cognitive limitations which are central to economics and finance. After a thorough literature review some interaction-based applications are run.

Keywords Social interaction · Evolutionary activities on networks · Complex adaptive systems · Agent-based models

Introduction

A distinctive feature of every economy is that it consists of a large number of interacting units who pursue their private interests in an uncertain environment within particular circumstances of time and space.[1] Even though it seems very complex

[1] There is a difference between uncertainty and risk. Although both are related to incomplete knowledge, uncertainty relates to the state in which outcomes and related probabilities are not known, while risk relates to the state in which all outcomes and their probabilities are known.

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from the outside, a lot of economic activity depends upon the very simple question of what individual members of a society know. Economic analysis has been traditionally done under the rationality assumption of, loosely speaking, perfect knowledge, perfect foresight and the agent's best possible selection, although the environment in which economic units make decisions is inherently the environment of incomplete and frequently contradictory knowledge that is dispersed among all economic units and which is evolving over time, as argued by Hayek (1937, 1945). Some presume that dispersion of beliefs is required for the market to function at all (Milgrom and Stokey 1982). Such decentralized systems represent the basis for extensive economic interactions among bounded rational individuals. In the environment which consists of many such individuals, the individual-specific characteristics and imperfections determine the peculiar problem of a rational economic order. A problem of a selection and allocation of available resources is thus not just a technical one that could be represented by the closed-form solution but a complex, making the economy a complex system. Generally, every complex system that is characterized by the repeated nonlinear interactions among its constituents, where one agent's decision affects and is affected by decisions of others, can induce coherent large-scale collective behaviors with a very rich structure that is impossible to foresee (Sornette 2004).

An agent-based approach in which bounded rational agents represent the driving force of the aggregate behavior that evolves over time takes this into account. Agents in an agent-based model are modeled as software entities and include data together with behavioral characteristics that act on these data. They are goal-oriented individuals of different characteristics who are able to learn over time and are subject to different constraints. Definition of an agent is not restricted only to human agents. Agents might also include social groupings, biological entities and physical entities. They can range from active data gathering decision-makers with sophisticated learning and cognitive capabilities to passive world features with no cognitive functioning (Tesfatsion 2002; Tesfatsion and Judd 2006). Then, the network consists of a group of mutually connected agents who may be very different in their structure. Starting from an initially specified system state and the rules of conduct, these agents are constantly engaged in local interactions by which they produce the outcome of the entire group, which in turn affects individuals' future behavior. Many of these applications borrow from the evolutionary game theory, which helps us examine how behavior of individual entities within a group changes over time according to behavior of their counterparts (Maynard Smith 1982; Weibull 1995).

The present chapter examines social networks in a social science context, especially in economics and finance. Early attempts of using networks in economics were due to Myerson (1977) and Kirman (1983, 1997). Since then, agent-based techniques have been increasingly used in economics. Our principal objectives were the following: to review the field of interaction-based models in economics and finance, to describe the properties of these models, and to present some applications.

Interaction-based approach provides a multidisciplinary tool for exploring many different phenomena and can easily incorporate elements from other fields, such as game theory, psychology, neurology, sociology, biology, which makes these models highly applicable. They have helped us understand many open questions from different research fields from within and out of economics, especially in cases of complex models that are mathematically intractable. At least, these models have provided us some complementary arguments to the questions at hand; not only about the solutions as such but also about how these solutions evolve over time and how they might change as the circumstances are slightly perturbed. In an interaction-based model, individual nodes, links or some other model attributes can be affected by different types of shocks while the modeler is able to examine the consequences. The modeler can also examine the model according to different rules of conduct and so on, which makes the agent-based approach well suited for examining very complex and evolving systems.

The chapter proceeds as follows. In section "Properties of the Interaction-Based Models", we present social networks and cellular automaton as two baseline models on which interaction-based games can be applied. They both represent an infrastructure which agents use to interact with each other and share information with one another. In addition, both of them are capable of encompassing many different theoretical aspects that might be relevant in interaction-based applications and agents' decision-making. The section ends with a brief discussion about the network types that would best fit the model's characteristics. Subsequent sections offer a thorough overview of interaction-based models in economics and finance by which we provide some ideas and the range of how these models can be used. We start with diffusion models because other interaction-based models borrow many concepts from this group. A special class of interaction-based models represents game theoretic models. In section "Applications Outside Economics", we review some interaction-based models from outside the economics and finance. Many solutions from these applications can be and have been very effectively used in examining various phenomena in economics and finance. Although very extensive, the review is far from being complete. Section "Simulation-Based Experiments" presents two examples of interaction-based applications, by which we present how these applications can be conducted and how (even minor) changes in parameter values or in the network structure end up in highly different outcomes of the entire system. These simulation experiments are then followed by a short discussion in Section "Discussion", while the last chapter concludes.

Properties of the Interaction-Based Models

The Network

A model for a social network is a graph. We can also say that a graph is a mathematical representation of a network. By definition, a graph $G = (V, E)$ is composed of a nonempty finite set of nodes (or vertices) V, representing the units, and a nonempty finite set of edges (or links) E, representing their pairwise relations (Fig. 10.1). Depending on the application, a node can be a single individual, a firm, a country, a group, or some

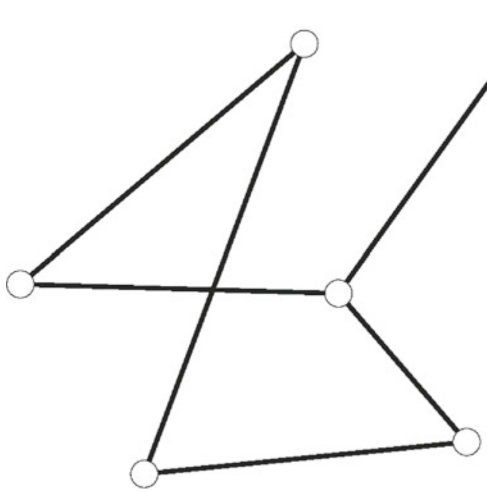

Fig. 10.1 A graph

other autonomous unit. Nodes and links may include a variety of properties, which may be numerical or qualitative. For instance, in a network of friends, nodes represent individuals and the links their friendship relations. In a banking network, nodes may represent different banks and the links the interbank exposures. Extensive reviews on social networks and the network-based models are given in Boccaletti et al. (2006), Goyal (2008), Jackson (2008, 2010), Wasserman and Faust (1994), Brock and Durlauf (2001a).

It is very common to denote the link between nodes i and j simply as $ij = 1$, and $ij = 0$ otherwise. Two nodes that are joined by a link are referred to as incident nodes or neighboring nodes or connected nodes. The presence of a link is a required condition for the information flow between the two nodes, but not also a sufficient. In an undirected graph, edges are unordered pairs of nodes, which means that if $ij = 1 \Leftrightarrow ji = 1$ and if $ij = 0 \Leftrightarrow ji = 0$. This applies to situations where two nodes are either in a relationship with each other or not. In a directed graph, edges have directions. An edge (i, j) allows us to move only from i to j but not also from j to i. We say the network is finite if it has a finite number of nodes.

In most applications, graphs do not contain loops or reflexive ties, by which single nodes would be linked to themselves, nor multiple edges by which a pair of nodes would be linked more than once. If such a structure exists, the elimination of a single link between the two nodes does not eliminate the link between them. A demonstration of a multiple-edge banking network would be the network, in which banks possess various different instruments from the same counterparty. Mixed graph is a graph with both undirected and directed links.

In a graph, individual nodes that are not directly linked may be reached through the sequence of nodes and links. A graph is connected if for every pair of nodes

(i, j) there exists a walk from node i to node j.[2] The distance $L(i, j)$ from node i to node j is equal to the length of the shortest path from i to j. Often, the shortest path between two nodes is referred to as a geodesic. If there is no path from i to j then $L(i, j) = \infty$. The eccentricity of a node is the largest geodesic distance between the node and any other node in the graph, i.e. $Ecc_i = \max_i L(i, j)$. Maximum eccentricity of any node is $(n - 1)$. A graph has a diameter D if every node in the graph can be reached by the maximum geodesic of a length $D = \max_{i,j} L(i, j)$. Diameter is the largest eccentricity. The term "degree of separation" is usually used in the context of diameter.

The degree of a node k is the number of edges incident with it. It represents the number of nodes linked to it. A node degree can range from 0 for an isolated node, to $n - 1$ for a node that is linked to every other node in the network. The set of nodes that are linked with node i is called the neighborhood of i. In directed graphs, every node has an in-degree that represents the number of incoming links, and an out-degree referred to as the number of out-going links. A bridge is a link in a graph such that its elimination splits the graph into several unconnected sub-graphs; i.e. components or islands. A node that connects two components is called a cutpoint. Connectivity is an important element in defining the network behavior and may induce different consequences when the network is used for different purposes. For instance, connectivity can work either contagiously or as a channel of risk-sharing in a financial network, while epidemiological networks do not have the risk-sharing potential.

The most basic topological characterization of a graph can be obtained in terms of the degree distribution $P(k)$. It relates to the statistical distribution of the nodes' degrees and is defined as the probability that an arbitrarily chosen node has degree k. Equivalently, it is defined as the fraction of nodes in the graph having degree k. In homogenous networks, such as random networks and small-world networks, nodes with degrees significantly higher than others do not exist. However, such networks are rather exceptional in reality. On the contrary, it has been argued that most real-world networks exhibit power-law distribution, which means that there exist some nodes with high degrees and a vast majority of nodes with small number of adjacent links (Albert and Barabasi 2002 and references therein). Such networks are referred to as scale-free networks. In scale-free networks, the rate at which individual nodes increase their degrees depends on their fitness to compete for the links of other nodes. This observation is very general, because fitness of a node may be determined by many factors, such as its degree, age, reputation, distance or some other competitive factor that attracts other nodes. By the same token, some nodes may avoid linking to some of the others. Not all nodes are identical in terms of fitness while each node increases the number of connections accordingly to the fitness it possesses over time. Barabasi and Albert (1999) have demonstrated that if the network develops according to this principle, which they call preferential attachment, it exhibits power law distribution. The preferential attachment and, under certain conditions, also the fitness

[2] A walk is a sequence of nodes and links, starting and ending with nodes, in which each node is incident with the links following and preceding it in the sequence.

models allow for the endless link formation, which is possible only theoretically. Most networks are subject to serious constraints, either due to the nodes' limited degree capacities or due to aging of nodes or links, for which the nodes' degrees have an upper bound (Amaral et al. 2000). In addition, communities of bounded sizes have been observed in reality.

One additional network-based characteristic that is very common in socio-economic networks is assortativity. It illustrates a phenomenon when nodes tend to be connected to other nodes that are similar to themselves. Kremer (1993) identifies such a pattern in a production process and argues that workers of the same skills are matched together in the equilibrium, which makes high-skill workers even more productive. Patterns of assortativity can be found in various grouping models, such as marriage models, models that build on trust, mating models, etc. Dissortativity (or negative assortative) is the opposite case, when, for instance, high degree nodes tend to be connected to low degree nodes. Similarly, one can find homophily in the networks, and heterophily, its opposite.

Although the notion of a node degree is compelling, it is by no means sufficient let alone exhaustive. The behavior of a network depends on the role, influence, and importance of single members within bigger communities. A node degree measures the number of nodes to which individual nodes are adjacent but this does not say anything about the importance of these links. A node can be linked to many unimportant nodes or to some highly important. There is no clear definition of a node's importance, which depends upon the structure of the network and the context as such. One measure is represented through the node's centrality, the other, when the directed networks are applied, through its prestige (see Ballester et al. 2006; Wasserman and Faust 1994). In Steinbacher et al. (2013), the node's importance is measured through the damage that its elimination causes to the system. Hence, each node is assigned an alpha-criticality index with the criticality level measuring the extent of the damage.

Cellular Automata

The alternative approach to the network-based experimentation represents cellular automaton (Wolfram 1983). Cellular automaton was originally introduced by von Neumann (1966). It is a discrete time and discrete state system in a D-dimensional lattice which consists of the cell space, cell size, neighborhood size and type, transition rules and temporal increments. In the schematic representation of Fig. 10.2, individual agents are colored black and move throughout the lattice according to the state on the lattice, their preferences and decision rules, and general rules of conduct.

Cellular automata on complex topologies are systems in which each agent can be in only one of a finite number of states. At each time step, the next state of each agent is computed as a function of its state and of the states of its neighbors on the network. Agents can move only in the neighborhood of the cell which they occupy. Agents' dynamics on the lattice could be limited by the outer bordering cells, but this does not have to be the rule. For instance, in a 2-dimensional lattice we can assume that

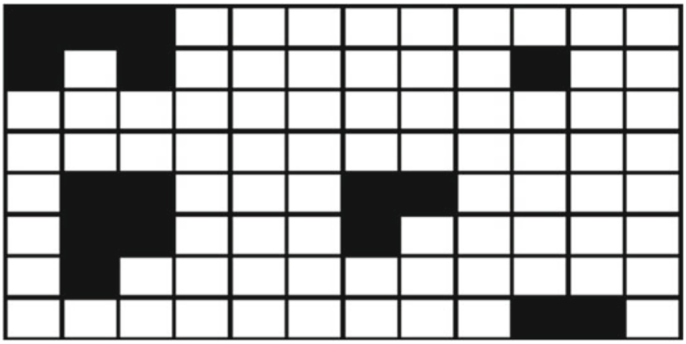

Fig. 10.2 Cellular automata model

the lattice is a representation of a globe. Therefore, an agent exiting of left enters the lattice on the right and each cell has eight neighboring cells no matter the position.

Applications of cellular automata are many. Cellular automata have been extensively used to simulate the evolution of self-organization systems and particularly for urban modeling. Langton (1990) finds that by manipulating the parameters of cellular automaton, the aggregate behavior of the system exhibits a phase transition between highly ordered dynamics to chaos. Some applications will be presented in the sequel.

The Game Structure

Many of the methodological concepts that we use are taken from the game theory literature (see Osborne 2002). By referring to games, we do not have necessarily the usual game theoretical framework in mind but computer-based experiments. Hence, the games on networks could alternatively be referred to as the activities on networks. We denote them games to highlight the connection to the games as we usually know. Each game consists of the finite set of $i = (1, 2, \ldots, N)$ agents, preferences and objectives P_i for each agent, payoffs $\Pi_i \in \mathbb{R}$ for each agent (or utility), a set of actions A_i for each agent and a set of rules R_i. In the network terminology, the properties of nodes are usually called attributes and the properties of links are called weights.

As we have said before, an agent may be an abstract version of a single individual, a firm, a country, a hub or some other autonomous unit, while a multiagent system is a system that contains multiple agents who interact with each other within the environment in which they live and, in some applications, also with the environment. Russell and Norvig (2010) define agents as anything that can perceive its environment through sensors and act upon that environment through effectors. The usual assumption here is that agents are heterogeneous and adaptive both in their attributes and the ways in which they react to the environment.

Because heterogeneous agents interact with one another, the system virtually always evolves over time dynamically and evolutionary, and often also stochastically. The agents' heterogeneity is an important factor which adds the complexity into the course of the game. And not only that, when using an agent-based framework, it becomes apparent that most of the problems we try to solve require heterogeneous agents.

The game usually proceeds as follows. The model is first constructed, then the initial conditions and the system states are specified and the rules of conduct defined. Afterwards, autonomous agents are constantly engaged in local interactions according to their characteristics and the predefined rules. During the game, agents affect the others and are affected by the others by which at every point in time the aggregate outcome of the entire group arises. Perpetual activity is thus integrated into the model structure, although the participants can also be just passive creatures.

Modeling Agents

Having defined the agents' attributes, they can be freely manipulated to meet the structure of the problem and to describe the agents' characteristics. Computational agents can be very broadly defined and can range from simple scalars to complex functions.[3] Agent-based approach is very robust in this respect and allows the modeler to define different types of agents with different knowledge and preferences, objective functions and endowments, or to model agents who follow different strategies and pursue different selection criteria, or agents who are omniscient or non-omniscient, rational or irrational, unsuspicious or suspicious, autonomous or subservient, far-minded or short-minded, patient or impatient, or conservative agents, etc. All this may bring the modeled agents much closer to how they look like in reality than the neoclassical.

A special class of games represents those in which agents refer to human beings. A lot of what has been said in the previous paragraph relates to the agents as human beings. Human agents differ from other types of agents in a cognitive component. When we talk about the cognitive component, we particularly have in mind the agents' preferences, their communication skills, knowledge and learning mechanisms, their abilities to set-up their goals and build expectations, to gather and process information, to maintain their (social) role in society and, finally, also their selection patterns. A cognitive component makes the decision making of human agents a complex task and their aggregate behavior a complex system of interdependent subjectivisms. Non-human agents are generally passive in nature and their actions instantaneous without the agents' control. However, this does not mean that passive agents are not

[3] Computational agents can also be referred to as the software agents, given that they are programmed as bits of computer data.

subject to different types of constraints. Electric hub, for instance, can transmit just a limited amount of electricity. On the other hand, not all the models of human-alike agents include a cognitive component.

In dynamic and evolutionary games, agents' initial attributes are set over a vector of characteristics for each agent. Agents usually also have some prior knowledge about the problem they face. If learning is applied, then the evolutionary dynamics for these attributes has to be specified. The ability to include learning is an important factor in interaction-based modeling, by which we make the agents the active units who are allowed to correspond reasonable to the changing circumstances. This gives the games a new dimension. Different situations spur different learning processes (Brenner 2006; Fudenberg and Tirole 1998; Russell and Norvig 2010). Agents can learn either when they are repeatedly faced with the same or related problem, or when they play the same or related game repeatedly, or from observing the changing environment. In the latter case, the dynamics inhibits learning processes. In the extreme case when the environment is chaotic, agents do not have many opportunities to learn. However, in the two-agent repeated game, each agent should also acknowledge that their actions may affect the future plays of their opponents. In the games that include learning, different types of reinforcement learning methods have been highly extensively used (Sutton 1988). In the regret matching model, agents decide upon the pain caused by the non-selected alternative as to the selected alternative, or upon experience-weighted attraction model, proposed by Camerer and Ho (1999), or copy actions of their neighbors (Erev et al. 1998), in Q-learning agents learn from delayed rewards. The method of temporal differences is a method of incremental learning in which learning occurs upon the difference between the predicted outcomes that are done upon past experience and the data, and actual outcomes. Reinforcement learning is a kind of feedback learning. In reinforcement learning games, agent's actions gradually approach the most efficient ones. Agents either try different actions over time or get information from their neighbors. In either case, this is either done through the trial-and-error search or through reward-based search.

Alternatively, Cowan and Jonard (2004) model knowledge diffusion as a barter process between agents, in which agents exchange different types of knowledge with their adjacent links. In their model, agents repeatedly meet their neighbors and trade if mutually profitable trades exist. In this way knowledge diffuses throughout the economy.

Anderlini and Ianni (1996) study the long-run properties of a class of locally interactive learning systems. A finite set of players at fixed locations play a two-by-two symmetric normal form game with strategic complementarities, with one of their neighbors selected at random. Their model exhibits emergent phenomena and a high degree of path dependence, which is induced by the endogenous nature of the model and the noise. Baumol and Benhabib (1989) argue that due to the huge sensitivity of an economic system to microscopic changes in parameter values, a chaotic system may reach the long-run at different points, producing very complex time paths, despite the simple and even deterministic relationships among its constituents as long as they are nonlinear. Such a local nature of search can explain price dispersion in a search model, wherein different agents sell the same good at different prices in a given market.

Although agents interact with their peers and exchange information with them, the interaction-based experiments are not bound only to such inter-personal links, but may also include information from the environment in which they live.

The last phase of a selection process is the selection as such. The selection is viewed as a mapping of agents' knowledge into a decision from a set of available alternatives, given the feasibility constraints. Although this is the universal description of the agent's problem, the assumptions about its constituents are the key to understanding agents' behavior. Traditionally, researchers have assumed that agents make perfect selections so as to maximize a utility function upon the perfect knowledge and time-consistent preferences.

The conceptual breakthrough that traced the path towards modeling cognitive agents was initiated by the work of Herbert Simon who substituted the optimization principle with the notion of satisficing agents (Simon 1957). The enormous literature on the psychological (or behavioral) economics which was later initiated by Kahneman and Tversky (1979), Tversky and Kahneman (1986) expanded the perception of the agent-based modeling, particularly in respect to individual agents. The behavioral economics research suggests that the expected utility theory does not adequately describe the agents' behavioral and selection patterns, because agents violate the fundamental axioms of the theory. Often, the "agent-based" agents are described as bounded rational agents. Rubinstein (1998) provides a thorough discussion on modeling bounded rational agents.

In the behavioral approach, agent's decision-making is subject to imperfections along the entire selection process from their preferences, set of alternatives, knowledge about the alternatives, data gathering, data processing, to the choice rules and the selection as such.[4] Nothing from this is static over time. It is standard to assume that agents face a hard budget constraint, while the behavioral literature also assumes that agents have incomplete and asymmetric knowledge about the available alternatives, that they do not have transitive preferences, violate rational expectations hypothesis, are time-inconsistent, learn, apply different learning rules, while their behavior is also affected by the behavior of others, etc. (see Barberis and Thaler 2003; Hirshleifer 2001). Agents' decision may be subject to various types of "errors" in the selection process, some of which might also be induced by confusion Selten (1975). In some instances, they gamble, speculate, use heuristics and even make blind guesses. In addition, agents can make decisions simultaneously or jointly as a part of different groups. Agents' decisions may either be perpetual solutions to their choice problems or one-shot actions. In order to include these agent-based specifics but not to make the model too precise which could easily make it very inappropriate, Steinbacher (2012) combines all the subjective specifics that are relevant for the agent's selection

[4] In the computational economics literature, the interaction-based models are often characterized as the behavioral models, although this does not adequately reflect the structure of these models. Behavioral aspect represents only one component in the interaction-based models, although very important one. The other is a social component.

into a residual variable which he denotes the level of suspiciousness. Each agent is thus allowed to make sub-perfect selections for whatever reason.

The emerging literature of a new approach that is commonly referred to as neuroeconomics introduces neuroscience, i.e. knowledge about brain mechanisms, into modeling economic agents' decision-making (see Camerer et al. (2005) for an overview). Neuroeconomics represents a very ambitious approach. The other ambitious area of research that is developing very fast relates to sentiment analysis and opinion mining (Pang and Lee 2008). This bunch of research is concerned with the analysis of what individuals think, and this could bring some new insights on how to give information an economic value and benefit from it.

Agent-based agents may either contain behavioral characteristics or be optimizers. The modeler is free to incorporate the neoclassical type of maximizing agents into the model. In fact, agents might range from zero-intelligence agents who make random guesses (Gode and Sunder 1993) to such who decide upon the very detailed procedure as in the ASM model where agents' rules include more than 100 parameters. Agents might have perfect recall, which means that they remember entire histories of their actions and the relevant data, or they may have an imperfect recall. In addition, agents' decision making and their expectations can be affected or even restrained by their cultural or religious characteristics, which have proved to be significant (see Guiso et al. 2006 for a survey). Culture is particularly relevant in relation to trust and economic activities that outdo the mere market mechanisms (Akerlof 1970).

Altogether, agents' selections might depart from the seemingly most promising alternatives if such ever existed. With such a shift away from a perfectly rational and omniscient agent, Homo Economicus becomes bounded and begins losing IQ, evolving into Homo Sapiens (Shiller 2000). Of course, it would be mistaken to think that computational agents do not try to think rationally or even to make optimal decisions if such existed. The behavioral approach gives the agents a "non-automata" and a human characteristic, in which their selections capture cognitive and social features.

Which Networks to Use?

In recent years, there has been a remarkable interest in the network structures. Over the years, many different networks have been developed and identified and, in the end, also used for different purposes. Some networks are very complex with a specific architecture, which gives them very specific and unique characteristics. Following Newman (2003), the networks can be divided into four broad categories: socio-economic, technological, information and biological networks.

From the economist's perspective, socio-economic networks are usually applied, especially in the game theoretic or agent-based applications. These networks consist of a set of people or groups of people who are connected together in pairs by links. Links signify some pattern of contacts or interactions between these individuals, and represent a channel which these individuals use to share their private information with one another. Because individual units usually communicate with each other, these networks usually take the form of communication networks. Technological networks include tangible objects. Typically, they are designed to represent the distribution of some commodity or resource. Some cookbook examples of technological networks include networks of roads, airline routes, pedestrian traffic, etc. In economics and finance, an example of a technological network would be a banking network in which banks are linked to each other through the interbank market of mutual exposures. Typical examples of the information networks are citation networks and the World Wide Web. Information networks are also referred to as knowledge networks. A class of preference networks in which people express their preferences on objects, e.g. book or stock recommendation, also belongs among information networks. Finally, biological networks model biological systems as a network and examine their activities in a network-based setting. Networks may include elements from different categories. The network of trading relationships among countries has elements of both socio-economic and technological networks (Jackson 2008).

Networks can be further classified into those in which objective data, such as risks, viruses or other events is transmitted and the networks where subjective beliefs, such as ideas and opinion are transmitted. The difference between the two is not just methodological but also technical, while applications of the latter are much more complex than that of the objective-data models.

Furthermore, when links are necessarily reciprocal it will generally be the case that mutual consent is needed to establish and maintain the link. Most economic applications fall under the reciprocal-link framework. In such a case, undirected networks should be adopted. When direction of a link is important, such as in credit risk models, directed networks should be applied. In weighted networks, nodes and links possess some attributes and weights. Usually, nodes possess some values whose dynamics is provided by the weights of the links. Granovetter (1973) used such a weighted socio-economic network, in which the links are given the strength of a friendship between two persons according to the closeness of existing link and the frequency of interaction. Hence, the interpersonal links can be either strong or weak with the latter illustrating casual acquaintances. Granovetter then demonstrates that although individuals get very useful information from their closest ties, the group homogeneity exhausts the level of unknowns within the group, which makes weak ties indispensable for the propagation of new information into highly homogenous group. Although Granovetter studied the job search market, weak ties can be instrumental elsewhere, as well. Goldenberg et al. (2001) argue that weak ties overcome the effect of the strong in all stages of the product life cycle. On the other hand, by definition, weak ties are less accessible than the strong and also less willing to share their knowledge and information, which may limit their value.

In addition, networks can be classified into the static and dynamic. In static networks, all nodes and links between them are fixed over time. In evolving networks, new nodes emerge over time and some of them die off, while nodes make new links and sever some of the existing. Many systems in reality would be best described by evolving networks. Models of evolving networks need to include a mechanics by which the network grows and develops (Albert and Barabasi 2002; Boccaletti et al. 2006; Jackson 2008, 2010; Newman 2003).

We do not exaggerate by saying that economics has developed into a highly interdisciplinary science with very broad research interests. Within this scientific development, social networks represent the additional methodological tool to examine the questions bottom-up from their substance. Furthermore, in tackling the complexity and to simplify the problem, economists have often used conformist assumptions in their models. Agent-based approach is, of course, not immune to such simplifications. On some occasions, a modeler uses simplifying assumptions in order to isolate the effects of particular factors of the model and simulate the model under these specific circumstances. In the other, increased complexity of the model structure seems redundant. Still, many models could not be solved without such simplifications. Finally, there are some open questions of which we still do not have a satisfying clue of how to tackle them successfully. In this respect, Gibbard and Varian (1978) argue that models can be either approximations which aim to describe reality, or caricatures which seek to give an impression that they describe reality. From this perspective, one could argue that it is not so much a question of which network-types are generally more appropriate for economics and finance, but which types better fit the specific problem at hand and satisfy the modeler's aims. This may be true, although the appropriate network is required if one would like to defend the argument.

In the following chapters we review the literature on agent-based models in economics with an emphasis on social interactions and discuss the models from several different aspects.

Diffusion Through the Networks

Spread of a Disease

We begin this overview of interaction-based models with the epidemic diffusion models for a simple reason; because they represent a platform for other diffusion models. They are also very intuitive and easy to understand, while the connection between the networks and the epidemic models is very straightforward.

Let us presume a group of people, which consists of a portion of infected individuals and the rest. The network can be formed if we imagine that nodes represent individual people and the links their pairwise connections along which the infection can spread (Newman 2002; Pastor-Satorras and Vespignani 2001). Each individual has a finite set of contacts, while over time these individuals interact with one another through interpersonal contacts. Individuals may make new contacts and sever some of the existing. The network structure allows the modeler to examine the epidemic dynamics over time. Namely, diseases spread by contact and go from the infected individuals to others and the modeler is able to monitor the speed and the extent of the progression under various circumstances. Applications of epidemic models usually include the propagation of human and electronic viruses or other diseases.

Generally, three different types of the epidemic model are examined. In a SIS model, individuals exist in one of the two discrete states: susceptible (S) or infected (I). At each time step, each susceptible node gets infected with some rate if it is connected to one or more infected nodes. A node which is connected to the bigger number of infected nodes has a higher probability that it will get infected. At the same time, infected nodes are cured and become susceptible with some rate. Susceptible individuals who become infected become potential virus transmitters.

The extended model includes a group of recovered individuals (R) and is thus referred to as a SIR model. Recovered (or dead) are those who have been infected and are immune for life (or are dead). Such individuals cannot be transmitters anymore. SIR models can be further extended to include the case when a recovered individual, if not dead, can again become susceptible and infected. This kind of model is usually referred to as a SIRS model.

The significant property of epidemic models is represented by the epidemic threshold, which marks the effective spreading rate of the infection. It is important because it gives us information whether the infection can become endemic or dies out. In SIS models, it is the quotient between the infection rate and the rate at which the infected nodes are cured.

Credit Contagion in Financial World

Epidemic models can easily be applied to the banking world to examine issues that relate to credit risk. Credit risk can be defined as a risk of changes in the value that is associated with unexpected changes in the credit quality of other counterparties. As such, a credit event spreads like a "virus" across the network. Credit contagion has been extensively studied within the network models (Allen and Gale 2000; Gai et al. 2011; Haldane and May 2011; Lelyveld and Liedorp 2006; Steinbacher et al. 2013, see Allen and Babus 2008 for a survey).

Financial system has a natural representation of a network, in which the nodes represent individual financial institutions (or banks) and the links the interbank positions. Banks may be modeled through their balance sheets. Financial network would usually be directed and weighted, reflecting the fact that banks are either debtors or creditors, and that interbank positions are of different sizes. Banks may be heterogeneous across the types and the size, which makes the banking network very complex. It has been argued that the network of major international financial institutions exhibits an increasing scale-free characteristic in which a few large banks interact with many others, although the system is strongly interdependent (Iori et al. 2008; Schweitzer et al. 2009). In the banking network, the bank capital serves as a cushion to absorb losses.

By using the network-based approach, the modeler is able to stress the model by either an idiosyncratic or macrostructural shocks and examine how these events affect the stability of the network and the banking system. The latter are considered systematic events because no particular bank that holds an asset that has been hit by the shock can avoid the consequences. The two events may be co-integrated and correlated.

When a credit event occurs and the borrowers are unable or unwilling to fulfill their obligations, the interdependent banking system may induce credit contagion, where failure of a single bank triggers the subsequent failures of counterparty banks. Contagion is a typical network effect and represents the counterparty risk. Following the bank default, adjacent banks are infected first and, if capital buffers are not sufficient to cover losses, the shock propagates through the chain of links. Contagion in the presence of a systemic event is different from that of the idiosyncratic in that the systemic shock itself reduces the capital of each bank, thus making them more vulnerable to additional writedowns due to the counterparty risk. Correlated exposures of banks to a common source of risk can propagate systemic risk through the banking system. In addition, the extent of a credit event depend upon the level of recovery rates and a time delay from the time the bankruptcy of a bank is acknowledged until the time when recovery rates are applied.

Hoarding models extend the perspective of the interbank market.[5] Liquidity hoarding refers to a situation when single institutions start to hoard liquidity from other banks which are exposed to them. In the best case, they hoard long-term liquidity and thus making the interbank market extremely short-termed. There are mixed views on the reasons for liquidity hoarding (see Acharya and Merrouche (2010) and Acharya and Skeie (2011) for the precautionary motive and Taylor and Williams (2009) for the counterparty risk). Hoarding model of Allen et al. (2009) includes the central bank, which could provide required liquidity to illiquid banks by using open market operations.

Risk propagation models can be further extended so that credit events exacerbate uncertainty, loss of confidence or panics. In addition, positive and negative news may also be transmitted from one place to another when they are not directly connected. Following the positive or negative news about certain entity, the relevant market participants may reassess their priors about entities that are similar to them or come from similar environments and make similar expectations as to those which they have examined. Historically, transmission of the Thai crisis of 1997 from Thailand to Brazil and Russia was largely psychological. In Russia, it induced the collapse of the stock market and then also of the ruble in 1998. Some other cases of international contagion are thoroughly examined in Kindleberger and Aliber (2011).

By the same token, credit contagion may also denote propagation of economic distress from one firm to another or from one country to another or from firms to banks and to countries' budgets and vice versa and so on. Such interdependence

[5] Interbank market is one of the most important factors for the financial system to work smoothly because it transfers liquidity from banks in abundance to banks with a deficit. As such, it is vitally important particularly for smaller banks which are usually short in liquidity and much more depend on the interbank market than big banks.

makes the effects of credit events nonlinear and complex, making the interaction-based approach even more appropriate.

The interbank market resembles the parallels to the epidemic networks; it acts as a transmission channel for spreading credit events from infected banks to their counterparties and hence across the network. However, credit contagion models differ from the epidemic models in transmissibility. As we have said before, the node connectivity may work as a channel for the risk propagation or risk-sharing in a banking world, while not also in epidemic networks, where single infected units do not have the risk-sharing potential and would infect the entire component.

Spread of Ideas and Opinion-Building

The third class of diffusion models relate to the propagation of ideas through a social network and the related opinion-sharing. It presumes that agents' beliefs are affected by the influence of others. Spread of ideas may also be referred to as information contagion. In these models, we implicitly assume that each agent makes a decision regarding some issue. Individual agents usually have some prior beliefs on the issue they decide about and they regularly update their knowledge (Bala and Goyal 1998; Banerjee 1992; Bikhchandani et al. 1992; Blume 1995; Castellano et al. 2009; Ellison and Fudenberg 1995; Steinbacher 2012). Technically, either undirected or directed networks may be applied. However, because correspondents are likely to respond differently to direct communication than to the indirect, the choice for either of the two network types may imply different consequences on how beliefs progress over time.

The usual framework is the following. Agents are represented by nodes and their pairwise connections by links. Agents are split into sub-groups of different priors. They meet (randomly or systematically) with each other and share their beliefs to one another. Depending on the agents' characteristics and that of the others, agents may get persuaded with some probability and adopt the priors of the fellows or remain with the same belief.

Heterogeneity in agents' attributes may not refer only to diversity in beliefs, but also to the magnitude. The priors may either be strong or weak which affects the evolutionary dynamics. Specifically, agents with stronger priors are more likely to convert other agents to their beliefs. Then, the size of the information cascade depends upon the network structure and the proportion of highly persuaded individuals who never change their initial beliefs. Glaeser et al. (1996) offer an interaction-based model to examine crime rates as a function of individuals' attributes and that of the neighborhood. In the model, there are individuals who influence and are influenced by their neighbors and those who influence their neighbors but who cannot themselves be influenced. Each individual faces a choice of whether or not to engage in criminal activity upon the behavior of his closest neighbors and the average behavior of the neighborhood. Although the network-based effects are identified in petty and moderate crimes, they are almost negligible in the most serious crimes. Golub and Jackson

(2012) apply the network-based approach to examine the effects of homophily to the speed of learning when agents apply best-response techniques.[6] They argue that when agents' beliefs or behaviors are developed by averaging what they see among their neighbors then homophily slows down the convergence to a consensus. Aral et al. (2009) examine peer effects in a dynamic network of social interaction and distinguish between the influence-based contagion and homophily-driven diffusion of ideas. A sort of the homophily-driven model was introduced by Mullainathan and Shleifer (2005) and Gentzkow and Shapiro (2011). The model of Mullainathan and Shleifer presumes that individuals prefer the news which is more consistent with their prior beliefs. This infers that individuals segment their audience according to their belief and prefer those that are likely to confirm their own views. Gentzkow and Shapiro claim that individuals with some prior belief may process information they receive very differently depending on the source and their priors. Similar to these is the network-based bounded confidence model introduced by Deffuant et al. (2000), in which agents can influence each other's opinion only if the two opinions are close enough. Agents start with some opinion, while at each time step an agent shares his opinion with a randomly selected neighbor. If the two opinions differ by more than a threshold parameter, both opinions remain unchanged; otherwise each opinion moves in the direction of the other. In the experiment, either both change their opinion or none. In the end, for the given difference in initial opinions, higher thresholds increase the probability that two opposing opinions converge towards an average opinion, while the low thresholds result in several opinion clusters. Yet, the ants' example of Kirman (1993) demonstrates that agents can change their beliefs autonomously with no influence of others.

In the classical model of learning and consensus formation that was proposed by DeGroot (1974), agents put weights on the opinion of others. At each time period, weights are assigned to each individual according to trust and the level of confidence an individual enjoys among other agents, while the opinion of each is then defined as the weighted average of the opinions of others. Holme and Newman (2006) offer an opinion model in which agents form their beliefs by either joining a group of individuals with a similar belief or by influencing each other's opinion which, as a result, is becoming similar. By controlling the balance of the two processes, they identify a phase transition, from a regime in which opinions are diverse to one in which most individuals hold the same opinion.

Acemoglu et al. (2010) examine how the presence of forceful individuals who influence beliefs of the others but are not willing to change their own, interferes with information aggregation. Their main result is that the worst outcomes are obtained when there are several forceful agents and forceful agents themselves update their beliefs only on the basis of information they obtain from individuals most likely to have received their own information previously. Watts and Dodds (2007) examine the "influentials hypothesis" and argue that large cascades of influence are driven not by

[6] Homophily relates to the observation, in which individual agents tend to associate disproportionately with individuals who are similar to them. In other words, node characteristics and the behavior of nodes are correlated with the network structure.

the opinion leaders but by a critical mass of individuals who can be influenced easily. Kreps and Wilson (1982) argue that in the multistage and imperfect information games, agents may try to acquire a reputation in the early stage of the game and use it as the game proceeds. Burnside et al. (2011) present a heterogeneous agent belief model to examine the housing market and explain variation in the housing prices. Agents have different priors about the long-run fundamentals, meet randomly and change their expectations following the interaction with others. The tighter the priors of an agent, the more likely it is that an agent will convert other agents to his beliefs. Sood and Redner (2005) examine the voter model and study its dynamics on heterogeneous graphs. Vazquez et al. (2003) examine a version of a voter model with three states: rightists, leftists and centrists, in which only the latter are involved into interaction and subject to the opinion change.

Hong et al. (2004, 2005) examine the effects of word-of-mouth information on individuals' stock market participation and find that local networks of "friends" affect their decisions. Goldenberg et al. (2001) use cellular automata to demonstrate how the presence of weak and strong ties contributes to the spread of information through the word-of-mouth and the acceptance of a new product. In their model, a purchase of a product by an agent induces the non-zero probability that an adjacent agent decides to purchase it as well, which makes the strength of ties highly effective in the product life-cycle after the introduction stage is over.

Opinion-sharing differs from the usual diffusion models in that it relates to the one's beliefs, whose dynamics depends on many factors, such as prior beliefs, knowledge and expectations, incentives, reputation of an agent who would like to spread his belief, willingness of an agent to change his prior belief. Many times the belief dynamics is contextual, subject to the changing circumstances of time and space, the general mood in society and similar. Often, after some new information arrives, the group of a predominant opinion is enlarged by the group of fast adopters who either have the least tight priors or have the most similar priors. Then the group is enlarged by those who decide upon the size of the group until the late adopters. Some individuals remain outside this box.

The most salient feature of this class of diffusion models is that agents refer to human agents with a strong cognitive component. This is an important distinction from the previous two classes of diffusion models.

Agent-Based Models in Finance

Agent-based framework is also highly applicable to finance. Financial markets are inherently occupied with issues that involve time and uncertainty. What is even more important, the market is characterized by the large number of micro agents who differ in many respects: knowledge, preferences, objectives, attitude towards risk, expectations building, learning capabilities, endowments, patience, friends, and the very subjective and mostly indeterminate factors such as daily mood, eureka, coincidence, the level of luck and similar. Additionally, agents on the markets are repeatedly engaged in local interactions and exhibit non-standard behavior by which they produce the aggregate outcomes that are path dependent with very complex time paths that go beyond the predicted outcomes.

A usual agent-based model of finance consists of a group of, presumably heterogeneous, agents who interact with each other and hence determine the dynamics of asset prices. The price dynamics is a perpetual activity caused by the agents' actions which in turn affects agents' future actions. In some cases, the aggregate behavior of the whole can induce huge oscillations on the markets, including highly unexpected outcomes such as market bubbles and crashes. As demonstrated by Lux (1995), these outcomes are attached to herding of interacting market participants and cause market instability. Kindleberger and Aliber (2011) provide a thorough historical overview of how manias, panics and crashes have shaped financial world over time. They also consider herding a central factor for price fluctuations. Therefore, financial markets are highly appropriate for modeling in an interaction-based fashion. Handbook of Computational Economics edited by Tesfatsion and Judd (2006) provides a thorough review of recent agent-based models in economics and finance.

One of the first agent-based models of financial markets with heterogeneous agents is attributed to Zeeman (1974). The model is populated with fundamentalists and chartists and explains switching phenomena in the proportion of the two types of traders between the bull and bear markets. Fundamentalists and chartists represent two typical groups of traders within the financial modeling. The first base their decisions upon market fundamentals, such as dividend return and economic growth, and the second upon the historical pattern of stock prices. There is no common rule of how to model fundamentals let alone the trend behavior. Zeeman argues that in a bull market the proportion of chartists who follow the trend increases, which pushes the prices even higher. The uptrend continues until fundamentalists perceive the prices too high and start selling, which in turn leads to price drops (bear market) and reduces the proportion of chartists, respectively. The downtrend provokes fundamentalists to start buying the stocks, which again turns the trend around. DeLong et al. (1990) used the finite horizon financial market model to demonstrate that a constant fraction of chartists may on average earn a higher expected return than fundamentalists and may survive in the market with positive probability. In the model of Day and Huang (1990), fundamentalists trade the more aggressively the farther the market price is from the fundamental value. In the models of Lux (1998) and Lux and Marchesi (1999) chartists pursue a combination of imitative and trend following strategies and switch between an optimistic (bullish) and pessimistic (bearish) mood, depending upon the majority opinion and the prevailing price trend. Boswijk et al. (2007) is among the recent heterogeneous agent models with fundamentalists and chartists.

A slight deviation from these models have been proposed by Kim and Markowitz (1989), whose simulated market contains two types of investors, rebalancers and portfolio insurers, and two assets, stocks and cash. This model is one of the earliest models of multi-agent dynamics. Hong and Stein (1999) propose a model, in which market is populated with newswatchers and momentum traders. Newswatchers make forecasts based on private information without conditioning on past prices, whereas momentum traders' forecasts are based on the most recent price change.

Brock and Hommes (1998) develop a discounted value asset pricing model with agents of heterogeneous beliefs. In the model, agents pursue an adaptive behavior and tend to switch towards strategies that have performed better in the past. Upon the parameter values, the resulting system is nonlinear and capable of generating the entire specter of complex behavior from local stability to high order cycles and even chaos.

Levy et al. (1994) present an early econophysics approach in finance. Their simulations exhibit rich phenomena which include cycles, booms, and crashes. Cont and Bouchaud (2000) develop a model of stock market returns by using tools from statistical physics. The model is constructed on interacting agents and it demonstrates how herding that is spurred by communication structure between agents and imitation induces heavy-tails in stock returns. Iori (2002) develops a model with heterogeneous agents, in which agents' interactions are restricted to nearest neighbors to examine large fluctuations in stock market returns and volatility clustering.

Artificial Stock Market model consists of an auctioneer, a risky and riskless asset, and the arbitrary number of traders (Arthur 1994; LeBaron et al. 1999; Palmer et al. 1994). At the beginning of each time period, each trader selects a portfolio to maximize his expected utility in the next period. Agents interact with each other, individually form their expectations of stock prices over time and continually introduce new rules into their decision-making. Agents' actions are a continuous activity. Each agent first monitors the stock price and upon the stock price submits bids and asks by which they jointly determine tomorrow's price. In the model, agents learn and modify their forecasting rules by a genetic algorithm, succeeded later by the method of swarms. Following these rules, they eliminate the worst-performing rules and replace them with new rules that are formed as variants of the retained rules.

Steinbacher (2012) proposes an interaction-based model that is run on a social network to study agents' portfolio decisions. In the model, stock prices are given and unknown to agents. At each point in time, agents interact with adjacent counterparts, share information with them and make regular decisions. Following the idea of Selten (1975), decisions of suspicious agents are subject to selection errors in a very selection phase, be they intentional or accidental. Agents' decisions are not just bound to the imperfect knowledge about asset prices, but also to imperfect selection. In the model, agents' inaction is also considered a decision that was done. The model is simulated under different circumstances, including bull and the bear markets.

Another category of agent-based models in finance represent the order book models. These are models of price formation, in which agents post their buy or sell orders (Rosu 2009 and the references therein). There are two classes of order book models. A limit order is an order to trade a certain amount of a security at a given price. A market order is an order to buy/sell a certain quantity of the asset at the best available price in the limit order book. The lowest offer is called the ask price and the highest bid is called the bid price. When a market order arrives it is matched with the best available price in the limit order book, and a trade occurs.

Table 10.1 The payoff matrix of the game

A/B	C	D
C	a, b	c, d
D	e, f	g, h

Game Theoretic Applications

Game theoretic models are another class of particularly appealing applications for the network-based approach. A game is an abstract formulation of an interactive decision situation with possibly conflicting interests. In a general form, it consists of the set of agents, payoffs for each agent and a set of rules and strategies for each agent (Osborne 2002). Traditional game theoretic application is given as a finite two-person simultaneous-move game in which each agent individually decides whether to cooperate (C) or to defect (D), while agents do not know what the other will do (Table 10.1).

Generally, for the given matrix structure, the game is named upon the values of these parameters in the matrix.

For instance, in a prisoner's dilemma game, defection yields higher payoff than cooperation. However, if both defect, both are worse off than if both had cooperated. On the contrary, in the stag-hunt game the player is better off doing whatever the co-player does (Santos et al. 2006). With the payoff matrix given, the usual game theoretic framework tries to answer a simple question of when should a person cooperate and when defect in an ongoing interaction with another person or a group of persons.

Evolutionary game theory extends these classical games with an evolutionary aspect such as uncertainty, learning, adaptation and a dynamic component. In the evolutionary games, the large populations of agents repeatedly engage in strategic interaction, which allows them to learn over time and change their behavior upon previous experience, communication with others and developments of individual games (Camerer 2011; Maynard Smith 1982; Weibull 1995). In the evolutionary setting, agents who are usually heterogeneous in nature adapt their behavior over the course of repeated plays. Some models are reviewed in Chakraborti et al. (2011), Goyal (2008), Jackson (2008, 2010), Szabo and Fath (2007).

The El Farol bar problem explores the dynamics of attendance (Arthur 1994). Each week agents independently decide whether to go to the popular bar or not, while the bar is enjoyable if it is not too crowded. The game could be denoted a prediction-based model, because agents, who are not allowed to communicate to each other, predict how many entered the bar the previous week. If an agent predicts more than a certain number will attend he stays home, otherwise he goes. Upon the success of their prediction, agents continuously adapt their predicting model and corresponding parameters. The game thus exhibits a non-linear behavior. The evolutionary perspective of the game was provided by Challet and Zhang (1997) and Challet et al. (2004).

Minority games have gained a widespread popularity. Chow and Chau (2003) propose a variation of the minority game where every player has more than two options. Bianconi et al. (2008) propose a version of a minority game in which agents may invest in different assets (or markets) and find that the likelihood that agents trade in a given asset depends on the relative amount of information available in that market, while agents prefer to play in the stock with less information.

A large portion of researchers have examined the evolution of cooperation among agents in evolutionary prisoner's dilemma games under different circumstances (Nowak and May 1992, 1993, see Szabo and Fath 2007 for an extensive overview).

Nowak and May argue that spatial version of the prisoners' dilemma game, with no memory among players and no strategic elaboration, can generate chaotically changing spatial patterns, in which cooperators and defectors both persist indefinitely. In these models, we can assume that the decision for cooperation or defection depends upon the given payoffs; as the reward for defection increases, the probability for cooperation decreases. However, Nowak (2006) has argued that cooperation can evolve by kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. Axelrod (1984, 1997a) has demonstrated that "Tit-for-Tat" is often the optimal strategy for iterated prisoner's dilemma. Abramson and Kuperman (2001) study an evolutionary prisoner's dilemma game, played by agents on different network topologies, in which agents change their strategies over time by imitating that of the most successful neighbor. They find that different network topologies produce a variety of emergent behaviors. Helbing and Yu (2009) argue that success-driven migrations help to establish cooperation and, besides the ability for strategic interactions and learning, play a crucial role for the evolution of large-scale cooperation and social behavior.

By using an *n*-person binary choice game, Axelrod (1986) has studied the emergence of behavioral norms in the game of bounded rational agents. He concludes that norms that have proved to be more effective are used more often in the future than the less effective. In the game, agents can choose either to cooperate or to defect. Young (1993) has examined a repeated *n*-person stochastic game to study the evolution of conventions and demonstrated that in an environment where agents' decisions are subject to mistakes, societies occasionally switch from one convention to another, while the society converges in probability to only one convention if the probability of mistakes approaches zero.

Evolutionary approach is applicable to include different stochastic elements into the usual game frameworks, such as errors in agents' decisions, signaling or screening, imperfect recall, impatience, reputation, learning methods, network topologies and similar. In the evolutionary perspective, agents may learn over time and modify their behavior as to the game developments.

Evolutionary Macroeconomics

Macroeconomic models have traditionally been analyzed by a top-down approach under the rationality condition and solved as optimization problems with constraints (see Ljungqvist and Sargent 2004). Although they considered the economy by a top-down approach, uncertainty and asymmetric information, imperfect competition and rivalry among many heterogeneous economic units, learning by doing and knowledge spillovers, (uneven) initial conditions, increasing returns, diffusion processes and imitation, incentives, strategic interaction, cooperation and collusion, transaction costs, institutional framework and social norms, heterogeneous economic environments and time component have been highlighted as important elements of a production process (see Barro and Sala-i-Martin (2004) for an overview of endogenous growth models). Sargent (1993) and Simon (1997) provide a survey on bounded rationality in macroeconomics. Research in behavioral science suggests that agents differ in their preferences, especially in relation to risk, expectations and time, and that their behavior is often time-inconsistent subject to errors, mistakes and regret. Simon argues that the optimization maxima, i.e. the choice of the best available alternative, that is a building-block of the standard approach is simply not feasible in most real-world situations and has to be substituted with that of satisficing, i.e. the choice of an alternative which meets specified criteria but is not necessarily the best. In their actions, individuals are often led by irrational exuberance or fads (Bikhchandani et al. 1992; Shiller 2005). As demonstrated by Schelling (1971), the outcome of a group of such interacting individuals with cognitive abilities can substantially differ from the outcome that would be aggregated upon their priors. Kirman (1992) provides a discussion against the use of the representative agent in economics.

Roots of the evolutionary approach to economic growth can be traced back at least into the late 18th century and Adam Smith's division of labor and the invisible hand dynamics, which transforms the environment of selfish individuals who interact with each other into an ordered system in time and space that goes beyond initial intentions of every individual. Hence, the market outcome of a decentralized economy is the intercept of individual self-interests of market participants with the price system being an integral part of the market order. In the 20th century, Joseph Schumpeter Schumpeter (1934, 1947) described the economy as a system that is characterized by perpetual creation of new ideas, products and firms and the decline of those existing that have proved to be less efficient. An entrepreneur has been put in the center of Schumpeterian economic development. Processes led by creative destruction and entrepreneurial experimentation make the economy inherently dynamic, stochastic and evolutionary. In the early 1980s, Nelson and Winter (1982) wrote a seminal book on the evolutionary approach to economic growth.

Delli Gatti et al. (2011) offer an agent-based approach to macroeconomics. Delli Gatti et al. (2010) model the economy as a network consisting of households, firms and banks, and simulate the behavior of the modeled economy for different parameter values. They explain cyclical behavior of the economy as a consequence of the complex interaction of the agents' financial conditions, and argue that a shock to the economy or to a significant group of agents in the credit network can be followed by a bankruptcy avalanche if agents' leverage is critically high. Gabaix (2011) and Acemoglu et al. (2012) examine the effects of productivity shocks that hit different sectors on a micro level to macro fluctuations and argue that firm-level idiosyncratic shocks translate into aggregate fluctuations when the empirical distribution of firms exhibits fat tail.

Within economics, social networks have been extensively used in job search models to explain many phenomena that were considered anomalies. A typical job search model consists of job postings and job candidates. Montgomery (1992) was among the first to study labor market as an evolutionary process and highlights the importance of social connections for the salary of employees. Calvo-Armengol and Jackson (2004, 2007) use the Granovetter's notion of weak ties (Granovetter 1973) to develop a model where agents get information about the job vacancies through the social interaction. Ioannides and Datcher Loury (2004) use social interaction to study job-market outcomes. Bramoulle and Saint-Paul (2010) build a model on the assumption that the probability of a new link formation is bigger between two employed individuals than between an employed and an unemployed individual, which generates negative duration dependence on exit rates from unemployment. Goyal and Moraga-Gonzalez (2001) examine the evolution of R&D networks of inter-firm collaboration on costly and human-capital intensive research and development activities.

Other Applications

In one of the earliest simulation-based models, Thomas Schelling applied cellular automata to demonstrate that an integrated society will generally turn into a rather segregated one although no individual agent strictly prefers this (Schelling 1971). This segregation seemed due to the spontaneous dynamics of the economic forces, with all individuals following their incentives to move to the most attractive locations. The model was later generalized by Fagiolo et al. (2007), who conclude that mild proximity preferences are an important possible explanation of segregation not only in regular spatial networks, but also in more general social networks.

Nagel and Schreckenberg (1992) use cellular automata to simulate freeway traffic and the related traffic congestion patterns. Epstein and Axtell's sugarscape model is an interaction-based model that is run on a lattice (Epstein and Axtell 1996). Each cell is filled with different amount of sugar. Sugar is the commodity that agents need to survive, while those who ran out of it die off. In addition, agents who reach the maximum pre-defined age die off as well. Agents move sequentially in random order from cell to cell by which they consume the sugar. Agents have different metabolic rates. Each cell can be occupied by at most one agent at a time. When an agent occupies a cell, he increases his sugar supplies by the amount of sugar from the cell. Sugar then grows on an empty cell at the given rate. Agents also have different lateral vision, which helps them to decide which cell to occupy. Agents move to the best available location. Interactions in the model are endogenous because they depend upon the moves of agents throughout the lattice. There is no learning in the game. The extended version of the game includes spice, which agents can trade with their neighboring agents. Agents can only interact and trade with their direct neighbors. How much sugar and spice agents trade with each other depends upon the utility functions of the two agents and the pre-defined bargaining rule. Additional extensions of the game include different replacement rules of the deceased agents, sex and the birth of offspring, credit relations between agents etc.

By using an interaction-based model, Föllmer (1974) was among the first to demonstrate that even simple interactions among individuals can generate sophisticated behavior at the macro level, including a breakdown of price equilibria. Currarini et al. (2009) develop a model of friendship formation in which individuals have different types and see type-dependent benefits from friendships. Bramoulle and Kranton (2007) analyzed networks in relation to public goods.

Axelrod (1997b) uses social networks to study cultural dynamics. Corominas-Bosch (2004) uses a bipartite network to study a repeated bargaining game between buyers and sellers who are connected by an exogenously given network. In the game, players can make repeated alternating public offers that may be accepted by any of the responders linked to each specific proposer. Chang and Harrington (2006) provide a survey of various agent-based models of organizations. Bramoulle et al. (2009) consider a model where interactions are structured through a social network to identify the peer effects. Brock and Durlauf (2001b) develop an externality model to examine aggregate outcomes when social interactions are embedded in individual decisions of the agents.

Applications Outside Economics

Social networks and interaction-based models have been extensively used to explain many phenomena from natural and social areas. Listed are just some of them.

Kirman (1993) uses it to examine behavior of ant colonies in exploiting two identical sources of food and characterizes a switching potential that is defined by the self-conversion probability and a probability of being converted. Pastor-Satorras and Vespignani (2001) use the network approach to study the spread of diseases, while Bullmore and Sporns (2009) to study complexity of brain's structural and functional systems. Barabasi and Oltvai (2004) use networks to study the cell's functional organization. Helbing (2001) uses it to examine the traffic dynamics and demonstrates that the behavior of panicking pedestrians in a smoky room leads to an inefficient use of available escape routes. The paper of Helbing also delivers an extensive review of the main approaches to traffic and related models. Leskovec et al. (2005) examine the dynamics of viral marketing. They observe the propagation of recommendations and the cascade sizes and analyze how user behavior varies within user communities defined by a recommendation network. Epstein (2001) presents two variants of an agent-based computational model of civil violence in which agents, who differ by their private level of grievance, and cops interact on a lattice. In the first a central authority seeks to suppress decentralized rebellion. In the second a central authority seeks to suppress communal violence between two warring ethnic groups. Nowak et al. (1999) extend the basic framework of the evolutionary game theory to examine the evolution of language. Christakis and Fowler (2007) use social networks to examine the spread of obesity over time and link it to social ties.

A special class of models represents those that study the evolution of social networks (see Boccaletti et al. 2006; Goyal 2008 and Jackson 2010 for a survey). Callaway et al. (2000) and Albert et al. (2000) examine the network fragility under different types of attacks on the networks and argue that only intentional attacks focused on the elimination of some of the most important nodes or links within the network can destroy the network. Marriage networks have been used to explain the rise of the Medici family in medieval Florence (Padgett and Ansell 1993).

Simulation-Based Experiments

In this section, we present some applications of agent-based games on social networks. The principal aim of this chapter is to demonstrate how these games can be conducted, while we also demonstrate how even small perturbations of different parameters might end in highly different outcomes. The first application is an example of the evolutionary game theory and examines the modified principal-agent inspection game. In the second, we propose a network-based model of credit contagion in financial markets and examine the effects of idiosyncratic and macroeconomic credit events to the banking system for various network topologies.

Game Theoretic application: Principal-Agent Inspection Game

Model

We extend the principal-agent inspection game of Dresher (1962) by introducing social interaction among agents. In the principal-agent game, the principal assigns a task to the agent for which the latter, if successfully accomplished, receives a payment. Because the two participants have opposite interests, the arrangement between them results in the principal-agent problem (Grossman and Hart 1983). In particular, while the employer wants his task accomplished, the employee tries to receive his payment with as little effort as possible. The dilemma is tackled by a costly inspection going at the expense of the employer and intended to reveal the true effort of the employee. If the employee is caught shirking he does not get paid. We extend this basic framework by adding a credible and powerful institution into the game, i.e. labor union, which warrants the shirking workers who are members of this institution

Table 10.2 The payoff matrix of the game

A_i/P	I	N
S	$0/-h$	$w/-w$
W	$w-g/v-w-h$	$w-g/v-w$
SU	$cw-f/-cw-h$	$w-f/-w$
WU	$w-g-f/v-w-h$	$w-g-f/v-w$

a partial pecuniary compensation. This institution does not have to be a labor union it can be any credible and powerful institution.

The game consists of a principal P who employs a finite set of employees (agents) A_i, $i = \{1, 2, \ldots, 1{,}000\}$, who are located on vertices of a small world network (Watts and Strogatz 1998). An average connectivity of the network is $k_i(g) = 6$ and randomness is $p = 0.1$. In every time period each agent simultaneously chooses between two discrete choices, either to work W or to shirk S. When working, each agent produces the output v for the principal, gets the payment w and bears some work-related costs g. To make it simpler, we assume that agents are homogeneous in this respect. A fraction u of agents is unionized, while the rest $1-u$ are not. Unionized agents are randomly placed among the others and principal does not know who they are. A unionized agent pays membership fee in the amount of f and gets a c part of the wage if found shirking.

On the other hand, the principal may opt to inspect $\{I\}$ the agents or not $\{N\}$. It is assumed that P cannot condition the wage on the observable outcome v. If P decides to inspect, this brings him additional cost h. In every time period, each agent is inspected with the given probability $r \in [0, 1]$, which agents do not know. Every agent, who is not found for shirking, gets payment w. A unionized agent who is found shirking gets a portion of the wage.

The time is discrete. During a single iteration of the game, each agent A_i plays the game with the principal P, where both choose their strategies simultaneously at the beginning of every time period, which means that they do not know what the opponent has selected. An agent A_i may have four different strategies available: shirking (S) and working (W), as well as shirking while being a union member (SU) and working while being a union member (WU). Principal chooses whether to inspect or not. Payoffs for each of them are given in the matrix from Table 10.2.

After each full iteration of the game, when P interacts with all A_i, agents compare their accumulated payoffs with a randomly chosen adjacent agent.

In every iteration, each agent A_i randomly selects one of the adjacent agents A_j and reports him the level of his wealth and the strategy he played. The two agents then compare the two strategies they have played and the accumulated wealth, e_i and e_j, and independently choose the strategy for the next period. Hence:

$$e_i(t) = s\sum_{h=0}^{t-1} q(h) + q(t) \tag{10.1}$$

where s is the workers savings rate, and $q(h)$ and $[q(t)]$ are the payoffs of A_i at iteration h and t, respectively. Agents' choice function is determined as:

$$\wp = \frac{1}{1 + \exp\left[\left(e_i - e_j\right)/\kappa\right]} \tag{10.2}$$

Parameter $\kappa \in (0, 1)$ represents the uncertainty parameter and denotes a nonnegative probability that an agent A_i will depart from adopting the most promising alternative of the two being compared. If $ran > \wp$, an agent keeps his alternative, otherwise an agent adopts the alternative of adjacent agent. Parameter $ran \sim U\ (0, 1)$ is a uniformly distributed IID random number (Press et al. 2007). In the model, the choice depends upon the expected benefit differential $\left(e_i - e_j\right)$ and the suspiciousness parameter κ. The scheme relates to the preferential attachment model where agents have a preference to "attach" to the most profitable alternative, but may fail to do it for different reasons. In general, the lower the κ the higher is probability that an agent adopts the most promising alternative, and vice versa. Agents also decide whether or not to get unionized. Principal's profit π depends upon the value produced by the workers and the expenditures for wages and inspection. In games, we examine the profit rate for a principal under different circumstances and the optimal inspection rate for a principal.

Results

All inspection games are iterated forward in time, using a synchronous update scheme. If not stated differently, we use the following values for corresponding coefficients. The output level of each agent v equals to 1 or zero if an agent shirks, while other figures are set relative to the level of v. Each agents earns $w = 0.4$ and bears work-related costs that are set at $g = 0.125$. Agents save 10 % of the wage, thus $s = 0.04$ and the union membership fee equals 5 % of the wage, thus $f = 0.02$. Inspection costs the principal $h = 0.16$. Unionization rate, where applicable, equals $u = 0.4$, and $\kappa = 0.1$. Parameters r and c may vary within [0, 1] with a 0.02 step. The outcomes presented in figures are average values after 10^5 iterations of 20 independent runs of the game.

Figure 10.3 shows how π varies in dependence on r and c. Color-palette on the heat-map visualization presents the profit of a principal. It is clear that what matters for the principal's payoff is the influence of the union that is directly correlated with its bargaining power c. In particular, as the authority of the union increases ($c \to 1$), the maximal average income of the firm per iteration (π) decreases steadily. By $c = 1$ the maximal π is obtained by $r = 1$, whereby then $\pi = 250$, which is slightly more than 50 % lower as the peak value of π in a no-union case at $c = 0$. The union without bargaining power cannot affect the performance of the firm but just lowers the net income of their members by f.

Results in Fig. 10.4 relate to the endogenous unionization rate, in which agents are allowed to adopt the status of an adjacent agent as well, not only the corresponding

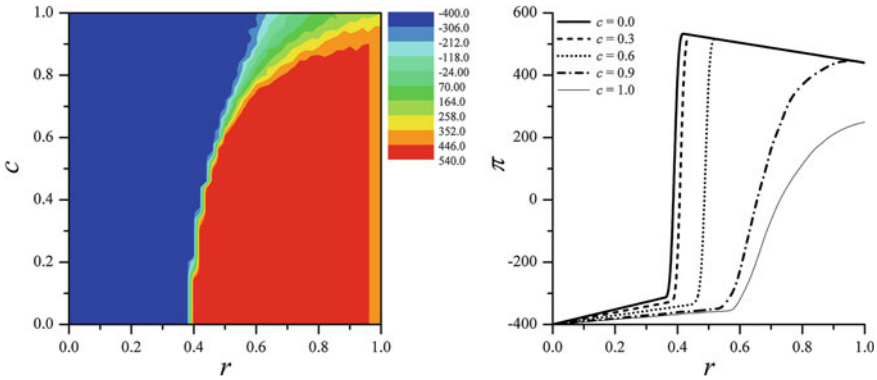

Fig. 10.3 Performance of the principal under exogenous unionization

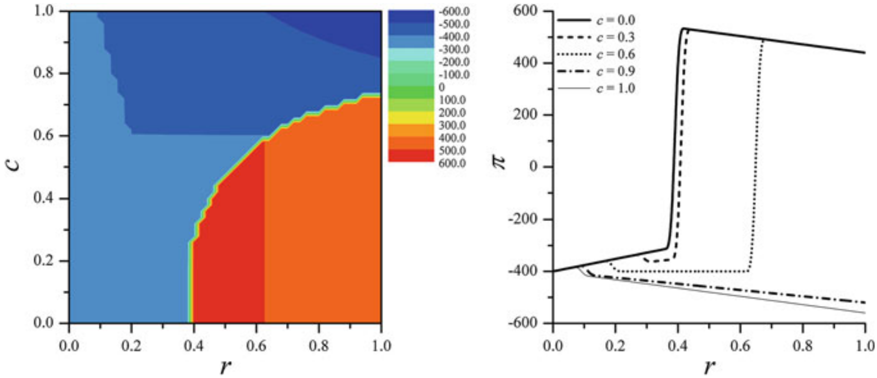

Fig. 10.4 Performance of the principal under endogenous unionization

strategy. The worst-case scenario, a loss of $\pi = -560$, is obtained at $c = r = 1$, when everyone is inspected with probability 1, no one works, recall that f is strictly less than w, while the principal is obliged to pay out full wages.

Inspection is a required condition for the principal to push agents to work and also a sufficient one in the no-union environment. If agents cannot be backed by the union, a principal does not need to inspect every agent in order to force them to work. On the other hand, inspection is neither required nor a sufficient in the environment of a powerful union and endogenous unionization. Albeit in a bit lesser extend, the union has an indirect effect even on non-members, in particular imposing a tendency to shirk when this is definitely not optimal neither desirable.

Epidemic Games: Credit Contagion

In this experiment, we examine the propagation of credit events throughout the system of interconnected financial institutions, which we call banks. Assume that financial system consists of a number of n banks, which are connected through the interbank market into a banking network. By definition, such network is weighted and directed, with the weighted links indicating the exposure of bank j to bank i, representing a counterparty risk for bank i with strength given by the weight. Exposures can also be both-sided.

Each bank that is exposed to other banks is vulnerable to losses of counterparty banks. While banks with many outgoing links make their financial positions very sensitive to the operations of other banks, banks with high in-degrees may provoke contagion if defaulted. The extent of out-degrees entails two opposing effects; it may work as a channel for the shock propagation or as a channel of risk-sharing. In addition to direct links, banks may be connected to each other through several different paths which all determine their status due to the credit event of a distant bank.

Model

The banking network consists of $n = 40$ banks which are numbered from 1 to 40. Banks retain the same number in all settings. Each bank is defined through its balance sheet. The sample includes 13 big banks with total assets exceeding 900 bn USD each. Total assets of 17 banks range from 100 to 700 bn USD each, while total assets of ten small banks do not reach 100 bn USD per bank. A cumulative initial value of banks' assets is 25951.16 bn USD. The banks represent real banks from different geographical regions. They were chosen arbitrarily.

We use the banks' 2011 Annual Reports to get the data on the banks' total assets and Tier 1 capital ratios as of December 31, 2011, from which we calculated each bank's initial Tier 1 capital level. Figure 10.5 plots banks' initial Tier 1 ratios to their total assets. The figure demonstrates that the smallest banks have both the lowest and the highest capital ratios, with the medium and the largest banks being in-between. Concentration towards the origin signifies that the sample consists of mostly undercapitalized banks. Some descriptive statistics of initial banks' positions are further provided in Table 10.3.

Time is discrete and defined over $t = \{1, 2, \ldots, 252\}$, which should resemble one business year. Financial position of each bank is defined and reflected in its balance sheet. The value of assets of bank i in time t is then given as:

$$A_{i,t} = H_{i,t} + B_{i,t} + N_{i,t} + \sum_{ij=1} IB^{j}_{i,t} \tag{10.3}$$

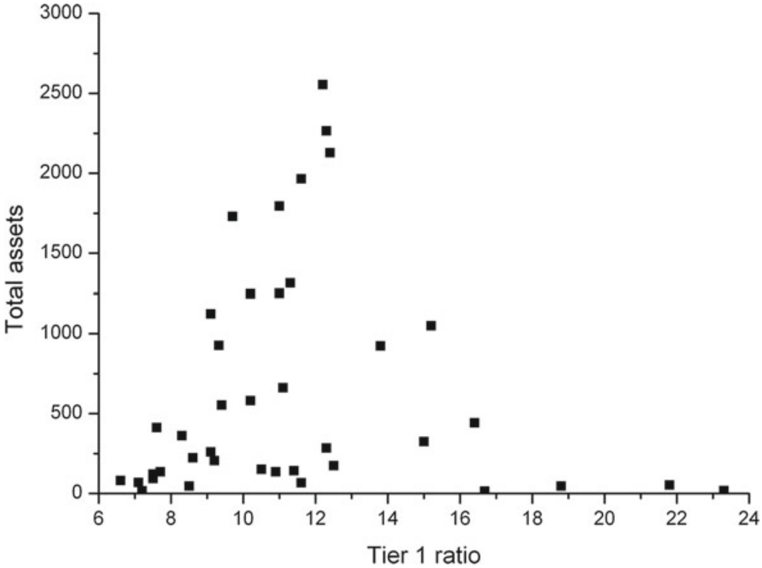

Fig. 10.5 Banks' total assets versus Tier 1 ratios

Table 10.3 Descriptive statistics of banks' initial positions

	Assets	Capital	Tier 1 ratio
Mean	648.78	73.66	11.40
Median	305.35	30.70	10.95
Maximum	2555.00	311.70	23.30
Minimum	13.80	1.20	6.60
Standard deviation	724.06	86.08	3.82
Skewness	1.17	1.30	1.39
Kurtosis	3.19	3.65	4.81
Number of banks	40	40	40

$A_{i,t}$, $H_{i,t}$, $B_{i,t}$ and $N_{i,t}$ denote the values of bank i total assets, mortgage loans, bonds and non-trading assets in time t, while $IB^{j}_{i,t}$ denotes the values of bank i assets of banks j in time t. In the equation, $ij = 1$ designates the link from bank j to bank i. Let us assume that the liabilities' side of each bank is confined to the level of its capital $C_{i,t}$. Capital of each bank promptly evolves according to the profit or loss $\Pi_{i,t}$ a bank generates on its trading part of assets as $C_{i,t+1} = C_{i,t} + \Pi_{i,t}$. The banks' assets develop over time according to the dynamics in the value of its equity portfolio and through dynamics of the interbank market.

Banks are not allowed to rebalance their balance sheets over time nor raise additional capital. Banks default when their Tier 1 capital ratio falls below 4 %. Bank capital thus represents its capacity for absorbing losses. Default of a bank deteriorates the balance sheet of its counterparties for the $(1 - RR)$ proportion of the exposure at default, where $0 \leq RR \leq 1$ designates the recovery rates assigned to each bank. For each bank RR is randomly taken from the uniform distribution on an interval 0.3 to 0.6 and is fixed for all repetitions and network topologies. Hence, the capital dynamics for each bank over time is thus given as:

$$C_{i,t+1} = C_{i,t} + \Pi_{i,t} - \sum_{ij=1 \mid C_j \leq 0} \left[\left(1 - RR_j\right) \cdot IB_{i,t}^{j} \right] \tag{10.4}$$

We test the model against idiosyncratic and systemic shocks. An idiosyncratic shock is represented as a sudden default of an individual bank. Generally, individual banks may default due to the failed business decisions, malpractice, fraud or any other bank specific event. A systemic shock is represented by a sudden drop in the value of mortgage loans. The two shocks induce different outcomes. Contagion in the presence of a systemic shock is different from that of the idiosyncratic in that the shock itself reduces the capital of each bank, by which each bank is more vulnerable to additional writedowns due to the counterparty risk later on. As the first banks default, the shock may be succeeded by a sequence of idiosyncratic events. All shocks are applied in t=10. They are unexpected events to banks, against which they cannot protect. We assume that the shock affects no other parameters.

Results

We first consider the consequences in the banking network after a sudden default of a bank which was given number 1. Bank 1 is a big bank with 2,129 bn USD in assets and initial Tier 1 capital ratio of 12.40. Figure 10.6 plots time evolution of the net defaulted assets within the banking system in 20 random network topologies. Network topologies determine the structure of the interbank market. We get net defaulted assets per scenario if we substract the benchmark evolution of the game in which the system is subject to no shock from the corresponding shock evolution framework. The figures thus represent pure differences in defaulted assets within the system that are only due to different network topologies.

Although the same shock magnitude has been applied in all network topologies, the plots clearly exhibit the differences in dynamics of banks' defaulted assets, which is a consequence of different credit contagion paths. This means that the effects of a bank default to the banking network depend upon the network topologies. In one case, interbank market works as a shock absorber, while it gets contagious in some other constellations.

We now examine the effects of a systemic shock. It is represented as a one-time drop in the value of housing for a specified percentage. Simulations start with a shock of a percentage point, while through the repetitions housing default rates progress with an increment of 1 % up to the 40 %. In addition to the direct effects of the shock, it can also become contagious if it induces bank defaults.

Again, we use 20 random network topologies. Heat-map visualizations in Fig. 10.7 present the amount of net defaulted assets within the banking system over time

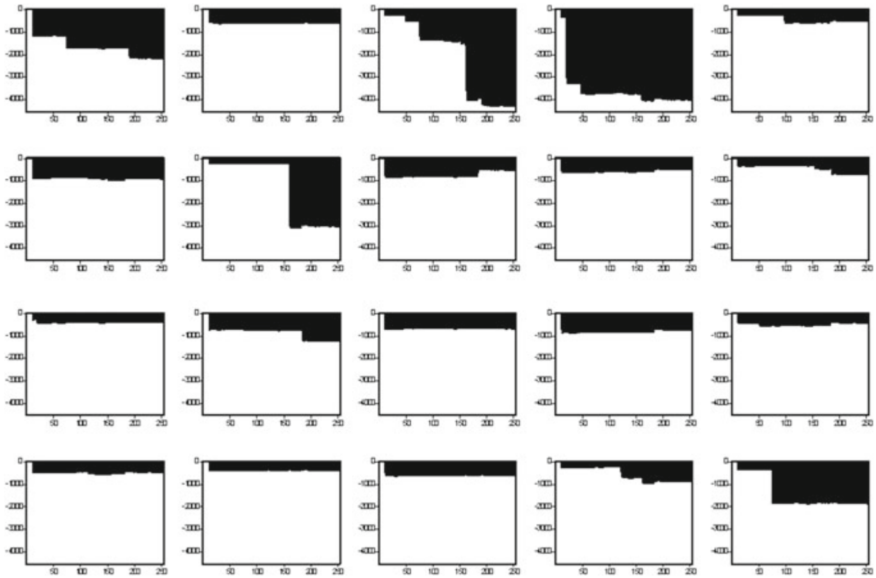

Fig. 10.6 Time evolutions of net defaulted assets after a default of bank number 1 in 20 random network topologies

(X-axis) by different levels of housing default (Y-axis). Color-palettes (Z-axis) progress from red (low value) to black (the highest value).[7]

Contagion in the presence of a systemic shock is different from that of the idiosyncratic in that the shock itself reduces the capital of each bank, by which each bank is more vulnerable to additional writedowns that arise either due to the counterparty risk or due to losses on the equity portfolio in the subsequent periods.

Discussion

In the preceding chapters, we have presented some arguments, theoretical, methodological and empirical, in favor of the agent-based approach in economics and finance. Obviously, the approach is very ambitious and gives us some novel techniques and methods to model and examine the old questions from a new perspective. Its multidisciplinary nature makes it highly applicable for exploring complex models that exhibit nonlinear dynamics.

[7] Red color designates the net value of defaulted assets lower than 3,125 bn USD, orange the net value of defaulted assets in range from 3,125 to 6,250 bn; yellow in range from 6,250 to 9,375 bn; darker green in range from 9,375 to 12,500 bn; lighter green in range from 12,500 to 15,630 bn; lightest blue in range from 15,630 to 18,750 bn; middle blue in range from 18,750 to 21,880 bn; darkest blue in range from 21,880 to 25,000 bn and black designate the net value of defaulted assets that exceed 25,000 bn USD.

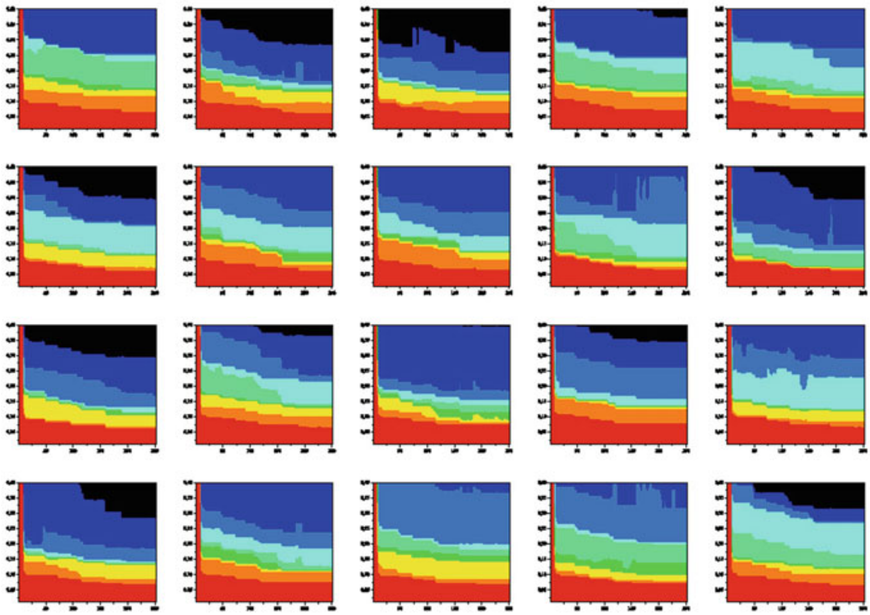

Fig. 10.7 Time evolutions of net defaulted assets after a default in housing for a specified percentage (Y-axis) in 20 random network topologies

A distracted individual in the sense of his multiple imperfections is put into the center of the agent-based approach. Although we refer to agents, an agent may not only represent a human agent but any entity, which possesses some data and is endowed with some behavioral rules.

A fundamental presumption of the agent-based approach pertains to decentralized markets which are populated with heterogeneous agents with cognitive abilities who interact with each other and also with the environment by which they regularly change the environment in which they live and adapt to these changes which they as a group create. Heterogeneous agents may respond differently to these developments, which may end in aggregate outcomes of very rich structure. This may induce highly extreme aggregate outcomes, such as market bubbles that are followed by crashes, the tragedy of the commons as argued by Hardin (1968), or segregation in urban communities as argued by Schelling (1971, 1978). Some of these are highly undesired and, very likely, contrary to personal interests of most of egoistic individuals. There is a "divergence between what people are individually motivated to do and what they might accomplish together" (Schelling 1971). Markets are thus considered as complex and adaptive systems in an uncertain environment and regularly exhibit nonlinearities.

Interaction-based techniques are more capable of explaining these outcomes than the equilibrium-based models which presume a representative agent. The latter models almost completely disregard the complex nature of economics which arises due to the microstructure, uncertainty, non-optimization, emergent behavior, etc. Although they have provided us many helpful insights and reduced the sensitivity of many models to the parameter estimates, equilibrium-based models have been subject to a severe critique. A huge dissatisfaction with inability of equilibrium-based models to explain some empirical facts could be reflected in the words of LeRoy and Werner (2001), who have called them the placid equilibrium-based models that “bear little resemblance to the turbulent markets one reads about in the Wall Street Journal” and have called for the improvements.

Interaction-based methods provide methodological improvements and include a great part of the micro-structure that was missing in previous models. Interaction-based approach offers a multidisciplinary tool for exploring many complex systems that are build on interacting units from different fields. They are especially useful for examining systems that consist of heterogeneous agents who exhibit non-standard behavior, or the systems that are characterized by evolution and path dependency. By using simulation-based experiments, we are able to observe and examine how autonomous agents behave over time, how egoistic agents cooperate with each other, and how they respond to different circumstances which their behavior creates. Significant features of interacting agents who are able to observe and imitate are herding and information cascades, which may induce large, unexpected and often also undesired aggregate outcomes.

Interaction-based models may include all the specifics of other computational models in which agents’ information sets include histories of observable and some hidden states. The related uncertainty is ingrained in the structure of agent’s selection. When agents make decisions under uncertainty, which is usually the case, they may rely on the probability which they assign to each alternative and then act according to the expected payoff. However, the behavioral theory firmly suggests that choices among risky alternatives exhibit the pattern which is inconsistent with the mere probability analysis. This is even more relevant when one adds the evolutionary perspective where certain events either occur or not. For, it has been demonstrated that events that occur induce much larger consequences from events that might have happened but have not.

From the methodological perspective, interaction-based approach applies a positive approach, addressing a question of which actions and strategies agents use not the one of which they should. Agents’ decisions are not considered as right or wrong, but as decisions that bring them lower or higher payoffs. Tversky and Kahneman (1986) argue that normative approaches are doomed to failure, because people routinely make choices that are impossible to justify on normative grounds, in that they violate dominance or invariance. The behavior of cognitive agents is nonlinear and can be characterized by thresholds, if-then rules, nonlinear coupling, memory, path-dependence, and hysteresis, non-markovian behavior, temporal correlations, including learning and adaptation. These assumptions are even more relevant given the very subjective nature of information, which is never (or extremely rarely) objective, and never available to everyone but is rather highly dispersed and dynamic. As argued by Hayek (1937), “it is important to remember that the so-called “data”, from which we set out in this sort of analysis, are (apart from his tastes) all facts given to the person in question, the things as they are known to (or believed by) him to exist, and not in any sense objective facts."

Methodological individualism and subjectivism together with interaction between heterogeneous economic agents go beyond the equilibrium which is so common to the economics society. The robustness of simulation-based modeling allows us to test, evaluate and challenge economic theories and models against different assumptions and data, which can be either real or imaginary. Models and theories always simplify. Usually, the assumptions on which they are built are very restrictive. The agent-based approach also simplifies. However, it allows us to examine robustness of these assumptions and the simplification factors as they may be relaxed, modified and challenged. Once the model is constructed, a modeler can very easily perturb (or stress) different parameters and then monitor and analyze the effects which may be remarkable or insignificant.

By using the interaction-based approach, we are not stuck in the equilibrium, but do not rule it out it in the long run, neither. If the equilibrium exists, we are able to see the adjustment process and examine the speed of convergence. The approach allows us to find the conditions under which these theories and theorems are supported and provide some arguments about the anomalies. In order to obtain reliable statistical estimations, equilibrium-based models regularly exclude extremes in spite of all the effects they produce and the content that they include. In this respect, we are able to identify critical points within the systems, whose elimination might well ruin the system as such (see Albert et al. 2000), or explain rare outcomes that occur under very specific circumstances which an econometrician, for instance, would simply regard as an outlier.

Similar to the interaction-based approach are laboratory experiments. Gode and Sunder (1993) and LeBaron et al. (1999) argued that the first are capable of isolating and monitoring the effects of individuals' various preferences, such as risk aversion, learning abilities, trust, habits, and similar factors, while this is nearly impossible in laboratory experiments. Even though the experimenter controls the procedure in laboratory experiments, those who take part in it are aware of the fictitious nature of the circumstances and are likely to adapt their responses accordingly. Such experiments do not necessarily reflect what individuals would do under the same circumstances in reality.

Although, methodologically, the interaction-based models reflect the real world more accurately than the equilibrium-based models, their efficiency is far from the absolute, be they approximations or caricatures. Sometimes we would like to bring the model as closer to reality as possible, the other time we would like to apply the "as if" assumption and examine the outcomes in a fictitious ideal world. In either case, by applying the interaction-based methods, complexity of the model behavior over time is usually not induced by a complex model, but by interaction of bounded rational agents who regularly make decisions upon the very simple behavioral rules. Of course, this does not prevent us from modeling agents with highly complex selection criteria.

Conclusions

The purpose of the chapter has been to present how the interaction-based methods can be used in economics and finance. Interaction-based approach encompasses micro behavior. It is rooted in methodological individualism and subjectivism which makes it applicable to various areas that involve agents and interaction. One key departure of interaction-based modeling from more standard approaches is that events are driven solely by agent interactions once initial conditions have been defined and the rules of conduct specified. Then, interacting economic agents are able to continually adjust their actions according to the changing environment which their actions produce. New opportunities that emerge over time impede the system to reach global optimum or general equilibrium, although the two are not ruled out a priori.

There is no doubt that the games on social networks or the activities on networks will be an important part of the future research in economics and finance as they represent a potentially highly useful instrument for conducting different kinds of agent-based experiments that are based on interaction. If the purpose of the model is to help us explain the questions which we come across or find them just intellectually challenging, and we think that this is the case, then interaction-based techniques represent an adequate and highly competitive tool for obtaining some of the answers. To represent at least a complementary method to the currently mainstream techniques if not supplementary. This comes true amid the fact that social networks are very robust and may easily include ideas from many different areas.

However, the future of economics and finance will to a great extent depend on how successful researchers will be in grounding the two fields on a psychological evidence about how people consider uncertainty and how they behave under different circumstances when they are faced with uncertainty. This is one of the major challenges in economic modeling. Simon (1997) has argued that the future challenge for economists relates to the question of how to "receive new kinds of research training, much of it borrowed from cognitive psychology and organization theory," and that they "must learn how to obtain data about beliefs, attitudes, and expectations."

With new methods that build on interaction among heterogeneous units, we are able to find better explanations for many problems that were before either considered intractable or were computationally too intensive or poorly calibrated.

On this trail for better models, the good news is that hardware and software solutions develop very fast, and that newly developed simulation techniques could allow for this data translation. The bad news is that no matter how good all these improvements are and will be in the future, given the capacity of people to communicate, think and adapt, human action will always be a couple steps ahead of the conceivable capabilities of researchers and financial economists to model and understand it. However, a good researcher will try to do his best.

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Chapter 11
Why Should Economics Give Chaos Theory Another Chance?

Victor A. Beker

Abstract Economic data provide little evidence -if any- of linear, simple dynamics, and of lasting convergence to stationary states or regular cyclical behavior. In spite of this the linear approach absolutely dominates mainstream economics. The problem is that mainstream economics is now in deep crisis. The recent financial crisis clearly showed that orthodox economics was quite unprepared to deal with it. Most mainstream economists not only did not foresee the depth of the current crisis, they not even consider it possible. It is well known since the famous contribution of Mandelbrot (1963) that many economic and financial time series have fat tails, i.e. that the probability of extreme events is higher than if the data-generating process were normal. However, the usual practice among orthodox economists has been to assume-implicitly or explicitly- a normal distribution. Orthodox economists represent the economy as a stable equilibrium system resembling the planetary one. The concept of equilibrium plays a key role in traditional economics. This approach is useful in normal, stable times. However, it is incapable of dealing with unstable, turbulent, chaotic times. The crisis has clearly showed this. Heterodox contributions shed much more light on what happens during these crucial periods in which a good part of the economy is reshaped; they provide powerful insights towards what policies to follow in those extraordinary circumstances. However, they remain as theories mainly suitable for those periods of instability and crisis. The challenge is to arrive at a unified theory valid both for normal and abnormal times. In this respect, the complexity approach with its use of non-linear models offers the advantage that the same model allows to describe stable as well as unstable and even chaotic behaviors. Although the results of chaos tests do not prove so far the existence of chaos in all economic variables they are consistent with its existence. The detection of chaos in economic time series faces three types of difficulties: (1) the limited number of

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observations such series contain; (2) the high noise level in economic time series; and (3) the high dimension of economic systems. However, topological methods for chaos detection seem to be a highly promising tool. On the other hand, in economics, there are no such things as crucial experiments. Economists seldom practice the falsificationism they preach. Confidence in the implications of economics derives from confidence in its axioms rather than from testing their implications. Therefore, non-linear dynamics and chaos theory should not be subject to more stringent rules than what is usual for the rest of economic theory.

Keywords Chaos · Determinism · Economic methodology · Nonlinearity · Predictability

Introduction

Economic data provide little—if any—evidence of linear, simple dynamics, and of lasting convergence to stationary states or regular cyclical behaviour. Irregular frequencies and amplitudes of economic fluctuations are persistent and do not show clear convergence or steady oscillations. In spite of this, the linear approach absolutely dominates mainstream economics.

It is well known since the famous contribution of Mandelbrot (1963) that many economic and financial time series have fat tails; that is, that the probability of extreme events is higher than if the data-generating process were normal. However, the usual practice among orthodox economists has been to assume—implicitly or explicitly—a normal distribution.

One of the few areas of economic analysis where non-linear models were used for some time is the one devoted to the study of economic fluctuations. The business cycle models by Hicks, Kaldor and Goodwin were pioneers in this regard. However, in the 60s there was a shift towards the use of linear models following the Slutzky-Frisch-Tinbergen approach of generating cyclic processes using stochastic shocks attached to low order linear difference equations. The fact that economic fluctuations appear as a sole product of exogenous shocks was in line with the dominant equilibrium approach in economic thought. In the absence of such shocks, the system would tend to a steady state, as different versions of the neoclassical model of optimal growth predicted.

Many assumptions in conventional economics—perfect rationality, identical representative agents, convexity—have been chosen over time not for their reality but to ensure an equilibrium, and an analytical solution. Schumpeter (1987, p. 935) clearly expresses this point of view:

"From the standpoint of any exact science the existence of a uniquely determined equilibrium is, of course, of the utmost importance, even if proof has to be purchased at the price of very restrictive assumptions; without any possibility of proving the existence of uniquely determined equilibria—or at all events, of a small number of possible equilibria—at however high a level of abstraction, a field of phenomena is really a chaos that is not under analytical control."

Orthodox economists represent the economy as a stable equilibrium system resembling the planetary one. The concept of equilibrium plays a key role in traditional economics. This approach is useful in normal, stable times, when what happened yesterday is the best guide to what will happen tomorrow. However, it is incapable of dealing with unstable, turbulent, chaotic times as the crisis has clearly showed.

Mainstream economics is now in deep crisis. The recent financial crisis clearly showed that orthodox economics was quite unprepared to deal with it. Most mainstream economists not only did not foresee the depth of the current crisis, they did not even consider it possible. For them, any departure from equilibrium should be just a temporary one: they not only assume that the economy tends towards an equilibrium but also that it is a stable one.

Heterodox contributions shed much more light on what happens during these crucial periods in which a good part of the economy is reshaped; they provide powerful insights towards what policies to follow in those extraordinary circumstances. However, as theories, they remain mainly suitable for those periods of instability and crisis.

The challenge is to arrive at a unified theory valid both for normal and abnormal times. In this respect, the complexity approach with its use of non-linear models offers the advantage that the same model allows us to describe stable as well as unstable and even chaotic behaviours.

However, the search for chaos in economics has been gradually vanishing, as very little empirical support for the presence of chaotic behaviour in economics has been found. In what follows we argue why economics should take advantage of the contribution that non-linear and chaotic models may provide to the development of the discipline.

There have been many definitions of chaos. Martelli et al. (1998) present many of them. Anyway, chaos has two most important attributes which are both necessary for the existence of a chaotic system; one, sensitive dependence on initial conditions, and two, complicated patterns of non-linear relationships (Bayar 2005, p. 2).The first condition means that a small change—no matter how small it is—at one place in a deterministic nonlinear system can result in large differences at a later state. The second one is necessary because without complex non-linearity, the system will usually be predictable and will not be oversensitive on initial conditions (Bayar 2005, p. 2).

What is a complex non-linear dynamical system? There are lots of definitions of "complexity" which led Horgan (1995) to complain that "we have gone from complexity to perplexity' as quoted by Rosser (1999, p. 2).

Barkley Rosser offers a "broad tent" definition, following Day (1994): a dynamical system is complex if it endogenously does not tend asymptotically to a fixed point, a limit cycle, or an explosion.

Sensitive dependence on initial conditions implies unpredictability. Beyond a certain temporal horizon predictions lose any reliability. Non-linearity magnifies any initial discrepancy no matter how insignificant differences in the starting value are.

The purpose of this chapter is to provide arguments from the methodological point of view in favour of a wider use in economics of the non-linear approach and chaos theory, emphasising that they should not be subject to more stringent rules than what is usual for the rest of economic theory.

The chapter is organised as follows. Section "Non-linearity, Increasing Returns and Path Dependence" analyses the implications of non-linearity in equilibrium analysis. Section "Non-linearity, Attractors and the Paradox of Chaos" focuses on the different type of attractors found in non-linear dynamic systems. Section "Determinism and Predictability" addresses the relationship between determinism and predictability in light of chaos theory. Section "Non-linearity and Chaos in Financial Markets" reviews some empirical studies which apply non-linear models in the area of financial markets. The difficulties that application of chaos theory faces in practice are described in Section "Chaos: From Theory to Applications". The methodological implications of non-linear dynamics and chaos for economic theory are analysed in Section "Methodological Implications for Economics". In section "Some Issues in the Chaos Research Agenda" some particular issues in the chaos research agenda are emphasised. Section "Conclusions" concludes.

Non-linearity, Increasing Returns and Path Dependence

Once non-linearity is admitted we are in the presence of positive feedback or increasing returns. Here multiple equilibria are the norm. Increasing returns studies tend to show common properties: a multiplicity of potential 'solutions'; the outcome actually reached is not predictable in advance; it is 'selected' by small events; it tends to be locked in; it is not necessarily the most efficient; itis subject to the historical path taken; and, while the problem may be symmetrical, the outcome is usually asymmetrical (Arthur 2010, p. 159).

Mainstream economic theory removed from most of its areas the assumption of increasing returns precisely for its tendency to generate the existence of multiple equilibria. Convexity was a necessary assumption to warrant uniqueness of equilibrium.

However, the existence of non-convexities and increasing returns are widely used assumptions in different areas of economic analysis. International trade theory, macroeconomics, economic growth, industrial organisation, regional economics and economics of technology are examples of it. Multiple equilibria are also a widespread result in game theory.

The selection problem—which equilibrium comes to be chosen—can be handled by modelling the situation in formation, by translating it into a dynamic process with random events. So it is possible to study the probability that a particular solution emerges under a certain set of initial conditions (Arthur 2010, p. 159). Arthur (1994) makes extensive use of the non-linear Polya process, a path-dependent process in probability theory.

The existence of increasing returns means that, once embarked on a certain path, cumulative advantages are generated in favour of one of the possible equilibria. The selected equilibrium is path dependent. Any random event which favours one alternative may be decisive to the outcome.

The multiplicity of equilibria means that there are many possible worlds. Which has finally resulted is the product of history: it is history dependent. Another dynamic trajectory may have led to another result. If the equilibrium is unique, history does not matter: sooner or later the system will arrive at that unique equilibrium. The process is ergodic: whatever the sequence of events, the outcome is always the same. On the other hand, if the process is non-ergodic, the path defines the result. From this perspective, the economy can be seen as a process of self-organisation: the system "chooses" between the options that are presented to it.

The influence of any factor—which in other circumstances would go unnoticed—can be decisive in the choice of the alternative to which the system moves. Once this singular event determines the system's path, this option closes the road to the others and becomes irreversible.

The result may not be the optimum, as evidenced by the case of the typewriter keyboard. A technology that generates small initial increases in returns but has a great potential in the long run can easily be discarded in favour of another one that has the opposite characteristics. Once the economy has opted for this one and is generating increasing returns because of the mass adoption of it, the other can hardly be adopted though it may be more efficient in a long-term perspective. The economy may be trapped in the inferior option.

The result is history dependent. There is not a single possible history from Adam and Eve until today, but there is one single *made* history. This is because time is irreversible: once an option is chosen, there is no possibility to retrace the path already travelled.

The evolution of a system cannot be understood out of its own history. That history combines necessity and freedom. The trajectory of a system passes through periods of stability, where uniquely predictable behaviours prevail, alternating with periods of instability, where it has "to choose" between various alternatives. Thus necessity and chance build history, and, of course economic history.

Non-linearity, Attractors and the Paradox of Chaos

The equilibrium approach in economics is interested in only one type of attractor: fixed point attractors. Most efforts are devoted to find out the conditions under which a unique and stable equilibrium exists. In fact, linear systems either converge to a fixed point or explode.

Non-linear dynamic systems may evolve towards other types of attractors such as limit cycle or periodic attractors, quasiperiodic attractors and chaotic attractors.[1]

A quasiperiodic attractor can be conceived as a mechanism consisting of two or more independent periodic motions. Orbits can look quite complicated, since the motion never exactly repeats itself.

Chaotic attractors possess the property of sensitive dependence on initial conditions and usually have a fractal structure; that is, they have a non-integer dimensionality.

Since sensitive dependence on initial conditions is the essential feature of chaotic dynamics, the measure of chaos is provided by the Lyapunov exponent, more precisely by the largest positive Lyapunov exponent. Kyrtsou and Serletis (2006) construct a standard error for the dominant Lyapunov exponent, thereby providing a statistical test for chaos.

Lyapunov exponents (L) measure how quickly nearby orbits diverge in phase space. Unpredictability is an intrinsic feature of chaotic systems. Chaos implies the existence of a temporal horizon—defined by the Lyapunov time[2]—beyond which predictions lose any reliability.

The paradox of chaos is that we are in the presence of unpredictable behaviour that is generated by a process completely deterministic. Deterministic is the opposite of random. When I throw a die, the result is random: it is the product of chance. The movement of the planets, on the other hand, is fundamentally deterministic. The fact that they are not subject to any significant random element is what allows us to know very precisely where the Moon, Venus, or Mars will be from now until next century. This led to identifying determinism with predictability. Laplace's demon is the symbol of this identification. The famous mathematician and astronomer of the 18th century imagined that, if there was an omniscient being able to know the exact location and speed of each of the objects in the universe at any given time as well as all the forces involved, she could from there deduct its past and future evolution.

The classic determinism conceived the universe like a high precision clock in which the present is simply the consequence of the past and the cause of the future. In a deterministic world, if all data were available there would be no difficulty to make accurate predictions. Failed predictions only show that there are missing data. For this reason, it has been argued that chance is the name we give to our ignorance; that is, to the variables that influence a phenomenon but that we have not been able to detect. Remove randomness and all predictions should be absolutely accurate.

[1] For some time the terms "strange" and "chaotic" attractors were used as synonymous. However, later on it was discovered that there are strange non-chaotic attractors—they have a fractal structure but do not possess the property of sensitive dependence on initial conditions—and non-strange chaotic attractors—they do not possess a fractal structure. For example, Starrett (2012) shows a chaotic dynamical system which has a one-dimensional attractor.

[2] The Lyapunov time (τ) is measured by the inverse of the Lyapunov exponent: $\tau = \frac{1}{L}$.

Determinism and Predictability

Yet chaos implies a quite new situation: a process absolutely deterministic that becomes unpredictable. Determinism and predictability are no longer equivalent. Chaos theory has shown that, even though we knew the values of all the variables involved in a phenomenon, unpredictability can arise from the impossibility of having absolute precision in our measurements.

Chaos and randomness share a common element: the limited cognitive capacity of the human being. In the case of chaos, it implies the impossibility of reaching an infinite degree of precision in our measurements. In the case of randomness, it reflects our lack of capacity for identifying every one of the multiple variables that have to do with complex phenomena.

However, whereas a purely random system has no structure, a chaotic system has a hidden one. Chaotic processes have a finite dimensional attractor whereas a truly random process should be infinite-dimensional. So the phase portrait of a truly random system would eventually totally fill up the whole phase space diagram since no region in phase space would be preferred over another. In chaotic systems the phase portraits are very intricate structures delimiting the dynamics to only circumscribed regions in their phase diagrams. This provides some measure of long-run predictability in chaotic systems because, as the attractors of the system are constrained to particular regions of phase space, it can be predicted, with a certain level of probability, that a certain trajectory will fall within a certain region. Beyond the Lyapunov time, probabilities replace determinism. We can only predict that a certain trajectory will fall within a certain region as if it were a truly random trajectory. Beyond the temporal horizon, statistics replaces mathematics.

Non-linearity and Chaos in Financial Markets

Economists' interest in non-linearity emerges from its potential aptitude to model fluctuations in the economy and financial markets. It offers more options beyond the linear model's binary alternative between a stable and an explosive path.

Financial markets constitute natural candidates to find the types of behaviour predicted by non-linear dynamics. It is not surprising, therefore, the preference shown towards this area of the economy by empirical studies inspired by chaos theory.

The traditional approach in this area has been based on the efficient market hypothesis, which argues that the price of financial assets should reflect all available information. To this approach contributes the assumption of identical investors with rational expectations; all this reasoning leads to sustain that price changes are independent of each other and follow a random walk.[3] There is, therefore, no opportunity for persistent speculative profits or for a profitable use of technical

[3] Strictly speaking, market efficiency does not necessarily imply a random walk model but the latter does assume market efficiency.

analysis. All news is immediately reflected in the prices and as these follow a random distribution, so do their variations (see Lucas 1978).

However, the view that emerges from this traditional approach contrasts with the widespread perception that financial markets offer opportunities for speculative profits, that the use of technical analysis can be profitable and that the operating volume is far from zero—as it should be according to the no trade theorems (Milgrom and Stokey 1982; Tirole 1982).

To accommodate this type of real world phenomenon, the literature has introduced an alternative approach based on the distinction between chartists—also called noise-traders—and fundamentalists. While the first extrapolate the past trends, the latter are rational investors who are governed by the fundamentals of the market. The interaction between both types of traders destabilises the prices of financial assets and contributes to their volatility. The price process is at least partially driven by an endogenous non-linear law of motion. The fact that these models are very successful in replicating the stylised facts of financial markets is seen as a kind of empirical validation.

An early version of such a chartist-fundamentalist model was formulated by Frankel and Froot (1990) and has been much refined by De Grauwe and Grimaldi (2006). Based on this, Altavilla and De Grauwe (2010) developed a simple theoretical model in which chartists and fundamentalists interact. The model predicts the existence of different regimes, and thus non-linearities in the link between the exchange rate and its fundamentals.The results suggest the presence of non-linear mean reversion in the nominal exchange rate process. Traditional linear rational expectations models cannot account for this except by introducing exogenous changes in regimes; that is, by leaving these switches unexplained. The non-linear structure of the model does not allow for a simple analytical solution. As a result the authors have to use numerical simulation methods. One drawback of this approach is that it is not easy to derive general conclusions. However, the authors present sensitivity analyses of the numerical solutions.

The most striking finding is that there appear to be two regimes: one in which the exchange rate follows the fundamental exchange rate quite closely and another one in which the fundamental does not seem to play any role in determining the exchange rate. Both regimes alternate in unpredictable ways; there are frequent switches between fundamental and non-fundamental regimes. As a result, the relation between the exchange rate and the fundamentals is an unstable one.

In the empirical part of the chapter, the authors examine the predictive power of various models for the euro-dollar rate of exchange dynamics. They find that the non-linear specification significantly improves forecast accuracy during periods when the deviation between exchange rate and fundamentals is large—for example 2001:1–2004:4. Conversely, when the exchange rate is close to its equilibrium value it tends to be better approximated by a naïve random walk.

These results are in line with other empirical studies that have so frequently found a disconnection between macroeconomic fundamentals and the exchange rate. They corroborate the advantages of using a non-linear approach which allows detecting the existence of more than one state.The switching nature of the exchange rate process is inconsistent with a linear representation of the relation between the exchange rate and its fundamentals.

In a more recent paper, De Grauwe and Rovira Kaltwasser (2012) introduce a distinction between optimist and pessimist fundamentalist traders, referring to traders that systematically overestimate or underestimate the fundamental rate respectively. They show that, even in the absence of chartists, chaos can govern the asset price dynamics. Furthermore, chaos can indeed be triggered by the presence of biased fundamentalist traders alone and also by the interaction between biased and unbiased fundamentalist traders. The model is extended introducing unbiased fundamentalists and chartists. The latter prove to have a destabilising influence: the larger the coefficient expressing the degree with which they extrapolate the past change in the exchange rate, the stronger their destabilisation power. The system exhibits a Neimark–Sacker bifurcation of the steady state that leads to a stable limit cycle of the market exchange rate. Increasing the value of the chartists' extrapolation coefficient eventually leads to a break of the limit cycle and the exchange rate is governed by a chaotic attractor. This feature of the model is a common result obtained in the literature of heterogeneous agent models in finance where the interaction between fundamentalists and chartists is analysed and the chartists act as a destabilising force in the market.

Finally, the authors perform a Monte Carlo simulation. The model replicates the widely observed phenomenon that exchange rate returns are not normally distributed, but on the contrary exhibit fat tails.

Along the same line of analysis, Li and Barkley Rosser Jr. (2001) studied the behaviour of a model of asset market dynamics with fundamentalists and noise traders. Complex dynamics and greater volatility are seen to emerge as certain parameters in the system are varied.

It is clear that once the Holy Trinity of the unbounded rational representative agent, efficient market and linearity hypotheses are put aside, new illuminating results are obtained.

The Friedman (1953) hypothesis, stating that non-rational agents will not survive evolutionary competition and will therefore be driven out of the market, provided support to a representative rational agent framework as a (long-run) description of the economy as well as to the market efficiency hypothesis. However, Blume and Easley (1992, 2006) and Beker (2004) have shown that the market selection hypothesis does not always hold and that non-rational agents may survive in the market. Agent-based models with evolutionary selection among many different interacting trading strategies in artificial stock markets showed that the market does not generally select for the rational, fundamental strategy (see Arthur et al 1997; Brock 1993, 1997; LeBaron 1999).

Several models have been introduced where markets are viewed as evolutionary adaptive systems with heterogeneous boundedly rational interacting agents. They match important stylised facts in financial time series such as fat tails and long memory in the returns distribution and clustered volatility. They exhibit interesting dynamics characterised by temporary bubbles and crashes. Hommes and Wagener (2008) reviewed some of these non-linear dynamic asset pricing models with evolutionary strategy switching and illustrated some of the key features present in the interacting agents literature. Simulations show that predictions from a linear, representative agent model versus a non-linear, heterogeneous agent model are quite different. In particular, extreme events with large deviations from the benchmark fundamental valuation are much more likely in a non-linear world. Asset price fluctuations are characterised by phases where fundamentalists dominate and prices are close to fundamentals, suddenly interrupted by possibly long lasting phases of price bubbles when trend following strategies dominate the market and prices deviate persistently from fundamentals.

Chaos: From Theory to Applications

The interest in chaos is motivated in economics and finance because of its ability to generate output that mimics the output of stochastic systems, thereby offering an alternative explanation for business cycles which does not mainly rely on ad-hoc introduced exogenous shocks.

However, it is difficult to distinguish between exogenous fluctuations produced by random shocks and endogenous fluctuations produced from the non-linear structure of the economy. In this respect, Barnett and Serletis (2000, p. 721) state that "there have been no published tests of chaos 'within the structure of the economic system', and there is very little chance that any such tests will be available in this field for a very long time. Such tests are simply beyond the state of the art". This is because "existing tests cannot tell whether the source of detected chaos comes from within the structure of the economy, or from chaotic external shocks, as from the weather" (Barnett and Serletis 2000, p. 721).

Boldrin points out the theoretical and empirical difficulties that the introduction of chaos theory faces in economics.

From the empirical side, most of aggregate economic series are non-stationary. Non-linear dynamical systems techniques require to first reduce them to mean stationary time series. But this causes major alterations in the properties of the data set and obscure its real structure (Boldrin 1997, p. 278).

On the other hand, changes in economic policy, institutional and political environments, market structures, technologies, regulations, etc. must affect the law of motion of the system. Therefore, it is very unlikely that a one or two-dimensional chaotic dynamical system can ever reproduce those movements in the aggregate economy (Ibid.).

From the theoretical point of view, individual rationality, decreasing returns, rational expectations and market completeness smooth the consumption/work/investment pattern and dampen endogenous oscillations. But the only thing this proves is that non-linear dynamics should not be searched within the boundaries of conventional economics. Unfortunately for orthodoxy, economic time series show irregular frequencies and amplitudes of economic fluctuations are persistent and do not show clear convergence or steady oscillations.

Faggini (2011) presents a review of tests for chaos in economic and financial series. The main conclusion is that there is ample evidence of the presence of non-linearities but a limited evidence of deterministic chaos. Although the results of chaos tests do not prove the existence of chaos in all economic variables they are consistent with its existence (Faggini 2011, p. 22).

The detection of chaos in economic time series faces three difficulties: (1) the limited number of observations such series contain; (2) the high noise level in economic time series; and (3) the high dimension of economic systems.

While in physics, chemistry or biology experiments involve working with tens of thousands to millions of observations, economics work with much smaller series. This prevents many of the non-linear dynamics tools from detecting intrinsic irregularities even when they are present.

The detection of chaos in meteorology has been achieved thanks to the huge number of observations collected through the network of meteorological stations and satellites devoted to that purpose. These instruments have made it possible to significantly improve the accuracy of weather forecasts in recent times. One should wonder what would happen if an equivalent investment were made for the collection of economic data. Economic and financial storms have proved to be at least as destructive as natural storms.

Additionally, another difficulty stems from the high noise level that exists in most aggregated economic time series. The presence of dynamic noise makes it extremely difficult to distinguish between (noisy) high-dimensional chaos[4] and pure randomness. One would need an extremely long time series to do so; the tests are highly sensitive to noise and this becomes worse when the dimension of the system increases. As Ruelle (1994, p. 27) stated, "the separation between noise and the deterministic part of the evolution is ambiguous, because one can always interpret 'noise' as a deterministic time evolution in infinite dimension".

On the other hand, concerning low-dimensional chaos, small noise easily causes the system to diverge to infinity in the chaotic parameter range. Hommes and Manzan (2006) show how the introduction of increasing levels of noise to a chaotic asset pricing model makes the Lyapunov exponent of the underlying chaotic skeleton model become negative due to the presence of even a small amount of dynamic noise. This may explain why there is weak evidence so far of low-dimensional chaos in economic and financial time series. Thus robust deconvolution techniques are needed. Until now, wavelet-based noise reduction techniques have found the broadest applications (see Daubechies 1992; Guégan and Hoummiya 2005). Since the wavelet transformation is an orthogonal operation, it preserves the probabilistic properties of the underlying system allowing reconstruction of the original attractor. Gao et al. (2010) propose a non-linear adaptive denoising algorithm, and compare their approach with a number of wavelet thresholding-based noise reduction approaches. Their approach demonstrates to be more effective in reducing noise in the cases they present.

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[4] Low-dimensional chaos is characterised by only one positive Lyapunov exponent while high-dimensional chaos by more than one such exponent.

Faggini (2010) argues in favour of using topological methods instead of the traditional ones to discover chaos in short noisy time series. She uses a topological tool, visual recurrence analysis, and tests some macroeconomic time series already analysed with traditional tests for chaos (correlation dimension and Lyapunov exponent) which did not show the presence of chaos. On the contrary, visual recurrence analysis detects the presence of chaotic behaviour. Faggini concludes that the topological approach can be more useful for economic analysis performed on noisy short time series.

Finally, another difficulty in economics is that the dimension of the economic system is much higher than the dimension of systems with which physical scientists usually work; so the techniques in mathematics and statistics that are available to test for chaos are more difficult to apply to economics in a convincing manner (see Barnett and He 2012).

Given the fact that the trajectories generated by high-dimensional chaotic systems are very similar to those arising from stochastic processes, one may ask if the analysis should focus primarily on thinking that chaos is an explanation for seemingly stochastic data or, alternatively, if the apparently chaotic behaviour of the observations should be attributed to stochastic processes.

This is an issue that has to do with the traditions of each discipline. For example, in fluid mechanics the phenomenon of stochastic appearance of turbulence has been addressed as a typical case of chaotic dynamics and the idea of attributing it to exogenous random shocks has had slight consideration in the discipline. On the contrary, in economics the stochastic process approach has prevailed.

In this respect, Barnett and He (2012, p. 10) argue that "since the hypothesis of chaos within the economic system has not been tested, we may instead wish to consider whether or not chaos is plausible on philosophical ground". The issue is then whether the economy should be viewed as a system which evolves naturally, as in the natural sciences, or as the product of intentional human design by economic "engineers" who design it to be stable.

Methodological Implications for Economics

Once we accept that many economic phenomena are of a non-linear nature, a change in the economic analysis approach is required. Non-linearity is a promising tool to analyse many of the disorders and illnesses in contemporary economies. But this requires a change in economists' priorities. Economic illness rather than economic health should be the main object of economists' efforts. This may sound rather obvious, but most of the orthodox economists' efforts are devoted to show the non-existence of economic problems. The bulk of their papers are aimed at showing how the market solves by itself any potential conflict or difficulty. If so, there is no economic problem to work on. Most of the scholars' effort is devoted to study "health" and very little to analyse "illness" in economics. But, of course, it is economic illness which causes concern to society. There is a lot of effort devoted to show why, most of the time, the economy works smoothly, and very little effort to the analysis of why, from time to time, the economic mechanism breaks down or—more importantly—what is needed to fix it. But these failures in the economic mechanism have huge economic and social costs.

The main message that non-linearity introduces is *quantity matters*, to use the style of the famous Friedman's assertion. Critical values play a crucial role in the natural sciences. For example, water boils at 100 °C and freezes at 0 °C. The physical laws that govern the universe contain many fundamental numbers; "the values of these numbers seem to have been very finely adjusted to make possible the development of life" (Hawkins 1988, p. 125). However, in economics the tendency to predict qualitative behaviour regardless of quantitative values has predominated. In particular, interest has been centred in determining the conditions under which there is a solution to an extreme-value problem and finding the sign of the first derivative of the solution values with respect to the variation of the parameters.[5]

Interest in finding qualitatively invariable behaviour has its cost: the need to impose extremely restrictive assumptions. It is, for example, what happens when the purpose is to guarantee the existence and uniqueness of a fixed-point attractor. When those heroic assumptions are relaxed it is found out that anything happens, as exemplified by the Sonnenschein-Mantel-Debreu theorem in the general equilibrium theory. An equivalent result provides the anti-Turnpike theorem when it proves that the neoclassical optimal growth model is compatible with any type of dynamics. If anything can happen, the theory lacks any predictive power. But this is only the other side of the fact that the excessive emphasis on obtaining unambiguous predictions may lead to having to use in many cases heroic assumptions in order to ensure unique results. This produces a theory for special cases instead of a general theory.

Perhaps it should be accepted that in the economy—as indeed happens in nature—behaviour varies depending on the numeric values of the parameters and that the study of such changes is an important objective of economic analysis. Non-linearity implies precisely that a system may have different types of behaviour based on the values that its parameters take. This provides relevance to the determination of some variables' critical values—for example, the debt/GDP ratio, the debt/export ratio—that generate such changes in behaviour (bifurcations). This could be a topic of particular interest to future economic and econometric analysis.

Equally important is the study of how the economy behaves under those different regimes. From the economic policy point of view, one thing is to be near the equilibrium and a quite different one to be far away from it.

The non-linear approach allows studying the economy as a complex dynamical system which evolves towards different attractors depending on the value of its parameters.

Non-linear dynamics paves the way to the study of cyclic, non-periodic and chaotic behaviour. Sensitive dependence on initial conditions is the essential characteristic of chaos. Its most important consequence is that, in chaotic systems, it is only short-term

[5] As Samuelson (1983, p. 21) points out, this method has been taken from equilibrium thermodynamics, which is based on linear relationships. It was the introduction of non-linear relationships which allowed the development of non-equilibrium thermodynamics.

prediction that is possible. This should also lead to a revision of the possibilities and limitations of prediction in economy. Also, perhaps, to reassess the relative weight that is given to it and to explanation in economics. The relationship between explanation and prediction is not symmetric or transitive. For example, we understand how earthquakes are generated but we cannot predict an individual episode. Conversely, the phases of an incurable disease can be predicted without having a thorough explanation of the causes of the ailment. As a result, in one case the centre of scientific activity is oriented to improve prediction; in the other, to know the causes.

Perhaps this may seem rather obvious. However, it would not be superfluous to remind that theory does not have as unique and exclusive purpose the formulation of predictions. The main purpose of science is explanation. If a theory explains, it helps understanding a phenomenon. If, additionally, it predicts, it is twice as useful. When an answer is not available, prediction is a good second best, but it is never a first best.

On the other hand, in economics there are no such things as crucial experiments. For example, given a certain econometric result, in many cases it is enough to just include another variable, or to slightly modify the model assumptions or the estimation method to get different, and even opposite, results. There are many examples in the economic literature in this respect. No matter how sophisticated the economic tools are and how detailed the set of data one deals with, very few robust relationships can be obtained.

As a matter of fact, economists seldom practice the falsificationism they preach. Confidence in the implications of economics derives from confidence in its axioms rather than from testing their implications (Hausman 1992, p. 1). Since economists are typically dealing with complex phenomena in which many simplifications are required and in which many interferences may appear, it does not seem rational to surrender a credible hypothesis because of predictive failure. Economists trust more in the implications deduced from the theory's axioms than in the negative results which may emerge from empirical testing. It is very unusual to disregard a theory because of an apparent disconfirmation.

In this respect non-linearity is by far a much more compatible assumption with most of the economic time series behaviour. So why should non-linear dynamics and chaos theory be subject to different—more stringent—rules than what is usual for the rest of economic theory?

Non-linear dynamics enables us to model change as a process of self-organisation, where a system far away from the stability region reaches a bifurcation point and undergoes a change of regime. In this sense, it may constitute a decisive contribution in the direction of filling the lack—denounced North (1994, p. 359) on the occasion of receiving his Nobel Prize—of a theory of economic dynamics comparable in accuracy to the theory of general equilibrium. It would give us an important analytical tool to understand critical issues such as evolution and economic change with the advantage of being more related to real phenomena than general equilibrium theory.

Although there are still serious problems to be solved that will require a lot of further research, the recent financial crisis may act as what I have called elsewhere Beker (2005) a "big social experiment"; that is, an event that discredits pre-existing ideas and demands replacing them with new ones. Chaos theory is a well-qualified candidate to be considered among them.

Some Issues in the Chaos Research Agenda

Since the existence of chaos implies unpredictability, a logical question that arises is whether it is feasible to control it. Working inside a chaotic attractor would permit controlling the system and being able to avoid explosions and strong volatility. Bala et al. (1996) raised the possibility of controlling a chaotic dynamic system by introducing a disturbance in its law of motion in order to achieve a fixed point.

Faggini and Parziale (2012) recall that Ott et al. (1990) proposed an ingenious and versatile method for controlling chaos. The key achievement of their paper was to show that control of a chaotic system can be made by a very small, "tiny" correction of its parameters. Faggini and Parziale go on pointing out that we can obtain a relatively large improvement in system performance by using small controls. They add that these considerations are particularly interesting in the applications of control of economic systems. They suggest that "the government may be able to manipulate some policy parameters in order to shift the economic system from a position of chaos to a fixed point outcome and in this way fulfil its stabilization goal" (Faggini and Parziale 2012, p. 5). So if authorities control some of the bifurcation parameters then they can manipulate their values in order to attain a region of fixed point stability.

As was mentioned in the introduction, many economic and financial time series have fat tails. Fat tails are defined as tails of the distribution that have a higher density than what is predicted under the assumption of normality.

Extreme value theory provides a framework to formalise the study of behaviour in the tails of a distribution. It aims at the probabilistic prediction of events of unusual intensity. It has traditionally been used for predicting risk.

Recent theoretical work has proved that extreme value laws also hold in certain chaotic deterministic dynamical systems. The statistical theory of extremes was originally developed for stochastic processes but substantial progress has been made recently in transferring this theory to chaotic deterministic systems.

While bifurcations are commonly associated with qualitative changes in dynamical behaviour, chaotic systems also may display smooth responses to parameter variation.

Vitolo et al. (2009) took first steps toward a theory of "robustness of extremes" and highlighted its potential usefulness for statistical inference and prediction in nonlinear systems. They investigate how the extremal properties of chaotic deterministic systems respond to parameter variation. They consider the statistical properties of extreme values that arise when the system enters asymptotically small regions of phase space.

A chaotic deterministic system is said to exhibit robust extremes under a given observable when the associated statistics of extreme values depend smoothly on the system's control parameters. Vitolo et al. formulated the notion of robust extremes, proved robustness of extremes for a simple class of systems, and demonstrated how knowledge of robust extremes can be used to improve inferences about the extreme values of complex systems.

This issue relates to the interpretation of extreme observations as inherently related to the dynamical behaviour of the model and has to do with the possibility of forecasting extreme events.

The authors discuss how to derive the dependence of a system's extremal properties on its control parameters, thereby determining whether or not the system exhibits robust extremes. Finally, they show how robustness can be used in interpreting and predicting non-stationary extremes.

An alternative approach for the analysis of extreme events is provided by Statistical Mechanics. This approach assumes that extreme events depend on the internal structure of the system (Salzano 2008, p. 200). Bak and Chen (1991) introduced the concept of self-organized criticality. Independent microscopic fluctuations can propagate so as to give rise to instability on a macroscopic scale. In a "subcritical" state, changes in one part of the system have a sufficiently weak effect upon neighbouring parts that the state in different regions of the system is correlated only over short distances. However, when the system reaches a "critical state" the correlation between parts of the system ceases to decay exponentially with distance, and even arbitrarily small external perturbations can have large effects upon the macroscopic state. The authors use as an example a sand pile which is generated by gradually adding grains of sand. Eventually, the slope of the pile reaches a critical value such that the addition of one more grain results in an "avalanche". The existence of the self-organized critical state is robust: the critical state is the system's attractor.

This class of dynamical systems are *weakly chaotic*; they exhibit zero Lyapunov exponents, meaning that the separation of nearby trajectories is weaker than exponential. Long-run prediction is therefore possible.

Scheinkmann and Woodford (1994) apply this approach to the analysis of economic fluctuations. They propose that the effects of many small independent shocks to different sectors of the economy do not cancel out in the aggregate, due to the presence of significant non-linear interactions between the different parts of the economy. The authors develop a model of production and inventory dynamics; they show how an "avalanche" of production can develop depending upon the asymptotic distribution for inventory configuration. They show the existence of a well-behaved limiting distribution for individual avalanche sizes with a fat tail, just as in the sand pile model.

Conclusions

Although economic data provide little—if any—evidence of linear, simple dynamics, and of lasting convergence to stationary states or regular cyclical behaviour, the linear approach absolutely dominates mainstream economics.

However, non-linearity is by far a much more compatible assumption with most economic time series behaviour. The non-linear approach allows study of the economy as a complex dynamical system which evolves towards different attractors depending on the value of its parameters. It paves the way to the study of cyclic, non-periodic and chaotic behaviour.

The detection of chaos in economic time series still faces some difficulties, mainly the fact that in economics we deal with short and noisy time series. In this respect, topological methods for chaos detection seem to be a highly promising tool.

On the other hand, economists seldom practice the falsificationism they preach. Confidence in the implications of economics derives from confidence in its axioms rather than from testing their implications. Therefore, non-linear dynamics and chaos theory should not be subject to more stringent rules than what is usual for the rest of economic theory

So, although there are still serious problems to be solved that will require a lot of further research, chaos theory is a well-qualified candidate to model fluctuations and other phenomena in economics and finance.

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Chapter 12
Disequilibrium Trade and the Dynamics of Stock Markets

Tönu Puu

Abstract The present work considers pricing and trade dynamics for stock commodity markets, which, unlike flow commodity markets have been little studied, if at all. Concepts and tools in economics are shaped to deal with flow markets, where commodities disappear in each period and then reemerge. This allows one to define unique demand and supply functions and their equilibria. A durable commodity, a stock, in contrast, remains on the market to the next period and may just change owner through exchange. This, however, changes demand and supply functions, and hence the equilibrium state to which a dynamic process may be heading. Dynamic processes are provided with memory of the actual exchange history. We also need to state how disequilibrium trade in stock markets takes place. This is another neglected issue, though a fact of reality. Using a case with only two traders of two stock commodities, and focusing pure trade, it is possible to specify the exact conditions for disequilibrium trade in each step of the dynamic process. In the end any of an infinity of equilibria can be reached, or trade can stick in some disequilibrium point while complex, even chaotic, price dynamics goes on.

Keywords Disequilibrium trade · Durable commodity markets · Complex dynamics · Multiple equilibria · Path dependence

Introduction

The present work focuses two different but related theoretical issues of economics, both concerning the markets for stocks or assets.

These concepts usually connote financial claims, such as shares, bonds, or cash, so, to avoid misunderstanding, it should be said from the outset that we presently

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intend something much more general: We just do not refer to flow commodities, such that disappear during one time period through consumption or as inputs in the productive process, but to any class of commodities that remain on the market period after period and that may change ownership through exchange.

Obviously, there are many such commodities, in fact any durable commodity that has a user value, either for the owner or a potential buyer. To fix ideas, just think of the housing market.

Digression on Real Estate Value Instability

This market is of special interest as recent turmoil in the western economy has been related to instabilities on housing markets. The explanations offered have referred to speculative bubbles. No doubt there exists some speculation through the intervention of real estate dealers, but, there are other issues at work which may explain at least some of the phenomena without any reference to speculation, which is difficult to model convincingly.

The present author tends to lean on Lord Keynes's pessimistic view that it is impossible, even if he, more than anyone, admitted that it is a regrettable fact of life. See Keynes (1936). For this reason, and for the reason that so (too?) much has already been written on so called heterogenous (chartist/fundamentalist) agent models, the present work will not touch upon this topic, nor on speculative behaviour at all.

So, if we disregard speculation, what else could there remain to blame the housing market instabilities on? As a matter of fact, there is something very fundamental, even trivial, but totally overlooked: Due to mobility and demographic change, housing space always changes hands.

Consider, on one hand, an old family living in a large villa with the children moved out, who would like to move to an inner city apartment, and, on the other, a newly married couple planning to have children and wanting some larger suburban living space.

As we do not live in a barter economy, prices drift up and down depending on how the real estate dealers conceive of the market. Suppose prices are on the move up; then people in city apartments get well paid, and can offer more for a suburban villa, and the villa owners too can afford to move to an attractive city space.

Everybody gets nominally richer, and the upgoing price trend is sustained. Of course, the bank owns most of the wealth, but as long as there are no constraints imposed on the banking system, it functions as a passive credit multiplier, and need not be taken in explicit account.

And, what about speculation? The present author would dare the claim that most people just want to live in their accommodations without too much regard to second hand value.

So why is this never mentioned? The most obvious reason is that economists simply do not have tools to analyze stock markets. The durability of stocks (unlike flows) makes demand and supply functions *change every time actual trade takes place*, so unique market equilibria, the dearest tools to the economist, can never be defined.

Related to this is the lack of a theory for trade in disequilibrium which no doubt is a fact of reality. The housing market always has a shortage or a superfluity of accommodation, but yet exchange does take place, without waiting for Kingdom come when the tatonnement has reached its equilibrium.

It is these two issues we are going to focus in what follows.

Stocks and Flows

Economic quantities can be classified as either stocks or flows, depending on whether they refer to time *points* or time *periods*. The stock of productive capital, the quantity of money, the labour force, inventories, etc. are all examples of stocks. Conceptually they are easier than flows, which assume a periodized background of evolving facts, and hence *imply some process in time*. Such quantities depend on the *length of period* chosen, and if one models evolution in continuous time, then they become something rather abstract, i.e., time derivatives. Examples of flows are commodities traded, hours of labour worked, periodized incomes, savings, investments, etc. The distinction holds on the micro as well as on the macro level.

The tools of economic analysis relate almost exclusively to phenomena for flows. To see this, consider any elementary microeconomics textbook. In one of the opening chapters there is a picture of consumer's preferences displayed as an indifference map. There is a budget line representing the possible choices, whose axis intercepts are the income divided by the respective price. The consumers seek maximum satisfaction, which, given the proper curvature, is obtained at the point of tangency of the budget line with an indifference curve. This explains how the consumers make their choices.

Next step is to see what happens if one price changes. The axes intercept that represents the other price remains fixed, but the budget line changes slope and so rotates around this fixed point, and this rotating movement sweeps out a curve of new points of tangency. To each price corresponds a (unique) quantity demanded, no matter how we rotate the budget line. By associating the demand quantity to the price that represents the slope, one derives a *unique* demand curve which can be aggregated over consumers, and used in the market setting to establish price and quantity traded at equilibrium, where demand equals supply.

As a further step consider a case where the consumer does not have an income in money (for instance a university professor of ancient times getting his salary in terms of quantities of firewood, salted pork, and the like). No problem! The fixed point is not on the axis, but somewhere out in the positive quadrant. If, given the current market prices, he finds he would be better off changing some firewood for more pork, he can do this on the market. Again, considering different relative prices, the budget line rotates and sweeps out a *unique* demand/supply function.

Irreversibility and Hysteresis

However, it is taken for granted that all firewood is burnt and all pork is consumed during the period. So, when the same situation arises anew next period, everything is just repeated. This is the case for flows.

But what if the commodities were durable, remaining from one period to the next? Then, if at some stage the consumers find it profitable to make an actual exchange, this will influence the future. After any such exchange, the budget line would rotate through a *new* point, so sweeping out a *different* demand/supply curve. The process would thus be provided with a memory of the actual trading, and neither demand nor supply functions would be unique. We encounter phenomena of irreversibility and hysteresis, which never show up in elementary economics textbooks; a sign as good as any that main steam economic theory is shaped for flows, not for stocks.

In the sequel, to the purpose of pinpointing the issue, we even take the total supply for given. In the case of the housing market, we thus disregard new production and scrapping due to wear, so total supply is fixed, and what appears as demand and supply on the market is just what individual owners want to sell or buy of different habitation types within given totals. The reader will forgive me for taking the discussion down to such elementary trivialities, but it was necessary for making the point.

Tools developed for flow markets were, however, gladly transferred to markets for stocks, for instance, the demand for money, such an important issue in the monetarism controversy. What if we *cannot define* a unique demand function? This question was never posed, not even by Lord Keynes (1936) in his liquidity demand function.

Disequilibrium Trade

When we are considering apartments and houses changing owners *within* a given total, we enter the other main issue; *disequilibrium trade*. As was noted above, trade as a rule *does* take place even when there is no equilibrium. In general, it is quite tricky to specify how such takes place, i.e., which consumers get fully satisfied and which do not, or only get partial satisfaction from exchange.

However, there is one case that provides us with a clear setting—the old Edgeworth box (Edgeworth 1894), provided we consider only two commodities and two agents. This box was never used in such connection, but its use provides a simple, even visual, start with an otherwise as messy as neglected issue.

The Edgeworth Box

So, imagine the story told above about indifference maps and rotating budget lines, and consider two agents, one's indifference map in normal position, the other rotated 180° and translated to a position such that the horizontal and vertical distances between the two origins equal the (fixed) totals of the commodities available for exchange. Now, the diagram is exactly the Edgeworth box, familiar to economics students from its application in international trade theory. One indifference set is concave, the other convex, and if we consider the intersection of two curves, they form a lens-shaped region in which every point is better for *both* agents than the intersection itself.

The lens collapses to a single point along the curve of points where the indifference curves, one from each set, touch. These are the Pareto efficient optimal points, all candidates for *equilibria*. The relative price ratio, or slope of the budget line, would then have to take the slope of the two touching indifference curves at that point.

But, what if we dealt with stocks? Take any disequilibrium point *not* on this curve of contact, and any announced price ratio. Would trade then be possible even if we do not end up at equilibrium? The answer is yes, quite as in real life, and in the simple setting chosen we can even state the precise conditions for this and for how trade takes place.

Any point in the box can represent an initial distribution of the total wealth of the two agents (equal to the sides of the box). A line through this point with some slope, corresponding to the price ratio, would be the *common* budget line for *both* agents.

The question then is how far on this line the agents would want to move. Sooner or later the budget line would touch an indifference curve from either map. These are the optimal points for the two agents. Any of them might be outside the box, but it does not matter as the process designed will never go outside the box. If the announced price ratio does not correspond to equilibrium, then there are three possibilities.

The optimal points are on the budget line to the same side of the initial point, to the right or left of it. Then one agent would like to exchange more than the other. That agent has no means to force the other to move further than she/he wants, but profits from proceeding even part of the way. Thus, disequilibrium exchange is possible, and profitable for both, but the *limit* is set by the agent wanting to exchange *least*. This results in two cases. In neither case, however, the exchange results in equilibrium.

As a third possibility, the optimum points may be on either side of the initial point. Obviously, then no trade is possible as both agents want to change the same good for another. When setting up the formal model we will go through all this in tedious detail (as six logically possible cases) .

Just give a thought to how much more complicated everything would be if we had three traders or three commodities. We are fortunate that the disequilibrium trade condition were so intuitively easy to set up for this two by two case.

Price Adjustment

If we add some price adjustment mechanism that, for instance, generates relative prices due to excess demand/supply, then the dynamic model is closed. As we will see, we can end up at infinitely many equilibrium points, or at infinitely many *dis*equilibrium points where no more trade is possible, but complicated price dynamics, periodic, quasiperiodic, or chaotic, goes on for ever in a vain search for an equilibrium.

Digression on Ex Ante and Ex Post

These issues go through all of economics. In the wake of the Keynesian macroeconomics, people started collecting data for the actual calculation of national income, which before had been a theoretical construct, such as periodic interest on the national wealth (see Lindahl 1939), and its components—consumption, saving, investment.

Soon the question was posed whether the equality of saving and investment was an equilibrium condition or an accounting identity.

The Stockholm School

The so called "Stockholm School" set out to clear these things up. Obviously, individual agents have plans, and in a modern economy saving (abstaining from consuming) and investment in capital for productive service are different actions dependent on different decisions of the individual agents. If their planned decisions (ex ante) match, then we are in a lucky and rare state of equilibrium.

However, if they do not, then in the national accounts they still balance (ex post). The trick is played by unintended saving by consumers who due to shortage were not able to buy the goods they wanted, and unintended investments in inventories of goods that could not be sold.

The Stockholm School never got further than coining the concepts ex ante and ex post, now absorbed by the entire economics profession. They did not manage to explain how these unintended savings and investments came about.

For the interested reader Palander's critique of all this (Palander 1941, 1953) cannot be too highly recommended. There are about five or six key works in this Stockholm School, but we only cite Myrdal's book (Myrdal 1939), as Palander's extensive article formally is a book review, though it is also a thorough critique of the confusion between stocks an flows and the total lack of any even rudimentary treatment of disequilibrium trade. This digression served to show how deep these issues cut even in macroeconomics. The merit of the Stockholm school was to shed some little light on these issues, especially in view of the fact that we afterwards used flow theory for stock markets and concentrated on equilibria to such an extent that disequilibrium trade was never dealt with.

The Model

Notation

Denote the commodity quantities for the first agent (x, y). As it makes no harm in a pure exchange model, let us normalize the totals of both commodities available on the market to unity. Hence, the corresponding quantities for the second agent are denoted $(1-x, 1-y)$. The Edgeworth box thus becomes a unit square. Any actual distribution of assets in denoted in upper case, (X, Y) for the first agent, $(1-X, 1-Y)$ for the second.

We only deal with the relative price, so normalize the price of the first commodity to unity, which just becomes a numéraire. The price of the second commodity is denoted p, which hence is a relative price.

Budget Constraints

The budget constraint for the first agent reads

$$x + py = X + pY \tag{12.1}$$

and accordingly for the second agent,

$$(1-x) + p(1-y) = (1-X) + p(1-Y) \tag{12.2}$$

However the latter is identical with (12.1), which we see if we subtract the expression $(1+p)$ from both sides of (12.2).

This almost is all notation we need. Let us just call the optimal points to which the agents would like to move (x_1, y_1) and (x_2, y_2) respectively, and we are finished. These two points obviously have to lie on the budget line (12.1) or (12.2), quite as the actual wealth distribution point (X, Y). The points (x_1, y_1) and (x_2, y_2) touch an indifference curve each from the preference map of either agent. All this is illustrated in Fig. 12.1. Obviously the system cannot move from (X, Y) to both optimal points. Below we will discuss to which of them the new wealth distribution point $\left(X', Y'\right)$ actually moves through trade.

Utility Functions

As for utility, take a Cobb-Douglas form for the utility function. Then, for the first agent the utility function is

$$U = x^{\alpha} y^{1-\alpha} \tag{12.3}$$

and for the second

$$V = (1-x)^{\beta} (1-y)^{1-\beta} \tag{12.4}$$

Obviously, the Cobb-Douglas exponents must be in the interval $0 < \alpha, \beta < 1$.

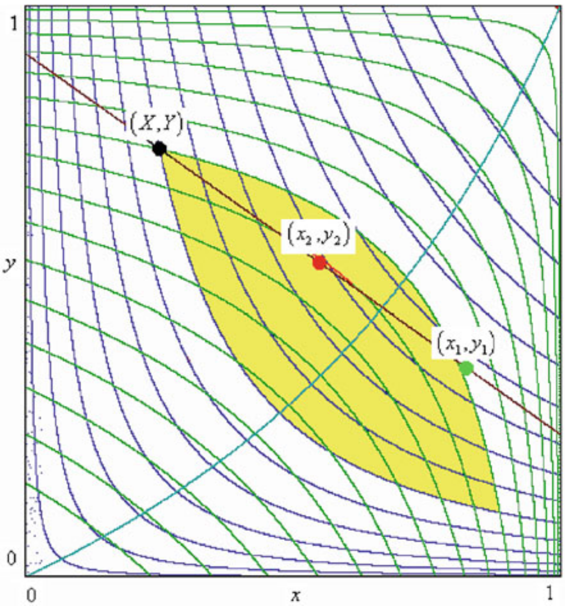

Fig. 12.1 Indifference *curve maps* for $U = x^{\alpha} y^{1-\alpha}$, $V = (1-x)^{\beta}(1-y)^{1-\beta}$, and the *budget line* $x + py = X + pY$. Initial point (X, Y), and the points (x_1, y_1) and (x_2, y_2), to which the agents would like to move, are displayed. Further on display is the *curve* for equilibria, along which the indifference curves touch, and the lense shaped area, where both agents are better off than in the initial point. *Parameter values* $\alpha = 0.6$, $\beta = 0.4$, $X = 0.25$, $Y = 0.75$, $p = 1.5$. Given these, from (12.5) $x_1 = 0.825$, $y_1 = 0.3\overline{66}$, and from (12.6) $x_2 = 0.55$, $y_2 = 0.55$

Individual Optima

The results of maximizing Cobb-Douglas functions, such as (12.3)–(12.4) subject to linear budget constraints, such as (12.1) or (12.2), are well known: Fixed shares of the budget, equal to α, $(1-\alpha)$ for the first agent and β, $(1-\beta)$ for the second are spent on the commodities x, y. The budgets are given by the right hand sides of (12.1) and (12.2) respectively. We only need to recall that for x the price is unity, so for that commodity quantity and value are identical. For commodity y one has to divide the budget share by price p in order to get the quantity demanded.

Hence the desired optima are

$$x_1 = \alpha (X + pY)$$
$$y_1 = (1-\alpha)\left(\frac{X}{p} + Y\right) \tag{12.5}$$

for the first agent, and

$$1 - x_2 = \beta\left((1 - X) + p\,(1 - Y)\right)$$
$$1 - y_2 = (1 - \beta)\left(\frac{1 - X}{p} + 1 - Y\right) \tag{12.6}$$

for the second. Note that, in addition to the exponents of the utility functions, (12.5) and (12.6) depend on relative price p and on the actual asset distribution X, Y, or $(1 - X), (1 - Y)$.

As a rule (x_1, y_1) and (x_2, y_2) are different; from each other, and from the actual asset distribution point (X, Y), as we see in Fig. 12.1. Only in one situation are they equal, as stated in the introduction, i.e. on the equilibrium curve which is also illustrated in Fig. 12.1.

This curve can be obtained in different ways. We can put $x_1 = x_2$ and $y_1 = y_2$ in (12.5), (12.6) and eliminate p; or we can calculate the locus of points where the indifference curves, one from each utility map, touch (through calculating and equating the implicit derivatives). The latter is the usual way way used in international trade theory where the equilibrium curve is called “cont(r)act curve”. Whatever the procedure chosen, the formula reads

$$Y = \frac{(1 - \alpha)\,\beta X}{\alpha\,(1 - \beta) + (\beta - \alpha)\,X} \tag{12.7}$$

which too is shown in Fig. 12.1. Note that if $\alpha = \beta$ then from (12.7) is a straight line (the diagonal).

The equilibrium relative price p in each point of (12.7) is well defined. It just equals the slope of the touching indifference curves in that point. Also note that along (12.7) there is a nondenumerable infinity of different possible equilibrium points.

Trade

Above it was stated that trade is limited to the *smallest* change that any agent wants to make, because the agent wanting to change more would then also benefit from moving part of the way towards his optimum, whereas he has no means to force the other agent to move further than he wants. Referring to Fig. 12.1, we see that, in the case portrayed, this means moving from (X, Y) to the new wealth distribution point $\left(X', Y'\right) = (x_2, y_2)$, distinguished by a dash.

Once this move has taken place, the budget line would pivot through this *new* point at any further change of the relative price p., thus changing the demand/supply functions.

Supposing price changes from p to p' and then back to p again, the original asset distribution point (X, Y) would *not* be retrieved, because after trading this point would not even be on the new budget line. For this reason one can neither speak of unique demand or supply functions in the case of assets, nor of unique equilibria.

Any asymptotically approached equilibrium state depends on the intervening trading process. As mentioned, there are infinitely many equilibria, and any of these can be the asymptotic state, depending on where the process starts and how the "tâtonnement" for pricing works.

Even worse, as we will see, there are also infinitely many *disequilibrium* fixed points off the curve of equilibria, where the process may stop, as further trade is not possible. This is the nucleus of the present argument.

The trade possibilities can now be classified in six distinct categories, based on how the points (X, Y), (x_1, y_1), and $(x_2.y_2)$ are ordered on the budget line, from left to right. Actually, we can just use either the x or the y coordinate for this ordering, so we go for the first option. This means that the trading map too can be set up in terms of x alone; a deceptive simplicity as the map (12.5)–(12.6) uses both, and so in one additional step brings both coordinates back in.

Six Cases

(i) $x_1 \le x_2 < X$: The first agent wants to sell $(X - x_1)$ but the second is only willing to buy $(X - x_2)$. There can be trade, but it is limited by the buyer's willingness. As a consequence one gets the new trade point $\left(X', Y'\right) = (x_2, y_2)$. There is left an excess supply amounting to $(x_2 - x_1)$, and a corresponding excess demand for the other commodity. Note for future use that for this case $(x_2 - x_1)(x_2 - X) \le 0$.

(ii) $x_2 \le x_1 < X$: The first agent wants to sell $(X - x_1)$ and the second wants to buy $(X - x_2)$, which is more Trade is now limited by the seller's, the first agent's willingness. The new trade point becomes $\left(X', Y'\right) = (x_1, y_1)$. There is left an excess demand amounting to $(x_1 - x_2)$, and a corresponding excess supply of the other commodity. Note that for this case $(x_1 - x_2)(x_1 - X) \le 0$.

(iii) $X < x_1 \le x_2$: The first agent wants to buy $(x_1 - X)$ and the second wants to sell $(x_2 - X)$, which is more. Trade is limited by the buyer's, i.e., the first agent's offer. The new trade point becomes $\left(X', Y'\right) = (x_1, y_1)$. There is left an excess supply amounting to $(x_2 - x_1)$. For this case $(x_1 - x_2)(x_1 - X) \le 0$.

(iv) $X < x_2 \le x_1$: The first agent wants to buy $(x_1 - X)$ and the second wants to sell $(x_2 - X)$, which is less. Trade is limited by the seller's, i.e., the second agent's offer. The new trade point is $\left(X', Y'\right) = (x_2, y_2)$. There is left an excess demand amounting to $(x_1 - x_2)$. For this case $(x_2 - x_1)(x_2 - X) \le 0$.

(v) $x_1 < X < x_2$: The first agent wants to sell $(X - x_1)$ and the second as well wants to sell $(x_2 - X)$. As both want to sell the same commodity, no trade is possible, so $\left(X', Y'\right) = (X, Y)$. There is left an excess supply amounting to $(x_2 - x_1)$. For this case $(X - x_1)(X - x_2) < 0$.

(vi) $x_2 < X < x_1$: The first agent wants to buy $(x_1 - X)$ and the second as well wants to buy $(X - x_2)$. As both want to buy the same commodity, no trade is possible. Again $\left(X', Y'\right) = (X, Y)$. There is left an excess demand amounting to $(x_1 - x_2)$. For this case too $(X - x_1)(X - x_2) < 0$.

The very simple argument is that if x_1 and x_2 are on either side of X, then trade is impossible (cases v and vi), because both agents want to buy/sell the same commodity. If x_1 and x_2 are on the same side of X, then there is one potential buyer and one seller, but the change is limited by the agent who wants to buy or sell least, agent 1 in cases ii and iii, or agent 2 in cases i and iv. The map formulated below thus boils down to three cases $\left(X', Y'\right) = (x_1, y_1)$, $\left(X', Y'\right) = (x_2, y_2)$, and $\left(X', Y'\right) = (X, Y)$, quite as suggested in the introduction.

The Trade Map

It just reads

$$\left(X', Y'\right) = \begin{cases} (x_1, y_1) & \text{if} \quad (x_1 - x_2)(x_1 - X) \le 0 \\ (x_2, y_2) & \text{if} \quad (x_2 - x_1)(x_2 - X) \le 0 \\ (X, Y) & \text{if} \quad (X - x_1)(X - x_2) < 0 \end{cases} \tag{12.8}$$

The application clauses exhaust all logical possibilities and are mutually exclusive as can be easily established. The two first rows of (12.8) represent trade corresponding to the limits set by agent 1 and 2 respectively, whereas the last row represents blocked trade because the agents want to buy and sell the same commodity. The weak inequality signs in the first two branches let us include the equilibria in the map.

Excess Demand

Let us just restate the definitions for the desired optimal points to be used in (12.8), (x_1, y_1) and (x_2, y_2). They were given in (12.5) and (12.6), though it is nicer to solve for y_1 and y_2 in explicit form. Hence

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \alpha\,(X + pY) \\ (1 - \alpha)\left(\frac{X}{p} + Y\right) \end{pmatrix} \tag{12.9}$$

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 - \beta\,((1 - X) + p\,(1 - Y)) \\ 1 - (1 - \beta)\left(\frac{1-X}{p} + 1 - Y\right) \end{pmatrix} \tag{12.10}$$

This almost completes the iterative map; only one item is missing, the setting of relative price p.

Price Adjustment

The case of two traders may seem to set the stage for bilateral monopoly, but we intend the model as a first stepping stone for generalization to more traders, so it is better to use some excess demand dependent price adjustment function of the type Samuelson suggests (Samuelson 1947). It formalizes Walrasian tâtonnement (Walras 1874–1877). Actually, Walras seems to deal with testing out equilibrium prices which come in effect only when equilibrium is reached, but Samuelson implies a dynamic process where excess demand/supply drives prices or down, with a force dependent on the size of the excess. When price increases due to excess demand, the latter is reduced. These prices seem to be conceived as real transaction prices in a transitory process As the demand and supply functions remain unchanged, we must conclude that a market for flows is intended. However, there seems to be no harm in choosing this most widely used type of price adjustment mechanism also for stock markets.

As we deal with relative price p, we can choose either excess supply on the market for x, or excess demand on the market for y, to trigger rises for the only variable price p, as there is a simple reciprocity between the two in the model. We go for the first alternative. From the six cases listed above we find that excess supply, or if negative, excess demand for x always equals $x_2 - x_1$.

In discrete time a linear price adjustment function could easily lead to negative prices which we want to avoid, so we choose the semilogarithmic,

$$p' = p \exp\left(\delta \left(x_2 - x_1\right)\right) \tag{12.11}$$

where δ denotes an adjustment step length. The choice of the map (12.11) has the advantage of symmetry with respect to the other relative price $1/p$, as the exponent then just changes sign.

The dynamic model we propose consists of (12.8) and (12.11), where (x_1, y_1) and (x_2, y_2) are as defined in (12.9)–(12.10). Despite its simple look, the model seems to be too complicated for further closed form analysis. We can, however, obtain much information through numerical experiment.

Numerical Analysis and Graphics

The Phase Plane

Trade Equilibria and Disequilibria

It is easy to run the map (12.8) and (12.11) with definitions (12.9)–(12.10) on the computer and display the results in a phase plane such as Fig. 12.1. One just needs to set the *parameters*, which are the exponents of the utility functions α, β, and the price adjustment step length δ, and further choose the *initial values* for the asset

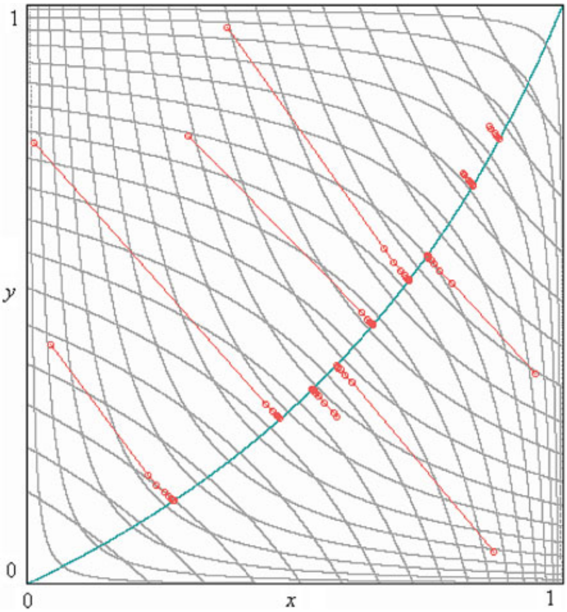

Fig. 12.2 *Orbits* of 10 randomly generated initial points (X, Y) using 100,000 iterations. *Parameters* $\alpha = 0.6, \beta = 0.4, \delta = 1$. Notably, the *orbits* converge very fast to the equilibrium *curve*, though to different points. Initial relative price is set at $p = 1.5$ in all iterations. Though p is adjusted in each iterate according to (12.11), it cannot be excluded that initial p like initial (X, Y) influence the *orbit* and the final equilibrium

distribution (X, Y) as well as for relative price p. Figures 12.2 and 12.3 show the orbits generated from ten randomly chosen initial points (X, Y) in the unit square. As in Fig. 12.1 we keep the parameter values $\alpha = 0.6$, and $\beta = 0.4$, and fix the initial relative price at $p = 1.5$. The system was run in 100,000 iterations in each case. The iterates are indicated by circles joined by line segments representing the jumps. Obviously, the first steps are giant, and the orbits in Fig. 12.2 converge rather fast on the final positions.

The difference between Figs. 12.2 and 12.3 is due to the step size, $\delta = 1$ in Fig. 12.2, $\delta = 5$ in Fig. 12.3. With the smaller step size all orbits converge to the equilibrium curve as displayed in Fig. 12.1, though to different points, thereby illustrating what was said about the dependence of equilibrium upon the dynamic adjustment process.

In Fig. 12.3, some orbits still converge to the equilibrium curve, but some stop at a distance from it. Visually this stopping in disequilibrium fixed points can occur in just few steps; the large number of iterations suggests that the process indeed does not leave these final disequilibrium fixed points

From the discussion above we know what these disequilibria signify—cases where both agents want to buy/sell the same commodity so that no further trading

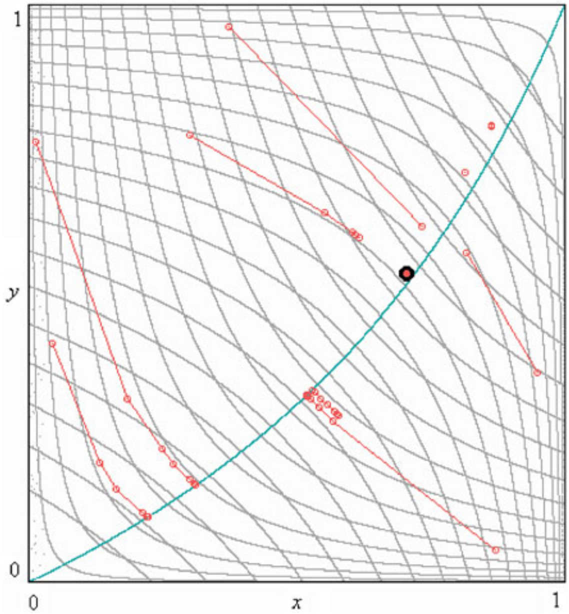

Fig. 12.3 *Orbits* of the same 10 initial points (X, Y) as in Fig. 12.2. Initial relative price is $p = 1.5$. *Parameters* $\alpha = 0.6$, $\beta = 0.4$, but now the step size is increased to $\delta = 5$. Again the *orbits* converge very fast, though to *disequilibrium* fixed points of the dynamic system where no further trade is posssible. One *orbit* which will be studied closer is marked with a *large dot*; this one never leaves the initial state. *Note* that it is not missing in Fig. 12.2, it just merges with another track

is possible. The excess demand triggered price adjustment process simply fails to reach an equilibrium point. Numerical experiment also indicates that in cases where the process seems to end up at the equilibrium curve, it only reaches a disequilibrium point in the close neighbourhood of an unstable equilibrium point.

Note that one of the disequilibrium fixed points in Fig. 12.3 is indicated by a larger dot. It represents a point where the price dynamics will be studied more closely in the sequel.

To get some more information about disequilibrium fixed points, instead of just generating a few initial phase points, as in Figs. 12.3 and 12.4, we next run the process from all initial phase points, packed as close as the resolution admits and mark just the final wealth distribution plane fixed points. As we see, they cover curves and areas in the phase plane. To make the computation manageable the number of iterations for each orbit was reduced to 5,000. The area of fixed points in Fig. 12.4 seems to gather around the equilibrium curve known from Figs. 12.1, 12.2 and 12.3, sometimes thin as a curve, sometimes swelling out to structures with nonzero area measure.

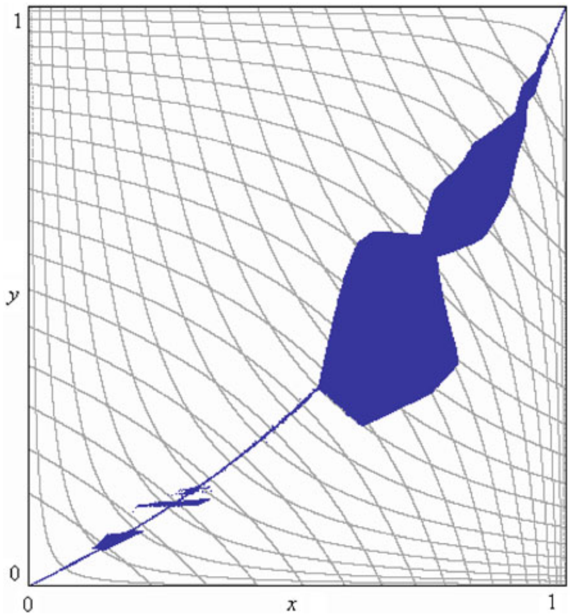

Fig. 12.4 Final fixed points of the *orbits* from all initial points (X, Y) in the *square*. The number of iterations from each initial point was reduced to 5,000. Initial relative price was again $p = 1.5$. *Parameters* $\alpha = 0.6$, $\beta = 0.4$, $\delta = 5$ as in Fig. 12.3. The *disequilibrium* fixed points agglomerate to the neighbourhood of the equilibrium *curve*, but occasionally swell out over considerable areas of the *square*

Price Oscillations

An interesting feature of the model is that the price adjustment process in a disequilibrium fixed point produces continued price dynamics, periodic or aperiodic. This is illustrated in Figs. 12.5, 12.6, 12.7 and 12.8, produced for the parameter combinations $\alpha = 0.6$, $\beta = 0.4$ quite as in Figs. 12.1, 12.2, 12.3 and 12.4. The initial price was again taken as $p = 1.5$, and the initial asset distribution as $(X, Y) \approx (0.74, 0.55)$, corresponding to the large dot indicated in Fig. 12.3, actually one of the randomly generated initial points in Figs. 12.2 and 12.3, from which the process does not take one single step.

Note that from (12.9)–(12.10) $x_2 - x_1 = (1-\beta) + (\beta - \alpha) X + p (1 - (1-\alpha) Y)$, which substituted in (12.11) $p' = p \exp(\delta (x_2 - x_1))$, gives an autonomous iterative map $p \to p'$, whenever (X, Y) is fixed, as it is in any disequilibrium fixed point.

The parameter that takes on different values in this series of illustrations is the step size parameter; $\delta = 5$ in Fig. 12.5, $\delta = 5.1$ in Fig. 12.6, $\delta = 5.2$ in Fig. 12.7, and $\delta = 5.25$ in Fig. 12.8. These pictures display the indifference maps and equilibrium curve in the phase plane; further the disequilibrium fixed point (X, Y) and a number

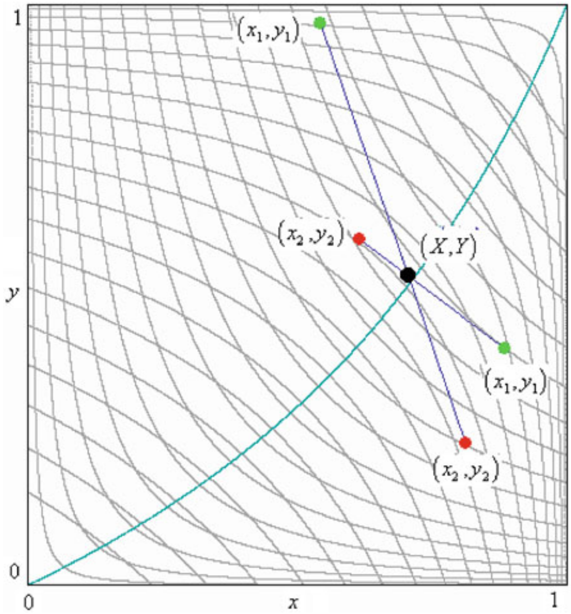

Fig. 12.5 2-period relative price oscillation. Fixed disequilibrium point (X, Y) as indicated by the *large dot*in Fig. 12.3, with initial relative price $p = 1.5$. *Parameters* $\alpha = 0.6$, $\beta = 0.4$, and $\delta = 5$. Shown are flipping *budget line* segments with endpoints (x_1, y_1) and (x_2, y_2). *Note* that they swap their positions relative to (X, Y)

of optimum points for the agents, (x_1, y_1) and (x_2, y_2). These come in pairs and are joined by line segments, two in Fig. 12.5, four in Fig. 12.6, and six in Fig. 12.7. Note that all line segments pass the point (X, Y). These line segments are actually segments of the budget lines.

In Fig. 12.8 the line segments are deleted and the end point pairs crowd dense along curves. These curves can be obtained in closed form through eliminating p in (12.5) and (12.6) respectively,

$$y_1 = \frac{(1-\alpha)\, Y x_1}{x_1 - \alpha X}, \tag{12.12}$$

$$y_2 = \frac{(1-\beta)\,(1-Y)\, x_2}{x_2 - \beta\,(1-X)} \tag{12.13}$$

These curves have been superposed on the numerically calculated trains of budget segment endpoints in order to show that this indeed is so.

The mechanism can be explained referring to Fig. 12.5. The relative price p oscillates between two different values and so the budget line flips between two different slopes. The endpoints (x_1, y_1) and (x_2, y_2), optima for the agents, are always on

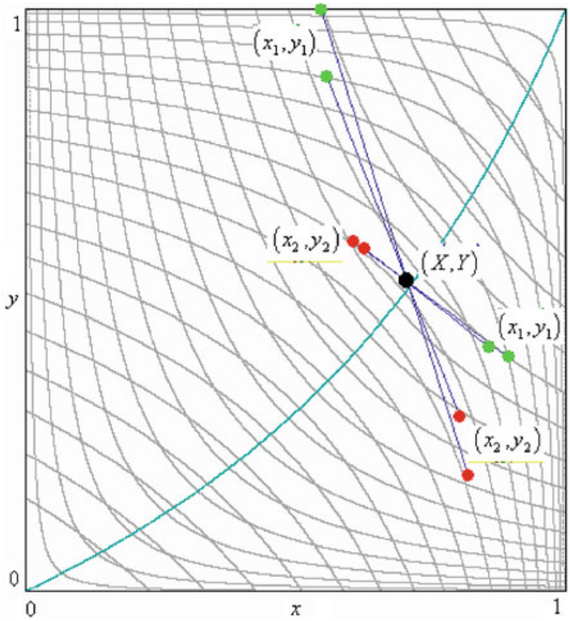

Fig. 12.6 The same case as in Fig. 12.5 but with a 4-period relative price oscillation when $\delta = 5.1$

either side of the fixed point (X, Y), but they switch positions when relative price oscillates; at one value both want to sell, at the other both want to buy the same commodity. Excess demand and excess supply alternate and the adjustment process for pricing always overshoots equilibrium. In Fig. 12.6 this 2-period oscillation has changed to a 4-period, in Fig. 12.7 to a 6-period, and in Fig. 12.8 to something aperiodic. Again the budget line flips between the different positions, and the endpoints swap their positions so that there is always excess demand or supply of the same commodity.

Bifurcation Diagrams

Step Size Bifurcations

The relative price dynamics displayed in Figs. 12.5, 12.6, 12.7 and 12.8 can be summarized by the bifurcation diagram shown in Fig. 12.9. We now display p versus δ. The initial asset distribution point (X, Y) as well as the initial relative price p were kept to the fixed values used in Figs. 12.5, 12.6, 12.7 and 12.8, as were the parameters α, β. At each value of $\delta \in [4, 6]$ the system was run for 10,000 iterations. The first 9,000 were trashed in order to get rid of transients, and the last 1,000 were then plotted. If there is a fixed point then the same p will eventually be hit over and over.

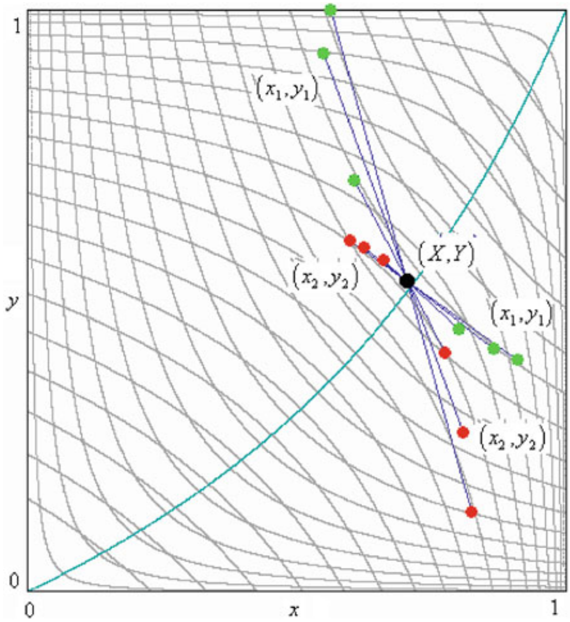

Fig. 12.7 6-period *orbit* when step size $\delta = 5.2$

We just see a point, or, considering different adjacent δ producing fixed points, a line or curve. Once the fixed point bifurcates to a 2-period cycle we see the curve split in 2 branches, and so on in a cascade, eventually seeming to cover entire areas.

We see that the case of $\delta = 5$ shown in Fig. 12.5 fits into the 2-branch region, whereas at $\delta = 5.1$ shown in Fig. 12.6, there has been a further period doubling to 4. Then, after a stretch of (possibly chaotic) intervals, at $\delta = 6.2$ there are clearly 6 curve branches complying with Fig. 12.7. For $\delta = 5.25$ a dense vertical stretch is shown in accordance with Fig. 12.8.

Bifurcations in the Utility Coefficient Plane

A different bifurcation diagram in parameter plane is produced in Fig. 12.10. Again we deal with the unit square, but now it is parameter space α, β and not phase space that is concerned. As we see the dominant shade is labelled 1, indicating fixed points. In the lower left corner there appears an irregularly concentric structure of periodicity "tongues" of a period adding appearance; 1, 2, 3, 4, 5, 6, with large gaps between indicating more complex dynamics.

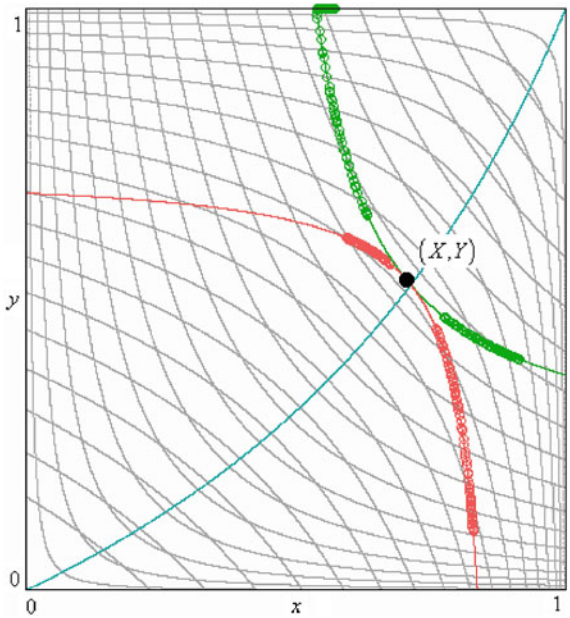

Fig. 12.8 When step size is increased to $\delta = 5.25$, the simple periodicity of relative price oscillation disappears. The points (x_1, y_1), (x_2, y_2) crowd densely on the *curves* (12.12) and (12.13), or on the edges of the box

Summary

To sum up, we suggested a unified model of stock market dynamics, the clue to which was the simple fact that stock commodities, unlike flow commodities, remain on the market from period to period. Through trade these are redistributed among the agents, which, however, changes the basis for future plans and actions. Due to this, unique demand/supply functions and market equilibria do not exist as they do in the case of flow commodities. *If* the system goes to an equilibrium, there are infinitely many to choose from, and the one on which it converges depends on the dynamic process itself. The system can also stick in *dis*equilibrium states from which it cannot move because the agents always want to sell or buy the same commodities.

Notable is that trade occurs in *dis*equilibrium states; the agents move towards higher satisfaction, but not all can reach their desired optima. In the simple two agent two commodities model, trade was limited to what the agent wanting to trade least in an actual asset distribution was willing to exchange. In this way the agent wanting to trade more could get part of the way to higher utility, lacking possibilities to force the other to exchange more than she/he wants.

Prices were assumed to be excess demand driven, and could overshoot unstable equilibrium points, resulting in complex price dynamics, periodic or aperiodic.

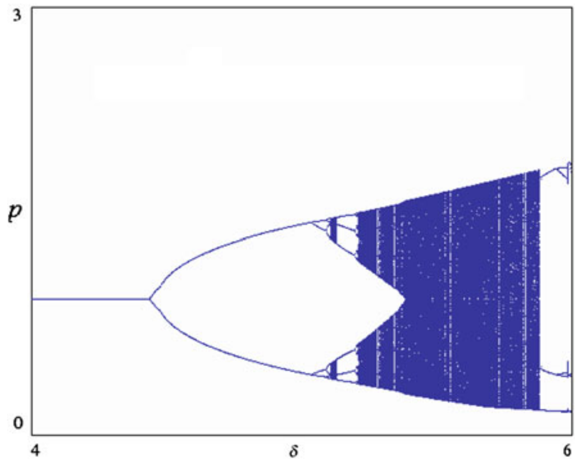

Fig. 12.9 Bifurcation diagram showing eventual relative price oscillations, periodic and aperiodic, as dependent on the step size parameter δ

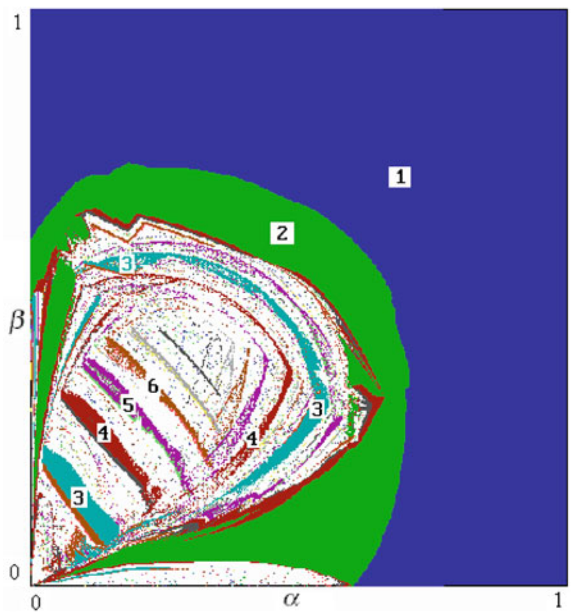

Fig. 12.10 This *picture* shows the bifurcation diagram in α, β parameter plane. The remaining parameter was fixed at $\delta = 5$, and an initial point in phase space $X = 0.25$, $Y = 0.75$ was chosen. For each combination of α, β, the system was run for 5,000 itertions after which the program checked for periodicities 1–15

A challenge to economists would be to set up a model with three (or more) agents, and three (or more) commodities.

The case for three commodities could be dealt with in a solid Edgeworth cube, with budget planes involving two relative prices. Trade possibilities could be studied in terms of the indifference surface projections on such budget planes, though it is no longer so obvious how to specify the conditions for trade. Likewise, things are much more complicated with three agents, as we need further assumptions on the success of the competitors. If there is excess demand and only one supplier, then it is clear that the supplier gets what she/he wants, but as for the demanders we must state who will come out more successful, and likewise for the other (now more than six) cases.

It seems to be important to make some advance on this neglected issues. It also is important to check the price generating process, which, after all, is responsible for the complex dynamic with overshooting. We took the traditional case of prices automatically dependent on excess demand/supply, but more realistic hypotheses concerning price formation would be highly desirable. This is as much neglected in economic theory as is disequilibrium trade.

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