Chapter 2
A Physicist's Musings on Teaching About Energy

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2.1 Introduction

Energy is at the same time a topic of high relevance for our everyday life and one of the deepest and most subtle ideas of science. When asked about examples of energy, some students list phenomena involving light, heat or electricity (e.g. Trumper 1990). Some may give examples such as energy stored in fuel (e.g. Lijnse 1990), food (e.g. Solomon 1983) or water (behind a dam) (e.g. Duit 1984). Adults may add terms like nuclear energy, solar energy, chemical energy or mechanical energy. Looking at such a list, it is very hard to see what all these diverse phenomena have in common, where they overlap and where they are distinguishable. Adding to this the fact that the way energy is described within different disciplines of science varies greatly – at times so much so that it is difficult to see connections between them – it is not surprising that energy is such a difficult concept for students to understand (e.g. Duit 1981; Driver and Warrington 1985; Liu and McKeough 2005; Neumann et al. 2013; see also Chap. 5 by Duit, this volume).

Students' need to know what energy is often leads to them being acquainted with a simple definition of energy (cf. Papadouris and Constantinou 2011). The teaching of simple definitions to students is based on the misconception that we learn words and concepts by being told their definitions. In fact we learn them by experiencing and applying them in multiple contexts (cf. Bransford et al. 2000). Any definition of energy at the macroscale level that would be general enough to be correct is either vague enough to be worthless, or contains a long list of "forms of energy" that seem so disparate that no concept can be abstracted from such a definition. This may lead to frustration in both teachers and students Perhaps it helps to discuss the fact that

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science did not arrive at the concept of energy by defining it, but rather by exploring it (see for example Coopersmith 2010), and that this the path that learners must take too, in order to understand it.

Different forms of energy are measured in different units because they were discovered and categorized at different times. Conservation of energy, first applied only for special and idealized cases (conservative systems), emerged as a more general principle as the relationships and transfers between the different types of energy and the conversion factors between their measures were recognized, and the deeper mathematics behind equations of motion explored. As always in teaching science, we need to untangle the ideas from their history, and decide when recapitulating the historical development of the idea is helpful to students and when it simply immerses them in confusions that they do not need to repeat to get reach a conceptual understanding of the topic being taught as it is understood today. In teaching about energy it is also important to make connections between the concepts related to energy used in different disciplinary contexts, as well as the everyday meanings of the word.

Physicists talk about kinetic and potential energy, using gravitational potential energy for most of the examples of energy transfer they introduce at the high school level, or perhaps elastic potential energy in a spring. Electrical potential differences are introduced in different units and used only to talk about electric circuits. What do they have to do with potential energy? Power is introduced with its own units; the fact that it is a rate of energy flow is not transparent. Energy concepts related to electric and magnetic fields are not discussed till advanced undergraduate courses. Mass-energy equivalence through $E = mc^2$ may be introduced in high school physics in the context of special relativity or nuclear processes, but the true generality of this relationship is seldom stressed. Physicists have adopted a convention that the term heat can only be used for energy transfers between systems, whereas for almost everyone else heat means thermal energy, whether or not it is being transferred. The deep inter-relationship between energy and forces is seldom introduced until advanced undergraduate courses, but the capacity of forces to transfer energy is stressed in introductory physics introducing the added concept of work, which is sometimes presented as a way to define energy (the capacity to do work) which is not particularly enlightening. Chemists talk about bond energy. Nuclear physicists use the term binding energy. Biologists and earth scientists talk about chemical energy, or food and fuel as sources or stores of energy. Engineers talk about electrical and mechanical energy and about energy conversion. Where in all this terminology is a student to develop a coherent concept of energy?

2.2 The Particle Physicist's View of Energy

As I am a particle physicist, the view of energy and matter at the smallest scales informs my thinking. I discuss it here, not because I think we can teach this view as the starting point for understanding energy, but because I think discussing this level of understanding energy allows us to think about what to teach, and when, in order for students to be moving over time towards a deeper and more consistent understanding of energy (i.e. it will prepare the ground for developing learning progression of energy).

At the level of quantum physics, or even advanced classical mechanics, we find that to define energy is to write the laws of nature. If we can define how the energy of a system depends on the relative positions and motions, and on the charges and masses, of the particles within a system, then we can predict (at least probabilistically) how that system will behave. The quantity (technically the Hamiltonian or the related quantity known as the Lagrangian) that describes and defines energy in a system is what determines the laws of physics (i.e. the equations of motion) for that system.

At the atomic or subatomic scale, energy has two basic components, it is either kinetic energy or energy stored in the interaction fields (electromagnetic, gravitational or subnuclear) between the particles. Electromagnetic radiation provides a tricky bridge between the two, because it can be described either as massless particles (photons) which nonetheless carry kinetic energy, or as time-changing and travelling electromagnetic fields carrying energy across space. Both descriptions say it carries energy from place to place, and which is most appropriate to use depends on the situation.

The energy of any system is built up out of these fundamental forms of energy, the motion and interaction energies of the fundamental particles it contains, just as matter is built up from those particles. At different scales it is convenient to describe both the structure of the system and the energy it contains in different ways. However, in the end, I think that, just as we cannot understand many properties of matter without atomic and sub-atomic understanding, we cannot clearly understand many of the commonly used terms for forms of energy until we break them down again and into the underlying particles and their interactions.

The fact that total energy is conserved is a fundamental theorem at this scale, closely linked by the magic of mathematics (Noether's theorem) to the fact physics does not depend on the time, location, or frame of reference. If we write a theory of matter and its interactions for which the function that describes energy has these desirable (and observed) invariance properties, it predicts conservation of energy and momentum among its consequences. However the mathematics that underlies these statements takes us well beyond high school mathematics, so the law of conservation of energy must be presented as a rule which has little empirical support. It is truly difficult to measure all forms and flows of energy, and so any demonstration of the law is at best approximate. While they may be able to see it as a limiting case, that is as true for an idealized system, students have no way to know that it how exact and general a law it is, except by being told it.

Perhaps the most widely recognized and least understood formula in all of science is $E = mc^2$. Most people, including Ph.D. level chemists and biologists, think it is something that only applies in nuclear physics. Instead it is a deep statement that says the quantity we call mass and the quantity we call energy are in fact indistinguishable. (The c^2 in the relationship is just an expression of the fact that we measure them in very different units.) The relationship tells us that, as viewed in the rest frame of the center of mass of any system, what we define and measure as the mass of a system is not just the sum of the masses of the particles that it contains. It includes all forms of energy within it. From outside the system, without probing inside it in some way, there is no measurement that can tell whether the system has a large mass because it contains some high mass objects, or because it contains less-massive but rapidly moving objects.

Indeed as we go to the most fundamental theories we find that most of the mass of protons and neutrons, which means most of the mass of any matter made from atoms, arises from the kinetic energy and interaction energy of the quarks within the protons and neutrons. The sum of the masses of the quarks is only a small fraction of the proton or neutron mass. (Even the quark masses appear as interaction energy. They are due to the interaction of the quarks with the omnipresent Higgs field.) Thus the notion that mass is anything other than an accounting of all energy within a system (when the center of mass of the system is at rest) disappears. Furthermore, for a moving object or system, the division of the energy of a moving particle into two parts, mass-energy (mc^2) and kinetic energy ($1/2\ mv^2$) turns out to be a low speed approximation to the more complete statement of Einstein's formula, which can be written as $E = mc^2/(1 - v^2/c^2)^{1/2}$. In this relationship mass-energy and kinetic energy for a moving system are not separable, but are inextricably intertwined.

While the equivalence of mass and energy is essential to gaining a fundamental understanding of energy, and of conservation of energy, it is irrelevant for most practical purposes, and certainly in the most of school science. In all but nuclear physics situations we do not need to discuss it. We simply leave out mass-energy in all our calculations of energy, because it is a large quantity that, if we are careful about the rest of the accounting, we can treat as a constant. This has an important consequence. Once we have excluded some energy we can never talk about total energy; we can examine only examine changes in energy. However if we are going to discuss conservation of energy as a system changes, we need to be sure we maintain a consistent definition for the energy we have excluded from the accounting.

Kinetic energy for a moving and unchanging object is relatively easy to describe, what is much harder for students to conceptualize is all the various forms of potential energy. In particle theories these all come down to energy stored in fields, relative to that in some reference situation. Theories of fundamental physics are built on a mathematical model in which the interactions between particles are mediated by fields. These fields are essential for modeling the mechanism of forces between distant objects and for modeling interaction energy, and the related concept of potential energy. The key idea is that these fields exist and vary across space, contain energy, and can transfer energy between distant objects. While they are invisible, their presence can be measured by their effect on a test charge or magnet, or in the case of gravitational fields, a test mass, placed in the field. The concept of a force field requires careful qualitative development. It can be introduced well before students are prepared to treat such fields mathematically. Even if students have a vague and science-fiction-based idea of an invisible force field (e.g. Adrian and Fuller 1997) this can be used as a starting point.. The concept can be refined and shaped as students experience phenomena, such as the effect of a magnet on iron filings, or "static electricity," that can be described and explained in terms of fields.

Without the concept of the fields, the interaction energy between the objects is not attached to anything and does not have any location that can be included in the students' mental models of phenomena. In this situation observations that masses speed up as they fall, and that magnets move things without touching them, appear to contradict the notion of conservation of energy. Students tend to conceptualize energy as a thing (e.g. Duit 1987). Physicists conceptualize it as a quantity that can be associated with things, and transferred from one thing to another, but which itself is not a substance[1]. Of course, force fields are not substances either, put they do have a detectable physical reality, that perhaps makes them more readily conceptualized than energy itself. This needs study. How can the concept of interaction energy as energy stored in the space between the interacting objects best be modeled for students? What experiences and activities help students develop this concept? At what stage can potential energy be conceptualized as a difference in interaction energy compared to a reference situation? When does the concept of a force field help, when is it just another meaningless set of words?

2.3 Descriptions of Various Types of Energy

I now examine many of the everyday terms used to describe energy. They overlap and are not generally well defined. It is useful to clarify what they represent and when they are useful. In most cases, as far as I can see, it is not useful to try to define them more precisely – when precision is needed we can achieve it without most of these terms.

2.3.1 Thermal Energy

Many students do not distinguish between heat and (thermal) energy (e.g. Kesidou and Duit 1993 or Maskill and Pedrosa de Jesus 1997). In strict physics definitions this is not acceptable, physics uses the term heat only for energy transfers, and not for energy present in a system. One reason for this is that, as discussed above, total energy present is not a useful concept in most situations, and furthermore it can be difficult to decide what part of that total energy in a given situation should be labeled as thermal energy.

[1] "... in physics today we have no notion of what energy is. We do not have a picture that energy comes in little blobs of a definite amount. It is not that way." (Feynman et al. 2011, pp. 4–1).

Indeed until one has a clear particulate understanding of matter, thermal energy cannot even be described. At the particulate level, it is often described as the energy of random translational motion of particles within a system; that is, as kinetic energy. However this description is only true for an ideal (non-interacting) monatomic gas. Whenever we have molecules or solid matter present, thermal motion also includes rotational motions of the molecules and vibrational motion of the atoms in a molecule. If we look more closely we see that the potential energy of interaction of atoms within the material is changing all the time as the atoms vibrate. Energy is constantly being transferred between the atomic motion and the potential energy between the atoms as the molecule stretches and contracts. A little thought makes it clear that if these changes in potential energy were not included in the definition of thermal energy, thermal energy would fluctuate as a molecule vibrates. That would be a most inconvenient definition. So, except in the ideal gas of non-interacting atoms, thermal energy must include some potential or interaction energy as well as kinetic energy.

As soon as we introduce interaction energy we are into the morass of defining energy relative to some fixed condition. Any set of interacting masses and charges has a total energy that depends on the relative positions and motions of the charges and masses, but we seldom need to know or care what that total energy is, in fact we only need to know how it changes when the positions and motions change. In principle we define absolute zero temperature (0 K) to be the temperature at which there is no thermal energy, but since we cannot actually get anything to that temperature that is more a theoretical statement than a practical one. For practical purposes we can relate changes in temperature to changes in thermal energy per unit volume, or per mass of material. With the exception of the ideal gas case, this relationship cannot be easily predicted but rather is extracted from measurements, and it is different for different substances.

The fact that it takes different amounts of heat to achieve the same change of temperature for the same mass of two different substances makes it clear that temperature cannot be measure of energy, or even of energy per unit mass. Students initially conceive of heat and temperature as much the same thing (Kesidou and Duit 1993), after all both have to do with getting hot! Learning to distinguish them and to understand their true relationship is an essential step in reaching a clear view of thermal energy. Many textbooks discuss the relationship only for an ideal gas, which elucidates only a part of the complex relationship.

The concept of heating as an increase in thermal motion clearly breaks down when we consider what happens as matter transitions from solid to liquid, where the energy of interaction between its constituent particles changes significantly. Ice at zero degrees has less energy than the same amount of water at zero degrees, as can be seen by the fact that it takes energy to melt the ice. The water molecules in ice are bound together into a solid. The energy needed to unbind them (that is to break the inter-molecular bonds) is called the latent heat of melting. This is amount of energy we must add to melt a given quantity of ice. This makes it a bit tricky to compare “thermal” energy of ice with that of water. The added energy has broken the bonds that formed the ice crystal. Likewise a change in interaction energy takes place as matter goes from liquid to gas, again energy is added without a change of temperature to achieve the change of state. This energy is called the latent heat of evaporation. So should we call those changes of state changes of thermal energy?

We simply do not need to try to answer to that question. It is a choice, just as defining what part of the energy we remove from the problem by calling it the mass of the system is a choice. Just like total energy as a whole, total thermal energy is not generally a useful concept. (Indeed to completely define the mass of a system you have to define not only its configuration, but also its temperature, because thermal energy too contributes to mass-energy. All of this is generally irrelevant for the problem at hand for K-12 students.) The idea of thermal energy is useful for talking about changes in a system, and where energy goes when it leaves a system, but not for calculating absolute quantities of energy.

2.3.2 Chemical Energy

In any chemical process the set of atoms present does not change, so the mass-energy of the atoms present is constant and thus irrelevant for any energy changes that do occur. Any chemical process takes a set of molecules and converts them to a different set, with different bonds between the atoms. With this in mind we understand why chemists focus on differences in total bond energies to explain energy released or captured in a chemical reaction.

All bond energies are negative because the stable molecule has less energy than the separated atoms. This can also be understood by looking at the electromagnetic fields due to the charged substructure of the atoms, and how the total energy stored in these fields can be reduced by bringing atoms together and "sharing" some of the electrons between them. Actually calculating such changes in energy from first principles is a complex quantum chemistry problem. The language of chemical bonds and bond energies is a useful shorthand to describe the results of such a calculation, or of measurements of energy differences. However it is completely wrong to talk about energy stored in a chemical bond – every chemical bond is a shortage of energy. So what do we mean by chemical energy?

Generally we mean some energy that has been, or could be, released in a chemical process. The energy captured or released in any chemical interaction is the difference between the sum of the bond energies before and after the reaction. Released energy typically manifests itself as increased thermal energy. The energy captured in the inverse process can come from thermal energy or from other forms such a sound energy or radiation. If energy is released, it is because the molecules after the reaction are more tightly bound than those before the reaction – the resulting molecules between them have a greater shortage of energy than the starting ones (compared always to the separated atoms). Thus the term chemical energy is, like thermal energy, not easily defined in any absolute way. All we care about are the changes. It is not meaningful to talk about total amounts of chemical energy, but it is meaningful to talk about the amount of energy released or captured in a particular chemical process.

In everyday language we say a battery converts chemical energy into electrical energy, or that food or fuel contains chemical energy (Calories), but neither of these statements is particularly precise. There is certainly stored energy in a battery, energy that can be released through a chemical process that occurs when the terminals are connected to a circuit. For rechargeable batteries energy can added to the battery by driving that process in the reverse direction with an external electric power source. Since we charged up the battery using an electric current should we now call the stored energy electrical energy, or have we converted it to chemical energy? Does it matter what we call it?

Biologists talk about food or biomass as having chemical energy, or as a source or a reservoir of energy, for example when discussing food webs or photosynthesis. We all do the same when we talk about fuel as a source of energy, or about the number of calories we eat, (A calorie is a unit of energy, defined as the amount of energy needed to heat 1 g of water by 1 °C, the ones we eat are actually Calories, that is to say kilocalories). In fact the food or fuel only provide energy by reacting with oxygen. Saying that the food provides the energy ignores the critical role oxygen plays in the energy balance of the chemical processes of combustion, respiration and photosynthesis. Oxygen (in the form O_2) is removed from the atmosphere (or from the ocean, lake or river) in reactions that release energy, and is added to them in photosynthesis, a process that captures energy from the sun to drive the reverse chemical reaction.

In calculating the energy changes in the processes that turn O_2 plus hydrocarbons into CO_2 plus water (combustion, respiration), or the reverse process (photosynthesis), the changes in the oxygen bonds are an important part of the energy balance. So technically it is incorrect to say the energy either comes from or is stored solely in the food or fuel. However, from a practical point of view, in an oxygen rich environment, the availability of food or fuel controls the availability of energy, so the language, while imprecise for understanding energy, is useful for understanding a food web or the societal needs for fuel. (Of course in oxygen- poor environments organisms rely on different set of chemical processes to release energy for their needs, but the principle that it takes a chemical process, not just one of the reactants, to provide the energy is the same.)

If the inter-dependence and competition between species in a food web can be understood by young students as interdependence in obtaining food, will it help them to discuss it in terms of energy when they really have little idea of what energy is? The stress on energy arises, I think, from the importance of the energy from sunlight for the development of biomass from air and water. Beyond that the food web model says little about energy, at least at the level it is presented to students. Much of the energy flow is at best implicit in the food web model.

For students to connect ideas about energy across the disciplines, in particular between chemistry and biology, it may help if the "food is energy" language were avoided. Can we discuss the food web as a biomass flow rather than an energy flow in the system? Can we say biomass or food provides organisms with access to energy for life functions, rather than that is or provides energy? Even very young students are likely to know they need to breathe as well as eat, when do we connect the need for oxygen with the need for energy? When do we introduce the idea of chemical changes as processes that can release or capture energy? That can be demonstrated as a phenomenon well before the atomic level chemistry is accessible to students.

What is puzzling, and indeed tricky, about the energy captured in photosynthesis is where the captured energy is stored. It is in both of the product materials, and is in fact part of their mass-energy. But as we are leaving mass energy out of the problem, all we can say is that the products of photosynthesis have more stored energy than the reactants did because the product molecules have less negative total bond energies. Calling the difference chemical potential energy is fine, because we see the energy can be released again in the reverse chemical interaction during cellular respiration, but saying any one substance has stored chemical energy eventually leads to confusion. Every chemical bond is a lack of energy. Those negative energies are a puzzle to most students, even at the high school level (e.g. Boo 1998). Certainly they are not needed to understand ecosystems. The shorthand of saying the energy is stored in the biomass simplifies the discussion of the ecosystem. However at some point it may begin to confuse the students. Probably somewhere in the middle school grades, discussion about differences in usage becomes important, acknowledging that, from the point of view of chemistry, the biological terminology about energy is imprecise. The differences in usage must be discussed in order for students to link their thinking about energy in chemical change to their thinking about energy in ecosystems and living organisms.

It is important to recognize that Ph.D. level biologists talk of biomass as energy in an ecosystem, and Ph.D. level chemists think conservation of mass is exact in chemical processes because they do not ever think in terms of the masses of molecules. These conventions are deeply embedded in the language of these disciplines and we cannot change them by changing how we teach at the K-12 level. However what we can do is be aware of the barrier to understanding that these differences across fields can create for students and help diminish that barrier by being explicit about these differences.

This conclusion, that one must discuss the fact that words have multiple meanings, and are used differently in different situations, is one of the major realizations for me in thinking about teaching energy across disciplines. A word may have very particular restricted usage and definition in a certain area of science, but we cannot say that is the only correct definition of the word. It has other meanings in everyday usage, and still others in other areas of science! Part of learning to "talk" science is learning to understand when the restricted definition is being applied, and when the word is being used in a related but less strictly defined fashion. All students can benefit from a discussion of language such as this, but it has particular value for those students whose home language is not the language of instruction.

2.3.3 Mechanical and Electrical Energy

Like chemical energy, mechanical energy and electrical energy are imprecisely defined though commonly used terms. Consider an operating machine with an electric motor (say an electric toothbrush) that is driven by a battery. Does it have mechanical energy, electrical energy, or chemical energy? Perhaps we can agree that it has some of each, but can we define how much of each? Rarely do we care! The brush moves, electric currents flow and the battery runs down through a process of chemical change. We could just as well say the system has motion energy and potential energy, we do not need to define the terms mechanical, chemical and electrical energy to describe it.

Any machine operates with some energy source, often either a chemical process or an electrical one, carries out some motions, and in the end stops, with some objects possibly moved to new locations and different stored energy. The term mechanical energy generally refers to the energy of the moving parts of the machine, but may include elastic or electromagnetic potential energy (such as that of a stretched spring) or even gravitational potential energy that plays a role in the cycles of that particular machine. In my opinion we really never need the term mechanical energy in a science class. Eliminating it is easier than defining it. Eliminating it does not mean ignoring it. As with all everyday terms that overlap with technical terms, students need to discuss the imprecise nature of everyday language in order to understand why scientists introduce and carefully define the new terminology, in this case the terminology of kinetic and potential energy.

Electrical energy arriving via the power grid seems to be one of the biggest mysteries for students (c.f. Stocklmayer and Treagust 1996; also see Bodzin 2011). When energy moves from the power plant to your house over the power grid, given that the grid is alternating current, electrons do not flow from one place to the other, they simply move back and forth in the wires. The kinetic energy of their motion is tiny. However because electrons carry electric charge, when they move the electric and magnetic fields around them change. These changing fields and their effect on matter or magnets are what heat your toaster, light your electric light, ring your doorbell, or drive the electric motors in your blender or can-opener. So we say that these devices are driven by electrical energy. Keeping track of where that energy resides when your appliances are turned off is a bit messy. Eventually it is transferred from the system that drives the generators at the power plant to the system you are using, and you pay for the amount that flows through your meter, without concerning yourself about where it was the moment before you flipped your switch. Modelling these systems and the fact that energy is transferred between them via the power grid is more useful than trying to model where the energy resides at any instant.

Students hear, learn and use all of these imprecise terms; for everyday uses they are quite adequate. The question for teaching about energy is whether and when it is important to define them or eliminate them – when does striving for precision add clarity, when does it just confuse? Clearly a transition to thinking about energy in terms of motion and interactions at the particulate level cannot precede the same transitions in thinking about matter. But can we use a little care and avoid reinforcing the misconceptions or contradictions of everyday language around energy? Can we discuss everyday terms without seeking to artificially define them to try to make them more scientific?

2.3.4 Conservation of Mass?

Historically and practically it is important to chemists to emphasize that mass is conserved in chemical processes. Well before anybody understood the variety of elements or the nature of their atoms, chemists had observed this fact. In trying to understand any process knowing that something is not changing is a very important step because it severely delimits possibilities. Even alchemists did not try to transmute light substances into gold, they knew that was impossible! With a modern atomic view we can see that the law of conservation of mass and the law of constant proportions in chemical processes can both be understood as consequences of the law of conservation of atoms in chemical processes. These empirically-discovered laws preceded, and helped lead to, our understanding of atoms. Furthermore we can readily measure masses of reactants and products but we cannot so readily observe atoms, so conservation of mass remains important as a phenomenon that students can observe.

However the statement that mass is conserved in all chemical processes contradicts the relationship $E = mc^2$ from physics. Conservation of energy and conservation of mass cannot both be exact in chemical processes. Kinetic energy changes in such a process. If mass does not change then some energy has appeared from nowhere. How can we resolve the discrepancy? Only by giving up conservation of mass as a principle.

To get a consistent view across disciplines, it is necessary to conclude that the mass of a molecule is actually a tiny bit less than the sum of the masses of the atoms it contains, by exactly the binding energy of the molecule divided by c-squared. Differences in binding energies are accompanied by differences in mass-energy, and thus in mass. However, the difference between the mass of a molecule and the sum of the masses of the atoms it contains is such a tiny fraction of the mass of the molecule that it is not measurable by any chemical balance. Furthermore the large difference in scale between the mass of the atoms and this mass difference makes it very inconvenient to discuss both in the same the units. Obviously, since atoms are conserved, the sum of the masses of the atoms is constant in any chemical process. Chemists therefore say mass is conserved and talk only about energy differences, that is differences in binding energy. They never actually discuss the mass of the molecules, or if they do, they treat it as being the same as the sum of the masses of the atoms, which it is to the accuracy of their measurements.

Even Ph.D. level chemists may be shocked by the idea $E = mc^2$ applies to molecules in this way, but eventually agree, that, while not measurable by their methods, this may be true in principle. From the chemist's perspective this is a totally irrelevant fact. From the perspective of gaining a common understanding of phenomena of different types I think it is critical. At the point when students are learning the meaning of $E = mc^2$ in physics, this issue needs to be discussed.

2.3.5 Energy Flows (Convection, Conduction and Radiation)

Energy moves from place to place in three generic ways, through movement of matter, through energy transport within matter without bulk movement of the matter (conduction), and through radiation. These mechanisms cannot be described with any precision before students have a particulate view of matter.

Whenever a local source heats a region the thermal energy so produced tends to be spread around by more than one of these mechanisms. Which one is the dominant effect depends on the situation. Students are often asked to say (or told) which of the three occurs in a sample situation, even though the situation, viewed in detail, actually involves more than one. Take for example a room warmed by a radiant space-heater –does convection or radiation dominate? – that probably depends on where in the room you are standing. Yet students are given this as an example of radiation. I think this kind of oversimplification confuses rather than clarifies. It would be much better to allow students to have a nuanced discussion to decide which type dominates than to present these as mutually exclusive options.

Obviously any moving object carries energy from place to place as it moves, since motion itself is a form of energy. In fluids energy can be moved around by a flow of hot fluid from one place to another within the fluid. When this occurs as a cycle driven by a heat source and gravity, and perhaps also by earth's rotation, we call the flow a convection current. Locally heated fluid rises because it is less dense than unheated fluid above it. Cooler fluid flows in from the sides to replace it, only to be heated in turn by the heat source, and thus to rise, setting up a flow pattern. In a spinning earth, its oceans or its atmosphere, earth's rotation also contributes to the patterns of the flows. The patterns of the winds, and of ocean currents, as well as the flow of fluidized rock deep within earth's crust are all important in earth's systems. Understanding and modeling these flows of matter and of energy are an important part of the earth sciences. Thus in earth sciences physical, chemical and even nuclear processes deep in the earth's core play complex and intertwined roles in understanding and modeling matter and energy flows. How and in what detail these phenomena can be treated depends on the order in which students are presented with the different disciplinary ideas, but whatever the order, if teachers do not make linkages across the disciplines and untangle different conventions for talking about energy within the disciplines, the students can not be expected to do so.

Radiation, the third type of energy flow, seems perhaps the most mysterious to students particularly when it is not visible (e.g. Libarkin et al. 2011). Any object is constantly radiating and absorbing electromagnetic radiation to and from the surrounding environment. If the object is hot enough we can see this radiation as a red glow, or hotter yet a "white hot" glow, but even objects that do not glow in the visible part of the spectrum are emitting radiation, just at longer wavelengths than those we see. Plants and animals glow in the infrared, as can be seen using infrared sensitive detectors or film. Night vision goggles take advantage of this effect.

The fact that light transports energy can be connected to the fact that we feel it as warmth when it is absorbed in our skin, but the relationship between visible light and other electromagnetic wavelengths is not obvious to young children and cannot be made so until they are well adapted to abstract models for scales that they cannot see. Models of matter at the scale of atoms and their substructure need to precede and inform models of how matter can produce and absorb electromagnetic radiation, and models for that radiation as it travels across space.

The term radiation carries a negative notion for many students because some radiation is both invisible and dangerous to our health (e.g. Millar 1994). This includes short wavelength electromagnetic radiation, where each photon carries enough energy to ionize atoms in our bodies. Most of the ionizing radiation from the sun is absorbed when it ionizes atoms in the upper atmosphere, but some ultraviolet penetrates to earth's surface and can cause sunburn and possibly skin cancer to those over-exposed to it. X-rays are even shorter wavelength and more dangerous ionizing electromagnetic radiation, and gamma rays are even more extreme.

Radioactivity introduces a different confusion around the word radiation. Some nuclear decays indeed produce ionizing electromagnetic radiation (gamma radiation). Other nuclear decay processes produce fast moving particles such as helium nuclei (alpha radiation), neutrons, or electrons (beta radiation). These are all matter particles, but when produced by nuclear transitions they are generically and confusingly referred to as nuclear radiation, and the source nuclei as radioactive. This terminology predates any understanding of the nature of the produced particles but persists in both everyday and nuclear physics usage today. Indeed, these energetic particles too can cause tissue damage and ionization, so from a medical perspective they are likewise described as radiation and assigned dose limits for safety. However from the point of view of trying to clarify different ways that energy is transmitted, these are massive moving particles, and the term radiation means electromagnetic radiation. How confusing is that? Again the contradictory terminologies cannot be avoided, so must be discussed.

2.3.6 Nuclear Energy

Nuclear energy is yet another poorly defined term. It is often used to mean electrical energy produced by a nuclear power plant. We could define it to mean energy released due to either nuclear fusion of nuclear fission processes. This energy first appears as motion of product particles or radiation, and then, in the power plant example, gets used as a way to heat water to drive a steam turbine to produce electric power.

All nuclear processes depend on one or other of the two nuclear interactions, strong and weak interactions. One characteristic of these processes that makes them notable is that the changes in stored energy are large enough that the changes in mass are a much larger fraction of the mass present than in a chemical reaction. Hence it is in nuclear processes that the equation $E = mc^2$ is usually introduced as an explanation of where the energy released came from – it came from a reduction in mass. But equally it can be described as coming from changes in interaction energy within the nucleus (e.g. in alpha decay), or even within the nucleons (in beta decay). Since this interaction energy is measurably included in our definition of the mass-energy, and hence the mass, of the nucleus or nucleon involved we are forced to say that mass changes in this case.

2.4 Key Energy Concepts for K-12 Science Education

I now turn to discussing the four key ideas about energy that I think can be taught to K-12 students. As in all science, one big part of this teaching must be to clarify and stress the distinction between technical usage of words and everyday usage of words. Indeed as discussed above, it turns out that to make connections across disciplines, you also have to understand that the term energy is used differently in different science disciplines, and so you also need to understand those differences as you try to understand the language (or rather languages) of science.

2.4.1 Only Changes in Energy Matter (Who Cares How Much You Have if Most of It Is Not Negotiable)

While energy is not a substance, it has one thing in common with matter as viewed at the K-12 level; both are conserved quantities – stuff we can neither make nor destroy. When we talk about energy transfers or energy flow, it can lead students to conceptualize energy as a material thing (Warren 1983), which it is not. Perhaps it would help to compare it to net worth, which can be held or transferred in many ways (of which currency is only one) and for which it is important to keep track of its coming and going through a system of book-keeping. The net worth of a school district includes the value of its physical plant, the schools and (usually) the land they stand on, but in deciding the budget for the coming year, most of that is fixed and not negotiable, so the total net worth of the school district does not matter, what matters is its projected income for the year, and its plan for spending. Keeping track of energy is like keeping track of a budget in that way. (Perhaps we could go even further with the analogy and think of kinetic energy as cash, and potential energy as money in the bank.)

When we discuss energy in any situation we are actually only concerned with changes in energy – how much it transferred between objects or systems, how much is captured or released during any change in the system. The absolute energy of the system never matters, unless we are trying to build the system, to create its massive matter, from energy alone. That only occurs in particle physics collisions where we collide and annihilate matter and antimatter at high energies and produce new particles and antiparticles with different masses. In any other situation we start with some matter, and it undergoes some processes, but the mass of the matter is not changed by a significant amount, except when nuclear processes occur. In all non-nuclear cases it is convenient to treat the mass-energy quite separately from other forms of energy, and to leave it out of the book-keeping for energy altogether. So, while the principle that mass is energy is general, and to my mind critical to full understanding, in most cases it makes sense to treat mass and energy as separate concepts. Mass-energy is not included in the definitions of energy for a train or a car. If we were to include it, it would be a large constant energy in any process. Then, in order to look at any other forms of energy we would be calculating differences that are tiny fractions of the whole. That is always inconvenient. Much better to take the large constant mass-energy out of the problem and deal only with changes.

Obviously, once we are leaving out one of the aspects of energy, we can never talk about the total energy of a system, only about changes in its energy. Even then, the way we describe and account for these changes depends on the scale at which we are describing the system. It also depends on the choice we make in order to define the mass of the system, that is the part of the energy that we want to remove from the equations. There is always arbitrariness to this choice. We must choose some reference situation, which we define to have zero potential energy. Whatever interaction energy, indeed whatever energy of any type, is present in this reference situation is to be included in the mass of the system and removed from the energy accounting problem. One consequence is that potential energy will sometimes be a negative quantity in our equations. This can be very confusing to students (Stephanik and Shaffer 2011). But what does negative energy actually mean? It simply means the system has less interaction energy than the reference system which we arbitrarily chose to define as the zero potential energy case.

As an example of this arbitrariness let us think about the system that consists of a mass hanging from a spring, bouncing up and down in the gravitational field at the surface of the earth. To study energy changes during the motion of this system we must consider how three things change – the kinetic energy of the mass, the gravitational potential energy of the mass, and the energy stored in the spring. The energy of the spring we separate into two parts, a constant mass-energy which we want to remove from consideration, and interaction energy differences relative to that, which we call the elastic potential energy. We have to pick a reference length of the spring to define its mass-energy and thus fix the zero value the elastic potential energy terms. We also have to pick a location for the mass at which we define its gravitational potential energy to be zero. There is no "right" choice.

Suppose we choose our reference position for both parts of the calculation to be the one where the spring has its relaxed, unweighted length. In that case the elastic potential energy will always be positive, because when it is stretched or compressed relative to this length the spring has added energy. However the gravitational potential energy will be sometimes be negative and sometimes positive relative to this position, depending on whether the mass is below or above it. Alternately, we could choose the lowest point of the motion as the reference point, the point that we pull the mass down to before we let it go to bounce up and down. Then gravitational potential energy will always be positive but elastic potential energy will be negative relative to this situation, because the spring is less stretched anywhere else in its motion. There are other possible choices. Each choice changes the equations we write, but not the basic underlying fact that changes in one type of energy are balanced by changes in the other two types, or by the loss of energy to the surrounding environment, which eventually will bring the mass to rest at a position that is different from the one where we let it go. We can even separate the two definitions, and define the elastic potential energy relative to one location and the gravitational potential energy relative to another – that may be confusing and certainly takes is careful book-keeping, but it is not wrong. In no case is it meaningful to talk about the total energy of the spring-plus-mass system, because it is interacting with the earth. Gravitational potential energy is part of a larger system spring-plus-mass-plus-earth.

It does not matter which choice we make – each has advantages and disadvantages. We just have to be clear about our choices and keep them consistent throughout our treatment of the problem. In principle, in each choice the spring-plus-mass system has a different mass-energy with the hanging mass held at rest at our reference point. Indeed, the larger system earth-plus-spring-plus-mass also has a different mass-energy at each choice of reference point. For either system the differences in mass between the different choices are too tiny to measure with any mass-measurement that we could make (and anyway for a system as artificial as "earth-plus spring-plus-mass" the mass of the system is never separable from lots of other mass and energy in the world around it). But we do need to recognize that for each different choice we make for defining the zero of any type of potential energy, we have decided to drop different parts of the energy out of our equations. While the changes in mass are a tiny fraction of the mass present, the changes in both gravitational and elastic potential energy are important for our problem, so we must carefully define the reference situation in order to write our equations for energy.

The idea that a reference system is needed to define what part of the energy we remove from our problem as a constant, and what part we treat as potential energy in our problem is seldom clearly introduced. Students are told that the zero of potential energy is arbitrary and can be chosen as they wish, but not that this is because they have in effect defined whatever energy is present in the reference situation as part of the mass of some object or system, to get it out of the way, and to avoid the irrelevant and highly complex question of total energy.

2.4.2 Any Change in Energy Is Balanced by Some Other Change in Energy (You Can't Make or Destroy Energy, Only Move It Around)

With total energy removed from consideration, conservation of energy becomes the statement that any change in energy is balanced by some other change in energy. In any system energy can be transferred between the components of the system, or between motion and interaction energy within the system, or it can be transferred into or out of the system. Keeping track of energy requires keeping track of all these things.

In everyday language we talk about producing and using energy –what we actually mean by producing energy is producing fuel that we can move around and burn to release energy when and where we choose, or generating electricity, which also serves to allow us to move energy around to use when or where we need it. To "use energy" means to use the fuel or electricity to provide energy to do whatever it is we want to do (move, keep warm, produce light). Once we have "used" the energy it is not gone. Energy always ends up dispersed into the surrounding environment as light, sound and heat and in waste materials. Diffuse energy in the environment is generally hard to capture and re-use, so we think of it as "gone" or "lost", but from a strict energy accounting point of view it is still there. Students can be confused by the contradiction between the common admonition that they should strive to conserve energy and the physics principle that it is always conserved. The differences in the meaning of the word conservation in these two cases merit some explicit discussion.

Obviously if matter flows into or out of the system it can carry energy. Even if there is no matter flow, energy can enter or leave the system as heat –either by radiation, or by conduction if there is any contact between the system being studied and anything else (for example the air around it). Physicists talk about the ideal notion of an "isolated system" but no system can actually be isolated in a way that prevents it from radiating energy, or absorbing radiation. The energy that leaves a system as heat is hard to measure. In general students cannot verify conservation of energy through their own measurements. There are a few examples, such as a collision of two different size pucks on an air table, where conservation of energy, together with conservation of momentum, can be used to predict outcomes, to the level of accuracy of the measurements. Even in these cases a student might observe that the collision made a sound, that friction, while reduced, is not zero, and that there is some drag on the objects from the atmosphere. So the conservation of energy in this situation is at best approximate. It took me many years of physics study before I realized that conservation of energy was a fundamental principle, not an idealization that would be dropped once I got to a deeper level of understanding. I do not know at what stage of a student's education it is worthwhile and meaningful to stress this difference.

Whenever we define a system that is not in fact isolated, there is another way that energy can be transferred into or out of it. That is through forces due to objects that we defined as external to the system, forces that act upon the system and change its state. Physicists define a quantity called "work", though it has little to do with our everyday concept of work. However the calculation of work done on a system allows us to keep track of the changes in energy due to forces acting on it. While very useful for physics, the physics concept of work is not natural to students (Gilbert and Osborne 1980) and is not much used in other disciplines. Physicists sometimes offer "the capacity to do work" as a definition of energy. In my opinion this definition is entirely useless for gaining a conceptual understanding of energy, and only meaningful in very restricted and idealized situations. Indeed it is logically circular, as the definition of work was arrived at by asking how much a force changes the energy of the object it is acting upon.

The concept of work is only one side of the relationship of forces to energy, the side that encodes the fact that forces acting on an object can change its energy. The other side of the coin is that any pair of (equal and opposite) forces acting between a pair of objects are an indication that there is interaction energy between them. This interaction energy would be reduced if each object moved in the direction of the force on it due to that interaction. Objects fall, or roll down hill because that reduces the gravitational interaction energy between the mass of the object and the mass of the earth. Like charges repel each other because moving them apart reduces the energy stored in electric fields between and around them, and unlike charges attract because the electric field between and around them, and thus the energy stored in that field, is reduced as they move closer together.

These phenomena that are usually learned as rules without explanation, yet they have explanations in terms of force fields and/or interaction energies. Potential energy (whether gravitational, electromagnetic or even nuclear) and negative chemical bond energy are other such phenomenon. Chemical bonds, elastic and tensile forces within matter, and contact forces between matter objects all depend on the charged substructure of atoms and the electromagnetic fields, and hence forces, between them. I would very much like to see some studies of whether (and at what stage) introducing the concept of fields can help students develop models that allow them to better model, interpret, apply and relate energy and force phenomena, and better understand chemical bonds and properties of bulk matter.

2.4.3 Energy Availability Governs What Can Happen (You Can't Do Anything Without Energy)

So if we always leave out some energy, and we cannot verify conservation of energy in any system students could observe, why do we talk about conservation of energy at all? The answer I think lies in the fact that it has important everyday consequences: access to energy controls and delimits what a system can do. Understanding energy flow and redistribution throughout a system is often a key to understanding the functioning of the system as a whole.

Conservation principles are useful precisely because they delimit possibilities. The fact that atoms are conserved in chemical processes allows us to do atom book-keeping to track matter through chemical reactions. It greatly reduces the set of possible processes, for example compared to those that could happen if mass were conserved but atoms could change type freely. Likewise the knowledge that energy is conserved restricts possibilities.

One consequence is that, in order for any system (whether natural or designed) to move anything it needs a way to capture or collect the energy needed for that task, at least temporarily. Generally at the end of any cycle that energy has been distributed into the surrounding environment as thermal energy and in waste materials, so the system needs a continued input of energy in order to continue operating. Thus to understand any system it is valuable to investigate how matter and energy is captured or provided to it, what it is needed for within the system, and how it is redistributed as a system functions. (This idea is highlighted as one of the "cross-cutting concepts" in the Framework for K-12 Science Education, for details see NRC 2012).

Another application of energy conservation seems trivial at the macro-scale but understanding has important consequences for understanding smaller scale examples. If two objects stick together then the combined object has less energy than the two objects separately – energy must be provided to pull them apart again. Thus we can explain why they do not fall apart as a consequence of conservation of energy: they cannot fall apart because they simply do not have enough energy to do so, just as ball sitting at the bottom of a hill cannot spontaneously roll up the hill. This idea seems obvious when we think about ripping apart Velcro or pulling up sticky tape, but can become a mystery to students when it applies at the atomic scale (Boo and Watson 2001). Perhaps emphasizing the parallel would help. A chemical bond is a lack of energy, any stable molecule has less energy than the set of atoms that it contains would have if they were widely separated. Chemists call this difference the bond energy. Analogies that liken the bond to a rubber band are confusing because the rubber band itself adds energy to, and becomes a part of, the system it holds together, even as it creates a combined system that is stable and cannot be taken apart without adding some more energy. A chemical bond is not an object, it is an interaction between objects, and one that lowers their combined energy compared to the situation when they are separated. The chemical bond is like the interweaving of the hooks in the Velcro, not like the Velcro itself.

Negative chemical bond energy is an example of negative potential energy. In both cases we are talking about differences in interaction energy relative to some reference situation. However, the parallel is rarely made, and the reference system is seldom mentioned. (It is neither the starting nor the ending set of molecules in a chemical process, but the hypothetical case of a collection of widely separated atoms.) Even students who understand the notion of negative potential energy in a physics example may become confused when they meet the binding energies in a chemistry class if neither the language of reference situation, nor that of potential energy, is introduced there. Conversely the student who may have grasped the chemical idea of negative bond energy is not necessarily encouraged to see that as an example of negative potential energy when they get to a physics class. It seems to me that these are the kinds of connections can help students integrate knowledge across disciplines. Why are they so rarely made?

2.4.4 Energy Tends to Spread Itself Around as Much as Possible

The final energy principle is perhaps the most mysterious when stated in its technical form – entropy tends to increase. The basic concept here (for the K-12 level) is that particle thermal motions and collisions, and thermal radiation tend to disperse energy throughout any system, and move it between systems. Energy concentrated in a small region of a system is unstable, because processes within the system tend to spread the energy throughout it, and to radiate it away from the system. Objects or regions that are hotter than their surrounding environment lose energy to that environment. Conversely cooler regions get heated. Without energy inputs systems evolve towards a condition of equal temperature throughout, which is a condition of maximally distributed thermal energy. Not only thermal energy, but interaction energies also tend to minimize local concentrations, as rocks fall, and the charged particles within matter move to find positions where the forces on them are balanced against each other. Indeed any large concentrations of stored energy can be dangerous if released rapidly, water behind a dam does incredible damage if the dam breaks, and batteries with high energy-density can catch fire or even explode.

Any process in a machine or living system always ends up heating the surrounding environment and thereby losing some energy. It is a fundamental law of physics that one cannot build a perpetual motion machine –one that goes on running forever with no input of energy – because of this effect. (The formal proof of this statement is not accessible to high school students, indeed many college physics majors struggle to comprehend it; despite that I think it is an important idea for students to learn and consider.) The unavoidable dissipation of energy means that machines need ongoing inputs of energy, and makes production of transportable energy- that is producing or extracting fuel (for combustion reactions with oxygen that release energy) or production of electric power – a major task in industrialized societies.

2.5 When and How Can Students Learn About Energy?

The challenge in all these detailed statements about how to describe energy comes down to the fact that everyday usage gives no way to unify diverse phenomena around energy, or even to define energy. The unifying ideas and technical definitions are all at the atomic or sub-atomic level. Only a limited and idealized set of cases can be treated quantitatively at the macroscopic level. This means that many of the concepts around energy cannot be made precise until students have a firm grasp of particulate models of matter. However, even at the high school or college level, there is no commonality in the way energy is discussed across disciplines, and the differences in definitions and language can leave students struggling to make connections.

Add to this the fact that the everyday usage of terms such as "having energy" or "feeling energetic" to describe the way a person feels or acts is quite a distinct concept from the technical meaning of the word energy, while at the same time it shares some aspects, for example the idea that more motion means more energy. This is the entry point into thinking about energy for young children, and it must be taken into account.

Everyday words that overlap but are not the same as technical words are not errors in usage, but they can lead to misconceptions about the technical meaning unless the differences are acknowledged and discussed as the technical usage is introduced. "Potential" energy has a different problem with everyday meanings. When we say "potential" in everyday language we mean something that might be, but does not yet exist – such as a potential partnership. Potential energy is actual energy stored in some interaction between objects, negative potential energy is a lack of such energy compared to a reference situation –in neither case is potential energy a possibility of energy yet to be realized. So the term potential energy brings its own confusions. Add to that the arbitrariness of the reference situation from which we calculate differences in interaction energies to determine the potential energy, and we see why students struggle to grasp the ideas around potential energy.

In the elementary grades student ideas about energy are necessarily going to be general rather than quantitative. Students experience energy-related phenomena – motion, heat, light, and sound, melting, evaporation, temperature changes. When should the language of energy be introduced? When is it needed? When does it clarify and when does it confuse? How do we help students connect ideas about energy across all the science disciplines? I do not have answers to these questions, but I am convinced that answering them requires both classroom research and a discourse across the disciplines as to how best to teach these ideas at various levels, including at the college level. That is why I am happy to contribute my thoughts to this volume.

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References

Adrian, B., & Fuller, R. (1997). *A qualitative investigation of college students' conceptions of electric fields*. Paper presented at joint meeting of the Arkansas-Oklahoma-Kansas and Nebraska AAPT Sections, Manhattan, KS. Retrieved June 17, 2013, from http://physics.unl.edu/~rpeg/Electric_fields.pdf.

Bodzin, A. M. (2011, April 3–6). *What do eighth grade students know about energy resources?* Paper presented at the NARST 2011 annual international conference in Orlando, FL, USA.

Boo, H. K. (1998). Students' understandings of chemical bonds and the energetics of chemical reactions. *Journal of Research in Science Teaching, 35*, 569–581. doi:10.1002/(SICI)1098-2736(199805)35:5<569::AID-TEA6>3.0.CO;2-N.

Boo, H. K., & Watson, J. R. (2001). Progression in high school students' (aged 16–18) conceptualizations about chemical reactions in solution. *Science Education, 85*, 568–585.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (2000). *How people learn: Brain, mind, experience, and school: Expanded Edition*. Washington, DC: National Academies Press.

Coopersmith, M. (2010). *Energy – The subtle concept*. Oxford: Oxford University Press.

Driver, R., & Warrington, L. (1985). Students' use of the principle of energy conservation in problem situations. *Physics Education, 20*, 171–176.

Duit, R. (1981). Students' notions about the energy concept – Before and after physics instruction. In W. Jung, H. Pfundt, & C. von Rhoeneck (Eds.), *Proceedings of the international workshop on 'problems concerning students' representation of Physics and Chemistry knowledge* (pp. 268–319). Ludwigsburg: Paedagogische Hochschule.

Duit, R. (1984). Learning the energy concept in school – Empirical results from the Philippines and West Germany. *Physics Education, 19*, 59–66.

Duit, R. (1987). Should energy be introduced as something quasi-material? *International Journal of Science Education, 9*, 139–145.

Feynman, R. P., Leighton, R. B., & Sands, M. (2011). *Feynman lectures on physics 1: Mainly mechanics, radiation, and heat*. New York: Basic Books.

Gilbert, J. K., & Osborne, R. J. (1980). Some problems of learning science. *School Science Review, 61*, 664–674.

Kesidou, S., & Duit, R. (1993). Students' conceptions of the second law of thermodynamics – An interpretive study. *Journal of Research in Science Teaching, 30*, 85–106. doi:10.1002/tea.3660300107.

Libarkin, J. C., Asghar, A., Crockett, C., & Sadler, P. (2011). Invisible misconceptions: Student understanding of ultraviolet and infrared radiation. *Astronomy Education Review, 10*, 010105–1. doi:10.3847/AER2011022.

Lijnse, P. L. (1990). Energy between the life-world of pupils and the world of physics. *Science Education, 74*(5), 571–583.

Liu, X., & McKeough, A. (2005). Developmental growth in students' concept of energy: Analysis of selected items from the TIMSS database. *Journal of Research in Science Teaching, 42*(5), 493–517.

Maskill, R., & Pedrosa de Jesus, H. (1997). Pupils' questions, alternative frameworks and the design of science teaching. *International Journal of Science Education, 19*(7), 781–799.

Millar, R. (1994). Students' understanding of key ideas radioactivity and ionizing radiation. *Public Understanding of Science, 3*, 53–70.

National Research Council [NRC]. (2012). *A framework for K-12 science education: Practices, crosscutting concepts, and core ideas*. Washington, DC: The National Academies Press.

Neumann, K., Viering, T., Boone, W. J., & Fischer, H. E. (2013). Towards a learning progression of energy. *Journal of Research in Science Teaching, 50*, 162–188. doi:10.1002/tea.21061.

Papadouris, N., & Constantinou, C. P. (2011). A philosophically informed teaching proposal on the topic of energy for students aged 11–14. *Science & Education, 20*(10), 961–979.

Solomon, J. (1983). Messy, contradictory and obstinately persistent: A study of children's out-of-school ideas about energy. *School Science Review, 65*(231), 225–230.

Stephanik, B. M., & Shaffer, P. S. (2011). Examining student ability to interpret and use potential energy diagrams for classical systems. arXiv:1109.2074 [physics.ed-ph].

Stocklmayer, S. M., & Treagust, D. F. (1996). Images of electricity: How do novices and experts model electric current? *International Journal of Science Education, 18*(2), 163–178.

Trumper, R. (1990). Being constructive: An alternative approach to the teaching of the energy concept. Part I. *International Journal of Science Education, 12*, 343–354.

Warren, J. W. (1983). Energy and its carriers: A critical analysis. *Physics Education, 18*, 209–212.