# Chapter 1
# Non-standard Packing Problems: A Modelling-Based Approach

The general subject of packing objects, exploiting the available volume, as much as possible, has represented, for centuries, or even longer, an extremely tricky task. This issue seems trivial, until one encounters it. The question arose, for instance, when dealing with cannon ball stowage in ancient vessels. It is not surprising at all that it gained the role of the packing issue par excellence, when Hilbert announced his eighteenth problem (to date resolved by computer-assisted proof, e.g. Gray 2000). It concerned the accommodation of equal spheres, attaining the maximum density.

Paramount effort has been carried out, and is ongoing, to dominate extremely challenging overall packing problems, from the theoretical point of view. Well-known directions of speculative investigations include infinite dimensional space issues. For instance, we could consider the packing of Platonic solids in the ordinary Euclidean space and of spheres in higher dimensions. Further examples concern finite space questions, such as those of placing squares/circles or cubes/spheres into regular figures (e.g. http://mathworld.wolfram.com).

An unquestionably much more practical slant is instead underlined in the operations research and computational geometry frameworks. In such a context, the role of the numerical approach to look into high-quality (albeit usually nonproven optimal) solutions to even more complex, real-world packing problems is emphasized. This is most definitely the point of view of this work.

There is vast specialist literature on multidimensional packing from a numerical optimization standpoint. It is, therefore, not intended to be surveyed here. The reader may refer to some comprehensive overviews (e.g. Cagan et al. 2002; Dyckhoff et al. 1997; Ibaraki et al. 2008). As is known, a significant part of the topical bibliography focuses on the orthogonal placement of rectangles/parallelepipeds into rectangles/parallelepipeds (e.g. Faroe et al. 2003; Fekete and Schepers 2004; Fekete et al. 2007; Martello et al. 2000; Pisinger 2002), even if several works also consider different typologies of packing issues (e.g. Addis et al. 2008a; Birgin et al. 2006; Egeblad et al. 2007; Gomes and Olivera 2002; Scheithauer et al. 2005; Teng et al. 2001).

G. Fasano, *Solving Non-standard Packing Problems by Global Optimization and Heuristics*, SpringerBriefs in Optimization,
DOI 10.1007/978-3-319-05005-8_1, © Giorgio Fasano 2014

Several versions of two-/three-dimensional packing problems can be differently classified, depending on the specific optimization criterion adopted. When, for instance, the number of containers is fixed and the total load has to be maximized (e.g. in terms of its volume/value), the relevant model is referred to as a *knapsack* problem (e.g. Caprara and Monaci 2004; Egeblad and Pisinger 2006, 2009; Fekete and Schepers 1997). It is reduced to the *single container* one, when only one container is available (e.g. Bortfeldt et al. 2012; Kang et al. 2010; Parreño et al. 2008).

The issue of loading a set of given objects, whilst minimizing the number of containers to utilize (or, more in general, their total volume/cost), is referred to as the bin packing problem (e.g. Lodi et al. 2010; Martello et al. 2000; Pisinger and Sigurd 2007).

Further questions concern the 'reduction' of the container. This is the case, in particular, of the *strip packing* problem (e.g. Iori et al. 2003; Kenmochi et al. 2009; Zhang et al. 2006), where a single dimension of the (rectangle/parallelepiped-shaped) domain has to be minimized. Another class of interesting issues consists of the (rectangle/parallelepiped-shaped) domain (area/volume) minimization problem (e.g. Li et al. 2002; Pan and Liu 2006). This is of importance in several applications, ranging from manufacturing and logistics to electronic design (e.g. *floor-planning* in very large scale integration, VLSI). Still related to the container volume minimization, it is worth mentioning the issue of the *sphere packing* in optimized spheres (e.g. Kampas and Pintér forthcoming).

Remarkable effort has been dedicated to tackling several kinds of packing problems algorithmically, often by adopting general *meta-heuristics* or dedicated heuristics (e.g. Allen et al. 2011; Bennell et al. 2013; Bennel et al. 2013; Bennell and Oliveira 2009; Bortfeldt and Gehring 2001; Bortfeldt et al. 2003; Burke et al. 2006, 2010; Coffman et al. 1997; Dowsland et al. 2006; Gehring and Bortfeldt 2002; Gomes and Olivera 2002; Gonçalves and Resende 2012; Hopper and Turton 2001, 2002; López-Camacho et al. 2013; Mack et al. 2004; Oliveira et al. 2000; Pisinger 2002; Ramakrishnan et al. 2013; Terashima-Marín et al. 2010; Wang et al. 2008; Yeung and Tang 2005). Nonetheless, modelling-based approaches have also been investigated (e.g. Allen et al. 2012; Chen et al. 1995; Chernov et al. 2010; Fasano 1989; Fischetti and Luzzi 2009; Hadjiconstantinou and Christofides 1995; Kallrath 2009; Padberg 1999; Pisinger and Sigurd 2005). These works refer to the overall context of *Mathematical Programming*, including *mixed-integer* (*linear*) *programming* (MILP, MIP) and *mixed-integer nonlinear programming* (MINLP).

The underlying theme examined here originates from a long-lasting research work devoted to tackling complex non-standard packing issues arising in space applications. These usually concern both design and logistics aspects. In this sector, the necessity of exploiting the spacecraft load capacity, as much as possible, presents the engineers with a paramount challenge. This is foreseen especially in the perspective of the extremely demanding missions that are going to be carried out in the near future.

Generally, the volume or the mass of the loaded cargo has to be maximized. Other optimization criteria, however, can also be stated, depending on the specific mission scenarios to deal with. In any case, very tough accommodation rules have to be taken into account, in compliance with demanding requirements relative to safety, ergonomic and operational concerns.

Tight balancing conditions, deriving from control specifications, are usually posed at an overall system level (i.e. the whole spacecraft). However, the requirement of considering them also when loading each single internal container (such as, for instance, racks or bags) is quite often needed. The space-system internal geometries are normally very intricate. As a consequence, in order to exploit each available volume, as much as possible, the shape of the adopted containers themselves can become quite peculiar. This occurs, for instance, when dealing with the cargo accommodation of the European Automated Transfer Vehicle (ATV, ESA, cf. http://www.esa.int). In such a case, some specific bags have curved surfaces to fit with the shape of the racks they have to be accommodated into that is, itself, determined by the cylindrical form of the carrier.

A specific class of 'hard' non-standard packing problems with additional conditions arises. All this is determined by balancing conditions, the shapes of domains and objects, the possible presence of internal *separation* planes, or *structural* elements, not to mention complicated accommodation rules. This situation can arise in space engineering and logistics. Despite the specificity of the context, however, it is, in more or less similar versions, susceptible to several real-world applications. This happens also in very different frameworks.

Balancing conditions and complex geometries, for instance, are increasingly important subjects in the high-speed transportation system sector. Complex engineering structures (e.g. oil rigs), even if related to quite different operational scenarios, pose similar problems. Non-standard packing issues have to be considered daily in manufacturing, even if not necessarily in the presence of balancing conditions. This occurs, often, just in a two-dimensional context (e.g. Electronic Design Automation, EDA and VLSI). This work discusses in depth some real-world packing scenarios, from an application perspective. It is aimed at introducing an efficient methodology to solve non-trivial problems in practice.

In several applicative contexts, items can often be assumed to be (rectangular) parallelepipeds, without significant loss of information. Nevertheless, generally, such an approximation does not hold, especially when dealing with large and complex items. Similar considerations can, moreover, be made, considering the container shape, since frequently it is not just a (rectangular) parallelepiped. The presence of additional geometric and operational conditions presents further challenges.

Remarkable works, concerning non-standard packing problems, are available. In the author's opinion, however, this topic definitely deserves much more commitment, also in consideration of the increasing demand generated by the real-world context. This is the essential motivation inspiring the drawing up of the present work.

When dealing with non-standard packing problems, with overall conditions, such as balancing, the simplistic approach (adopted by several packing algorithms) of placing items one at a time is scarcely efficient. A strongly nonlocal viewpoint is therefore highly desirable, also in consideration of the outstanding results recently achieved in the framework of *global optimization* (GO, consult, e.g. Addis et al. 2008b; Castillo et al. 2008; Floudas et al. 2005; Floudas and Pardalos 1990, 2001; Floudas et al. 1999; Horst and Pardalos 1995, 1997; Horst and Tuy 1996; Kallrath 1999, 2008; Liberti and Maculan 2005; Locatelli and Raber 2002; Pardalos and Resende 2002; Pardalos and Romeijn 2002; Pintér 1996, 2006, 2009; Rebennack et al. 2009).

A modelling-based philosophy, as opposed to a pure algorithmic one, has been looked into, characterizing the whole approach followed hereinafter. GO represents therefore a first highlight. The packing problems in general, moreover, even when posed in quite an elementary version (e.g. the placement of simple boxes in a container box, without any additional conditions) are well known for being *NP-hard*. As a consequence, no deterministic methodology to successfully solve the problem to optimality is expected. An overall heuristic point of view is therefore a second key characteristic of this volume. In particular, a joint use of GO, based on MIP/MINLP formulations, and heuristic procedures is emphasized.

The concept of tetris-like item is introduced, representing a fundamental reference paradigm for the generic approach proposed here. It generalizes the original idea of tetris item, deriving, in turn, from that of *polyomino* (see Golomb 1994). Either three- or two-dimensional tetris-like items are considered, and, differently from the original concept, they are not supposed to have integer side lengths.

This notion is, by itself, quite interesting, as it is adequate to represent a wide range of real-world objects, in a streamlined but sufficiently realistic way. In several applications, indeed, one has to deal with quite complex objects, characterized by intricate shapes. Considering items, as a whole, just in terms of their smallest enclosing boxes would clearly result, in most cases, extremely restrictive. In any accommodation practical problem, where quite an efficient exploitation of the overall volume available is a mandatory task, this would be of no use at all. Substituting complex objects with tetris-like items, i.e. clusters of mutually perpendicular cuboids (rectangular parallelepipeds), could make their representation much more realistic, even if simplified and thus still approximate.

The original object is therefore partitioned into parts, enclosing each in a cuboid. Obviously, the bigger (i.e. refined) the partition is, the more realistic the representation results. Figure 1.1 shows a (not too sophisticated) tetris-like approximation of a real-world object. Similarly to the case concerning the items to load, structural elements, equipment/devices and clearance/accessibility regions, inside the container, may well benefit from this representation, as illustrated by Fig. 1.2, referring to the internal part of a space module. Reinforcements of the cylindrical structure are present, together with some electronic devices that are supposed to be protected by forbidden zones.

In addition to what mentioned above, the relevant modelling features of the tetris-like representation are quite suitable for MIP formulations that provide a

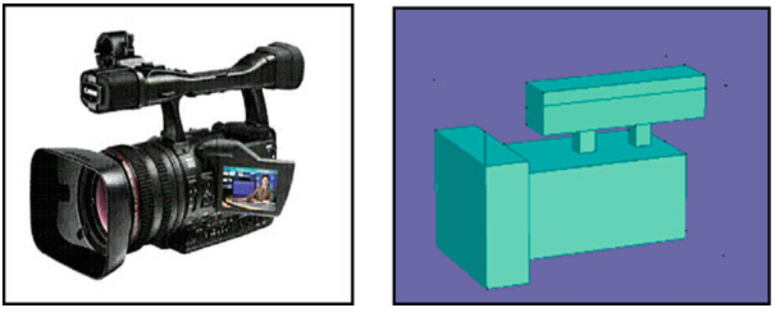

**Fig. 1.1** Representation of complex objects with tetris-like items

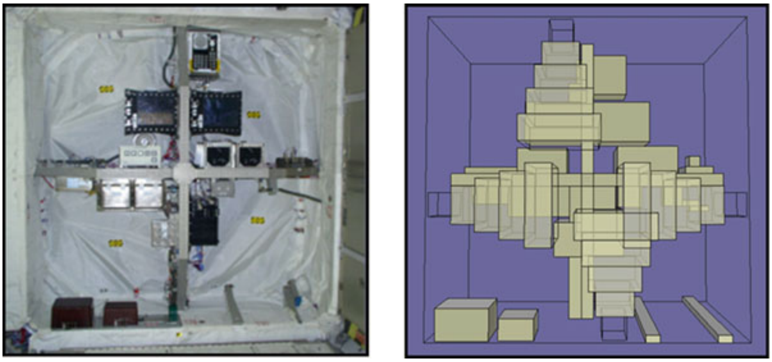

**Fig. 1.2** Internal devices and forbidden zones approximated by tetris-like items

linear-based overall structure, obviously beneficial to a GO approach. Moreover, its MIP formulation is able to cover a non-negligible number of additional conditions.

This volume both reviews the author's previous works (e.g. Fasano 2008, 2013) and introduces new research outcomes, establishing a basis for further investigation and development.

The second chapter discusses, at quite a detailed level, a general mathematical model for the orthogonal packing of three-dimensional tetris-like items within a convex domain (polyhedron). Some critical aspects are pointed out, suggesting how it is quite easy to overcome them. A number of additional conditions are looked into, including the prefixed position/orientation of subsets of items, the presence of 'holes' or forbidden zones as well as of *separation* planes and *structural* elements, relative distance bounds and *static*/*dynamic* balancing requirements.

The corresponding *feasibility* subproblem is discussed in the third chapter. It consists of the special case taking place when no optimization criterion (e.g. the total volume maximization) is selected a priori, and all items have to be loaded. This situation can be profitably exploited by introducing an ad hoc *objective* function, aimed at facilitating the resolving process in finding *integer-feasible* solutions. Both linear and nonlinear readjustments of the general MIP model are considered. The third chapter also outlines the issue of *tightening* the general MIP model, by introducing implications and *valid* inequalities, suitable, in particular, for a dedicated *branch-and-cut* approach.

As the general MIP model is extremely tough to solve, even when not too large-scale instances are involved, an MIP-based heuristic point of view is described in the fourth chapter. There, the basic concept of *abstract configuration* is enucleated. It essentially consists of a set of item-item relative positions, feasible in any unbounded domain. The *feasibility* sub-models are profitably adopted to generate ‘good’ *abstract configurations*. The heuristic approaches delineated in this chapter are founded on recursive generations of these.

The fifth chapter is devoted to the experimental results, obtained to date, relevant to a real-world application framework. The sixth explores both extensions of the general MIP model and nonlinear (MINLP) formulations, in order to tackle two further non-standard packing issues. The first concerns the creation of possible *virtual* items, to exploit the empty spaces of a container, already partially loaded with tetris-like items. This aspect is of importance in several applications. The second issue deals with the non-orthogonal placement of polygons with (continuous) rotations in a convex domain (polygon). Also in this case, a GO-based heuristic approach is proposed. It is aimed at finding a ‘good’ approximate solution susceptible to further local refinement by more sophisticated formulations, such as the one based on the Stoyan’s $\Phi$-functions (e.g. Stoyan et al. 2004). The tetris-like item model is advantageously exploited to provide the MINLP solution process with a ‘good’ starting solution.

The last chapter concludes the volume providing some insights on prospective enhancements, in terms of further experimental analysis needed, but also from the modelling and development point of view, including extended applications (one in particular dealing with scheduling problems).