Transforming Professional Practice in Numeracy Teaching

Merrilyn Goos, Vince Geiger, and Shelley Dole

Abstract The development of numeracy, sometimes known as quantitative literacy or mathematical literacy, requires students to experience using mathematics in a range of real-world contexts and in all school subjects. This chapter reports on a research study that aimed to help teachers in ten schools plan and implement numeracy strategies across the middle school curriculum. Teachers were introduced to a rich model of numeracy that gives attention to real-life contexts; application of mathematical knowledge; use of representational, physical, and digital tools; and positive dispositions towards mathematics. These elements are grounded in a critical orientation to the use of mathematics. Over one school year, the teachers worked through two action research cycles of numeracy curriculum implementation. The professional development approach included three whole-day workshops that supported teachers' planning and evaluation and two rounds of school visits for lesson observations, teacher and student interviews, and collection of student work samples. During workshops, teachers also completed written tasks that sought information about their confidence for numeracy teaching and how they were using the numeracy model for planning. Drawing on data collected during workshops and school visits, we demonstrate how teachers' instructional practices changed over time as they progressively engaged with the numeracy model.

Keywords Numeracy • Mathematical literacy • Quantitative literacy • Teacher development • Instructional practice • Contexts • Dispositions • Tools • Critical orientation

M. Goos (✉) • S. Dole
School of Education, The University of Queensland, St Lucia, QLD 4072, Australia
e-mail: m.goos@uq.edu.au; s.dole@uq.edu.au

V. Geiger
School of Education, Australian Catholic University, P.O. Box 456, Virginia, QLD 4014, Australia
e-mail: Vincent.Geiger@acu.edu.au

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Numeracy is a term used in many English-speaking countries, such as the UK, Canada, South Africa, Australia, and New Zealand, whereas in the USA and elsewhere, it is more common to speak of *quantitative literacy* or *mathematical literacy*. Steen (2001) described quantitative literacy as the capacity to deal with quantitative aspects of life and proposed that its elements included confidence in mathematics, appreciation of the nature and history of mathematics and its significance for understanding issues in the public realm, logical thinking and decision-making, use of mathematics to solve practical everyday problems in different contexts, number sense and symbol sense, reasoning with data, and the ability to draw on a range of prerequisite mathematical knowledge and tools. The OECD's (2004) PISA program offers a similarly expansive definition of mathematical literacy as:

> an individual's capacity to identify and understand the role mathematics plays in the world, to make well-founded judgments, and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (p. 15)

Steen (2001) maintains that, for numeracy to be useful to students, it must be learned in multiple contexts and in all school subjects, not just mathematics. Although this is a challenging notion, a recent review of numeracy education undertaken by the Australian government (Human Capital Working Group, Council of Australian Governments 2008) concurred, recommending:

> That all systems and schools recognise that, while mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment. (p. 7)

The cross-curricular importance of numeracy is endorsed by the recently introduced national curriculum for Australian schools, which identifies numeracy as a general capability to be developed in all subjects (Australian Curriculum, Assessment and Reporting Authority 2012).

This chapter reports on a yearlong research study that investigated approaches to help teachers plan and implement numeracy strategies across the curriculum in the middle years of schooling (grades 6–9). The study was informed by a rich model of numeracy that was introduced to the teachers as an aid for their curriculum and instructional planning. The chapter addresses two of the research questions that guided the project:

1. To what extent did teachers' instructional practices change over time as they progressively engaged with the numeracy model?
2. How effective was the professional development approach in building teachers' confidence in numeracy teaching?

The first section of the chapter outlines the theoretical framework for the study, which comprises the numeracy model and the professional development approach for working with teachers. The second section describes the research design and methods. In the third section, we describe how teachers developed new strategies for numeracy instruction, drawing on an analysis of the whole group's developmental trajectories through the numeracy model and a case study of one individual teacher. The final section evaluates the effectiveness of the professional development approach in terms of changes in teachers' confidence in numeracy teaching.

Theoretical Framework

Numeracy Model

Current definitions of numeracy, quantitative literacy, and mathematical literacy share many common features and may usefully inform curriculum development and national or international assessments of students' educational achievement. However, they do not provide direct guidance to teachers on how to plan instruction with a rich numeracy focus. Goos (2007) has also argued that a description of numeracy for new times is needed to better acknowledge the rapidly evolving nature of knowledge, work, and technology. She developed the model shown in Fig. 1 to represent the multifaceted nature of numeracy in the twenty-first century. This model was intended to be readily accessible to teachers as an instrument for planning and reflection.

According to this model, numeracy development requires attention to real-life *contexts*; the application of *mathematical knowledge*; the use of representational, physical, and digital *tools*; and positive *dispositions* towards the use of mathematics. A further important and overarching element of the model is a *critical orientation* to the use of mathematics. Table 1 provides a succinct summary of the elements of the numeracy model, each of which is elaborated below.

A numerate person requires *mathematical knowledge*. In a numeracy context, mathematical knowledge includes not only concepts and skills but also problem-solving strategies and the ability to make sensible estimations (Zevenbergen 2004).

A numerate person has *positive dispositions* – a willingness and confidence to engage with tasks, independently and in collaboration with others, and apply their mathematical knowledge flexibly and adaptively. Affective issues have long been held to play a central role in mathematics learning and teaching (McLeod 1992), and the importance of developing positive attitudes towards mathematics is emphasized in national and international curriculum documents (e.g., National Council of Teachers of Mathematics 2000; National Curriculum Board 2009).

Being numerate involves using *tools*. Sfard and McClain (2002) discuss ways in which symbolic tools and other specially designed artifacts "enable, mediate, and shape mathematical thinking" (p. 154). In school and workplace contexts, tools may be representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners), physical (models, measuring instruments), or digital (computers, software, calculators, Internet) (Noss et al. 2000; Zevenbergen 2004).

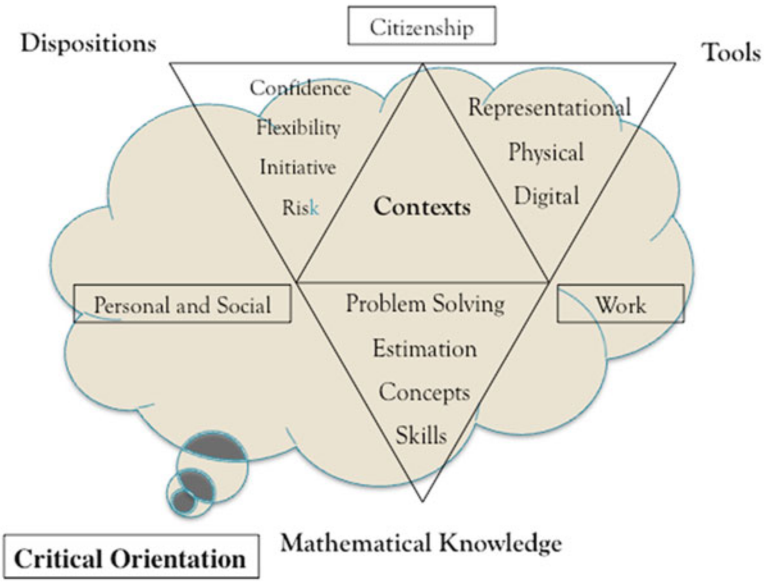

Fig. 1 A model for numeracy in the twenty-first century (Goos 2007)

Table 1 Descriptions of the elements of the numeracy model

Element	Description
Mathematical knowledge	Mathematical concepts and skills, problem-solving strategies, and estimation capacities
Contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings
Dispositions	Confidence and willingness to engage with tasks and apply mathematical knowledge flexibly and adaptively
Tools	Use of physical (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables), and digital (computers, software, calculators, Internet) tools to mediate and shape thinking
Critical orientation	Use of mathematical information to make decisions and judgments, add support to arguments, and challenge an argument or position

Because numeracy is about using mathematics to act in and on the world, people need to be numerate in a range of *contexts* (Steen 2001). All kinds of occupations require numeracy, and many examples of work-related numeracy are specific to the particular work context (Noss et al. 2000). Informed and critical citizens need to be numerate citizens. Almost every public issue depends on data, projections, and the kind of systematic thinking that is at the heart of numeracy. Different curriculum contexts also have distinctive numeracy demands, so that students need to be numerate across the range of contexts in which their learning takes place at school (Steen 2001).

This model is grounded in a *critical orientation* towards numeracy since numerate people not only know and use efficient methods, they also evaluate whether the results obtained make sense and are aware of appropriate and inappropriate uses of mathematical thinking to analyze situations and draw conclusions. In an increasingly complex and information-drenched society, numerate citizens need to decide how to evaluate quantitative, spatial, or probabilistic information used to support claims made in the media or other contexts. They also need to recognize how mathematical information and practices can be used to persuade, manipulate, disadvantage, or shape opinions about social or political issues (Jablonka 2003).

Professional Development Approach

In working with teachers, we integrated four professional development strategies recommended by Loucks-Horsley et al. (2003). The first strategy involved formation of *collaborative partnerships* between participating teachers, university researchers, and curriculum support officers from the state Department of Education, which commissioned the study. Collaborative structures provide opportunities for professional learning around topics negotiated and agreed upon by the group, thus ensuring common goals. Collaborations are more contextualized to the teachers' setting than most other forms of professional learning. The emphasis on collegiality and communication provides a forum for teachers to discuss specific issues related to their classrooms in an environment in which the discussion is valued by their colleagues.

The second strategy was to *examine teaching and learning* using action research. We conducted a series of project workshops and school visits to support teachers through two action research cycles of plan, act, observe, and reflect in order to replan and continue through the next cycle. Additional elements of this strategy included inviting teachers to contribute to or formulate their own questions, linking teachers with sources of knowledge and stimulation from outside their schools, and documenting and sharing the learning from research.

Third, we provided teachers with *immersion experiences* that included numeracy-based learning opportunities and examples of numeracy investigations and assessment tasks. Successful use of immersion experiences as a strategy for professional learning requires two key elements: qualified facilitators and long-term experiences. Both of these elements were embedded in the design of the project. The research team has extensive mathematical and numeracy knowledge, many years' experience as classroom teachers, and in-depth understanding of the goals and challenges of implementing numeracy pedagogy. In addition, unlike one-off professional development models, the immersion experiences occurred at every teacher meeting. Hence many of the drawbacks of immersion experiences – lack of time and resources, mismatch with individual teachers' learning sequence, and lack of connection to direct classroom practice – were reduced.

Fourth, we expected *curriculum implementation* by requiring teachers to develop and implement units of work that targeted numeracy demands of the diverse curriculum areas from which they were drawn. Most short-term workshops that demonstrate innovative materials do not provide support (or even the expectation) for the teachers to trial these ideas in their own classrooms. As a result, the ideas provided in these one-off workshops are rarely put to use in the classroom (Cohen and Hill 2001). By including the requirement of implementation of the units as part of the project, we were able not only to ensure that the teachers would use the ideas with their students but also to allow them critical support and time for reflection on the experience. In addition, because the teachers completed at least two cycles of curriculum implementation, they were able to put into practice elements that were learned and refined after the first attempt.

Research Design and Methods

Teachers were recruited from ten schools with diverse demographic characteristics: four primary schools (kindergarten–grade 7), one secondary school (grades 8–12), four smaller schools in rural areas (grades 1–12), and one school that combined middle and secondary grades (grades 6–12). Each school nominated two teachers, thus ensuring that participants were able to connect with a colleague from their own school. They included generalist primary school teachers who taught across all curriculum areas as well as secondary teachers qualified to teach specific subjects (mathematics, English, science, social education, health and physical education, design studies).

There were three elements to the research design: (1) an audit of the middle years curriculum to identify the numeracy demands inherent in all curriculum areas (see Goos et al. 2010); (2) three whole-day professional development workshops that brought together all participants to *examine teaching and learning* and provide *immersion experiences*; and (3) two daylong visits to each school to evaluate *curriculum implementation* via lesson observations, collection of planning documents and student work samples, and audio-recorded interviews with teachers and students. The overall research plan is summarized in Table 2 to show the timeline for the project, key activities in the professional development approach, and data sources that informed our research into changes in teachers' instructional practices.

At the first project workshop, teachers were introduced to the numeracy model and the action research approach, and the findings of the curriculum audit were shared and discussed. The aim of the audit was to draw teachers' attention to the numeracy demands within all curriculum areas and hence to encourage them to accept responsibility for developing students' numeracy capabilities in the subjects they taught. We provided immersion experiences that were designed as cross-curricular numeracy investigations suitable for use with middle year students. These included investigations of Barbie dolls' physical proportion (with direct links to the health and physical education curriculum; see Fig. 2), the occurrence of the Golden

Table 2 Research design

Time	Activity	Data sources
February	Curriculum audit: identify numeracy demands in all curriculum areas	
March	Professional development workshop: introduce numeracy model; share findings from curriculum audit; try out numeracy teaching strategies and tasks; plan for implementation	Survey of numeracy teaching confidence
June	School visits: observe and evaluate implementation	Lesson observations, interviews with teachers and students, collection of teaching materials
August	Professional development workshop: evaluate implementation; share teaching resources and strategies; plan for implementation	
October	School visits: observe and evaluate implementation	Lesson observations, interviews with teachers and students, collection of teaching materials
November	Professional development workshop: evaluate implementation; reflect on professional learning	Survey of numeracy teaching confidence Map trajectories through the numeracy model

Rectangle in art, design, and nature (linked to the arts and design studies curricula), and planning for participation in the Tour Down Under, a bicycle race similar to the Tour de France (with links to the social education curriculum). At the end of this day, teachers were also asked to complete a survey that asked them to assess their confidence in various aspects of numeracy teaching.

The focus of the second workshop was on evaluating the implementation of the initial numeracy unit from the perspective of participating teachers and students and on setting goals and planning for the second action research cycle. All teachers were asked to bring evidence of one idea, activity, or unit they had tried with their class, to describe to the whole group how well (or not) it had worked, and to explain what they learned from this experience and how they would use this evaluation to plan subsequent lesson sequences. Time was also allocated to revisiting the numeracy model and the curriculum audit and to provide feedback on observations from the first round of school visits. We found very little evidence of a critical orientation in any of the lessons we had observed in the first round of school visits, despite opportunities in these lessons for including critique of real-life situations or actions. In interviews with teachers, it emerged that they were unsure about how to embed this element of the numeracy model into their planning and practice. Therefore, at the second workshop, we presented a range of stimulus materials drawn from print and digital media sources and asked teachers to work together to develop these into lessons that would promote a critical orientation in their students, without losing sight of the other elements of the numeracy model. (See Fig. 3 for an example of stimulus material.)

Barbie and Body Measurements

1. Measure your:

- height
- arm span
- length of index finger
- length of nose (bridge to point)
- head circumference
- wrist circumference.

(Work in pairs or small groups to make the measuring easier.)

2. Record females' personal measurements in the left column of the table below.

Name:			**Barbie**		
Body part	**Measurement**	**% of Height**	**Body part**	**Measurement**	**% of Height**
height			height	30.0 cm	
arm span			arm span	24.0 cm	80.0%
index finger			index finger	1.0 cm	3.3%
nose			nose	0.5 cm	1.7%
head			head	10.0 cm	33.3%
wrist			wrist	2.0 cm	6.7%

3. Now calculate and record the **ratio** of each measurement to height, and convert this to **percentage** of height. **Record** this personal information in the "% of height" column.

4. Compare the % of height data for female members of your group.

5. We will also use **spreadsheet formulae** to calculate **ratios** of body parts to heights for the whole class (using decimal or percentage representation).

6. Now we'll use **spreadsheet formulae** to find the **mean** (average) of each body ratio for the class. What similarities and any differences do you notice? What is the physical meaning of these?

7. Make similar **measurements** for **Barbie** and record these in the table above. Calculate her body **ratios** (express also as percentage of height) and record these. **Compare** her proportions with the average proportions of human females calculated from our class set of data.

Is Barbie a realistic representation of human proportions?

What would Barbie look like if she were scaled up to human height?

Fig. 2 Barbie activity used in first teacher workshop

At the third workshop that concluded the project, the research team began by reporting on students' perceptions of numeracy (taken from interviews with students on school visits); how well they liked mathematics (or not); where they saw numeracy in other learning areas, outside school, and in future careers; what advice they would give mathematics teachers to make learning mathematics more effective and enjoyable; and what they thought their teachers learned through

Readers of the *Courier-Mail* newspaper are invited to write in with questions such as the one below for the cookery expert (a well known chef).

Q. I am planning to make a small Christmas cake in a six-inch tin (15 cm) and would like to know how to calculate the quantities of ingredients needed if my recipe is for a larger tin.

A. Just break down the recipe accordingly; for example, if your cake recipe is for a 12-inch tin (30cm), then halve the recipe.

Is this answer good advice? What questions could you ask to help your students take a critical orientation to this information?

Fig. 3 Critical orientation stimulus material used in second teacher workshop

participation in this project. We next provided teachers with copies of the numeracy model and asked them to map their trajectory through the model throughout the project. They did this by identifying the element of the model that represented their entry point to the project, together with other elements of the model that became more meaningful or significant to them over time. As a result of our analysis of school visit data, we invited four teachers who exemplified different types of professional learning trajectories to report on their experiences. (One of these is the teacher whose case study is reported in a later section of this chapter.) We also immersed the teachers in a final numeracy investigation that used temperature data we had collected from inside a car at 10-min intervals over 2 h. Teachers were asked to sketch out a numeracy activity driven by questions that would support a critical orientation to the data and the contexts in which it might be collected and interpreted. At the end of the workshop, we readministered the numeracy self-assessment survey to enable us to track any changes in confidence in numeracy teaching.

The data used in this chapter are drawn from the following sources:

1. Field notes from lesson observations and interviews with teachers (research question 1)
2. Annotated copies of the numeracy model that were made by teachers at the last project workshop to map their developmental trajectory (research question 1)
3. Teachers' responses to the numeracy confidence survey that was administered at the first and last project workshops (research question 2)

The data were analyzed using qualitative and quantitative methods. To address research question 1, changes in teachers' instructional practices were analyzed by identifying how teaching plans and actions aligned with the elements of the numeracy model as the project progressed. For research question 2, concerning teacher confidence, Likert-style responses to the survey were converted to scores, and the score totals were compared at the beginning and end of the project to identify any changes in confidence levels.

New Numeracy Teaching Strategies

This section discusses the developmental trajectories of all teachers in terms of the numeracy model that guided the study and illustrates new numeracy teaching strategies via a teacher case study.

Teacher Trajectories Through the Numeracy Model

At the final project workshop, teachers were provided with a copy of the numeracy model and asked to annotate it in a way that indicated their changing understanding of numeracy over the duration of the project (Geiger et al. 2011). For example, Karen annotated her copy of the numeracy model as shown in Fig. 4.

Karen's annotations show that her initial entry point to promoting students' numeracy was to improve their mathematical knowledge, or specifically their skills. However, her annotations show her growing awareness of elements of numeracy through her participation in the project. Following her annotations counterclockwise appears to reflect her trajectory: she wanted to develop mathematical knowledge but also increase links to real-world contexts. For example, she came to recognize the importance of using real data. She also noticed increased student confidence and initiative in relation to dispositions. Her note about taking a risk reflects her own change in perspective in relation to students having confidence in their mathematical skills rather than just being able to demonstrate the skills. Her comment at the top of the diagram is an overall reflection on the growth of her own conceptualization of numeracy.

Of the 20 teachers involved in the project, 18 completed the mapping task in the way we requested. We examined every annotated copy of the model to identify the entry point or starting element indicated by each teacher and then the sequence of elements in which they claimed to have developed interest and understanding as the project progressed. The resulting teacher trajectories are shown in Table 3.

Of the 18 valid responses, 8 people indicated that they had entered the project with a concern for students' *dispositions*. Their annotations suggested that they were uneasy with students' negative feelings towards mathematics and wanted to devise units of work that would have a positive impact on dispositions. Seven teachers indicated that their starting point had been students' *mathematical knowledge* and skills, and their annotations suggested that they believed that if students had appropriate mathematical knowledge and skills, they would be successful in applying these as required in context. Only three teachers indicated that they started the project with an emphasis on *contexts*, stating that through contexts, students could apply their mathematics knowledge in meaningful situations. None of the teachers indicated that they came to the project with a primary interest in the use of *tools* or a *critical orientation*.

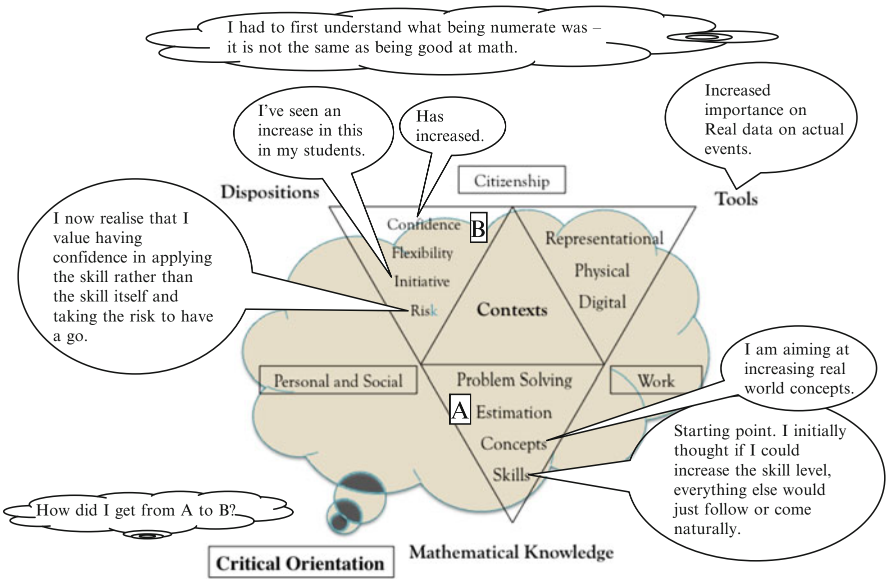

Fig. 4 Karen's trajectory through the numeracy model

Table 3 Teacher's self-identified trajectories through the numeracy model

Starting element	Trajectories		
Dispositions (D)	D → K/T/C	D → C	
	D → K/T → C	D → C → T	
	D → K/T → C/CO	D → C → K (2 teachers)	
	D → K/T/C → CO		
Knowledge (K)	K → D (2 teachers)	K → T → D (2 teachers)	K → C → D
	K → D/C		
	K → D → T		
Context (C)	C → K → CO	C → All	
	C → K → D → T		

D dispositions, *K* knowledge, *T* tools, *C* context, *CO* critical orientation

Although varied, teachers' trajectories through the model showed some patterns of similarity (see Table 3). *Knowledge* to *dispositions* (K → D) and *dispositions* to *knowledge* (D → K) were common patterns, possibly indicating teachers' beliefs about the connection between success in using mathematical knowledge and a positive disposition. Only four teachers indicated that they considered the *critical orientation* aspect of the numeracy model, and this was their end point. Although the teachers identified different starting points and trajectories through the numeracy model, at least half of the valid responses to the mapping task indicated they had attended to four of the model's five components during the life of the project: 16 teachers annotated *knowledge*, 16 *dispositions*, 13 *contexts*, and 9 *tools*.

Challenge: The Roman Colosseum

Ancient Rome was said to be founded in 753BC.

1. How long after Rome was founded was the Colosseum commissioned?
2. How long after Rome was founded was the Colosseum finished?
3. How long after the Colosseum was finished was the Arch of Constantine built?

The Colosseum is elliptical in shape. The area of an ellipse is given by the formula $A = \pi ab$, where a is the "long radius" and b is the "short radius".

4. What is the area of the Colosseum?
5. Would the Colosseum fit into the Melbourne Cricket Ground (MCG)?

"It cost more to visit the Colosseum today than to see the football at the MCG!", said a disgruntled Australian visitor.

7. Is this statement true, or is he just grumpy because his football team won the wooden spoon again this year?

Fig. 5 The Roman Colosseum challenge

Case Study of a Secondary School Mathematics Teacher

Maggie was one of the teachers in the project who explored every aspect of the numeracy model as she developed new numeracy teaching strategies. She taught mathematics and science at a large secondary school in a rural town. She was an early career teacher only in her second year of teaching. The class with which she worked for this project was a grade 8 mathematics class.

First School Visit

Initially, Maggie struggled to come to grips with how to highlight the *numeracy* within mathematics, but she decided to focus on teaching mathematics in real-life contexts that would be of interest to her students. She was supported in her planning by the school's mathematics coordinator, who was an experienced teacher. Together they planned an investigation based on the television program *The Amazing Race*. They decided that students would need 2–3 weeks in the computer laboratory to complete the investigation, which was based on organizing an adventure holiday around the world, given an itinerary and a budget of $10,000. Along the way students had to complete a number of challenges for which they earned an additional $2,000 each. The challenges, which included *Diving with Sharks* in Cairns, *Skiing* in Switzerland, and visiting *The Roman Colosseum*, focused on using directed number in context, a topic that students had studied in the previous weeks. In *The Roman Colosseum* challenge, students were also required to use formulas in the context of comparing areas of the Colosseum and the Melbourne Cricket Ground, as well as looking at exchange rates and converting between currencies (see Fig. 5). Students were expected to use the Internet to find information about flights, accommodation, and places they would be visiting. Maggie felt nervous about the first lesson because she was unsure of how the class would react. At first students seemed somewhat daunted by the size of the investigation, but Maggie and the mathematics coordinator decided that it was well structured enough to be tackled in small chunks.

Members of the research team observed the second lesson of this unit. The lesson took place in the computer laboratory that Maggie had booked for the duration of this numeracy investigation. Students entered the room and went straight to work without any prompting by Maggie. Although there were enough computers for each student, most collaborated with a partner on the tasks. Students appeared motivated and well prepared, and they were able to explain the investigation to us when we questioned them. Maggie noted that some previously disengaged students were interested in the investigation, while a few others remained aloof. Some students seemed so engaged in the task that they acted as though it was real; for example, when Maggie asked one boy "Where are you up to?," he replied, "I'm on my way to Paris!"

This lesson placed mathematics in the real-life *context* of an adventure holiday. It targeted *mathematical knowledge* of directed numbers and operations with integers (formulae, money calculations), using digital (Internet) and representational (charts, tables) *tools*. We did not observe teacher actions that promoted positive *dispositions* towards numeracy, but students were clearly motivated and confident in tackling the investigation and trying out different combinations of flights and accommodation bookings that would fit within their budget. A *critical orientation* does not seem to have been built into this investigation. However, this orientation could be promoted via teacher questioning, such as that we observed when Maggie helped a student compare advantages and disadvantages of booking cheap backpackers' accommodation.

At this stage of the project, Maggie thought she had changed the way she approached teaching numeracy in mathematics by placing more emphasis on using "bigger" tasks without a purely mathematical focus. She also realized that tasks she thought were routine, such as extracting data from tables, posed numeracy challenges for students that she had previously taken for granted.

Second School Visit

Since our first visit, Maggie had been reflecting on what she had learned as a teacher from her previous investigation – *The Amazing Race*. She observed that any student who had attempted the task had done something well, but overall the performance by students on the task was uneven. Maggie attributed this unevenness to absenteeism, for some students, who found it difficult to catch up on the work they had missed and so lost momentum with the larger task, and, for other students, difficulty with maintaining focus on the task for its 3-week duration.

Maggie used *The Amazing Race* experience when planning her next task – an investigation into the relationship between the heights and walking speeds of

Does your height influence your walking speed?

In this activity you will investigate whether there is any relationship between a person's height and their walking speed.

Using Microsoft Excel and the data we have collected, construct a scatterplot for our class data, with height in cm on the horizontal axis and speed on the vertical axis. Also construct scatterplots for the male data and the female data separately.

Comment on whether there is a trend in the data. Compare all three graphs. What are the similarities and differences?

Based on your findings is it possible to predict the walking speed of the following people?

Staff Member	Teaching Area	Height (cm)	Predicted Speed
Miss G	Mathematics	156	
Mr D	Science	196	
Mr M	Principal	180	
Miss J	English	178	

If so, explain the process you have used to make the predictions. If not, explain your reasons for not being able to make a prediction.

Fig. 6 Predicting walking speeds

students in her class, which was part of a bigger theme titled *Approaches to a Healthy Lifestyle*. Within this theme, Maggie included many smaller tasks that she hoped would make it easier for students to maintain interest and to catch up if they were absent.

The mathematics embedded in the *height versus walking speed* investigation included elements of collecting, representing, reducing, and analyzing data. Students were required to learn how to calculate the mean, median, and mode of a data set, represent data using line and scatterplots, and use representational tools such as graphs to make predictions about an individual's walking speed, given their height (see Fig. 6 for part of this activity). As part of the preparation for the task, Maggie had explicitly taught the underlying mathematical concepts and skills required by the task.

In the lesson we observed, students were to make scatterplots using Excel in order to determine whether there was a pattern in the data they had collected on height and walking speed. In earlier lessons, they had collected height data and calculated the mean, median, and mode. In another lesson, students had marked out a 40 m section of a 100 m running track and then found the time it took to walk this distance. With this information, students had calculated their walking speeds in meters per second, meters per minute, and kilometers per hour.

Students worked in the computer room in much the same way as we had previously observed. All appeared engaged with the task and each group or individual produced a scatterplot, although the appearance of the graphs varied between each group and individual depending on the scales chosen or on the choice of variable for the x and y axes. Most students were able to describe a general trend in the data and use this to make a prediction about what might be Maggie's (Miss G's) or the school principal's walking speeds, based on their heights (see Fig. 6). Interestingly, many students gave most attention to their own data point within the scatterplot with comments such as "This is me" (pointing at the appropriate data point) and "This is how tall I am and how fast I walk." Using personal data seemed to be effective for engaging students with the task. From a student's perspective, the activity was about them and how they compared to the rest of the class.

Students expressed surprise that the scatterplot was not linear, so that taller people did not necessarily walk faster. Maggie spoke to each group and challenged them to explain why this should be the case. Some groups suggested that alternative variables – with associated alternative hypotheses – should be explored, including, for example, the relationship between walking speed and leg length or between walking speed and stride rate. One group suggested there might be a stronger relationship between a person's height and their maximum walking pace rather than their natural walking pace.

Maggie chose an engaging *context* that made use of students' personal details to introduce the *mathematical knowledge* that was used in this lesson. The use of personal data encouraged positive *dispositions* towards involvement in and completion of the task. This task required knowledge of how to produce a scatterplot from a data set using Excel and the capacity to make predictions from trends in the data. Maggie asked students to use *representational tools* such as scatterplots and *digital tools* in the form of computers and Excel. By challenging students to explain the variance in their data from the anticipated linear relationship, Maggie introduced a *critical orientation* to the task.

Maggie's Trajectory Through the Numeracy Model

When we asked what the key factors in developing Maggie's new understanding of teaching numeracy were, she said she began with a desire to improve her teaching by increasing her focus on embedding student learning in engaging *contexts*. She believed this was a vital precondition to helping students understand why they needed to gain *mathematical knowledge*. Through the course of the project, Maggie noticed her increased focus on developing activities that provided a *critical orientation* towards the use of mathematics. Only later did she realize the role that *dispositions* played in encouraging students to try approaches to solving a problem for themselves rather than expecting her, as the teacher, to simply provide solutions. By the end of the project, Maggie said she had also increased her use of digital *tools* because she could see there were advantages in using these to explore and analyze authentic contexts.

When Maggie reflected on how she had changed and what she had learned during the course of the project, she identified her readiness to make use of more extended tasks when teaching mathematics. However, she tempered this view by arguing that tasks needed to be made up of self-contained subtasks that allowed students to move towards smaller achievable goals. Structuring tasks in this way also meant that students who had been absent were not intimidated by what they needed to do to cover work that took place while they were away.

In the future, Maggie aims to implement two extended units per semester like *The Amazing Race* and *Approaches to a Healthy Lifestyle*. For her, the level of engagement she observed while students were working on thematic activities was a compelling case for their inclusion within mathematics classes. Together with her teaching partner in this project, she was invited to speak at a staff meeting about her involvement in the project. She hopes that once other teachers understood the benefits for students of working with context-driven, extended tasks, there might be opportunity to work across a broader range of subject areas.

Effectiveness of the Professional Development Approach

Changes in teachers' confidence in numeracy teaching provide an indicator of the effectiveness of the professional development approach that was used in this project.

At the first and last project meetings, teachers completed a survey that asked them to assess their confidence in various aspects of numeracy teaching. The survey was based on the *Numeracy Standards for Graduates of Pre-Service Teacher Education Programs* published by the Queensland Board of Teacher Registration (2005). Two sets of standards were published: one for teachers of mathematics (early years and primary teachers, specialist mathematics teachers in the middle and senior years of schooling) and another for teachers of disciplines other than mathematics (specialist teachers in the early and primary years, as well as teachers of subjects other than mathematics in the middle and senior years of schooling). Participating teachers were asked to identify themselves as belonging to one of these two categories in order to complete the relevant survey.

The Numeracy Standards draw on the *Standards for Excellence in Teaching Mathematics in Australian Schools* formulated by the Australian Association of Mathematics Teachers (2006). They address three domains:

1. Professional knowledge: knowledge of students, of numeracy, and of students' numeracy learning
2. Professional attributes: personal attributes, personal professional development, and community responsibility
3. Professional practice: learning environment, planning, teaching, and assessment

Standards statements were available for each domain and sub-domain. These were turned into survey items for which teachers were asked to indicate their level of confidence, using a 5-point Likert scale, where a score of 1 corresponded to very unconfident, 2 to unconfident, 3 to unsure, 4 to confident, and 5 to very confident.

At the first and last project meetings, 15 and 12 teachers, respectively, completed the self-assessment as teachers of mathematics, while 4 and 5, respectively, completed the self-assessment as teachers of disciplines other than mathematics. Some teachers were present at only the first or last meeting, while two changed the way they identified themselves (as teachers of mathematics or other disciplines) between the first and last meetings. Because we are interested in change over time, we report only on data obtained from teachers who attended both the first and last project meetings and who completed the same version of the survey on both occasions. These respondents included nine teachers of mathematics and three teachers of disciplines other than mathematics.

To analyze survey responses, score totals were first calculated for each survey item for both groups of teachers who completed the surveys at the first (pre) and last (post) project meetings. To examine changes in confidence between the beginning and end of the project, shifts in the total scores of at least 4.5 and 1.5 were considered to be of interest for the two groups, respectively, because this was equivalent to half the group changing their level of confidence by 1 point on the Likert scale (e.g., from unsure to confident). The magnitude of score totals was also of interest, with score totals of at least 36 or 12 indicating confidence (i.e., an average score of 4) across the respective groups.

The complete data set is provided in Table 4 for teachers of mathematics and in Table 5 for teachers of disciplines other than mathematics. The tables also provide the results of the pre-post analysis using the criteria described above: score totals on items for which teachers indicated they felt confident are presented in bold type, while items signaling a change in confidence over the duration of the project are identified by shaded cells.

Table 4 indicates that at the start of the project, the teachers of mathematics (primary teachers as well as specialist teachers of mathematics in the middle and senior years of schooling) felt confident that they possessed the personal attributes and commitment to professional learning required for numeracy teaching, which vindicated their selection as participants. They also had confidence in some aspects of their professional knowledge (knowledge of the diversity of students' numeracy needs, of the pervasive nature of numeracy, of numeracy learning opportunities across the curriculum). However, they lacked confidence in their ability to establish an appropriate numeracy learning environment, plan for numeracy learning, and demonstrate effective numeracy teaching and assessment strategies. By the end of the project, these teachers felt confident in almost every aspect of numeracy teaching, apart from their ability to foster risk taking and critical inquiry in numeracy learning and to cater for the diversity of mathematical abilities and numeracy needs of learners. According to the analysis criteria identified earlier, their confidence levels had risen substantially on 16 of the 32 survey items, most notably on those in the domains of professional knowledge and professional practice. We were also interested to note increased confidence in areas not explicitly targeted by the project, such as theories of how students learn mathematics and the use of multiple representations of mathematical ideas.

Table 5 suggests that the small group of teachers of disciplines other than mathematics expressed greater confidence in numeracy teaching than their colleagues at the start

Table 4 Confidence scores for teachers of mathematics (pre-post); $n=9$

Domain	Standard statement	Score totals	
Professional knowledge	**Teachers will**	**Pre**	**Post**
Students	Understand the diversity of mathematical abilities and numeracy needs of learners	**38**	**38**
Numeracy	Exhibit sound knowledge of mathematics appropriate for teaching their students	35	**37**
	Understand the pervasive nature of numeracy and its role in everyday situations	**37**	**40**
	Demonstrate relevant knowledge of the central concepts, modes of inquiry, and structure of mathematics	32	**36.5**
	Establish connections between mathematics topics and between mathematics and other disciplines	35	**41**
	Recognize numeracy learning opportunities across the curriculum	**36**	**42**
Students' numeracy learning	Understand contemporary theories of how students learn mathematics	29	**36**
	Possess a repertoire of contemporary, theoretically grounded, student-centeredteaching strategies	33	**36**
	Demonstrate knowledge of a range of appropriate resources to support students' numeracy learning	32	**36.5**
	Integrate ICTs to enhance students' numeracy learning	29	**36.5**
Professional attributes	**Teachers will**		
Personal attributes	Display a positive disposition to mathematics and to teaching mathematics	**40**	**42**
	Recognizethat all students can learn mathematics and benumerate	**37**	**42.5**
	Exhibit high expectations for their students' mathematics learning and numeracy development	**37**	**40**
	Exhibit a satisfactory level of personal numeracy competence for teaching	**40**	**40**
Personal professional development	Demonstrate a commitment to continual enhancement of their personal numeracy knowledge	**40**	**41**
	Exhibit a commitment to ongoing improvement of their teaching of mathematics	**41**	**44**
	Demonstrate a commitment to collaborating with teachers of disciplines other than mathematics to enhance numeracy teaching and learning	**38**	**37**
Community responsibility	Develop and communicate informed perspectives of numeracy within and beyond the school	34.5	**36**
Professional practice	**Teachers will**		
Learning environment	Promote active engagement in numeracy learning	35.5	**39**
	Establish a supportive and challenging numeracy learning environment	33.5	**37**
	Foster risk taking and critical inquiry in numeracy learning	32.5	31
Planning	Highlight connections between mathematics topics and between mathematics and other disciplines	34	**38.5**
	Cater for the diversity of mathematical abilities and numeracy needs of learners	34	35
	Determine students' learning needs in numeracy to inform planning and implementation of learning experiences	31.5	**36**
	Embed thinking and working mathematically in numeracy learning experiences	28.5	**36**
	Plan for a variety of authentic numeracy assessment opportunities	31.5	**36**
Teaching	Demonstrate a range of effective teaching strategies for numeracy learning	32.5	**37**
	Utilize multiple representations of mathematical ideas in mathematics and in other curriculum areas	28.5	34
	Sequence mathematical learning experiences appropriately	35.5	**39.5**
	Demonstrate an ability to negotiate mathematical meaning and model mathematical thinking and reasoning	30.5	**36**
Assessment	Provide all students with opportunities to demonstrate their numeracy knowledge	31	**39**
	Collect and use multiple sources of valid evidence to make judgmentsabout students' numeracy learning	30	**36**

Table 5 Confidence scores for teachers of disciplines other than mathematics (pre-post); $n = 3$

Domain	Standard statement	Score total	
Professional knowledge	**Teachers will**	**Pre**	**Post**
Students	Recognize the numeracy knowledge and experiences that learners bring to their classrooms	**12**	**12**
	Understand the diversity of numeracy needs of learners	**12**	**12**
Numeracy	Understand the pervasive nature of numeracy and its role in everyday situations	**13**	**13**
	Understand the meaning of numeracy within their curriculum area	**12**	**13**
	Recognize numeracy learning opportunities and demands within their curriculum area	8	**14**
Students' numeracy learning	Demonstrate knowledge of a range of appropriate resources and strategies to support students' numeracy learning in their curriculum area	11	**12**
Professional attributes	**Teachers will**		
Personal attributes	Display a positive disposition to supporting students' numeracy learning within their curriculum area	**13**	**13**
	Recognize that all students can be numerate	11	10
	Exhibit high expectations of their students' numeracy development	**12**	11
	Exhibit a satisfactory level of personal numeracy competence for teaching	**13**	**13**
Personal professional development	Demonstrate a commitment to continual enhancement of personal numeracy knowledge	**13**	**14**
	Exhibit a commitment to ongoing improvement of their teaching strategies to support students' numeracy learning	**13**	**14**
	Demonstrate a commitment to collaborating with specialist teachers of mathematics to enhance their own numeracy learning and numeracy teaching strategies	**13**	**14**
Community responsibility	Develop and communicate informed perspectives of numeracy within and beyond the school	**12**	**13**
Professional practice	**Teachers will**		
Learning environment	Promote active engagement in numeracy learning within their own curriculum context	**13**	**13**
	Establish a supportive and challenging learning environment that values numeracy learning	**13**	**12**
Planning	Take advantage of numeracy learning opportunities when planning within their own curriculum context	**13**	**14**
	Display willingness to work with specialist teachers of mathematics in planning numeracy learning experiences	**13**	**14**
	Determine students' learning needs in numeracy to inform planning and implementation of learning experiences	11	**13**
Teaching	Demonstrate effective teaching strategies for integrating numeracy learning within their own curriculum context	**12**	**14**
	Model ways of dealing with numeracy demands of their curriculum area	**12**	**14**
Assessment	Provide all students with opportunities to demonstrate numeracy knowledge within their curriculum area	**12**	**13**

of the project (score totals equivalent to "Confident" on 18/22 items). Yet, despite this high starting point, reasonable gains in confidence over the life of the project were recorded on items related to recognizing the numeracy learning opportunities and demands in their own curriculum areas, determining students' numeracy learning needs to inform planning, demonstrating effective numeracy teaching strategies, and modeling ways of dealing with the numeracy demands of their curriculum area.

Expressions of increased confidence provide indirect evidence of the effectiveness of the professional development program and need to be interpreted in the light of observations of classroom practice and other data collected from teachers, such as the trajectories through the numeracy model discussed in a previous section. Teachers reported feeling confident about many aspects of numeracy teaching before the project started, and so we were surprised to see endorsement of even higher confidence levels at the end of the project. However, the responses to the confidence survey are consistent with other data in that we observed changes in teachers' planning and classroom instruction as well as changes in their understanding and use of the numeracy model.

Conclusion

In highlighting the positive outcomes of this project, we do not wish to imply that all participating teachers made significant changes to their practice, even though most claimed to have gained more confidence in numeracy teaching and a better understanding of what numeracy means. We have not reported here on our observations of the varying levels of commitment to the project displayed by the teachers with whom we worked. Many became fully immersed in exploring and implementing the numeracy model to the extent that some commented that the model had changed the way they thought about teaching. However, a few teachers, according to their students, only made an effort to incorporate numeracy into their lessons on occasions when the researchers were visiting their schools. There are many possible reasons why teachers might engage with, ignore, or even resist changes in instructional practice promoted by teacher educators. Pedagogical beliefs, planning skills, and constraints within the school environment that limit access to resources or support for new ideas are all factors that need to be considered when designing research that aims to transform teaching practice.

There are many challenges in planning for and promoting numeracy learning across the school curriculum. This study demonstrated that it is possible to plan for numeracy learning, but teachers also need to be alive to serendipitous moments for promoting numeracy as opportunities occur during lessons, for example, by "seeing" the numeracy embedded in current events or students' personal experiences. Effective numeracy teaching also requires that teachers have a rich conception of numeracy themselves. The numeracy model provided a framework for attending to and valuing numeracy in a holistic way. Teachers seemed most comfortable with incorporating the *knowledge*, *dispositions*, and *contexts* components of the model into their thinking about numeracy. Development of a *critical orientation* occurred to a lesser extent, and in general teachers continued to express low confidence in this aspect of their practice. Even those individuals who eventually incorporated a critical orientation into their planning did so only after exploring and becoming comfortable with the other elements of the numeracy model. Perhaps teachers still lacked a clear understanding of how a critical orientation could be embedded into numeracy teaching, or they may not have felt ready to address this aspect of the model until their understanding of other components was secure. Further research is needed to explore how teachers can be supported in developing personal conceptions of numeracy, as well as numeracy teaching practices, that value a critical orientation, since this perspective is vital to educating informed and aware citizens.

References

Australian Association of Mathematics Teachers. (2006). *Standards for excellence in teaching mathematics in Australian schools*. Retrieved June 5, 2012, from http://www.aamt.edu.au/content/download/499/2265/file/standxtm.pdf

Australian Curriculum, Assessment and Reporting Authority. (2012). *General capabilities in the Australian Curriculum*. Retrieved March 24, 2012, from http://www.australiancurriculum.edu.au/GeneralCapabilities/Overview/

Board of Teacher Registration, Queensland. (2005). *Numeracy in teacher education: The way forward in the 21st century*. Brisbane: Author.

Cohen, D., & Hill, H. (2001). *Learning policy: When state reform works*. New Haven: Yale University Press.

Geiger, V., Goos, M., & Dole, S. (2011). Teacher professional learning in numeracy: Trajectories through a model for numeracy in the 21st century. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and (new) practices* (Proceedings of the 23rd biennial conference of the Australian Association of Mathematics Teachers and the 34th annual conference of the Mathematics Education Research Group of Australasia, pp. 297–305). Adelaide: AAMT & MERGA.

Goos, M. (2007). *Developing numeracy in the learning areas (middle years).* Keynote address delivered at the South Australian Literacy and Numeracy Expo, Adelaide.

Goos, M., Geiger, V., & Dole, S. (2010). Auditing the numeracy demands of the middle years curriculum. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 210–217). Fremantle: MERGA.

Human Capital Working Group, Council of Australian Governments. (2008). *National numeracy review report*. Retrieved January 7, 2011, from http://www.coag.gov.au/reports/docs/national_numeracy_review.pdf

Jablonka, E. (2003). Mathematical literacy. In A. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Second international handbook of mathematics education* (pp. 75–102). Dordrecht: Kluwer.

Loucks-Horsley, S., Love, N., Stiles, K., Mundry, S., & Hewson, P. (2003). *Designing professional development for teachers of science and mathematics* (2nd ed.). Thousand Oaks: Corwin Press.

McLeod, D. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York: Macmillan.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston: NCTM.

National Curriculum Board. (2009). *Shape of the Australian curriculum: Mathematics*. Retrieved June 5, 2012, from http://www.acara.edu.au/verve/_resources/Australian_Curriculum_-_Maths.pdf

Noss, R., Hoyles, C., & Pozzi, S. (2000). Working knowledge: Mathematics in use. In A. Bessot & J. Ridgeway (Eds.), *Education for mathematics in the workplace* (pp. 17–35). Dordrecht: Kluwer.

Organisation for Economic Cooperation and Development. (2004). *Learning for tomorrow's world: First results from PISA 2003*. Paris: OECD.

Sfard, A., & McClain, K. (2002). Analyzing tools: Perspectives on the role of designed artifacts in mathematics learning. *The Journal of the Learning Sciences, 11*(2&3), 153–161.

Steen, L. (2001). The case for quantitative literacy. In L. Steen (Ed.), *Mathematics and democracy: The case for quantitative literacy* (pp. 1–22). Princeton: National Council on Education and the Disciplines.

Zevenbergen, R. (2004). Technologising numeracy: Intergenerational differences in working mathematically in new times. *Educational Studies in Mathematics, 56*, 97–117.