Transformation of Japanese Elementary Mathematics Textbooks: 1958–2012

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Abstract Quality of teaching is a major factor in students' mathematics learning. Stigler and Hiebert (1999) showed that mathematics teaching in Japanese schools is significantly different from what is typically observed in US classrooms. However, Japanese mathematics educators claim that Japanese mathematics teaching has transformed significantly over the last 50 years. Although teaching is influenced by a variety of factors, textbooks play a significant role in what mathematics is taught and how it is taught. In other words, textbooks may significantly influence students' opportunities to learn. Thus, six editions of a Japanese elementary school mathematics series since 1958 were analyzed to identify any change that might indicate the transformation of mathematics instruction in Japan. The analysis revealed that the features included in the series have changed over the years to support more explicitly the problem-solving-based mathematics instruction described by Stigler and Hiebert (1999).

Keywords Elementary school mathematics • Japan • Historical analysis • Textbook analysis • Problem-solving-based instruction

Introduction

There is a general consensus that teaching is the most critical in-school factor influencing students' learning (e.g., National Council for Accreditation of Teacher Education 2010). Therefore, continuously improving mathematics teaching is a major focus of mathematics educators, both practitioners and researchers.

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Mathematics teaching is, however, a complex activity and is influenced by many factors. It is unlikely that changes in one single factor would completely transform mathematics teaching either individually or collectively. On the other hand, the effects of changes in several factors may not be purely additive – the whole may be more than just the simple sum of the parts. Therefore, it is important that we continue to work on those factors we do know influence mathematics teaching.

One important factor that has been shown to influence teaching is textbooks. Shimahara and Sakai (1995) argued that elementary school teachers in both Japan and the United States heavily depend on their textbooks to teach mathematics. Textbooks are the essential bridge between the intended curriculum (such as a national course of study in Japan and the Common Core State Standards (CCSSI 2010) in the United States) and the implemented curriculum. Thus, textbooks influence both what and how mathematics teachers teach, which in turn influence students' opportunities to learn mathematics.

Stigler and Hiebert (1999) characterized Japanese mathematics instruction as "structured problem solving" (p. 27). In this form of teaching, a lesson starts with a teacher posing a problem without showing students how to solve it. After students tackle the problem independently for several minutes, the teacher will have them share their solutions, often both correct and incorrect. The teacher will then orchestrate a whole class discussion, carefully analyzing the shared ideas to lead the class to an understanding of new mathematics. The lesson ends with a brief period in which the teacher, often with the students, summarizes what was learned in the lesson. According to a survey conducted by the Japan Society of Mathematical Education (2001), more than 95 % of Japanese teachers surveyed felt that this style of mathematics teaching that centers on problem solving is a generally effective teaching model. In the same survey, about 60 % of the teachers responded that they either regularly or frequently utilize this style of teaching. An additional 37 % of the teachers responded that they occasionally implement problem-solving-based lessons.

Watanabe's (2001) examination of Japanese elementary school mathematics textbooks and the accompanying teacher's manuals revealed that the textbooks are organized to support structured problem solving. In the Japanese elementary mathematics textbooks, the beginning of a lesson is signified by a problem. The teacher's manual will often include anticipated students' responses, including common misconceptions. The manual also provides a mathematical evaluation of some of those responses, which may be useful as teachers orchestrate the whole class discussion. In addition, the teacher's manual includes *hatsumon* which are key questions teachers can pose to facilitate students' mathematical explorations.

Although the current Japanese elementary mathematics textbooks may be organized to support structured problem solving, some Japanese mathematics educators argue that the shift to the problem-solving-based mathematics instruction is a fairly recent event, strongly influenced by the publication of the NCTM's *Agenda for Action* in 1980 (e.g., A. Takahashi 2001, personal communication). Several other influential writings on problem solving, including George Polya's *How to Solve It*, were translated and published in Japan in the 1970s and 1980s, which Japanese mathematics educators examined and tested their ideas through lesson study to gradually transform their instruction.

Therefore, if the shift to structured problem solving is a recent event and textbooks are one of the critical influences of mathematics instruction, a natural question to ask is how Japanese mathematics textbooks have changed over the years. To explore that question, 6 editions of a Japanese elementary mathematics textbook series from 1958 to present were analyzed. This chapter reports the findings from the analysis of these editions and discusses the potential implications.

Methodology

Textbooks

Currently, there are six commercial publishers who produce elementary school (grades 1 through 6) mathematics textbooks. The textbook series examined for this study is published by Tokyo Shoseki. Historically, the series has been one of the two most widely used elementary mathematics textbooks in Japan. These two series are used in about 70 % of Japanese schools. The 1989 and 2008 editions have been translated into English. All textbooks used in Japanese schools must be reviewed and approved by the Ministry of Education, Culture, Sports, Science, and Technology to ensure their alignment to the national courses of study (COS). Since the original COS, which was published after the World War II, the COS has been revised eight times. Specifically, the editions examined in this study were approved for six different revisions – 1958, 1968, 1977, 1989, 1999, and 2008.[1]

Mathematical Focus of the Analysis

Examining the entirety of the textbooks was not feasible. Therefore, the analysis focused on two topics: area of triangles and quadrilaterals in grade 5 and multiplication and division by fractions in grade 6. These two topics were selected because they were two of the critical foundations for algebra identified by the National Mathematics Advisory Panel (2008). In addition, the grade-level placement of these topics remained constant across all revisions of the COS. By focusing on the topics that were consistently discussed at the same grade level, the difference in grade placement could be eliminated as a potential reason for modifications. Finally, these topics remain challenging both for teachers to teach and for students to learn. These topics can easily be taught by simply giving students the formulas or the

[1] Because some of the old editions obtained for the analysis did not include the publication years, in this manuscript these editions are referenced by the corresponding COS years.

algorithms. Yet, such a procedural focus is far from sufficient in light of recent recommendations and standards (e.g., NCTM 2000; CCSSI 2010). Therefore, understanding how Japanese textbooks transformed the teaching of these topics may be informative for teachers from other countries.

Textbook Analysis

Because the current study is examining the changes in Japanese elementary school mathematics textbooks in light of the structured problem-solving approach to mathematics teaching, the analysis needed to focus on the important features of this teaching approach. Those features include:

- A lesson focus on one (or a few) problem(s)
- An invitation for students to share their own ideas
- Critical examination of solution strategies by students to synthesize a new idea and/or a procedure (Stigler and Hiebert 1999; Takahashi 2011)

Thus, even though a lesson centers on a problem, the solution of the problem is not the focus. Rather, it is the reasoning process of solution strategies and collective critical reflection on those strategies that are the central features of instruction. Furthermore, visual representations play an important role for both teachers and students (Nunokawa 2012). Therefore, a decision was made to focus the analysis on problems and visual representations in these editions. In addition, we attempted to identify and examine any other features that might influence the way teachers might teach mathematics with these textbooks.

The analysis of these editions took place in two stages. In stage one, the focus was identifying features of the textbook. Thus, during this stage, all problems as well as their locations in these editions were marked. The problems were then counted and examined to determine their natures – for example, if the question was just asking for a specific numerical answer or asking for an explanation. The problem context for all word problems was also noted. Likewise, all visual representations in the units were marked, and their types were recorded.

In the second stage of the analysis, the findings identified in the first stage were compared and contrasted across different editions. For example, a probe was made into the use of the same problems, or problems in the same context but different numerical values, in different editions. If a problem found in one edition was not in other editions, the body of the textbook in other editions was examined to see if the same question, or a similar one, was being discussed in the narrative. Another example of the comparison made is the nature of worked-out solutions. If a complete solution to a problem was presented in one edition, the other editions were examined to see if a comparable problem was also worked out. As those worked-out problems were compared, it was also noted that some editions would attribute those solutions to hypothetical elementary school students and ask students who are using the textbook to think about the solution strategy. Yet, in another edition, alternative solutions were presented, and students were asked to compare them.

Similar comparisons were made with respect to the visual representations identified in the first stage. For example, if a particular type of visual representation was used in an edition, the other editions were checked to see if the same type of representation was also used with similar problems and what other representations preceded or followed the representation. For example, most editions used double number line diagrams to represent multiplication of fractions, but in some editions, the representation was presented later in the unit than in others. Finally, as different editions of the series were compared and contrasted, modifications of some features in these editions were noted.

Findings

Problems

Table 1 summarizes the number of problems in these 6 editions of the textbook series. As for the number of problems, the oldest edition (1958) appears to include a slightly smaller number of problems than the other five editions, both in terms of the total numbers and in terms of the average number per textbook page. This difference becomes more distinct when we consider where these problems are found. In a textbook chapter, whether we are looking at a Japanese textbook or a US textbook, we often find special sections that are composed of collections of problems. Those sections are often titled "Exercises," "Practices," "Unit Problems," etc. The 1958 edition differs from the other five editions in that it contains many more problems proportionally in those special sections than the other 5 editions do. Thus, when only the main body of the unit is considered, the 1958 edition contains, on the average, only one problem per page, much fewer than the other five editions, as it can be seen in Table 1.

On the surface, a fewer number of problems may appear to be more consistent with the problem-solving teaching often attributed to Japanese mathematics teaching. However, there is another difference in where problems appear in the 1958 textbook

Table 1 Number of problems and their distributions

COS year	1958	1968	1977	1989	2000	2008
Area						
# of problems total	43	64	71	53	60	88
# of problems/page	3.1	4.6	4.4	3.3	4.3	4.2
# of problems in special sections	26	27	15	13	19	14
# of problems in the body/page	1.0	4.1	4.3	3.6	4.1	4.1
Fractions						
# of problems total	68	115	147	80	82	86
# of problems/page	3.4	4.4	4.6	3.8	4.3	3.7
# of problems in special sections	44	48	45	16	25	19
# of problems in the body/page	2	3.7	4.1	3.8	3.8	3.5

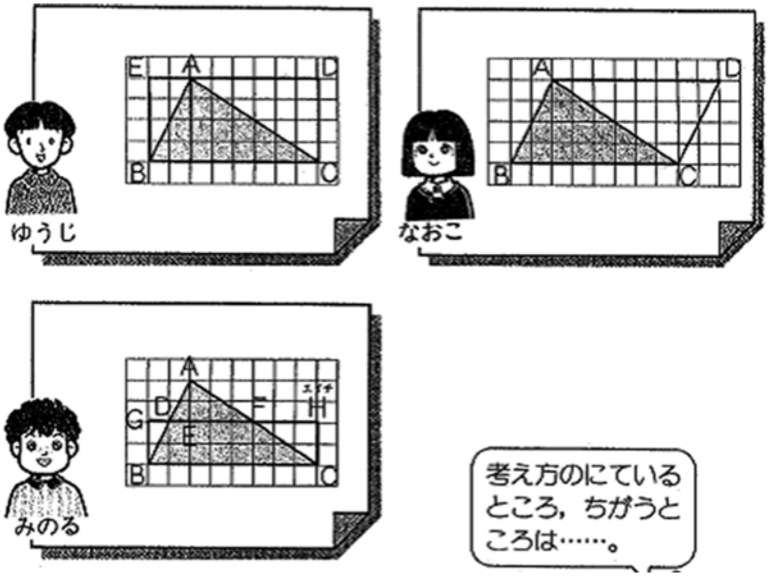

Fig. 1 Three hypothetical students' ideas about how to find the area of the triangle (*shaded*) from the 1989 edition of the textbook (p. 73)

compared to the other edition. In the 1958 edition, problems often follow explanations. For example, in the area unit, the 1958 textbook opens with an explanation of how a parallelogram may be transformed into a rectangle by cutting and rearranging a triangular section from one end to the other. Then, the question is posed to find the area of this parallelogram. In contrast, starting with the 1968 edition, students are first presented with the task, "Let's think about ways to find the area of this parallelogram." Thus, although the 1958 edition may contain a fewer number of problems, the way those problems are posed in the textbook does not appear to be consistent with the structured problem-solving approach in which students are asked to tackle a problem without first being shown how such a problem may be solved.

Another way the 1958 edition is different from the other editions is the number of open-ended problems. Many – in fact, a majority – of the problems in all of these editions of the textbook series ask for one specific numerical answer, such as the area of a triangle with specific dimensions or how much 1 m of wire weighs when the weight of 1 1/3 m of the same wire is given. However, there are also questions that do not have a specific numerical answer. For example, in the area unit of the 1989 edition, students are asked to "explain ways 3 students found the area of the given triangle" (see Fig. 1).

In the multiplication of fractions unit of the same edition, students are asked to think about ways to calculate $4/5 \times 2/3$. For the purpose of this analysis, these types of problems were labeled "open" problems. As it can be easily seen in Table 2, the number of open problems dramatically increased starting in the 1968 edition. The increase in open problems is more drastic in the units on fraction multiplication

Table 2 Number of open problems found in the body of the textbooks across the six editions

COS year	1958	1968	1977	1989	2000	2008
Area						
# of open problems in the body of textbook	3 (18 %)	12 (32 %)	13 (23 %)	16 (40 %)	12 (29 %)	16 (22 %)
Fractions						
# of open problems in the body of textbook	0 (0 %)	12 (18 %)	9 (9 %)	15 (23 %)	20 (35 %)	27 (40 %)

% in the parentheses indicates the proportion of open problems in the body of textbooks

and division. In the 2000 and 2008 editions of the textbook, more than a third of problems in the body of the textbook are open problems.

Although the five editions – 1968, 1977, 1989, 2000, and 2008 – share many similarities that contrasted with the 1958 edition, the three most recent editions are different from the 1968 and 1977 editions in important ways. Although it is very common for textbooks to attribute an idea or a solution of a problem to a hypothetical student, starting in the 1989 edition, this series also began including cartoon drawings of those students. Moreover, for some problems, the textbook includes two (or more) students' ideas and asked students (readers) to examine, compare, and contrast those ideas. For example, Fig. 1 above shows a problem from the 1989 edition (5A p. 73) that asks students to explain how Yuji, Naoko, and Minoru thought about finding the area of the given triangle. Figure 2 comes from the fraction multiplication unit in the most recent (2008) edition. The textbook poses the following problem as the opening problem in the unit (translation is by the author throughout this chapter):

With 1 deciliter of paint, we can paint 4/5 m^2 of boards. How many square meters of boards can we paint with 2/3 deciliters of this paint?

Then, solutions by Yumi and Hiroki are shown, and students are asked to compare the final equations in these two solution approaches.

In the 1958, 1968, and 1977 editions, there are no instances in which the textbooks presented more than one student's ideas simultaneously to be examined. Having students examine multiple solutions to a given problem is a key step in the structured problem-solving instruction. Thus, starting with the 1989 edition, this series seems to include that step of instruction explicitly.

Representations

The analysis of representations used in these editions of the textbook focused on the fraction multiplication and division units in grade 6. The area units contained many drawings, but they are of the figures whose area must be determined. Therefore, they were not considered "representations."

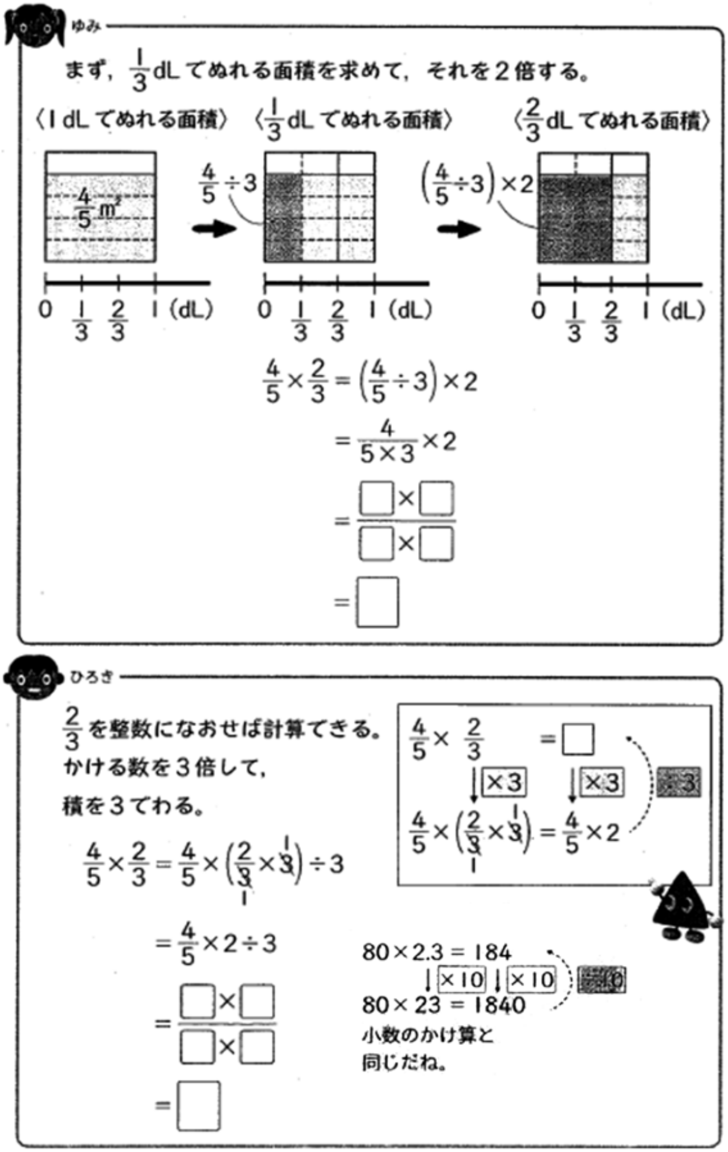

Fig. 2 Two hypothetical students' ideas about how to calculate 4/5×2/3 from the 2008 edition of the textbook (p. 25)

Once again, representations – both in types and how they are used – in the 1958 edition are different from the other five editions. In the 1958 edition, the unit on fraction multiplication and division opens with a story in which students are trying to determine the area of a flowerbed at their school. The rectangular flowerbed

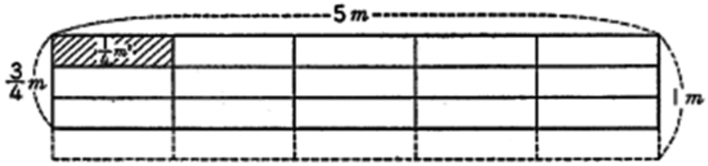

Fig. 3 Area model presented in the opening section of the 1958 unit on fraction multiplication and division (p. 5)

measures 5 m by 3/4 m. The book goes on to describe how a student, Yoshiko, thought of this situation as $5 \times 3/4$,[2] which is multiplying a fraction by a whole number, the idea they studied in grade 5. The textbook carries out the calculation and concludes that the area of the flowerbed is 3 3/4 m^2.

The textbook then presents the reasoning of another student, Tadashi. Tadashi, unlike Yoshiko, thought of the situation as $3/4 \times 5$, multiplication of a whole number by a fraction, something they had not yet studied. The book then presents the area model shown in Fig. 3 and explains how $3/4 \times 5$ can be calculated.

The textbook explains that, from the diagram, we can see that the flowerbed is made up of 15 small rectangles with areas of 1/4 m^2 each. Therefore, the total area of the flowerbed is 3 3/4 m^2. Thus, the 1958 edition uses the area model to illustrate multiplication by fractions, and the diagram is used as a tool for the authors to explain the procedure.

In the other five editions, unlike the 1958 edition, the unit opens with a problem. Although the problems in these five editions all involve area, the mathematical nature of the problems is different from the problem in the 1958 edition. The problems in the five later editions are as follows:

A tractor can plow 3/5 ha of fields in 1 h. How many hectares of fields can you plow in 3/4 h? (1968)

With 1 deciliter of paint, you can paint 3/5 m^2 of boards. How many m^2 can you paint with 3/4 deciliters of this paint? (1977)

With 1 deciliter of paint, you can paint 4/5 m^2 of boards. How many m^2 can you paint with 2/3 deciliters of this paint? (1989, 2000, and 2008)

Although these problems involve the area of a rectangular region, the factors are no longer the dimensions of the rectangle. Rather, these problems are rate problems. Therefore, these five editions use a slightly different representation which is a combination of the area model with a number line (see Fig. 4, from the 1977 edition).

[2] In the Japanese convention, the first factor in a multiplication expression represents the multiplier. In this textbook series, multiplication (and division) of fractions by whole numbers is discussed before the unit on multiplication by fractions, sometimes in grade 5 and sometimes in grade 6, depending on the COS. This is done so because students can continue to use the equal group interpretation as long as the multiplier is a whole number. When the multiplier becomes something other than a whole number, students must expand their interpretation of the multiplication operation, in addition to thinking about the calculation process.

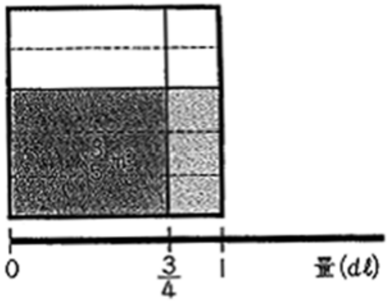

Fig. 4 Representation of the opening problem in the fraction multiplication unit in the 1977 edition (p. 5)

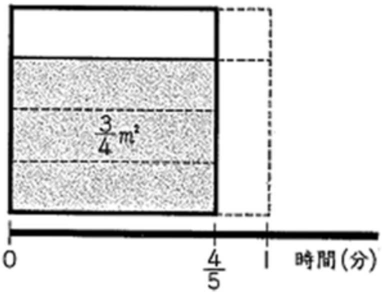

Fig. 5 Combined area-number line representation for a partitive division problem from the 1977 edition

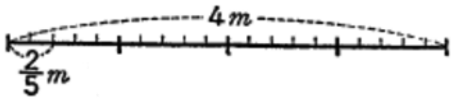

Fig. 6 The representation of the opening division problem in the 1958 edition (p. 11)

Although the end result may be similar to the typical area model representation, this combined area-number line representation may be used with partitive division problems. For example, in the 1977 edition, the unit on fraction division opens with the following problem:

It took 4/5 min for Akira's father to paint 3/4 m^2 of boards. How many m^2 can you paint in 1 min?

This problem is then represented as shown in Fig. 5.

An area model cannot truly represent this problem situation, as 4/5 is not the dimension of the rectangle. However, an area model may be used to represent the calculation, $3/4 \div 4/5$ by drawing a rectangle with the area of 3/4 m^2 and 4/5 m as one of the dimensions. However, such a drawing is of little help to actually find the quotient. In fact, the 1958 edition, the division of fraction section starts with the situation in which a student cuts out 2/5 m segments from a 4 m tape, a quotitive division situation. The textbook then uses a segment model shown in Fig. 6 to represent the situation.

Another feature that is common in all but the oldest (1958) edition is the use of equations with words. In these 5 editions, after the problem is posed to the student, the initial emphasis is that the problem situation can be represented by a multiplication equation with a fraction multiplier. In order to help students understand

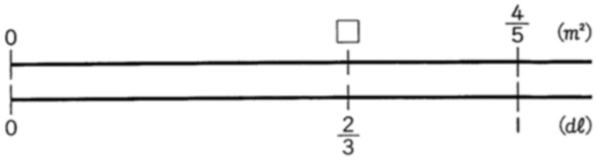

Fig. 7 A double number line representation for the introductory problem on multiplication by fractions in the 1989, 2000, and 2008 editions (Taken from the 2000 edition (p. 63))

that idea, these 5 editions use an equation with words. The problem situations for the 1989, 2000, and 2008 editions are identical, and the textbooks include the following equation:

$$[\text{Area of boards that can be painted with 1 deciliter}] \times [\text{Amount of paint (deciliter)}] = [\text{Area of boards that can be painted}]$$

In each of these five editions, the textbook develops the idea that the problem can be solved by the calculation $3/5 \times 3/4$ (in the 1968 and 1977 editions) or $4/5 \times 2/3$ (in the 1989, 2000, and 2008 editions). Then, and only then, the textbook asks students to think about how this calculation may be completed.

Although the five editions since 1968 use the same combined area-number line representation and an equation with words to introduce multiplication and division by fractions, the three most recent editions (1989, 2000, and 2008) also use a double number line representation (see Fig. 7) that does not appear in the 1968 and 1977 editions. In fact, in these three editions, the double number line representation is presented immediately after the problem statement, before the combined area-number line model and the equation with words.

This model, unlike the area model or the combined area-number line model, does not necessarily help students find the product. Rather, it represents how the quantities in the problem situation are related. However, as Watanabe et al. (2010) noted, this form of representation is used to represent the multiplication and division of decimal numbers in grade 5. Thus, it appears that the intention of this model is also to help students understand the multiplicative nature of the problem situation based on the relationships of the quantities. In these three most recent editions, as well as the 1968 and 1977 editions, the combined area-number line model is used to illustrate how the calculation may be completed.

The 1968 and 1977 editions use a similar representation – double-sided number line – later in the units. For example, in these editions, after the calculation method for fraction multiplication is developed, special cases (e.g., multiplying mixed numbers) are considered. Then, the 1977 edition explores the relationship between the multiplier and the size of the product in relationship to the multiplicand through the following problem:

1 m of cloth costs 360 yen. What is the price of 1 1/3 m of the same cloth? What is the price of 2/3 m?

Fig. 8 A double-sided number line representation from the 1977 edition (p. 9)

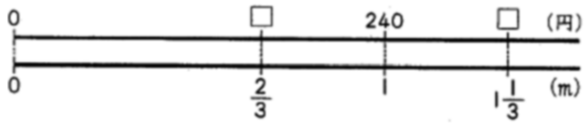

Fig. 9 A double number line representation of a similar problem from the 1989 edition (p. 11)

To illustrate this problem situation, the textbook includes the following model (Fig. 8).

Readers can easily see that the basic structure of this model is the same as that of a double number line. In the 1989 edition of the book, a similar problem (the price of 1 m of cloth is 240 yen) is represented as shown below (Fig. 9).

Thus, it is quite possible to include a double-sided number line representation with the introductory problem in the 1968 and 1977 editions, as the double number line is used in the 1989, 2000, and 2008 editions. However, it is clear that the authors of the 1968 and 1977 editions chose not to do so, while the authors of the 1989, 2000, and 2008 editions intentionally included it as the first model of the problem situation.

General Features

As we examined the general features of these six editions of the series, we noted that the three most recent editions (1989, 2000, and 2008) shared some similarities that are distinct from the previous three editions. For example, in the 1989 through 2008 editions, the opening problems in the units (for both area and multiplication and division of fractions) appear on the right-hand page of the book. All of these problems are worked out; however, because of this layout, the initial pages only show the problems, with the solutions on the following pages. The 1989 and 2000 editions include a 1-page review problem section so that the division of fraction units starts on the right-hand page. Since the units start on the left-hand page in the previous three editions, this choice appears to be intentional.

Another distinct feature of the three most recent editions is the inclusion of cartoonlike characters. The inclusion of cartoon drawings of hypothetical elementary school students was already discussed above. However, in addition to these cartoon- children characters, these three editions include various avatars offering comments and questions. Some of the comments offered by these avatars suggest possible ways of reasoning for the given problem. For example, in the fraction multiplication unit of the 1989 edition, an avatar comments, "What if the amount of the paint used were 2 deciliters…" beside the question asking students to write an equation to represent the problem situation. Thus, the avatar's comment leads students to think about what they have already learned. In the area unit of the 2000 edition, after the textbook asks students to consider ways of determining the area of a parallelogram, an avatar comments, "If we change the shape to something for which we already know how to calculate the area …." Once again, the avatar suggests thinking about ways to use prior knowledge.

Another type of comment offered by these avatars is summaries of mathematical explorations. For example, in the 1989 edition, after students explore the relationship between the multiplier and the size of the product in relationship to the multiplicand, a different avatar offers the summary in a balloon:

$$\text{Multiplier} > 1 \rightarrow \text{Product} > \text{Multiplicand}$$
$$\text{Multiplier} < 1 \rightarrow \text{Product} < \text{Multiplicand}$$

In the 2000 edition, after students discuss various ways to find the area of the given parallelogram by transforming it into rectangles, an avatar comments, "Even though the shapes have changed, their areas are the same, aren't they?"

Discussion

From these six editions of the series, we get the sense that problem solving has been an essential feature of each edition of the textbook. However, problem solving in the oldest edition (1958) appears to play a different role than it does in the other five editions. In the 1958 edition, each unit opens with an inquiry situation. For example, the unit on fraction multiplication begins with a question statement, "How many square meters is the area of a flowerbed at Tadashi's school if it is a rectangle with the length of 3/4 m and the width of 5 m?" However, this question is not marked as a question for students. Instead, the textbook immediately states that the area can be calculated using $5 \times 3/4$ (already learned) or $3/4 \times 5$ (not yet learned). Then, the book goes on to explain how $3/4 \times 5$ can be calculated utilizing the area model. Problems that are clearly marked for students follow the explanation. In contrast, in the 1968 through 2008 editions, each unit opens with a problem that is clearly intended for students. Thus, in the 1958 edition, problems are included to help students practice the ideas that have been explained. In contrast, in the other five editions, problem solving is an important step of mathematics learning.

Although each unit opens with a problem in the five more recent editions, the way the problem is handled is different in the three most recent editions (1989, 2000, and 2008) from how it is handled in the 1968 and 1977 editions. In the 1968 and 1977 editions, the opening problem is completely worked out and explained. For example, in the fraction multiplication unit of the 1977 edition (see above for the problem), the textbook explains that even when the amount of paint used becomes a fraction, like 3/4, we still use multiplication to find the total area painted. Then, they state, "Let's think about how we can calculate $3/5 \times 3/4$." However, this statement is immediately followed by an explanation: "We can determine the amount of area that can be painted with 3/4 deciliters by tripling the amount that can be painted with 1/4 deciliter." Then, the textbook presents the following two tasks to guide students to an answer for the original problem:

Determine the amount of area that can be painted with 1/4 deciliter by calculating $3/5 \div 4$.

Based on the amount of area that can be painted with 1/4 deciliter, determine the amount of area that can be painted with 3/4 deciliters.

The progression in the area unit is similar. After the opening problem, which asks students to think about ways to calculate the area of the given parallelogram, the book immediately instructs the students to change the given parallelogram to a rectangle, as shown in the figure. Thus, in the 1968 and 1977 editions, although the textbook starts with a problem for students, a solution is clearly specified and demonstrated.

On the other hand, the opening problems in the 1988, 2000, and 2008 editions are followed by another question or a less suggestive comment by an avatar. Thus, in these three editions, it is the students who must come up with the multiplication expression, $4/5 \times 2/3$, instead of being given the expression. Moreover, the inquiry task "Let's think about ways to calculate!" is posed clearly as a task to students. Similarly, in the area unit of the 1989 edition, an avatar asks, "How can we change the parallelogram into a rectangle?" Then, instead of the textbook presenting a way to transform the parallelogram into a rectangle, the 1989 edition includes two hypothetical students' ideas and asks students to explain how those two students might have thought about the problem.

Thus, the textbook series overall seems to be moving toward the expectation that students do more reasoning. Perhaps this trend is part of the reason that the average number of problems per page is about the same in the more recent editions compared to the 1968 or 1977 editions, even though the newer editions are dealing with fewer problem situations. Some of the questions worked out in the 1968 and 1977 editions are posed as tasks for students in the newer edition, thus increasing the number of problems.

Although the differences in the oldest edition to the most recent edition are striking, the changes between two successive editions seem to be relatively small in general. The exceptions are between the 1958 and 1968 editions and between the 1977 and 1989 editions. The shift between the 1958 and 1968 editions seems to suggest a significant shift in teaching philosophies. In the 1958 edition, the image of instruction presented in the textbook is that of teacher demonstration, followed by student practice. However, starting with the 1968 edition, this particular series seems to put more emphasis on students' problem solving as the main mechanism of teaching and learning instead of teacher (or textbook) explanation – an image of mathematics instruction more consistent with the structured problem-solving approach described by Stigler and Hiebert (1999).

Although the shift between the 1958 and 1968 editions may indicate the beginning of a shift in instruction, the images of mathematics teaching surmised from the textbook in the 1968 and 1977 editions are still different from structured problem solving. In those two editions, as discussed earlier, a particular approach to solve the given problem is often discussed immediately after the problems are presented. Although some of the ideas may be attributed to a hypothetical student, a mathematics lesson illustrated in the textbook does not include critical examination of a variety of solution processes, an essential component of the structured problem-solving style of teaching. In that perspective, the shift between the 1977 and 1989 editions may be more significant.

As discussed earlier, starting with the 1989 edition, this series began including more than one approach to the opening problem in a unit. Students are then asked to explain the reasoning – an important step in comparing and contrasting the various approaches. Those solution strategies seem to serve as possible examples of students' reasoning that teachers may expect from their students. Furthermore, subquestions and comments by avatars seem to suggest possible teachers' questions and comments spoken while students are solving the opening problem or during the class discussion. Thus, these features in the more recent editions are written just as much for teachers as for students, and the newer textbook seems to support the structured problem-solving approach to mathematics teaching much more explicitly than older editions do.

Even what appear to be superficial changes, like the presentation of the opening problem on the right-hand page, may be significant support for teachers in implementing a problem-solving-based lesson. Although Japanese teachers may rely on their textbooks to teach mathematics lessons, we have also witnessed many lessons in which teachers tell the students to put their books away at the beginning of the lesson. The teachers then present the problem from the textbook for students to think about. The presentation of the problem can be easily done with a document camera or an enlarged copy of the textbook page. If the page contains the solution, the teachers must make sure that the undesired part is covered up.

Although the newer edition of the series appears to be in alignment with the structured problem-solving approach described by Stigler and Hiebert (1999), there are still some aspects of such a style of teaching that is not fully present in the textbook series. For example, in a problem-solving-based lesson, students' incorrect reasoning plays a significant role. However, even the most recent edition of the series does not include incorrect solutions. For example, we know that many students think that the area of a parallelogram may be calculated by multiplying the lengths of two adjacent sides. Such a misconception may play an important and useful role during an actual lesson. However, because it is not included in the textbook, teachers are left to determine how to incorporate it productively in a lesson. Perhaps the teacher's editions provide some suggestions; they were, unfortunately, not available for this analysis.

Closing Remarks

The analysis of the textbook pages presented in this chapter generally supports the claim by some Japanese mathematics educators that the transition to more problem-solving-based mathematics teaching happened gradually over the years. However, the current study has several limitations. First, the analysis only examined units on area of polygons and fraction multiplication and division. Although these are two mathematically significant topics, they occupy only about 10 % of the textbook pages in those two grades.

Furthermore, both of these topics are discussed in the upper elementary level. Might there be differences in the way the textbook is organized in the primary grades versus in the upper elementary levels? A cursory glance through the 2008 edition of the 2nd grade textbook shows that most units start on the right-hand pages. Furthermore, there are a number of problems for which multiple ideas from hypothetical students are presented. Thus, the general patterns observed in this study may indeed be generalized to the whole textbook series. However, a more comprehensive analysis might be useful.

This chapter addresses the potential influences of curriculum, and textbooks in particular, on transforming mathematics instruction. However, the study reported in this chapter is limited in at least two ways. First, we do not really know whether Japanese mathematics instruction transformed over the last half century. The Japanese teaching described in Stevenson and Stigler (1992) was based on observations in the late 1970s and the 1980s. The description appears to be reasonably consistent with the description given in the Stigler and Hiebert (1999), based on the observations made in the 1990s, supporting the idea that teaching is a cultural activity and much of it remains constant across generations (Stigler and Hiebert 1998). Unfortunately, we do not have any data about what Japanese mathematics instruction was like in the 1960s, or earlier. However, we have heard from many Japanese mathematics educators that mathematics teaching in Japan in the 1960s was teacher centered and teacher driven – teaching that is much more consistent with the 1958 edition of the series.

Another obvious limitation is that this study does not involve analysis of actual instruction. Although textbooks may be an important bridge between the intended curriculum and the implemented curriculum, it is still not the implemented curriculum. However, we believe that textbooks do present a vision of mathematics instruction espoused by the authors. We can also anticipate what a lesson might look like if a teacher were to teach from the textbook.

In spite of these limitations, the findings of the study provide some insights into the transformation of mathematics instruction through school curriculum changes, particularly changes in textbooks. Although the current study did not examine actual instruction incorporating this textbook series, it is safe to conclude that the series continues to change to accommodate more and more of the vision of mathematics instruction espoused by Japanese mathematics educators (e.g., Takahashi 2011). For example, the newer editions include more alternative solution approaches to be compared and contrasted during the whole class discussion phase of structured problem solving. Sub-questions following the main problem help teachers establish students as, at least, cocreators of new knowledge. Reflective comments and suggestions offered by cartoon characters provide a model of mathematical habits of mind.

Brown (2009) points out that textbooks can influence teachers' actions through their affordances and constraints. The changes in this Japanese textbook series demonstrate how textbooks can incorporate affordances and constraints to promote a particular approach to mathematics teaching – namely, structured problem solving. These changes adopted by the publisher may be a contributing factor in the spread of this teaching approach, which is now spread to the point that a majority of Japanese teachers practice it frequently.

References

Brown, M. W. (2009). The teacher-tool relationship: Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 17–36). New York: Routledge.

Common Core State Standards Initiative. (2010). *Common core state standards for Mathematics*. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Japan Society of Mathematical Education. (2001). *Sansuujugyou no houhou ni kansuru chosa no kekka*. (Results of a survey of elementary school mathematics lesson format.) *83*(2), 31–42 (in Japanese).

National Council for Accreditation of Teacher Education. (2010). *Transforming teacher education through clinical practice: A national strategy to prepare effective teachers*. Report of the Blue Ribbon Panel on Clinical Preparation and Partnerships for Improved Student Learning. Washington, DC: The Council.

National Council of Teachers of Mathematics. (1980). *Agenda for action*. Reston: The Council.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston: The Council.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Nunokawa, K. (2012). Multi-relation strategy in students' use of a representation for proportional reasoning. *Eurasia Journal of Mathematics, Science, & Technology Education, 8*(4), 167–182.

Shimahara, N. K., & Sakai, A. (1995). *Learning to teach in two cultures: Japan and the United States*. New York: Garland Publishing.

Stevenson, H. W., & Stigler, J. W. (1992). *The learning gap: Why our schools are failing and what we can learn from Japanese and Chinese education*. New York: Summit Books.

Stigler, J. W., & Hiebert, J. (1998). Teaching is a cultural activity. *American Educator, 22*(4), 4–11.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: The Free Press.

Takahashi, A. (2011). The Japanese approach to developing expertise in using the textbook to teach mathematics. In Y. Li & G. Kaiser (Eds.), *Expertise in mathematics instruction: An international perspective* (pp. 197–219). New York: Springer.

Watanabe, T. (2001). Content and organization of teachers' manuals: An analysis of Japanese elementary mathematics teachers' manuals. *School Science and Mathematics, 101*, 194–205.

Watanabe, T., Takahashi, A., & Yoshida, M. (2010). Supporting focused and cohesive curricula through visual representations: An example from Japanese textbooks. In B. Reys & R. Reys (Eds.), *Mathematics curriculum: Issues, trends, and future directions* (NCTM seventy-second yearbook, pp. 131–144). Reston: National Council of Teachers of Mathematics.