Tracking Performance of the Blind Adaptive LMS Algorithm for Fading CDMA Systems

Zahid Ali and Ahmad Ali Khan

Abstract. Reliable and accurate time delay estimation is an important signal processing problem and is critical to a diverse set of applications. Multi-user receivers in asynchronous Code Division Multiple Access (CDMA) systems require the knowledge of several parameters including delay estimates between users. In this paper, we address this problem by proposing a novel approach based on blind least mean squares (LMS) based early-late delay tracker. Analytical expressions have been derived and simulation results of the proposed delay tracker are compared with the classical delay locked loop (DLL) approach in a multipath fading scenario. These results show that the proposed delay tracker provides very good tracking performance in challenging cases of multipath delays.

Keywords: multiple access interference (MAI), delay locked loop, code acquisition, tracking, least mean square algorithm.

1 Introduction

CDMA time delay estimation has received much attention in current literature and many different approaches have been proposed. Joint estimation techniques for single and multiuser case have been also addressed in the literature [1] and as well as multipath scenario in [2]. Filtering methods based on Kalman filter and particle filter have also been investigated [3, 4]. Other techniques based on super resolution methods [5] and expectation maximization have also been researched [6].
The synchronization task can be divided into initial acquisition of relevant delays and subsequent tracking of acquired delays. Acquisition is used to coarsely align the received signal with the locally generated PN code to within one chip duration and then tracking is initiated to minimize the delay offset to maintain

Zahid Ali · Ahmad Ali Khan
KFUPM, Saudi Arabia
e-mail: zawali@ud.edu.sa, s201074140@kfump.edu.sa

S.M. Thampi, A. Gelbukh, and J. Mukhopadhyay (eds.), *Advances in Signal Processing and Intelligent Recognition Systems*, Advances in Intelligent Systems and Computing 264,
DOI: 10.1007/978-3-319-04960-1_16,

synchronization between the signals. Code tracking based on delay-locked loop (DLL) and the tau-dither loop (TDL) [7], have extensively been used. Other variations of the basic DLL and TDL have also been proposed.

The DLL is suitable for code tracking under additive white Gaussian noise (AWGN) channels, but it suffers severe performance degradation in the presence of MAI and multipath fading. The discriminator characteristic or S-curve of the DLL is distorted and randomly biased by the time-varying multipath and MAI. This results in a tracking bias harming the tracking capability of the loop and thus degrading receiver performance.

In this paper we present a modified DLL in a multiuser environment that employs an early and late channel with LMS-type algorithm for delay update.

2 The System Model

We consider here an asynchronous DS-CDMA system with BPSK modulation. The transmitted k[th] user's data signal $s_k(t)$ in a DS/CDMA channel can be modeled by an equivalent complex baseband representation as

$$s_k(t) = \sqrt{2P_k} b_k(i) c_k(t - iT_b) \cos \omega_c t \tag{1}$$

where ω_c is the phase of the carrier, P_k is the power and $b_k(i)$ is the i[th] information bit transmitted by the k[th] user given by

$$b_k(t) = \sum_{i=-\infty}^{\infty} b_k(i) p_{T_b}(t - iT_c)$$

and $c_k(t)$ is the spreading waveform of the k[th] user,

$$c_k(t) = \sum_{m=0}^{N_c} c_k(m) p_{T_c}(t - mT_c)$$

where p_{T_c} is a pulse waveform of length T_c, and N_c is the number of chips in one spreading code period given as $N_c = T_b / T_c$. The spreading sequence is binary, i.e. $c_k(m) \in \{-1,1\}$, τ_k is the transmission delay of the k[th] user. It is assumed that τ_k is independent and uniformly distributed over $[0, T_b]$. The combined transmitted signal due to all K users in the channel is thus given by

$$s(t) = \sum_{k=0}^{K} s_k(t - \tau_k) h_k(t - \tau_k) \tag{2}$$

where $h_k(t)$ is the channel response associated with the k[th] user and is given by
$h_k(t) = a_k(t)e^{j\phi_k(t)}\delta(t)$

Where $a_k(t)$ is the amplitude response of the channel and $\phi_k(t)$ is the phase response associated with it. The received signal is given by

$$
\begin{aligned}
r(t) &= s(t) + n(t) \qquad (3)\\
&= \sum_{k=0}^{K} s_k(t-\tau_k)h_k(t-\tau_k) + n(t)\\
&= \sum_{k=0}^{K} \sqrt{2P_k}\, b_k(i_k)h_k(t-\tau_k)c_k(t-iT_b-\tau_k)\cos(\omega_c t(t-\tau_k)) + n(t)
\end{aligned}
$$

where $i_k = \left\lfloor \frac{(t-\tau_k)}{T_b} \right\rfloor$ and $n(t)$ is complex base band additive white Gaussian noise with zero mean and pass band two-sided power spectral density $N_o/2$.

3 Early Late Delay Tracking Algorithm

In this work we propose an algorithm based on early late delay tracking by introducing a block for LMS for update of the delay τ_k as shown in figure 2. The two channels, an early and a late channel, are used for the purpose of delay adjustment. Each channel has a bank of matched filter (MF) and a multiuser interference estimation block shown in figure 1.

The top channel is called the early channel as the relative delay to the MF bank is "earlier than" the estimated delay $\hat{\tau}$. Similarly, the other channel is called the late channel as the relative delay to the MF bank is "delayed than" the estimated delay $\hat{\tau}$.

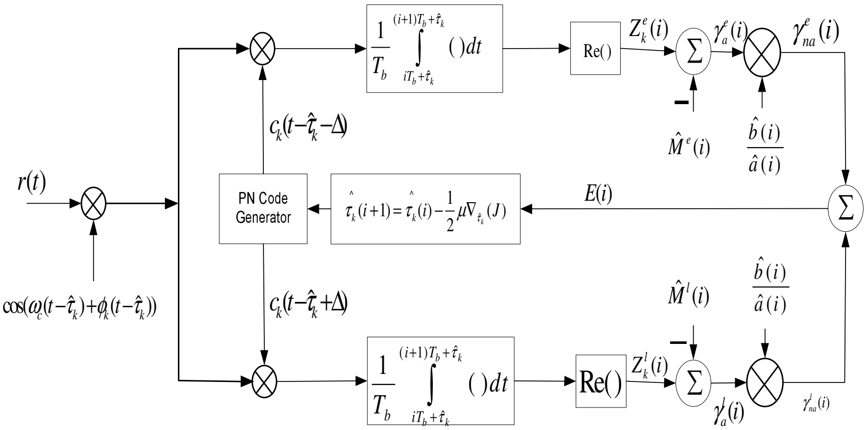

Fig. 1 Proposed Early Late delay tracking structure

Figure 2 shows how estimated delay $\hat{\tau}$ can be used to generate $\hat{M}^{e}(i)$ and $\hat{M}^{l}(i)$, where $\hat{M}^{e}(i)$ and $\hat{M}^{l}(i)$ are the early and late estimated interference from K-1 users, respectively.

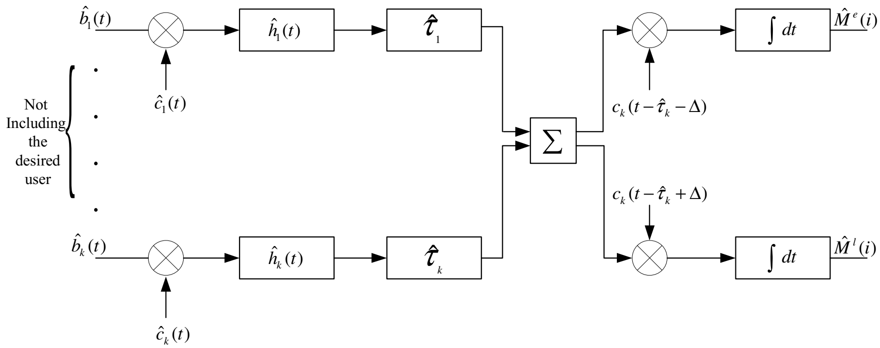

Fig. 2 Proposed $\hat{M}^{e}(i)$ and $\hat{M}^{l}(i)$ estimation block

Let $Z_k^e(i)$ and $Z_k^l(i)$, respectively, represent the output of early and late matched filters as shown in figure 2. Thus, $Z_k^e(i)$ for the k[th] user can be obtained by taking real part of the matched filter operation using k[th] user's spreading waveform as follows:

$$
\begin{aligned}
Z_k^e(i) &= Re\left\{\frac{1}{T_b}\int_{iT_b+\hat{\tau}_k}^{(i+1)T_b+\hat{\tau}_k} r(t)c_k(t-\hat{\tau}_k+\Delta)\cos(\omega_c(t-\hat{\tau}_k)+\phi_k(t-\hat{\tau}_k))\,dt\right\} \qquad (4)\\
&= \sqrt{\frac{P_k}{2}}b_k(i_k)a_k(i_k)R_{kk}^e(i)+\sum_{\substack{m=0\\ m\neq k}}^{k}\sqrt{\frac{P_m}{2}}b_m(i_m)a_m(i_m)R_{mk}^e(i)+n^e(i)
\end{aligned}
$$

where $i_k = \left\lfloor \frac{t-T_b}{\tau_k} \right\rfloor$. Here $R_{kk}^e(i)$ and $R_{km}^e(i)$, respectively, represent the early auto correlation of the k[th] user and early cross-correlation of the k[th] and m[th] user, that is,

$$
R_{kk}^{l}(i)=\frac{1}{T_b}\int_{iT_b+\hat{\tau}_k}^{(i+1)T_b+\hat{\tau}_k} c_k(t-\tau_k)c_k(t-\hat{\tau}_k+\Delta)\cos(\omega_c(t-\tau_k)+\phi_k(t-\tau_k))\cos(\omega_c(t-\hat{\tau}_k)+\phi_k(t-\hat{\tau}_k))dt
$$

$$
R_{mk}^{e}(i)=\frac{1}{T_b}\int_{iT_b+\hat{\tau}_k}^{(i+1)T_b+\hat{\tau}_k} c_k(t-\tau_m)c_k(t-\hat{\tau}_k+\Delta)\cos(\omega_c(t-\tau_m)+\phi_m(t-\tau_m))\times\cos(\omega_c(t-\hat{\tau}_k)+\phi_k(t-\hat{\tau}_k))dt
$$

Similarly, for the late channel, $Z_k^l(i)$, $R_{kk}^l(i)$ and $R_{km}^l(i)$ can be obtained as follows:

$$Z_k^l(i) = Re\left\{\frac{1}{T_b}\int_{iT_b+\hat{\tau}_k}^{(i+1)T_b+\hat{\tau}_k} r(t)c_k(t-\hat{\tau}_k-\Delta)\cos(\omega_c(t-\hat{\tau}_k)+\phi_k(t-\hat{\tau}_k))\,dt\right\} \tag{5}$$

$$= \sqrt{\frac{P_k}{2}}b_k(i_k)a_k(i_k)R_{kk}^l(i) + \sum_{\substack{m=0\\m\neq k}}^{k}\sqrt{\frac{P_m}{2}}b_m(i_m)a_m(i_m)R_{mk}^l(i) + n^l(i)$$

$$R_{kk}^l(i) = \frac{1}{T_b}\int_{iT_b+\hat{\tau}_k}^{(i+1)T_b+\hat{\tau}_k} c_k(t-\tau_k)c_k(t-\hat{\tau}_k-\Delta)\cos(\omega_c(t-\tau_k)+\phi_k(t-\tau_k))\cos(\omega_c(t-\hat{\tau}_k)+\phi_k(t-\hat{\tau}_k))dt$$

$$R_{mk}^l(i) = \frac{1}{T_b}\int_{iT_b+\hat{\tau}_k}^{(i+1)T_b+\hat{\tau}_k} c_k(t-\tau_m)c_k(t-\hat{\tau}_k-\Delta)\cos(\omega_c(t-\tau_m)+\phi_m(t-\tau_m))\times\cos(\omega_c(t-\hat{\tau}_k)+\phi_k(t-\hat{\tau}_k))dt$$

According to the figure 2, the adjusted matched filter output for the early channel, $\gamma_a^e(i)$, may be expressed as

$$\gamma_a^e(i) = Z_k^e(i) - \hat{M}^e(i) \tag{6}$$

$$\approx z^e(i) + \sqrt{\frac{P_k}{2}}b_k(i_k)a_k(i_k)R_{kk}^e(i)$$

Similarly, for the late channel, we have

$$\gamma_a^l(i) = Z_k^l(i) - \hat{M}^l(i) \tag{7}$$

$$\approx z^l(i) + \sqrt{\frac{P_k}{2}}b_k(i_k)a_k(i_k)R_{kk}^l(i)$$

Let $\hat{a}^e(i)$ and $\hat{a}^l(i)$ be the estimated complex amplitudes for the early and late channels, with $\hat{b}^e(i)$ and $\hat{b}^l(i)$ be the estimated symbols, then normalized adjusted output is

$$\gamma_{na}^e(i) = \frac{\gamma_a^e(i)}{\hat{a}^e(i)}$$

and

$$\gamma_{na}^l(i) = \frac{\gamma_a^l(i)}{\hat{a}^l(i)}$$

The error signal between early and late estimate of the desired symbol is given by

$$E(i) = \gamma_{na}^{e}(i)\hat{b}^{e}(i) - \gamma_{na}^{l}(i)\hat{b}^{i}(i) \tag{8}$$
$$= \sqrt{\frac{P_k}{2}} b_k(i)\hat{b}^{e}(i)\frac{a_k(i)}{\hat{a}_k(i)}R_{kk}^{e}(i) - \sqrt{\frac{P_k}{2}} b_k(i)\hat{b}^{l}(i)\frac{a_k(i)}{\hat{a}_k(i)}R_{kk}^{l}(i) + \tilde{n}(i)$$

if the amplitude and data bits are estimated close enough for each channel, then

$$\frac{a^{e}(i)}{\hat{a}^{e}(i)} \approx 1, \qquad \frac{a^{l}(i)}{\hat{a}^{l}(i)} \approx 1$$

$$b^{e}(i)\hat{b}^{e}(i) \approx 1, \quad b^{l}(i)\hat{b}^{l}(i) \approx 1$$

so that

$$E(i) = \gamma_{na}^{e}(i) - \gamma_{na}^{l}(i)$$
$$\approx \sqrt{\frac{P}{2}} R_{kk}^{e}(i) - \sqrt{\frac{P}{2}} R_{kk}^{l}(i) + \tilde{n}(i)$$

where

$$\tilde{n}(i) = n^{e}(i) - n^{l}(i)$$

In the next section we derive expression for the delay update based on LMS algorithm.

4 Derivation of Blind Nonlinear LMS

This algorithm is termed as blind LMS algorithm as it does not have desired output available to calculate the update error and it has non-linearity because the delay is inside the cosine function. The LMS algorithm is most commonly used as an adaptive algorithm because of its simplicity and a reasonable performance. According to the well-known steepest descent approach [8], the LMS-type algorithm for the update of the delay estimate can be set up as follows [11]

$$\hat{\tau}_k(i+1) = \hat{\tau}_k(i) - \frac{1}{2}\mu\nabla_{\hat{\tau}_k}(J) \tag{9}$$

where μ is defined as the step size and J represents the cost function to be minimized which is chosen as the square of the error signal *E(i)* and is given by

$$\mathrm{J} = \mathrm{E}[E^2(i)] \tag{10}$$

$$\begin{aligned}\nabla_{\hat{\tau}_k} \mathrm{J} &= \frac{\partial}{\partial \hat{\tau}_k} \mathrm{J} \\ &= 2E(i)\frac{\partial}{\partial \hat{\tau}_k} E(i) \\ &= 2E(i)\left[R_{kk}^{e\,'}(i) - R_{kk}^{l\,'}(i) \right]\end{aligned} \tag{11}$$

Now, for the above differentiations, we have used the following relation [9]

$$\frac{d}{da}\int_{\psi(a)}^{\varphi(a)} f(x,a)dx = f(\varphi(a),a)\frac{d\varphi(a)}{da} - f(\psi(a),a)\frac{d\psi(a)}{da} + \int_{\psi(a)}^{\varphi(a)} \frac{d}{da} f(x,a)dx$$

Moreover, in the differentiation of cosine terms we have used the approach of [10]

$$R_{kk}^{e\,'}(i) = \frac{\partial}{\partial \hat{\tau}_k} R_{kk}^{e}(i)$$

$$= \frac{\partial}{\partial \hat{\tau}_k}\left\{ \frac{1}{T_b} \int_{iT_b+\hat{\tau}_k}^{(i+1)T_b+\hat{\tau}_k} c_k(t-\tau_k)c_k(t-\hat{\tau}_k+\Delta)\cos(\omega_c(t-\tau_k)+\phi_k(t-\tau_k))\cos(\omega_c(t-\hat{\tau}_k)+\phi_k(t-\hat{\tau}_k))dt \right\}$$

$$= \frac{1}{T_b}\left\{ \begin{aligned} & c_k((i+1)T_b+\hat{\tau}_k-\tau_k)c_k((i+1)T_b+\Delta)\cos\left(\omega_c((i+1)T_b+\hat{\tau}_k-\tau_k)+\phi_k((i+1)T_b+\hat{\tau}_k-\tau_k)\right)\cos\left(\omega_c((i+1)T_b)+\phi_k((i+1)T_b)\right) \\ & -c_k(iT_b+\hat{\tau}_k-\tau_k)c_k(iT_b+\Delta)\cos\left(\omega_c(iT_b+\hat{\tau}_k-\tau_k)+\phi_k(iT_b+\hat{\tau}_k-\tau_k)\right)\cos\left(\omega_c(iT_b)+\phi_k(iT_b)\right) \\ & +\int_{iT_b+\hat{\tau}_k}^{(i+1)T_b+\hat{\tau}_k} c_k(t-\tau_k)\cos\left(\omega_c(t-\tau_k)+\phi_k(t-\tau_k)\right)\frac{\partial}{\partial \hat{\tau}_k}\left[c_k(t-\hat{\tau}_k+\Delta)\cos\left(\omega_c(t-\hat{\tau}_k)+\phi_k(t-\hat{\tau}_k)\right)\right]dt \end{aligned} \right\}$$

In order to evaluate the derivative $\frac{\partial}{\partial \hat{\tau}_k} c_k(t-\hat{\tau}_k-\Delta)$ we employ the methodology of [10] to arrive at

$$\frac{\partial}{\partial \hat{\tau}_k} c_k(t-\hat{\tau}_k+\Delta) = \operatorname{sign}\left(c_k\left(t-\left\lceil \hat{\tau}_k \right\rceil+\Delta\right) - c_k\left(t-\left\lfloor \hat{\tau}_k \right\rfloor+\Delta\right)\right)$$

As a result, the derivative of $R_{kk}^{e}(i)$ is found to be

$$R_{kk}^{e'}(i)=\frac{1}{T_b}\left\{\begin{array}{l} c_k((i+1)T_b+\hat{\tau}_k-\tau_k)c_k((i+1)T_b+\Delta)\cos\left(\omega_c((i+1)T_b+\hat{\tau}_k-\tau_k)+\phi_k((i+1)T_b+\hat{\tau}_k-\tau_k)\right)\cos\left(\omega_c((i+1)T_b)+\phi_k((i+1)T_b)\right) \\ -c_k(iT_b+\hat{\tau}_k-\tau_k)c_k(iT_b+\Delta)\cos\left(\omega_c(iT_b+\hat{\tau}_k-\tau_k)+\phi_k(iT_b+\hat{\tau}_k-\tau_k)\right)\cos\left(\omega_c(iT_b)+\phi_k(iT_b)\right) \\ +\displaystyle\int_{iT_b+\hat{\tau}_k}^{(i+1)T_b+\hat{\tau}_k} c_k(t-\tau_k)\cos\left(\omega_c(t-\tau_k)+\phi_k(t-\tau_k)\right)\left[\begin{array}{l} \operatorname{sign}\left(c_k\left(t-\left\lceil\hat{\tau}_k\right\rceil+\Delta\right)-c_k\left(t-\left\lfloor\hat{\tau}_k\right\rfloor+\Delta\right)\right)\cos\left(\omega_c(t-\hat{\tau}_k)+\phi_k(t-\hat{\tau}_k)\right) \\ -c_k\left(t-\hat{\tau}_k+\Delta\right)\sin\left(\omega_c(t-\hat{\tau}_k)+\phi_k(t-\hat{\tau}_k)\right)\left(-\omega_c+\dfrac{\phi_k(t-(\hat{\tau}_k+\Delta\hat{\tau}_k))-\phi_k(t-\hat{\tau}_k)}{\Delta\hat{\tau}_k}\right)\end{array}\right]dt \end{array}\right\}$$

where $\Delta\hat{\tau}_k=\hat{\tau}_k-\hat{\tau}_{k-1}$

Similarly,

$$R_{kk}^{l'}(i)=\frac{1}{T_b}\left\{\begin{array}{l} c_k((i+1)T_b+\hat{\tau}_k-\tau_k)c_k((i+1)T_b-\Delta)\cos\left(\omega_c((i+1)T_b+\hat{\tau}_k-\tau_k)+\phi_k((i+1)T_b+\hat{\tau}_k-\tau_k)\right)\cos\left(\omega_c((i+1)T_b)+\phi_k((i+1)T_b)\right) \\ -c_k(iT_b+\hat{\tau}_k-\tau_k)c_k(iT_b-\Delta)\cos\left(\omega_c(iT_b+\hat{\tau}_k-\tau_k)+\phi_k(iT_b+\hat{\tau}_k-\tau_k)\right)\cos\left(\omega_c(iT_b)+\phi_k(iT_b)\right) \\ +\displaystyle\int_{iT_b+\hat{\tau}_k}^{(i+1)T_b+\hat{\tau}_k} c_k(t-\tau_k)\cos\left(\omega_c(t-\tau_k)+\phi_k(t-\tau_k)\right)\left[\begin{array}{l} \operatorname{sign}\left(c_k\left(t-\left\lceil\hat{\tau}_k\right\rceil-\Delta\right)-c_k\left(t-\left\lfloor\hat{\tau}_k\right\rfloor-\Delta\right)\right)\cos\left(\omega_c(t-\hat{\tau}_k)+\phi_k(t-\hat{\tau}_k)\right) \\ -c_k\left(t-\hat{\tau}_k-\Delta\right)\sin\left(\omega_c(t-\hat{\tau}_k)+\phi_k(t-\hat{\tau}_k)\right)\left(-\omega_c+\dfrac{\phi_k(t-(\hat{\tau}_k+\Delta\hat{\tau}_k))-\phi_k(t-\hat{\tau}_k)}{\Delta\hat{\tau}_k}\right)\end{array}\right]dt \end{array}\right\}$$

5 Simulation Results

For the purpose of simulation we have considered a typical asynchronous DS-CDMA reverse link with a Rayleigh fading channel for four-users-two path scenario. PN code synchronization follows a procedure of combined tracking/reacquisition tracking after a previous initial acquisition to within half a chip. The value of the update is chosen to be $\mu = 0.0125$ and the first arriving timing epoch has been detected for the estimation purposes.

First we consider the accuracy of DLL-based TOA estimation as shown in Figure 3 in the form of the histogram of the residual timing error at the mobile

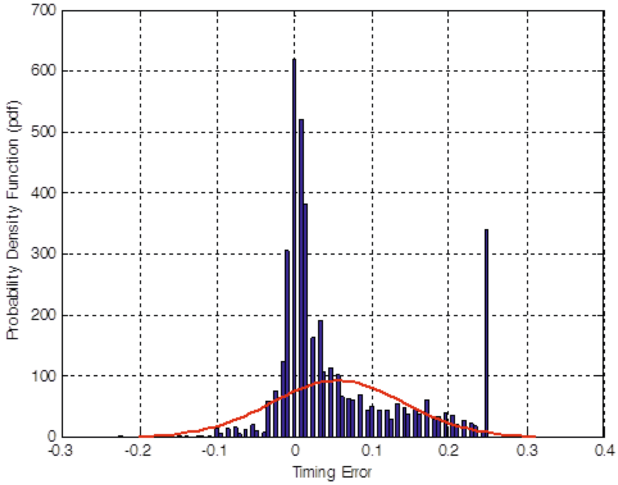

Fig. 3 Histograms for PDF's of DLL timing error (equal power users)

serving base station. It can be seen that the DLL timing error is affected with timing error distributed over $\pm T_c/2$ which is the same as initially assumed after the acquisition stage.

If we compare DLL estimate with the proposed delay tracker we immediately see improved results as shown in Figures 4 where the timing error axis has been zoomed in for error histogram. The timing error for both equal and unequal power

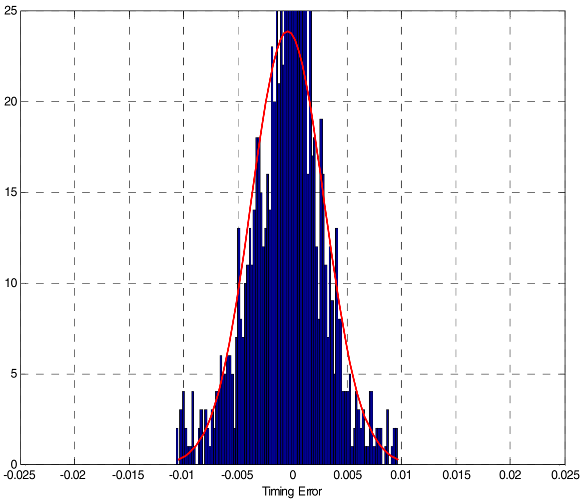

Fig. 4a Histogram of the timing error for LMS based delay tracker for equal power users

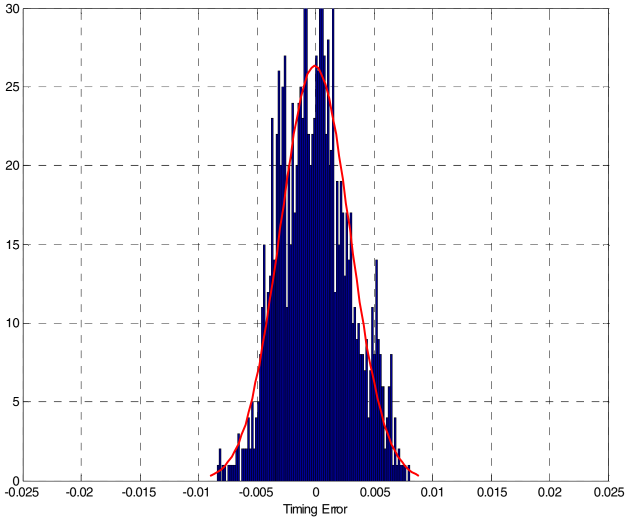

Fig. 4b Histogram of the timing error for LMS based delay tracker for unequal power users

users shows significant improvement converging to zero. It is also clear that the variance of timing error for the proposed structure is also less than the classical DLL structure.

6 Conclusion

In this paper we show that the realization of code synchronization is a challenging problem in DS-CDMA systems. We proposed a new synchronization method using a blind nonlinear LMS approach. Analytical expressions have been derived with simulation results showing that the proposed method performs better than the classical DLL approach in a multipath fading channel.

References

1. Iltis, R.A.: A DS-CDMA tracking mode receiver with joint channel/delay estimation and MMSE detection. IEEE Trans. Communications 49(10), 1770–1779 (2001)
2. Burnic, A., et al.: Synchronization and channel estimation in wireless CDMA systems. In: IEEE 9th International Symposium on Spread Spectrum Techniques and Applications, pp. 481–487 (2006)
3. Flanagan, B., et al.: Performance of a joint Kalman demodulator for multiuser detection. In: Proceedings of the 2002 IEEE 56th Vehicular Technology Conference, VTC 2002-Fall, vol. 3. IEEE (2002)

4. Ghirmai, T., et al.: Joint symbol detection and timing estimation using particle filtering. In: Proceedings of the 2003 IEEE International Conference on Acoustics, Speech, and Signal Processing, ICASSP 2003, vol. 4. IEEE (2003)
5. Ge, F.-X., et al.: Super-resolution time delay estimation in multipath environments. IEEE Transactions on Circuits and Systems I, 1977–1986 (2007)
6. Masmoudi, A., Billili, F., Affes, S.: Time Delays Estimation from DS-CDMA Multipath Transmissions Using Expectation Maximization. In: 2012 IEEE Vehicular Technology Conference (VTC Fall). IEEE (2012)
7. Glisic, S.G.: Adaptive WCDMA Theory and Practice. John Wiley & Sons (2003)
8. Yuan, Y.X.: A new step size for the steepest descent method. Journal of Computational Mathematics 24(2), 149–156 (2006)
9. Grandshteyn, I.S., Ryzhik, I.M.: Table of Integral, Series, and Products, 7th edn. Academic Press, Elsevier (2007)
10. Lim, T.J., Rasmussen, L.K.: Adaptive symbol and parameter estimation in asynchronous multiuser CDMA detectors. IEEE Transactions on Communications 45(2), 213–220 (1997)
11. Ali, Z., Memon, Q.A.: Time Delay Tracking for Multiuser Synchronization in CDMA Networks. Journal of Networks 8(9), 1929–1935 (2013)