Chapter 39
Financial, Real, and Quasi Options: Similarities and Differences

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39.1 Introduction

“Es sei hier nur noch erwähnt, dass die Bachelierschen Betrachtungen jeder mathematischen Strenge gänzlich entbehren” [17, p. 417]. This quote refers to a comment of Andrei Nikolajewitsch Kolmogoroff on the works of Louis Bachelier. What is remarkable about this quote is that it has been published in 1930s but it took more than 60 years to recover the contribution of Bachelier [9] to the evaluation of financial and real assets and to appreciate the contributions of among others Albert Einstein, Adriaan Fokker, Andrei N. Kolmogoroff, Max Planck, and Norbert Wiener for evaluating financial and real options. They laid the foundations for evaluating the movement of particles under uncertainty. The interest of Albert Einstein was not to describe the precise place of a molecule but the probability that a molecule would be at a certain place at a certain time considering its initial position [12]. The mathematics have been further developed by Max Planck and Adriaan Fokker. The Fokker-Planck equation describes the evolution of a probability distribution over time. A similar result has been obtained by Kolmogoroff and known as the Kolmogoroff forward or backward equation. These equations have become a central tool for deriving analytical as well as for developing numerical solutions for investments under uncertainty (e.g. [10, 38]). An important building block of models has been the Wiener Process, named after Norbert Wiener, who formalized

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random walks more rigorous than Einstein did,[1] while first known applications of real options at least date back to the ancient Greeks [6].

While all these developments did happen in the field of mathematics and physics it took until the late 1960s that these methods had been picked up by economists to first evaluate the prices of financial assets under uncertainty where the price of an asset can be seen as being equivalent to a particle in physics. It took again about 10 more years before a number of papers did appear to use the same mathematical tools to evaluate real instead of financial assets. In the early 1970s Kenneth Arrow and Anthony Fisher did publish their seminal paper on valuing environmental preservation under uncertainty and irreversibility [2]. In the same year Claude Henry published his paper on investment and uncertainty and the irreversibility effect [13]. Both, the Arrow and Fisher as well as Henry contribution point out that irreversibility effects create a bias towards delayed investment in comparison to assessments that do not consider uncertainty, irreversibility, and flexibility in decision making explicitly. Arrow and Fisher call the size of the bias the quasi option value while Henry calls it the irreversibility effect.

The main message it that even so the expected value of an investment under uncertainty is positive, the value of the investment considering postponement might be even larger—implying that the profit maximizing strategy is to postpone the investment. This is similar to the evaluation of a financial call option. Exercising a call option might be profitable, the option is "in the money", but further waiting to exercise the option can increase profits.

In the following the three approaches, the financial, real, and quasi option approach will be presented in a discrete time discrete state model. In Sect. 39.3 the three approaches will be compared. The differences and similarities will be illustrated using a numerical example. Section 39.4 discusses applications and challenges for modeling in particular with respect to the bioeconomy as well as an outlook for future research while Sect. 39.5 concludes.

39.2 The Three Approaches

39.2.1 The Financial Call Option

A financial call option gives the holder of the call option the right but not the obligation to buy a financial instrument S, $S : [0, T] \to \mathfrak{R}$ at time t_0, expiring at time T, $T \in \mathfrak{R}^+$ with an exercise price K, $K \in \mathfrak{R}$, T at a given price, C, today, t_0 [21]. The call can only be exercised at maturity date T (European Call Option), the price movement can either be up, u, or down, d, with probabilities q

[1]The Wiener process is a Markov process with a normal distributed variance that increases linear in time.

and $1 - q$. Therefore,

$$\begin{array}{lll} & C_u = \max[0, uS - K] & \text{with probability} \quad q \\ \nearrow & & \\ C & & \\ \searrow & & \\ & C_d = \max[0, dS - K] & \text{with probability} \quad 1 - q. \end{array}$$

The question to be answered in the context of this paper is how much a potential holder of the call option should pay for the call option today.

Following Cox et al. [8] the "fair price" of the call option will be:

$$C = [\rho C_u + (1 - \rho)C_d]/(1 + r), \tag{39.1}$$

with $\rho \equiv \frac{(1+r)-d}{u-d}$ and $1 - \rho \equiv \frac{u-(1+r)}{u-d}$, r the riskless interest rate, $r \in \Re^+$ over the period $0 \rightarrow T$, $u - 1 > 0$ the upward move of the stock price and $d - 1 < 0$ the downward movement of the stock price, $C_u = \max[0, uS - K]$ and $C_d = \max[0, dS - K]$, while the probability of an up-ward move is q and the probability of a downward move is $1 - q$. The result of Eq. 39.1 is obtained by assuming that risks in the movement of the financial instrument can be hedged using a portfolio of riskless bonds and n shares of the financial instrument S. Since the value of the portfolio depends on S, it matches the risk of the call. ρ and (1- ρ) change if a dividend equivalent to $nS(r - 1)$ will be paid. In that case $\hat{\rho} \equiv \frac{1-d}{u-d}$ and $1 - \hat{\rho} \equiv \frac{u-1}{u-d}$. The remarkable result of Eq. 39.1 is that the "fair price" of the call option is independent of the probabilities of the upward, q, or downward $(1 - q)$ movement of the price of the financial instrument.[2] If investors agree on the size of the upward and downward movement, and, the riskless interest rate is the same everyone, then all investors would price the call the same, independently of their attitudes towards risk. This is a noteworthy property which will be relevant when the real option and quasi option value approach will be discussed.

39.2.2 The Real Option Value

The valuing of call options on financial instruments has been translated to the valuation of investments under uncertainty and flexibility. An investment opportunity has properties similar to those of a call option. An investor has the right but not the obligation to invest. The question is whether or not to exercise the option immediately, or to postpone and decide at a later point in time whether or

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[2]This not necessarily applies to dividend paying financial instruments.

not to invest. To introduce the problem, consider a simple investment where the value of the investment option depends on the movement of the net product price, $p, p \in \Re^+$, of the product to be produced. Investment option I, $I : [0, 1] \rightarrow \Re$ at time t_0, expiring at time t_1, $T \in \Re^+$ and $T \rightarrow \infty$. The investment option can only be exercised today, t_0, or at maturity t_1. The product price p_0, can either move up with size u, $u - 1 > 0$ and probability q, or move down with size d, $d - 1 < 0$ and probability $1 - q$. The value of the immediate investment is $V_0 = p_0 + q\frac{up_0}{r} + (1 - q)\frac{dp_0}{r}$. The value of the investment opportunity if one has to invest at t_0 is $\Omega_0 = \max[V_0 - I, 0]$. The value of postponed investment to t_1 is $V_1^u = \frac{up_0}{r}(1 + r)$ or $V_1^d = \frac{dp_0}{r}(1 + r)$ and the value of investment at t_1 is $F_1 = \max[V_1 - I, 0]$. V_1 and F_1 are random variables form the perspective at t_0 and the value at t_0 is $E[F_1] = \{(q \max[V_1^u - I, 0] + (1 - q) \max[V_1^d - 1, 0])/(1 + r)\}$ and the optimal decision to be taken at t_0 is $F_0 = \max[V_0 - I, E(F_1)]$. Assuming $I < \frac{dp_0}{r}(1 + r) + p_0$ it pays to invest immediately. In case $\frac{dp_0}{r}(1 + r) + p_0 \leq I < \frac{up_0}{r}(1 + r)$ it pays to delay and decide after uncertainty has been resolved whether or not to invest. Gains from waiting arise as long as $\frac{dp_0}{r}(1 + r) + p_0 \leq I < \frac{up_0}{r}(1 + r)$ as in this case by Jensen's Inequality $-I + p_0 + q\frac{up_0}{r} + (1 - q)\frac{dp_0}{r} < q(-\frac{I}{1+r} + \frac{up_0}{r})$.

39.2.3 The Quasi Option Value

The quasi-option value approach originates from the paper by Arrow and Fisher [2]. The basic question being asked if whether or not converting a piece of land with amenity values in a different form of use such as e.g. housing when future benefits from preservation as well as development are uncertain but uncertainty be resolved over time generates opportunity costs that are not captured by standard cost-benefit-analysis using expected values of uncertain future benefits from preservation as well as development. They show a bias towards development exists, if the assessment will be based on expected values. The bias reduces the opportunity costs of development and Arrow and Fisher name the bias *quasi option value*. The bias is a result of ignoring that as time passes new information arrives and uncertainty about future states of nature will be reduced. The model is a bit more complex than the real option model presented as benefits from development say investment as well as benefits from preservation say non-investment are considered while in the basic real option model presented benefits from non-investment are zero.

Two future states of nature are considered, A_1 and A_2 with the future denoted as t_1. If A_1 occurs development is the better option while if A_2 occurs preservation is the better one. If A_2 occurs but development has been chosen at t_0 the decision cannot be reversed to preservation. The benefits and costs from development include onetime development costs either c_0 or c_1 and present and future benefits b_{d0} and b_{d1}. The preservation option includes only present and future benefits b_{p0} and b_{p1}. The net-present-value of the opportunity to either preserve or develop at t_0, NPV_0 includes the following mutually exclusive payment streams where all values are expressed in present values:

$$\max NPV = \begin{cases} b_{d0} - c_{d0} + b_{d1} \\ b_{p0} + b_{d1} - c_{d1} \\ b_{p0} + b_{p1}. \end{cases} \tag{39.2}$$

Immediate development would take place if $b_{d0} - c_{d0} - b_{p0} + b_{d1} - \max\{(b_{d1} - c_{d1}), b_{p1}\} > 0$. As the future will be uncertain, benefits and costs at t_1 can be replaced by their expected value resulting in $b_{d0}-c_{d0}-b_{p0}+E[b_{d1}]-\max\{E[(b_{d1}-c_{d1})], E[b_{p1}]\} > 0$. This approach would be appropriate as the standard method applied in cost-benefit analysis, if new information is unavailable. If the arrival of new information can be used, the maximum of the following two alternatives will be the optimal decision:

$$\max NPV = \begin{cases} b_{d0} - c_{d0} + E[b_{d1}] & \text{develop immediately} \\ b_{p0} + \max\{E[(b_{d1} - c_{d1})], E[b_{p1}] & \text{postpone decision.} \end{cases} \tag{39.3}$$

The development option will be chosen if $b_{d0} - c_{d0} + E[b_{d1}] - b_{p0} - \max\{E[(b_{d1} - c_{d1})], E[b_{p1}] > 0$. Now the decision under uncertainty and irreversibility including and excluding future information can be compared. The difference yields: $\max\{E[(b_{d1} - c_{d1})], E[b_{p1}] - \max\{E[(b_{d1} - c_{d1})], E[b_{p1}]$. Again, by Jensen's Inequality this difference is larger than or equal to zero. This difference is the quasi option value that needs to be considered for an appropriate assessment of an investment that includes irreversibilities and uncertainties.

39.3 A Comparison

For comparing the three models a numerical example illustrating similarities and differences will be used. The numerical examples use the parameter values of Chap. 2 in [10] used as well in [11] and [25]. For the purposes of comparison, the *nomenclatura* by Dixit and Pindyck will be used. The equivalent *nomenclatura* used either within financial economics or the quasi option literature has been listed in Table 39.1. The irreversible investment costs I are 1,600. The current price p at $t = 0$, p_0 is 200. The future price after one period at $t = 1$, p_1,is either in the case of an upward jump, u, $p_1^u = 300$ or in the case of a downward jump, d, $p_1^d = 100$. For simplicity, it is assumed that p reflects the net-price, revenues minus reversible costs and that the prices will stay constant until infinity after the end of period one. The probability of an upward jump q is equivalent to the probability of a downward jump $1-q$ with $q = 1-q = 0.5$. The discount rate r will be constant with $r = 0.1$. Hence, the value of the investment, if exercised today will be

Table 39.1 Numerical example comparing the three approaches

Numerical example	FOV	ROV	QOV
1,600	Exercise price	Irreversible investment	Development costs
2,200	Current stock price	Current project value, V	Benefits from immediate development
3,300	Future stock price (underlying) high	Future Price of underlying asset high	Future benefits from development high
1,100	Future stock price (underlying) low	Future Price of underlying asset low	Future benefits from development low
773	Value of the call	Value of the option to invest	Value of the development opportunity considering irreversibility and arrival of additional information
600	Intrinsic option value	Value of immediate investment, $V - I$	Benefits from immediate development
173	Option time value	Value of waiting/flexibility	
227			QOV

$$V_0 = p_0 + q\sum_{t=1}^{\infty}\frac{p_1^u}{(1+r)^t} + (1-q)\sum_{t=1}^{\infty}\frac{p_1^d}{(1+r)^t}$$

$$= p_0 + q\frac{p_1^u}{r} + (1-q)\frac{p_1^d}{r} = 200 + 1500 + 500 = 2,200.$$

The value of the investment at $t = 1$ in the case of a price increase will be $V_1^u = \sum_{\tau=0}^{\infty}\frac{p_1^u}{(1+r)^\tau} = 3{,}300$ and in the case of a price decrease $V_1^d = \sum_{\tau=0}^{\infty}\frac{p_1^d}{(1+r)^\tau} = 1{,}100$. The expected value of an immediate investment is $NPV_i = V_0 - I = 600$. The expected value of a postponed investment valued at $t = 0$ is $NPV_{p,i} = q(V_1^u - I)/(1+r) = 733$). The difference $NPV_{p,i} - NPV_i = 173$ is the value of waiting or in the terminology of financial options the option time value.

In the quasi-option approach of Arrow and Fisher the quasi option value is $E[\max\{0, NPV_{p,i}^u\}] - \max\{0, E[q(NPV_{p,i}^u) + (1-q)(NPV_{p,i}^d)]\}$ yielding $0.5 \cdot 1{,}545.45 - 0.5(1{,}545.45 - 454.54) = 227.23$.[3] The quasi option value is higher than the value of waiting. The difference between the quasi option value and the value of waiting yields: $(1-q)NPV_{p,i}^d - (qNPV_{p,i}^u - NPV_i)$. Collecting terms and simplifying yields $QOV - VOW = \frac{I}{1+r} - I + p_0$. Hence, the difference is the difference in foregone benefits and costs by a postponed investment as pointed out by Mensink and Requate [20]. This includes two components, the benefits arising when immediate investments being made p_0 but reduced by the savings on

[3] Note, the opportunity costs in this example are considered to be zero.

investment costs that can be made by investing later, $\frac{rI}{1+r}$. The foregone benefits and costs have been considered within the real option value approach but not explicitly in the quasi option value approach. In case there are no immediate benefits, i.e. $p_0 = 0$ and decisions will be made continuously, $t \to 0$, *ROV* and *VOW* will be equivalent. In case decisions will be considered at incremental time steps the *QOV* needs to be reduced by foregone benefits and costs to yield the same result for the irreversibility effect within the real option value and quasi option value approach. The foregone benefits and costs can be negative in case $p_0 < \frac{rI}{1+r}$. In this case, the *VOW* will be larger than the *QOV*. For the numerical example, setting $p_0 = 0$ results in a value of waiting of about 418 while the quasi option value remains the same.[4] The foregone benefits as well as the savings in investment costs would be captured by standard benefit-cost-analysis comparing delayed with immediate investment and what matters are the gains from additional information.

Applying the financial option pricing, ρ and $(1 - \rho)$ change if a dividend equivalent to nSr will be paid. In that case $\hat{\rho} = \frac{1-d}{u-d}$ and $1 - \hat{\rho} = \frac{u-1}{u-d}$. Using the evaluation of financial options with $u = 0.5$, $d = 1.5$, $i = 0.1$, $\hat{\rho} = (1 - \hat{\rho}) = 0.5$, $C_u = 3{,}300 - 1{,}600 = 1{,}700$, and $C_d = 0$, yields a value of $C = 1{,}700 \cdot 0.5/1.1 = 773$. If dividend payments will be included, then the value of the call as well as the real option value will depend on the size of q as q has an effect on the expected rate of price changes. A higher q in this case will increase the probability of immediate investment.

39.4 Modifications, Applications, and Outlook

The three approaches discussed have been presented in discrete time discrete state. Most applications deviate from discrete time, discrete state by analyzing investments in a continuous time, continuous state framework. Model applications include not only irreversible costs but also irreversible benefits, optimal abandonment, entry and exit, uncertainty over several variables such as reversible and irreversible costs and benefits, discount rates, and many more as discussed in more detail in recent reviews by Mezey and Conrad [22] and Perrings and Brock [23]. In the following, the discrete time discrete state model discussed in Sect. 39.3 will be presented in continuous time continuous state by choosing as an example the introduction of a new technology.

39.4.1 An Illustrative Case: Introducing Transgenic Crops

Consider a decision maker or a decision making body, similar to an EU Agency, or, the United States Environmental Protection Agency (USEPA) that has the authority

[4]Please not in this case $V_p^u = 3{,}000$ and $V_p^d = 1{,}000$.

to decide whether or not a particular transgenic crop, e.g. a toxin producing crop like Bt-corn,[5] should be released for commercial planting. The agency can approve an application for release or postpone the decision. The objective of the agency is to maximize the welfare of consumers living in the economy and ignore positive and negative transboundary effects. The supply for all transgenic crops is perfectly elastic and demand perfectly inelastic per unit of time. Ex-ante effects of the decision to release transgenic crops on the up-stream sector of the economy are ignored by the agency. The welfare effect of releasing a specific transgenic crop can be described as the net-present-value from T until infinity of the additional net benefits at the farm level, V, which will be further defined below, minus the difference between irreversible costs, I, and irreversible benefits, R. R and I are assumed to be known and constant.

This is a useful simplification for two reasons. Firstly, not much is known about the magnitude of irreversible costs I. As will be shown later, the model can be solved for the irreversible costs and provide information about an acceptable level, which can then be compared with available information. Secondly, information about the irreversible damages from pesticide use on a per hectare level, which are the irreversible benefits of planting transgenic crops, R, are available and can easily be included into the model.

As the agency has the possibility to postpone the decision on whether or not to release the transgenic crop, the agency has to maximize the value resulting from this decision, $F(V)$, to maximize the welfare. This objective can be described as maximizing the expected value from releasing the transgenic crop:

$$\max F(V) = \max E[(V_T - (I - R))\, \mathrm{e}^{-\rho T}] \tag{39.4}$$

where E is the expectation operator, T the unknown future point in time when the transgenic crop is released into the environment and ρ the discount rate.

As the release of a transgenic crop has almost no effect on the fixed costs, the net-benefits from a transgenic crop at farm level for a specific region are the total sum of gross margins over all farms. The welfare effect at farm level, hence, is the difference between the sums of gross margins from transgenic crops minus the total sum of gross margins from the alternative non-transgenic crop (further called conventional crop). From now on this difference will be called the additional net-benefit from transgenic crops B. The instantaneous additional net-benefit, B, at time T, B_T, is then the difference in gross margin between the transgenic and traditional crops. The gross margin for each crop type is defined as the difference between the revenues and variable costs at T. Other additional benefits arising from the application of the new technology, such as, e.g., through "peace of mind", are assumed to be balanced by concerns about the new technology, on average, and, therefore are ignored.

[5]Modified corn that produces the δ-endotoxins of the soil bacterium Bacillus thuringiens which control the European Corn Boxer.

The benefits and costs used to calculate B_T are those that are reversible. The instantaneous additional net-benefits under the given level of information are known with certainty. Future additional net-benefits are uncertain as new information about prices, costs and yields arrives continuously. The uncertainty can be modelled by choosing a stochastic process that describes the future development path of the additional net-benefits B_T.

The geometric Brownian motion has frequently been used to model uncertain returns from agricultural crops [28], returns from pig-raising [24] and on-farm investments [16, 29, 39]. Richards and Green [30] suggest decomposing returns from agricultural crops. They model crop prices as a geometric Brownian motion and crop yields as a geometric Brownian motion combined with a Poisson process, where the geometric Brownian motion represents "normal" years and the Poisson process years with extreme yields. Because additional net-benefits are chosen as the stochastic variable, we assume that extreme yields are smoothed, and, hence, a decomposition of prices and yields is not necessary.

A mean-reverting process could also model additional net-benefits where it is assumed that additional net-benefits decrease over time. The decrease could be explained e.g. by the observation that pests are becoming resistant to plant produced pesticides and weeds to broadband herbicides. Wesseler [34] compares the results of modelling additional net-benefits with a geometric Brownian motion and a mean-reverting process and shows that the different processes could result in different decisions.[6] This leads to the problem of identifying the relevant process. The identification of the relevant process based on time series data is difficult, as the results are ambiguous [27]. Dixit and Pindyck [11] therefore recommend identifying the process based on theoretical arguments.

This case uses the geometric Brownian motion to model the uncertain future additional net-benefits, for the following two reasons: firstly, it is reasonable to assume that technical change for a transgenic crop will be continuous and secondly, the process is analytically tractable.

The geometric Brownian motion is a non-stationary continuous time stochastic process with Markov properties where α is the constant drift rate, σ the constant variance rate and dz the Wiener process, with $E(dz) = 0$ and $E(dz)^2 = dt$:

$$dB = \alpha B dt + \sigma B dz. \tag{39.5}$$

The geometric Brownian motion is the limit of a random walk [7]; hence it is consistent with assuming log-normality of the stochastic variable with zero drift. The expected value of this process grows at the rate α. The use of the geometric Brownian motion also assumes that B_t will not turn negative, which is similar for continuous differentiation of the process at the boundary of zero (see [11], Eq. (39.17), p. 191). This assumes that growers will immediately stop (start) planting without having to bear additional costs as soon as they realize the gross

[6] See also [31] for similar results.

margin will turn out to be negative (positive). Which is a reasonable assumption, as farmers can and do easily move from one crop variety to another.

When today's additional net-benefits, B_T, are known, follow a geometric Brownian motion until infinity and are discounted at μ (the risk adjusted rate of return derived from the capital asset pricing model (CAPM), then the expected present value of additional net-benefits from transgenic crops, V_T at time $t = T$ is:

$$E[V_T] = B_T \int_{t=0}^{\infty} e^{(\alpha-\mu)t} dt = \frac{B_T}{\mu - \alpha}. \tag{39.6}$$

As V_T is a constant multiple of B_T, also V_T follows a geometric Brownian motion with the same drift parameter α and variance parameter σ. If speculative bubbles are ruled out and as $V(0) = 0$, Eq. (39.3) will also be the value of releasing transgenic crops into the environment. Equation (39.4) can then be rewritten:

$$\max F(B) = \max E\left[\left(\frac{B_T}{\mu - \alpha} - (I - R)\right) e^{-\rho T}\right].$$

As the irreversible costs I and the irreversible benefits R are assumed to be constant, the option pricing approach using contingent claim analysis as described by Dixit and Pindyck (1994, Chap. 5)[11] can be applied. This results in the standard second order differential or Fokker-Planck equation, which has to be solved:

$$\frac{1}{2}\sigma^2 B^2 F''(B) + (r - \delta)BF'(B) - rF(B) = 0. \tag{39.7}$$

A solution to this homogenous second order differential equation is:

$$F(B) = A_1 B^{\beta_1} + A_2 B^{\beta_2}. \tag{39.8}$$

Solving Eq. (39.8) according to the boundary conditions (Dixit and Pindyck 1994) provides the following solutions:

$$B^* = \frac{\beta_1}{\beta_1 - 1}\delta(I - R) \tag{39.9}$$

$$A_2 = 0 \tag{39.10}$$

$$A_1 = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{(I - R)^{\beta_1 - 1}(\delta\beta_1)^{\beta_1}} \quad \text{with} \tag{39.11}$$

$$\beta_1 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \tag{39.12}$$

where B^* is the optimal level of additional net-benefits B, r the risk-free rate of return, δ the convenience yield, which is the difference between the risk adjusted discount rate μ and the growth rate α, σ the variance parameter of the geometric Brownian motion of equation (39.2), and β_1 the positive root of the quadratic equation (39.7), in the following called β for short.

Equation (39.9) says it is optimal to release a transgenic crop into the environment immediately, if the additional net-benefits B_T are equal to the with δ annualised difference between irreversible cost and irreversible benefits multiplied by the so called "hurdle rate" or option multiplier $\beta/(\beta - 1)$.

Equation (39.9) can be rearranged to:

$$I^* = R + \frac{B_T}{\delta} / \frac{\beta}{\beta - 1} = R + \frac{B_T}{\delta} - \frac{B_T/\delta}{\beta}, \tag{39.13}$$

where I^* are the maximum incremental social tolerable irreversible costs of releasing a transgenic crop into the environment. This shifts the attention from the additional net-benefits to the irreversible cost. The irreversible costs are now the critical variable, whereas the additional net-benefits are assumed to be known. This is a more reasonable expression, as far more information is available about the additional net-benefits from field trails, releases of similar crops or from other countries. Equation (39.13) can be formulated as a rule that the agency should follow when it has to decide whether or not a transgenic crop should be released:

> Postpone the release of a transgenic crop into the environment, if the irreversible costs are higher than the irreversible benefits plus the present value of an infinite stream of instantaneous additional net-benefits, using the convenience yield as the relevant discount rate, divided by the hurdle rate.

This rule has two important properties, which result out of the use of the contingent claim analysis (see Appendix). Firstly, future costs and benefits have been discounted using rates provided by the market. No individual discount rates have been used. Secondly, uncertainty about the additional net-benefits has been included by using a riskless hedge portfolio and, hence, the evaluation of the benefits is independent of attitudes towards risk, which reduces the impact of risk-preferences on decision-making.

The last formulation of the maximum incremental social tolerable irreversible costs in Eq. (39.13) illustrates the effect of waiting due to uncertainty and irreversibility. The first two terms, R and B/δ, illustrate the results of the orthodox approach. Without recognizing explicitly irreversibility and uncertainty, the benefits are the sum of the irreversible benefits plus the present value of infinite additional net-benefits. By including irreversibility and uncertainty, a proportion of the present value of infinite additional net-benefits, $\frac{B_T}{\delta}/\beta$, has to be deducted. This proportion in this context can be interpreted as the economic value of uncertainty and irreversibility of releasing transgenic crops.

The maximum incremental social tolerable irreversible costs as explained in Eq. (39.13) will change over time with new information about additional net-benefits. These changes will not only consist of changes in yields but also of changes in product prices and variable costs due to regulatory and other polices.

These policies will have either an increasing or decreasing effect on I^*. An increase (decrease) in I^* can be seen as an increase (decrease) in the likelihood to release transgenic crops earlier, as the higher (lower) the maximum incremental social tolerable irreversible costs are the lower (higher) the chances that they will be crossed. The impact of changes in the growth rate α and the standard deviation σ on the annualised hurdle rate are illustrated in Table 39.1. An increase in the growth rate α increases the maximum incremental social tolerable irreversible costs as the first derivative of I^* with respect to α is positive (proof in Appendix 2):

$$\begin{aligned}\frac{\partial I^*}{\partial \alpha} &= \frac{\partial (B/\delta)}{\partial \alpha}\frac{\beta}{\beta-1} + \frac{B}{\delta}\frac{\partial((\beta-1)/\beta)}{\partial \alpha} > 0 \\ &= \frac{\partial (B/\delta)}{\partial \alpha}\frac{\beta}{\beta-1} + \frac{B}{\delta}\beta^{-2}\frac{\partial \beta}{\partial \alpha} > 0.\end{aligned} \tag{39.14}$$

The overall effect can be decomposed into two effects. The first term on the right-hand-side of Eq. (39.14) shows the impact on current additional net-benefits B, which is positive. An increase in α reduces the discounting effect, increases total benefits, increases the maximum incremental social tolerable irreversible costs and hence, increases the probability of an earlier release. The second term on the right-hand-side reduces the effect, as the partial derivative of β with respect to α is negative. This is the effect of a higher growth rate on the option value. An increase in the growth rate increases the value of releases in the future, which increases the value of the option to release at a later point in time and hence increases the probability of a later release. As the effect on the present value is greater then the effect on the future value the overall effect is positive as mentioned earlier.

An increase in the uncertainty of additional net-benefits has the opposite effect, as the impact of an increase in the variance parameter σ on I^* is negative as the partial derivative of β with respect to σ is negative (proof in Appendix 2):

$$\begin{aligned}\frac{\partial I^*}{\partial \sigma} &= \frac{B}{\delta}\frac{\partial((\beta-1)/\beta)}{\partial \sigma} < 0 \\ &= \frac{B}{\delta}\beta^{-2}\frac{\partial \beta}{\partial \sigma} < 0.\end{aligned} \tag{39.15}$$

An increase in uncertainty decreases the likelihood of an early release, as the future benefits increase while future losses can be avoided by waiting. This is the standard result from the literature on financial economics.

A change in the risk-free rate of return, r, also has a negative impact on the maximum incremental social tolerable irreversible costs I^* (proof in Appendix 2):

$$\begin{aligned}\frac{\partial I^*}{\partial r} &= \frac{B}{\delta}\frac{\partial((\beta-1)/\beta)}{\partial r} < 0 \\ &= \frac{B}{\delta}\beta^{-2}\frac{\partial \beta}{\partial r} < 0.\end{aligned} \tag{39.16}$$

The decreasing effect of an increase in the risk-free rate of return can be explained by the decrease of the opportunity costs of the option to release transgenic crops (see Appendix 1).

Also of interest is a simultaneous change in the growth and the variance rate. Considering Young's theorem this can be modelled by getting the derivative of I^* with respect to α and σ (proof in Appendix 2):

$$\frac{\partial^2 I^*}{\partial\sigma\partial\alpha} = \frac{\partial(B/\delta)}{\partial\alpha}\beta^{-2}\frac{\partial\beta}{\partial\sigma} + \frac{B}{\delta}\frac{\partial\left(\beta^{-2}\frac{\partial\beta}{\partial\sigma}\right)}{\partial\alpha} < 0. \tag{39.17}$$

The first term of Eq. (39.17) shows the change the growth rate α has on the current additional net benefits, which is positive and multiplied by the negative effect of σ on β. Hence, the total effect of the first term on I^* is negative. This negative effect is augmented by the second term also being negative. The overall impact of a simultaneous marginal change is a decrease in the maximal tolerable irreversible costs I^*. The positive effect of an increase in the growth rate on the likelihood of an earlier release is surpassed by the negative effect of an increase in uncertainty on the likelihood of an earlier release.

The continuous time continuous state result of the simple investment problem presented above provides well-known results (see e.g. [11, 19]). While considering uncertainty over one or more variables as long as they follow the same stochastic process can often still be solved analytically, most models have to be solved numerically. One of the major building blocks has been the Wiener process. Other processes include jump processes to consider drastic environmental changes or ex-post liabilities, mean-reverting processes for deviations from long-term equilibria, Brownian bridges, and more as discussed in detail in a number of text books such as [1, 14, 32] or [11].

Problems can be solved either by using a dynamic programming or a contingent claim approach. The major problem within the dynamic programming approach is the right choice of the discount rate or discount rates. This is not a trivial issue as the debate about climate change policies illustrates. Within the contingent claim analysis this less of a problem as market are used, but the problem will be the identification of the appropriate matching portfolio that replicates the uncertainty of the investment under consideration, i.e. the quasi option value. Nevertheless within the debate about environmental problems such as the conservation of biological diversity this would be a promising approach as it would allow to identify the "fair" market price of biological diversity.

In general, for applications related to the bioeconomy whether at the micro or at the macro level benefits and costs have to be differentiated between reversible and irreversible benefits and costs. Also, for many assessments a differentiation between private and external benefits and costs is useful and in particular if also sustainability issues are of concern [37].

In particular the postponement of investments has in recent years become an issue of importance. The costs of delays caused by regulations can be substantially undervalued if forgone benefits are irreversible [36]. Economists in general agree that the optimal level of regulations is where marginal benefits equal the marginal costs of regulations [3]. The calculation of marginal benefits and marginal costs will be complicated if uncertainties and irreversibilities need to be considered, which holds for almost all regulations in food production. At the micro level in particular regulatory issues such as labeling requirements and different production standards become increasingly important resulting in new forms of contractual arrangements [33]. These arrangements generally include ex-ante regulations as well as ex-post liability rules in case non-compliance happens. The optimal design of those arrangements often includes irreversible ex-ante compliance costs while ex-post liability follows a jump process. Both can be combined to model the economics of contractual arrangements allowing for more detailed insights about incentives to participate in new contractual arrangements as well as the incentives to comply with the arrangements (see e.g. [5]). One of the major insights from that literature is that irreversible ex-ante regulatory costs provide incentives to delay adoption of more stringent regulations as irreversible investment costs can be delayed and costs avoided if future benefits will be low but that also the size of the firm will be important if irreversible regulatory costs increase nonlinear with firm size.

At the macro level during the last decade the concept of genuine investment as an indicator for sustainable development has been proposed [4]. Yet, the concept does not consider possible irreversible benefits and costs and uncertainty of genuine investments explicitly. This will be another fruitful area of research. First attempts in that direction can be found in [37] and [35]. Within a real option framework not the value of the economy as measured by genuine investment but the value of an economy as measured by the option value and hence changes in option values would be the relevant indicator for sustainable development. In this context future opportunities become important and for policy makers the major question will be if their policies increase or decrease the option value.

Some criticism has been raised against stressing the importance of irreversibilities as in the end all costs are irreversible. This will be correct if decisions are made within continuous time but this is hardly the case. Take agriculture as an example. Decisions about the quantity of hog production are made on about a 6 month basis. These decisions are reversible while the specific investment in the pig barn cannot be reversed after a 6 month period. There will be substantial losses involved if the barn has been constructed to last for several years bit production closes after 1 year. In crop production on annual basis seed expenditures can be considered reversible as the crops have been harvested within a year and production choice can be adjusted if economic circumstances change in favor corn instead of wheat. Investments in seeds and pesticides can be considered irreversible within the cropping season. Decisions on pesticide use are made under uncertainty as future pest and disease problems are not know with certainty. They depend on future whether conditions, what neighboring farmers are doing and more. The expenditures for pesticides within a cropping season can be considered as irreversible. Analyzing optimal pesticide use within a real option framework helps to explain why farmers rationally use less pesticides than standard benefit-costs-analysis would suggest [18].

39.5 Conclusion

Three approaches have evolved to model investments under uncertainty, irreversibility, and flexibility. The methods allow considering irreversibilities and uncertainties of a decision explicitly and enable researchers to recognize the risk associated with the investment at the theoretical level. Not only irreversible costs but also irreversible benefits matter as discussed by e.g. [37] and [26]. Including irreversible benefits and costs into the benefit-cost framework results in a different decision rule in comparison to the standard deterministic neoclassical framework. This is now well known in the literature on real options.

A comparison between the real and quasi option approach shows the quasi option value does not include foregone benefits and costs of a delayed investment, but is a measure of the economic value of uncertain information.

The decision rule for investments using contingent claim analysis allows solutions to be derived that are independent of risk and time preference. Individuals that are highly concerned about a new technology and those who are not, but both want to maximize their income, would come to the same conclusion about the timing of introduction. The risk-adjusted rate of return μ derived from the CAPM in the example depends on the risk free interest rate r and the market price of risk; hence the optimal decision to release transgenic crops is not independent of changes in interest rates.

The effects of policies on the timing of investments were analysed in a two-step procedure. First, the impacts on model parameters were identified and then the effect of the parameter changes on the maximum incremental social tolerable irreversible costs. The most counterintuitive result was the increase in the likelihood of an earlier investment with a decrease in additional net-benefits. This is explained by the opposite impact a simultaneous change in the growth rate and the variance rate has on the maximum incremental social tolerable irreversible costs.

Future applications within the evaluation of genuine investments and analysing the effect of changes in government regulations on compliance incentives as well as the optimal design of regulation are fruitful areas for future research in this domain.

Appendix 39.1: Solving for F(B) Using Contingent Claim Analysis as Explained by Dixit and Pindyck (Chap. 5, pp. 150–152)

Assuming that an asset or a portfolio of assets exists that allows the risk of the additional net-benefits to be tracked, the arbitrage pricing principle can be applied to value the portfolio that includes the additional benefits from transgenic crops [25]. In this case, a portfolio can be constructed consisting of the option to release transgenic crops into the environment, $F(B)$, and a short position of $n = F'(B)$ units of the additional benefits of transgenic crops. The value of this portfolio is

$\Phi = F(B) - F'(B)B$. A short position will require a payment to the holder of the corresponding long position of $\delta F'(B)Bdt$. The total return from holding this portfolio over a short time interval $(t, t + dt)$ holding $F'(B)$ constant will be:

$$d\Phi = dF(B) - F'(B)dB - \delta BF'(B)dt. \tag{39.18}$$

Applying Itô's Lemma[7] to $dF(B)$, equating the return of the risk less portfolio to the risk free rate of return $r[F(B) - F'(B)B]dt$ and rearranging terms results in the following differential equation:

$$\frac{1}{2}\sigma^2 B^2 F''(B) + (r - \delta)BF'(B) - rF(B) = 0. \tag{39.19}$$

A solution to this homogenous second order differential equation is:

$$F(B) = A_1 B^{\beta_1} + A_2 B^{\beta_2}. \tag{39.20}$$

As the value of the option to release transgenic crops into the environment is worthless, if there are no additional net-benefits, A_2 has to be 0. The other boundary conditions are the 'value matching', Eq. (39.8), and 'smooth pasting', Eq. (39.9), conditions[8]:

$$F(B^*) = V(B^*) - I + R \tag{39.21}$$

$$F'(B^*) = V'(B^*). \tag{39.22}$$

Solving Eq. (39.7) according to the boundary conditions provides the following solutions:

$$B^* = \frac{\beta_1}{\beta_1 - 1}\delta(I - R) \tag{39.23}$$

$$A_1 = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{(I - R)^{\beta_1 - 1}(\delta\beta_1)^{\beta_1}} \quad \text{with} \tag{39.24}$$

$$\beta_1 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1. \tag{39.25}$$

[7] See, e.g., Sect. 22 [15] for an introduction of Itô-stochastic processes.

[8] The value matching condition sustains that the value of the option to release the transgenic crop is equivalent to the value of immediate release. The smooth pasting condition says that at the point of value matching a marginal change in the value of the option to release the transgenic crop has to be equal to a marginal change in the value of immediate release [11, pp. 130–132].

Appendix 39.2: Proof of the Results of the Partial Derivatives

To improve the readability of the equations the following notation will be introduced:

$$\upsilon = \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{39.26}$$

$$\chi = \left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right) \tag{39.27}$$

$$\theta = \frac{r-\delta}{\sigma^2}. \tag{39.28}$$

The following assumptions will be made for the proofs:

1. $B, r, \mu, \sigma > 0$.
2. $\mu - \alpha = \delta > 0$.
3. $\beta = -\chi + \upsilon > 1$.

Proof. 1: $\frac{\partial I^*}{\partial \alpha} > 0$:

$$\frac{\partial I^*}{\partial \alpha} = \frac{B}{\delta^2} - \frac{B}{\delta^2}\beta^{-1} - \frac{B}{\delta}\beta^{-2}\frac{\partial \beta}{\partial \alpha} > 0 \tag{39.29}$$

$$\frac{B}{\delta^2} - \frac{B}{\delta^2}\beta^{-1} - \frac{B}{\delta}\beta^{-2}\frac{\partial \beta}{\partial \alpha} > 0$$

$$\Rightarrow \frac{1}{\delta} - \frac{1}{\delta\beta} > \frac{1}{\beta^2}\frac{\partial \beta}{\partial \alpha}. \tag{39.30}$$

The left-hand-side of Eq. (39.30) is positive as δ is positive and $\beta > 1$. Equation (39.29) would be correct if the right-hand-side of Eq. (39.30) is negative, that is if $\partial\beta/\partial\alpha < 0$.

$$\frac{\partial \beta}{\partial \alpha} = -\frac{1}{\sigma^2} + \frac{\chi}{\upsilon\sigma^2} < 0 \tag{39.31}$$

$$-\frac{1}{\sigma^2} + \frac{\chi}{\upsilon\sigma^2} < 0$$

$$\Rightarrow -\chi + \upsilon > 0$$

$$\Rightarrow \beta > 0.$$

□

Proof. 2: $\frac{\partial I^*}{\partial \sigma} < 0$:

$$\frac{\partial I^*}{\partial \sigma} = \frac{B}{\delta}\beta^{-2}\frac{\partial \beta}{\partial \sigma} < 0. \tag{39.32}$$

Equation (39.32) will be correct if $\partial\beta/\partial\sigma < 0$:

$$\frac{\partial \beta}{\partial \sigma} = \frac{2(r-\delta)}{\sigma^3} - \frac{2\chi\left(\frac{r-\delta}{\sigma^3}\right) + \frac{2r}{\sigma^3}}{\upsilon} < 0. \tag{39.33}$$

Equation (39.33) can be rearranged to:

$$\frac{(r-\delta)\upsilon - \chi(r-\delta) - r}{\upsilon\sigma^3} < 0. \tag{39.34}$$

Equation (39.34) will be correct if the nominator is negative as the denominator is always positive:

$$\begin{aligned} & (r-\delta)\upsilon - \chi(r-\delta) - r < 0 \\ \Rightarrow \quad & (r-\delta)(-\chi+\upsilon) < r \\ \text{or} \quad & \beta(r-\delta) < r. \end{aligned} \tag{39.35}$$

This has to hold for the case $(r-\delta) < 0$ and $(r-\delta) > 0$.

For the case $(r-\delta) < 0$ follows that $\beta > \frac{r}{r-\delta}$ is correct as $\beta > 1$.

For the case $(r-\delta) > 0$ Eq. (39.35) can be rearranged to:

$$\upsilon < \frac{r}{(r-\delta)} + \chi. \tag{39.36}$$

Equation (39.36) can be rearranged to:

$$\left(\frac{r}{r-\delta}\right)^2 + \frac{r}{\sigma^2} - \frac{r}{r-\delta} > 0. \tag{39.37}$$

Equation (39.37) is correct if $\frac{r}{r-\delta} > 1$. This holds if $(r-\delta) > 0$. □

Proof. 3:

$$\frac{\partial^2 I^*}{\partial\alpha\partial\sigma} < 0:$$

Using the result of Eq. (39.32) and differentiation according to σ provides:

$$\frac{\partial^2 I^*}{\partial\alpha\partial\sigma} = \frac{B}{\delta^2}\beta^{-2}\frac{\partial\beta}{\partial\sigma} + \frac{B}{\delta}\left(\frac{\partial^2\beta}{\partial\alpha\partial\sigma}\beta^{-2} - 2\beta^{-3}\frac{\partial\beta}{\partial\alpha}\frac{\partial\beta}{\partial\sigma}\right) < 0. \tag{39.38}$$

Equation (39.38) can easily be rearranged to

$$\frac{B}{\delta} > 2\beta^{-3}\frac{\partial\beta}{\partial\alpha}. \tag{39.39}$$

This is correct if $\partial\beta/\partial\alpha < 0$ which has already been proven. □

Proof. 4: $\frac{\partial I^*}{\partial r} < 0$:

$$\frac{\partial I^*}{\partial r} = \frac{B}{\delta}\beta^{-2}\frac{\partial\beta}{\partial r} < 0. \tag{39.40}$$

Equation (39.40) is correct if $\frac{\partial\beta}{\partial r} < 0$.

$$\begin{aligned}
\frac{\partial\beta}{\partial r} &= -\frac{1}{\sigma^2} + \frac{\partial\upsilon}{\partial r} < 0 \\
&-\frac{1}{\sigma^2} + \frac{\partial\upsilon}{\partial r} < 0 \\
\Rightarrow \quad &-\frac{1}{\sigma^2} + \frac{\chi+1}{\upsilon} < 0 \\
\Rightarrow \quad &\chi + 1 < \upsilon \\
\Rightarrow \quad &\beta > 1.
\end{aligned}$$

□

References

1. Amram, M., Kulatilaka, N.: Real Options. Harvard Business School Press, Boston (1999)
2. Arrow, K., Fisher, A.: Environmental preservation, uncertainty, and irreversibility. Q. J. Econ. **88**, 312–319 (1974)
3. Arrow, K.J., Cropper, M.L., Eads, G.C., Hahn, R.W., Lave, L.B., Noll, R.G., Portney, P.R., Russell, M., Schmalensee, R., Smith, V.K., Stavins, R.N.: Is there a role for benefit-cost analysis in environmental, health, and safety regulation? Science **272**, 221–222 (1996)
4. Arrow, K.J., Dasgupta, P., Goulder, L.H., Mumford, K.J., Oleson, K.: Sustainability and the measurement of wealth. Environ. Dev. Econ. **17**, 317–353 (2012)
5. Beckmann, V., Soregaroli, C., Wesseler, J.: Ex-ante regulation and ex-post liability under uncertainty and irreversibility: governing the coexistence of gm crops. economics: the open-access. Open-Assessment E-J. (2010). http://dx.doi.org/10.5018/economics-ejournal.ja.2010-9
6. Bernstein, P.L.: Capital Ideas: The Improbable Origins of Modern Wall Street. Wiley, Hoboken (2005)
7. Cox, D., Miller, H.: The Theory of Stochastic Processes. Chapman and Hall, London (1965)
8. Cox, J.C., Ross, S.A., Rubinstein, M.: Option pricing: a simplified approach. J. Fin. Econ. **7**, 229–263 (1979)

9. Davis, M., Etheridge, A.: Louis Bachelier's Theory of Speculation: The Origins of Modern Finance. Princeton University Press, Princeton (2005)
10. Dixit, A., Pindyck, R.S.: Investment Under Uncertainty. Princeton University Press, Princeton (1994)
11. Dixit, A., Pindyck, R.S.: The options approach to capital investment. Harv. Bus. Rev. (May–June) 105–115 (1995)
12. Einstein, A.: Concerning the motion, as required by the molecular-kinetic theory of heat, of particles suspended in liquids at rest. Annalen der Physik **17**, 549–560 (1905)
13. Henry, C.: Investment decision under uncertainty: the irreversibility effect. Am. Econ. Rev. **64**, 1006–1012 (1974)
14. Hull, J.C.: Options, Futures and Other Derivatives: Global Edition, 8th edn. Pearson, Essex (2011)
15. Kamien, M.I., Schwartz, N.L.: Dynamic Optimization. Amsterdam, North-Holland (1991)
16. Khanna, M., Isik, M., Winter-Nelson, A.: Investment in site-specific crop management under uncertainty: implications for nitrogen pollution control and environmental policy. Agr. Econ. **24**, 9–21 (2000)
17. Kolmogoroff, A.N.: Über die analytischen Methoden in der Wahrscheinlichkeitsrechnung. Mathematische Annalen **104**(1), 415–458 (1931)
18. Ndeffo Mbah, M.L., Forster, G., Wesseler, J., Gilligan, C.: Economically optimal timing of crop disease control under uncertainty: an options approach. J. R. Soc. Interface **7**(51), 1421–1428 (2010)
19. McDonald, R., Siegel, D.: The value of waiting to invest. Q. J. Econ. **101**, 707–728 (1986)
20. Mensink, P., Requate, T.: The Dixit-Pindyck and the Arrow-Fisher-Hanemann-Henry option values are not equivalent: a note on Fisher. Resource Energy Econ. **27**, 83–88 (2000)
21. Merton, R.C.: Application of option pricing theory: twenty-five years later. Am. Econ. Rev. **88**, 323–349 (1998)
22. Mezey, E.W., Conrad, J.M.: Real options in resource economics. Annu. Rev. Resour. Econ. **2**, 33—52 (2010)
23. Perrings, C., Brock, W.: Irreversibility in economics. Annu. Rev. Resour. Econ. **1**, 219–238 (2009)
24. Pietola, K.S., Wang, H.H.: The value of price- and quantity-fixing contracts for piglets in Finland. Eur. Rev. Agr. Econ. **27**(4), 431–447 (2000)
25. Pindyck, R.S.: Irreversibility, uncertainty, and investment. J. Econ. Lit. **29**, 1340–1351 (1991)
26. Pindyck, R.S.: Irreversibilities and the timing of environmental policy. Resource Energy Econ. **22**, 233–259 (2000)
27. Pindyck, R.S., Rubinfeld, D.: Econometric Models and Economic Forecasts, 3rd edn. McGraw-Hill, New York (1991)
28. Price, J., Wetzstein, M.: Irreversible investment decisions in perennial crops with yield and price uncertainty. J. Agr. Resource Econ. **24**(1), 173–185 (1999)
29. Purvis, A., Boggess, W.G., Moss, C.B., Holt, J.: Adoption of emerging technologies under output uncertainty: an ex-ante approach. Am. J. Agr. Econ. **77**(3), 541–551 (1995)
30. Richards, T.J., Green, G.P.: Economic hysteresis in variety selection. J. Agr. Appl. Econ. **35**, 1–14 (2003)
31. Slade, M.E.: Valuing managerial flexibility: an application of real-option theory to mining investments. J. Environ. Econ. Manag. **41**, 193–233 (2001)
32. Trigeorgis, L.: Real Options. MIT, Cambridge (1996)
33. Weaver, R., Wesseler, J.: Monopolistic pricing power for transgenic crops when technology adopters face irreversible benefits and costs. Appl. Econ. Lett. **11**(15), 969–973 (2005)
34. Wesseler, J.: Assessing the risk of transgenic crops - the role of scientific belief systems. In: Matthies, M., Malchow, H., Kriz, J. (eds.) Integrative Systems Approaches to Natural and Social Sciences - Systems Science, pp. 319–327. Springer, Berlin (2001)
35. Wesseler, J. The Santaniello theorem of irreversible benefits. AgBioForum **12**(1), 8–13 (2009)
36. Wesseler, J., Zilberman, D.: The economic power of the Golden Rice opposition. Environ. Dev. Econ. (2014). doi:10.1017/S1355770X1300065X

37. Wesseler, J., Scatasta, S., Nillesen, E.: The Maximum Incremental Social Tolerable Irreversible Costs (MISTICs) and other benefits and costs of introducing transgenic maize in the EU-15. Pedobiologia **51**(3), 261–269 (2007)
38. Wilmott, Paul.: The Mathematics of Financial Derivatives. Cambridge University Press, Cambridge (1995)
39. Winter-Nelson, A., Amegbeto, K.: Option values to conservation and agricultural price policy: application to terrace construction in Kenya. Am. J. Agr. Econ. **80**(2), 409–418 (1998)