Chapter 6
A Brief History of 30 Years of Model Updating in Structural Dynamics*

François M. Hemez and Charles R. Farrar

Abstract Since the development of the Finite Element (FE) method at the University of California Berkeley and the Boeing Company in the 1960s, the question of appropriateness of a model has always preoccupied developers and practicing engineers. Because of the early focus on predicting the linear vibrations of coupled systems for aerospace and civil engineering applications, test-analysis reconciliation initially consisted in updating the FE matrices such that their eigen-properties reproduce the identified resonant frequencies and mode shape vectors. As the FE method increased in sophistication in the following decades, and computational resources became widespread, test-analysis reconciliation evolved beyond optimal matrix updating to include sensitivity and residual-based methods that attempted to calibrate individual element matrices or design parameters. Fueled by an ever-increasing diversity of applications, FE model updating expanded beyond the correlation of modal response to handle frequency response functions, static deflections, and time-domain waveforms. Component mode synthesis concepts were progressively integrated to handle the spatial mismatch between measurement points of a structure and the FE discretization where the spatial information is predicted. This publication briefly overviews the first 30 years of FE model updating development, from the mid-1960s to the mid-1990s, because most of the technology currently available originates in this period. FE model updating methods are categorized into broad categories that each offer their own benefits and limitations. Potential growth areas, such as application to nonlinear dynamics, are discussed.

Keywords Finite element model updating • Calibration • Test-analysis correlation • Verification and validation • Uncertainty quantification

6.1 Introduction

Since it was first developed in the 1960s, Finite Element (FE) modeling has become an essential analysis method in computational physics and engineering. Areas of application range from disciplines of conventional mechanical engineering (solid mechanics, structural dynamics, fluid dynamics, etc.) to material science, the medical field, transport applications and problems featuring coupled physics. The desire to "validate" models and quantify the agreement between physical measurements and code predictions has paralleled the development of the FE method since its early beginnings. This chapter develops a brief discussion of techniques developed for test-analysis correlation in the context of FE modeling, also referred to as *model updating*.

Because the history of model updating is closely linked to the development of the FE method, Sect. 6.2 starts by overviewing how the idea of finite elements came about. This discussion is not meant to be an exhaustive history of the FE technology. A few ideas are presented that are useful to understand the potentials and limitations of FE model updating.

*This chapter is dedicated to Dr. David Zimmerman, great researcher and educator who made significant contributions to the discipline of finite element model updating.

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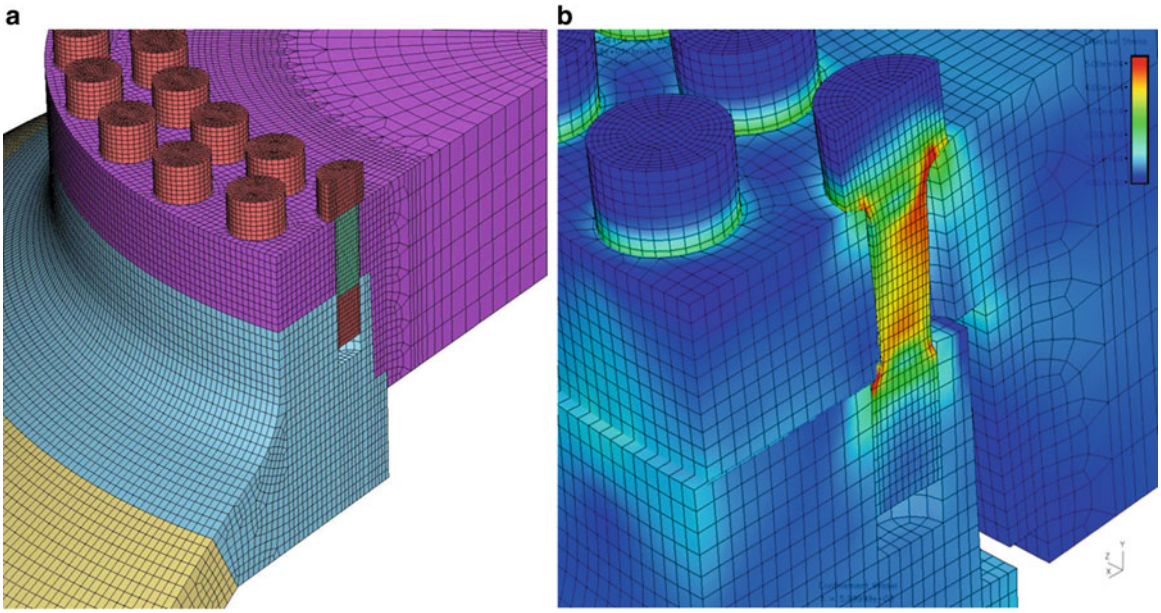

Fig. 6.1 Illustration of a FE mesh for the simulation of pressure vessel dynamics. (**a**) Finite element discretization. (**b**) Stress prediction in a bolt (credit: J. Pepin, LANL, and B. Thacker, Southwest Research Institute)

Section 6.3 organizes the vast landscape of model updating into broad categories, motivates each category and briefly presents their advantages and drawbacks. Again, the discussion is not meant to be exhaustive and the chapter is not attempting to fully review the discipline of model updating. Our intent, instead, is to promote a better understanding of why FE model updating has evolved as it has.

Figure 6.1 illustrates the FE method with a dynamic calculation of stress for a pressure vessel. The element discretization is shown in Fig. 6.1a, while the cut-away view of Fig. 6.1b illustrates the stress distribution of a bolt as well as the relative displacement between the vessel wall and top door.

A characteristic of the FE method, not always observed in other computational approaches, is that the physical interpretation is closely coupled to the mathematical theory. Indeed, the two have been developed in synergy. Literature on the subject parallels this observation. Oden [1], for example, discusses the finite element theory, while [2, 3] develop somewhat more practical-oriented explanations. Hughes [4] introduces the main techniques for analysis of linear problems, which currently still comprise the majority usage of the FE method.

The need to calibrate, or update, FE models stems from the desire to generate predictions that reproduce, to the extent possible, the available measurements. In linear mechanics, these are static deflections and vibration properties of structures (frequency response functions, resonant frequencies, mode shapes). Due to the prominence of experimental modal analysis in structural dynamics, these measurements represent the overwhelming majority of data used to update FE models today. However, we will discuss that the early focus of FE model updating was not so much to "validate" a model. It was to match the available measurements. Said differently, a model was found to be "validated" if its predictions were able to reproduce the measurements, which begs the first question: are the right answers obtained for the right reasons?

The second question is: what gets calibrated? Due to limited computing resources, the early developments of FE model updating emphasized optimal matrix updates to estimate mass and stiffness matrix corrections, ΔM and ΔK, for the free-vibration equation. With the arrival of more powerful computing resources, then, came the development of methods based on Taylor series expansion of the equation-of-motion, as well as iterative solvers and optimization algorithms. Combined with sensitivity analysis, these techniques made it possible to apply corrections to individual material properties or geometrical attributes of finite elements.

The third question, that we wish to briefly address in this chapter, is: to what extent is model updating able to handle the practical difficulties of engineering applications? Irrespective of how sophisticated, or elegant, a correction method might be, it remains an academic exercise if it cannot handle "real" situations. Engineering applications offer many complications, including the fact that nodes of a FE discretization have no reason to be collocated with measurement points, the fact that experimental variability needs to be accounted for in the updating process, and the potential for nonlinear dynamics. We will briefly discuss the extent to which the early FE model updating technology was able to meet these challenges.

The chapter is organized as follows. Section 6.2 discusses a few concepts of the FE method that are useful to understand the history of model updating. In Sect. 6.3, updating methods are organized in three broad categories. The logic behind each category is presented in order to explain their respective strengths and weaknesses. Section 6.4 discusses factors that complicate the development of model updating techniques, and their application to "real-life" problems.

6.2 Genesis and Concepts of the Finite Element Method

In this section, a few concepts of the FE method are briefly discussed. The discussion is not meant to provide a particularly complete overview of the technology; the aspects emphasized, instead, are useful to understand the history of model updating.

6.2.1 Early Genesis of the Finite Element Method

What marks the unofficial "birth" of the FE method in 1955, approximately, is the research performed at Boeing (Seattle, Washington) on "structural elements" to analyze the vibrations of large-aspect-ratio Delta wings. Two basic formulations are then studied, the first one based on the concept of stiffness, while the second one uses a flexibility approach. (For more about this, see Sect. 6.2.3). Prior to the 1960s, these analyses of vibration dynamics were performed "by hand" using back-of-the-envelope, 1D beam-like models. Despite being based on highly simplified models and somewhat crude approximations, this approach nevertheless provided accurate-enough predictions for design and analysis. So why seek a new approach?

New design concepts, such as Delta wings, make it necessary to develop a general-purpose capability for predictions because structural stress distributions can no longer be estimated with confidence from beam-like approximations. This lack-of-confidence prompts the paradigm shift from closed-form derivations to numerical methods. It coincides with the fast development of computer science that opens the door to novel and faster methods to assess prototype designs.

The work of Professor Edward Wilson, University of California Berkeley (Civil Engineering), makes him what many consider the "founding father" of the FE method [5–7]. Professor B. Fraeijs de Veubeke, University of Liège (Belgium), although not as well-known in the U.S., is the other key figure [8, 9]. In the early 1960s, applied mathematicians and engineers make the link between Boeing's "structural elements" and variational principles [10]. The original FE method is re-discovered as a Ritz-Galerkin weak formulation of the equations-of-motion using piece-wise linear shape functions. In 1966, Carlos Felippa, now Professor at the University of Colorado at Boulder, publishes a doctoral thesis on the calculation of stresses in dam structures using finite elements [11]. It is the 2nd thesis ever published demonstrating what the method could achieve.

Throughout the 1970s, the main trends are, first, the development of computational software that implements the FE method and, second, the justification by applied mathematicians of its scientific rigor. Theoretical work leads to advances in the stability and convergence properties of finite elements. The equivalence is established between strong and weak solutions, and all the pieces are in place to promote the development of the method as the main analysis tool in solid mechanics and structural dynamics. Fast forwarding through five decades of development, one can evoke many theoretical breakthroughs and practical applications in disciplines such as a posteriori error indicators, mesh adaptation, fluid–structure interaction, particle and geo-physics transport methods, stability analysis, vibration, acoustics, and nonlinear dynamics.

6.2.2 The Equations-of-Motion Considered

The general-purpose Equation-Of-Motion (EOM) considered in this work is the equation of linear dynamics written as:

$$\left(\mathrm{M}\frac{\partial^2 \bullet}{\partial t^2} + \mathrm{D}\frac{\partial \bullet}{\partial t} + \mathrm{K}\right)\ \mathrm{U}\,(\mathrm{t}) = \mathrm{F}_{\mathrm{Ext}}\,(\mathrm{t}) \tag{6.1}$$

where M, D, K are the master (assembled) mass, viscous damping, and stiffness matrices of the FE representation, respectively; $\mathrm{F}_{\mathrm{Ext}}(\mathrm{t})$ is a time-varying vector of applied forces and moments; and U(t) is the resulting, time-varying vector of displacement and rotation Degrees-Of-Freedom (DOF). With the assumption of time-space decoupling, modal superposition can be applied to Eq. (6.1), leading to the well-known linear equation of non-damped, free vibration:

$$\mathrm{K}\,\Psi = \mathrm{M}\,\Psi\,\Omega^2 \tag{6.2}$$

Table 6.1 Comparison of attributes of the stiffness and flexibility FE methods

Criterion	Stiffness method	Flexibility method
Are element matrices easy to compute?	Easy	Difficult
Are master matrices easy to assemble?	Easy	Difficult
How easy is it to handle rigid body modes?	Difficult	Easy
Is the numerical solver expensive?	Expensive	Cheap
What is the master (linear) equation-of-motion?	$Z(s)\,U(s) = F_{Ext}(s)$	$U(s) = H(s)\,F_{Ext}(s)$
How is the master matrix stored?	Sparse	Full
What is the algorithm to solve the equations?	Matrix inversion	Matrix product

where Ψ is the matrix of mode shape deflections (one vector per column), and the diagonal matrix Ω stores the corresponding resonant frequencies. Unless otherwise noted, the present discussion is based on the equation of vibration Eq. (6.2), written compactly for a single mode as:

$$Z(\omega_k)\ \ \Psi_k = 0 \tag{6.3}$$

where $Z(s) = K–s^2M$ is the dynamic stiffness matrix at frequency "s." [Note that Eq. (6.3) is written with $s = \omega_k$ for the k^{th} resonant mode.] Even though it is not used here, the discussion can also be applied to the linear EOM in statics, where a vector U of generalized displacements results from applied forces and moments F_{Ext} according to:

$$K\,U = F_{Ext} \tag{6.4}$$

FE model updating in the context of nonlinear dynamics, where the EOM is described by the momentum equation where inertia forces and internal F_{Int} forces balance the externally-applied forces F_{Ext}, such as:

$$M\frac{\partial^2 U(t)}{\partial t^2} + F_{Int}(t) \ = \ F_{Ext}(t) \tag{6.5}$$

is not discussed for two main reasons. First and foremost, the early development of FE updating had an almost exclusive focus on the equation of linear dynamics [Eq. (6.2)]. Second, it is the authors opinion that generalizing FE updating concepts, such as those overviewed herein, to nonlinear dynamics has not yielded significant achievements, except in the case of limited-scope, "niche-like" applications. For a discussion of nonlinear model updating, see [12].

6.2.3 *Basic Concepts of the Finite Element Method*

The basic concept of the FE method, or most other computational methods as a matter of fact, is that the EOM can be solved analytically for elemental geometries with simple-enough boundary conditions. Here, "analytical" refers to a solution of the continuous EOM that can be derived in closed-form or with high accuracy using, for example, Gauss-point quadrature rules. The geometry of the original problem, therefore, is discretized into these elemental geometries. The *continuous* problem is then decomposed into a multitude of similar, yet, *discrete* problems formulated on elemental geometries that are the finite elements.

There are two formulations of the FE method that are dual of one another, and can be referred to in broad terms as the *stiffness* and *flexibility* methods. Table 6.1 summarizes the main attributes of these two approaches. In the stiffness method, the applied forces are known and the momentum equation, that is, $F_{Ext} = M\,d^2U/dt^2$, is converted to a displacement-based equation by assuming representations of, first, the kinetic condition and, second, the material behavior within each element. In statics (for simplicity), it gives Eq. (6.4). In comparison, the flexibility method calculates the displacements using a matrix–vector multiplication such as:

$$U = \ H\,F_{Ext} \tag{6.6}$$

Historically, both approaches were developed in parallel. Dr. Richard McNeal, who investigated the stiffness approach, moved on to fame and glory by creating the NASTRAN™ software. Who remembers the name of the other engineer tasked with developing the flexibility method? It is nevertheless interesting to note that, in dynamics, the flexibility method yields a representation, $U(\omega_k) = H(\omega_k)\,F(\omega_k)$ for the k^{th} resonant mode, analogous to vibration measurements. Adopting a

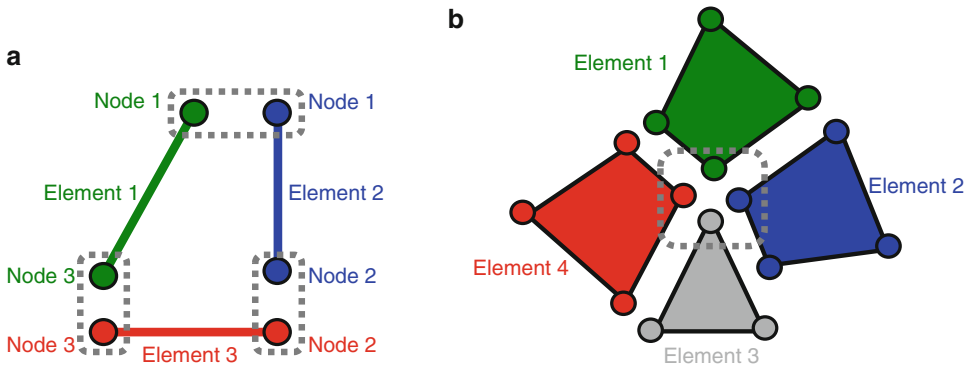

Fig. 6.2 Illustrations of the two fundamental rules of the stiffness FE method. (**a**) Assembly of three truss elements. (**b**) Assembly of four shell elements

flexibility representation, as opposed to the conventional stiffness approach, would make it much easier to compare vibration measurements to FE predictions. Discussion presented in the remainder focuses on the stiffness method.

The stiffness method is articulated around two fundamental rules:

Rule 1: The summation of all forces contributed by finite elements that share a common node, or DOF, is equal to the externally applied force.
Rule 2: Displacements contributed by different finite elements at the same node, or DOF, are equal.

These rules are illustrated on the left side of Fig. 6.2 in the case of truss (or bar) elements. The right side of Fig. 6.2 suggests that similar rules apply to continuous elements such as shell (or membrane) finite elements.

The aforementioned rules are briefly illustrated for the three-truss example of Fig. 6.2a, which is useful to highlight properties of FE matrices that are discussed later in the context of model updating. The first rule provides the principle to equilibrate the forces at the k^{th} node, or DOF, of the FE discretization. For the three-truss example, it can be written as:

$$\text{Rule 1} \;\rightarrow\; F^{(1)} + F^{(2)} + F^{(3)} = F_{\text{Ext,k}} \tag{6.7}$$

Representations of the kinetic condition and material behavior, discussed further in Eqs. (6.11)–(6.13) below, lead to a force-displacement equation expressed within each element as:

$$F^{(e)} = k^{(e)}\, U^{(e)} \tag{6.8}$$

The second rule converts the generalized displacement unknowns contributed by each element, denoted by $U^{(e)}$, into a single unknown per DOF:

$$\text{Rule 2} \;\rightarrow\; U^{(1)} = U_k,\; U^{(2)} = U_k,\; U^{(3)} = U_k \tag{6.9}$$

Inserting Eq. (6.8) in the summation [Eq. (6.7)] for each element, then, replacing element-specific displacements $U^{(e)}$ with the global DOF unknown U_k from Eq. (6.9), leads to the definition of the master stiffness matrix as:

$$\left(k^{(1)}\, U^{(1)}\right) + \left(k^{(2)}\, U^{(2)}\right) + \left(k^{(3)}\, U^{(3)}\right) = F_{\text{Ext,k}} \tag{6.10a}$$

$$\left(k^{(1)} + k^{(2)} + k^{(3)}\right) U_k = F_{\text{Ext,k}} \tag{6.10b}$$

$$\underbrace{\sum_{e=1\cdots N_E} k^{(e)}}_{K}\, U = F_{\text{Ext}} \tag{6.10c}$$

It is noted that, for simplicity, Eqs. (6.7)–(6.10) are derived symbolically in the local orientation frame of each element and the transformation to the global frame-of-reference is not included. For completeness, this omission is corrected in Eq. (6.14).

What remains to complete this description of the stiffness method, is a brief discussion of three so-far-overlooked aspects. The first two are assumptions made to describe, first, the kinetic relation that connects the element-level displacements to

strain deformations and, second, the constitutive law implemented to describe material behavior. Irrespective of how these conditions are defined, they lead to equations that can be written as:

$$\varepsilon^{(e)}_{\text{Local}} = \text{B}^{(e)}\ \text{U}^{(e)} \tag{6.11a}$$

$$\sigma^{(e)}_{\text{Local}} = \text{C}^{(e)}\ \varepsilon^{(e)}_{\text{Local}} \tag{6.11b}$$

Equation (6.11a) converts the element-level displacements and rotations in a strain vector, while Eq. (6.11b) expresses the constitutive behavior that transforms the element-level strains into stresses. Equation (6.11b) is written for linear, isotropic elasticity and additional terms would be added to express a nonlinear strength model with, for example, plastic deformation.

The internal energy is, by definition, equal to the contracted tensor product ε:σ integrated over the volume of the finite element. Substituting Eq. (6.11), and using the vector notation $\varepsilon^{\text{T}}\sigma$ for simplicity, one can verify that the element-level stiffness matrix is obtained as:

$$\left(\text{U}^{(e)}\right)^{\text{T}}\ \text{k}^{(e)}\ \text{U}^{(e)} = \int_{\Omega^{(e)}} \left(\varepsilon^{(e)}_{\text{Local}}\right)^{\text{T}}\ \sigma^{(e)}_{\text{Local}}\ \text{dV} \tag{6.12}$$

where:

$$\text{k}^{(e)} = \int_{\Omega^{(e)}} \left(\text{B}^{(e)}\right)^{\text{T}}\ \text{C}^{(e)}\ \text{B}^{(e)}\ \text{dV} \tag{6.13}$$

Finally, the master (assembled) stiffness matrix can be written by summing the element-level stiffness contributions for all finite elements of the geometry discretization, as previously shown in the case of the three-truss example of Fig. 6.2a:

$$\text{K} = \sum_{e=1\cdots \text{N}_\text{E}} \left(\text{T}^{(e)}\right)^{\text{T}} \left(\int_{\Omega^{(e)}} \left(\text{B}^{(e)}\right)^{\text{T}}\ \text{C}^{(e)}\ \text{B}^{(e)}\ \text{dV} \right)\ \text{T}^{(e)} \tag{6.14}$$

It is emphasized that the definition of master stiffness matrices in Eqs. (6.10c) and (6.14) are analogous. The main difference is that Eq. (6.14) introduces the transform matrix, $\text{T}^{(e)}$, used to convert the local coordinate system (x; y; z), defined in the frame-of-reference of the element, to the global (or laboratory frame) orientation (X; Y; Z).

One important aspect of Eqs. (6.11) and (6.13) is that only the constitutive law $\text{C}^{(e)}$ depends on material properties such as a modulus of elasticity, E, or Poisson's ratio, ν. The matrix $\text{B}^{(e)}$ that discretizes the kinetic relation might depend of geometric attributes, such as a length or shell thickness, but it does not depend on material properties. The local-to-global orientation matrix $\text{T}^{(e)}$ is a transform that, again, does not depend on the material behavior. These observations are important to consider when the calibration of material properties is sought.

6.2.4 *Discussion of Implications for FE Model Updating*

The previous discussion of FE modeling has implications for model updating. It is important to account for these considerations when implementing a semi-automatic correction method. This was not always the case, however, in the early developments of model updating. We briefly address four important implications.

1. *The sparsity of a FE stiffness matrix represents the structural load path*. It implies that a calibration procedure should not be allowed to make corrections to this load path, otherwise, the definition of the problem is fundamentally changed. One exception might be structural damage detection, where the presence of damage can severely alter the ability of a component to carry loads from one point to another. But under no circumstance should a non-existing load path be created by calibration; this would be a non-physical calibration of FE matrices.
2. *Rigid-body modes are essential properties of the FE representation*. Whether they are calibrated or not, FE matrices should preserve the rigid body dynamics and other fundamental characteristics, such as the center of mass and moments

of inertia of the "real" structure. These constraints, unfortunately, are usually not accounted for when formulating FE model updating.

3. *Constitutive properties are defined for elements of the discretization*. In the FE method, the principle of decomposing the geometry into simple elements is foundational. The element is the point-of-focus that implements descriptions of the kinetic condition, such as Eq. (6.11a), and material behavior, such as Eq. (6.11b). It implies that modeling an energy dissipation mechanism that would dissipate *across* element boundaries is nearly impossible within linear dynamics. A consequence is that model updating is fundamentally limited to adjust for modeling errors that would take their origin in these dissipation mechanisms.
4. *A FE model "flows" information from local elements to global responses*. Equation (6.14) indicates that the FE representation assembles element-level information into the master matrix. Hence, information "flows" from local elements to global responses, such as static deflections or mode shape eigenvectors. Model updating, on the other hand, is an *inference* that starts from global-level measurements and attempts to correct the element-level discretization. It raises the question of sensitivity versus error localization. Corrections brought to the model can be caused by the influence that specific elements exercise on predictions, instead of the modeling error.

6.3 A Classification of Finite Element Model Updating Methods

To discuss the history of FE model updating, a classification of methods is proposed. The classification is not exhaustive; it helps, however, to map the vast body of literature and discuss the rationale for its evolution in the last five decades. Early developments of FE model updating are discussed in Sect. 6.3.1. Section 6.3.2 introduces three categories: optimum matrix updating methods, small perturbation methods, and iterative sensitivity-based methods. Each category is then briefly discussed in the subsequent Sects. 6.3.3–6.3.5.

6.3.1 Early Developments of Finite Element Model Updating

The main driver for the development of semi-automated correction approaches, applicable to the FE method in the early 1960s, is the U.S. aerospace industry. Novel analysis techniques are sought to address challenging designs, such as the Delta wing concept, and to support the space launch industry. Model updating simply follows in the wake of these new techniques, with the desire to demonstrate that model predictions are capable to reproduce the measurements.

This early history is illustrated in Fig. 6.3 that depicts the Saturn-V launch vehicle (Fig. 6.3a) and adjustments brought to a 1D beam model of vibration dynamics (Fig. 6.3b). This example is extracted from [13]; it focuses on lateral vibrations of the Saturn-V. The FE model, back in 1974, consisted of 28 beam elements whose shear and bending stiffness coefficients were calibrated using experimental modal data acquired with 19 accelerometers. The updating was formulated as a Bayesian calibration method. The procedure reduced the prediction error of resonant frequencies from $\approx$5 % error, before calibration, to less than 0.05 %, after calibration. Collins et al. [13] was a milestone for its ability to apply updating to a "real-life" problem and, more importantly, for being the first technique, to the best of the author's knowledge, to account for experimental variability in the calibration of an aerospace FE model.

The other important driver of the 1960s early model updating technology, is the development of closed-loop controllers for guidance and navigation. For the most part, these applications also originate from the aerospace industry (missile guidance, satellite navigation, etc.). In a typical controller, the structure is described by low-frequency resonances and mode shape deflections; they constitute the target modes that the control algorithm attempts to stabilize. The need for a simple model, that reproduces these (measured) target modes with as-high-as-possible fidelity, had the consequence to orient model updating towards techniques that bring global adjustments to the mass and stiffness matrices.

A formulation can be, for example, to seek corrections $\Delta\mathrm{M}$ and $\Delta\mathrm{K}$ of the master matrices such that the equation of free vibration is verified when the measured dynamics (Ψ_{Test}; Ω_{Test}) are substituted to the FE-based eigen-solutions:

$$(\mathrm{K} + \Delta\mathrm{K})\ \Psi_{\mathrm{Test}} - \ (\mathrm{M} + \Delta\mathrm{M})\ \Psi_{\mathrm{Test}}\ \Omega^2_{\mathrm{Test}} = 0 \tag{6.15}$$

Formulations such as Eq. (6.15) lead to numerous model updating schemes, with little-to-no consideration for constraints such as preserving the load path representation of the original FE model, or even preserving the symmetry of FE matrices. The

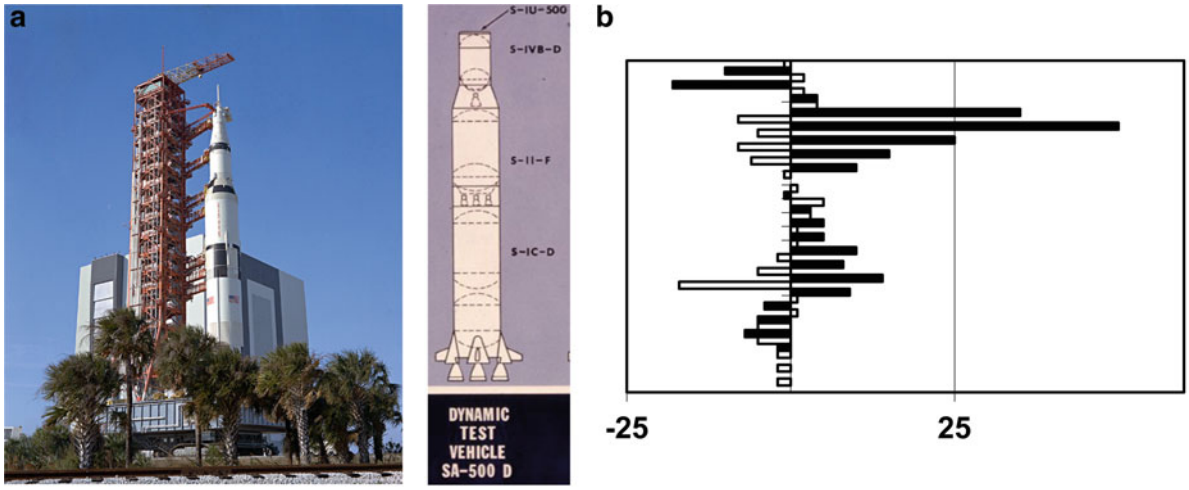

Fig. 6.3 FE model updating of a beam vibration model of the Saturn-V rocket, 1974. (**a**) Rocket (*left*), conceptual model (*right*). (**b**) Bending and shear stiffness updating [credit: NASA archive (**a**) and [13] (**b**)]

goal of these techniques, identified below as "optimal matrix updating," was simply to develop a model whose predictions match the measurements.

6.3.2 Three Broad Categories of Model Updating Techniques

At a high level, updating is nothing but the calibration of a model. If so, why not simply select a parameterization and formulate it as a "standard" calibration problem? Doing so would entail defining a cost function J(p) to minimize and, possibly, constraints C(p) to satisfy:

$$\min_{\{p\}} J(p) \quad \text{subject to constraints } C(p) \geq 0 \tag{6.16}$$

In Eq. (6.16), the cost function and constraints depend on parameters $p = \{p_1; p_2; \ldots p_N\}$ of the model. The cost function can be, for example, the error between measured and predicted frequencies or out-of-balance residuals defined from Eqs. (6.2) and (6.4). The constraints can be defined from the lower and upper bounds of model parameters. It would not be desirable, for example, to allow a negative-valued material density or shell thickness.

Early FE model updating techniques developed in the 1960s and 1970s did not formulate the problem in this way, first, because of a lack of adequate computing resource required to solve an optimization problem and, second, due to the fact that standard calibration tends to treat the model as a "black box." Instead, early developments strived to take advantage of fundamental FE properties, such as the local-to-global assembly procedure. Today's availability of efficient numerical optimization solvers, that rely on parallel processing, renders the aforementioned "plain vanilla" calibration of Eq. (6.16) much more attractive.

To organize the overview of early model updating, a classification into three broad categories is proposed. The first category is the optimum matrix updating (Sect. 6.3.3), where closed-form solutions are derived for global matrix corrections such as those shown in Eq. (6.15) of free vibrations. The second category is the small perturbation updating (Sect. 6.3.4) that consists in writing Taylor series-like expansions to linearize the optimization problem. The third category generalizes this last approach to iterative, sensitivity-based updating (Sect. 6.3.5). It is noted, once again, that this classification is not meant to be a review of existing literature. Imregun and Visser [14], Mottershead and Friswell [15] and Friswell and Mottershead [16] are examples of survey articles, recommended to complement our discussion.

6.3.3 The First Category: Optimum Matrix Updating (OMU)

The first category of updating methods is the Optimum Matrix Updating (OMU) that derives closed-form solutions of global matrix corrections, ΔM and ΔK. As alluded to previously, the emergence of this approach was due to the combination of limited computing power and desire to develop mass-stiffness representations that reproduce the measured data for applications such as launcher-payload coupled loads analysis, guidance and vibration control.

Amongst the early "founding fathers" of OMU, Dr. Alex Berman, Kaman Aerospace Corporation, and Professor Menahem Baruch, Technion (Israel), can be cited [17, 18]. In [18], the updating problem is defined as the estimation of global corrections ΔM and ΔK such that the adjusted FE model minimizes a user-defined error criterion, $J(\Delta M; \Delta K)$:

$$\min_{\{\Delta M;\Delta K\}} J(\Delta M; \Delta K) \text{ subject to constraints } C(\Delta M; \Delta K) \geq 0 \tag{6.17}$$

An updating method is an OMU if it achieves a direct solution, that is, no iteration is required, by analytically writing and solving the Kuhn–Tucker conditions of the optimization problem [Eq. (6.17)]. The set of equations to solve is shown next, assuming for simplicity that only a stiffness correction is sought (that is, $\Delta M = 0$), and using Lagrange multipliers, λ, to account for constraints, $C(\Delta K)$:

$$\frac{\partial}{\partial \Delta K}\left(J(\Delta K) + \lambda C(\Delta K)\right) = 0, \quad \frac{\partial}{\partial \lambda}\left(J(\Delta K) + \lambda C(\Delta K)\right) = 0 \tag{6.18}$$

In the OMU formulation [Eq. (6.17)], constraints are used to enforce fundamental properties, such as the symmetry, positive-definiteness, and sparsity pattern that represents the load path, of FE matrices. Examples of OMU schemes, that implement different combinations of constraints, are given in [19–22]. When simple-enough cost functions and constraints are defined, and further approximations are made (discussed below), the system of Eq. (6.18) can be solved in closed-form, hence, providing global correction matrices ΔM and ΔK whose evaluation requires no computationally expensive, iterative solver or optimization solver.

What about the cost function? Because the early OMU focus was on "matching measurements," cost functions were generally defined as error norms between the physical measurements and model predictions. For example, reproducing the resonant frequencies can be written as:

$$J(\Delta M; \Delta K) = \left\| \Omega^2_{\text{Test}} - \Omega^2 \right\| \tag{6.19}$$

where, even though the notation is simplified, the FE-derived resonant frequencies stored in Ω^2 depend on matrix corrections $(\Delta M; \Delta K)$. Another commonly encountered approach is to define the cost function using the norm of residuals obtained from the EOM. Equation (6.2) of free vibrations is used for illustration. Because of the presence of modeling error, the equation is not verified when resonant frequencies and mode shape deflections predicted by the FE model are replaced by their measured counterparts:

$$K\ \Psi_{\text{Test}} \neq\ M\ \Psi_{\text{Test}}\ \Omega^2_{\text{Test}} \tag{6.20}$$

This inequality can be taken advantage of to define out-of-balance residual forces, R:

$$R = K\ \Psi_{\text{Test}} -\ M\ \Psi_{\text{Test}}\ \Omega^2_{\text{Test}} \tag{6.21}$$

The global matrix corrections $(\Delta M; \Delta K)$ are sought such that the vibration equation is verified, as proposed in Eq. (6.15). Collecting the unknown terms in the left-hand side, leads to:

$$\Delta K\ \Psi_{\text{Test}} - \Delta\ M\ \Psi_{\text{Test}}\ \Omega^2_{\text{Test}} =\ -R \tag{6.22}$$

Equation (6.22) is the basis for many model updating schemes of the OMU and other categories.

Most OMU methods pay particular attention to preserving convexity of the optimization problem. This is because, with a convex problem, positivity of the Hessian does not need to be verified, and the Kuhn–Tucker conditions [Eq. (6.18)] are sufficient to calculate the minimum solution. Clearly, adopting the L^2 norm, that is, $\|y\|_2{}^2 = y^T y$ or $\|A\|_2{}^2 = \text{trace}(A^T A)$, is advantageous in this regard.

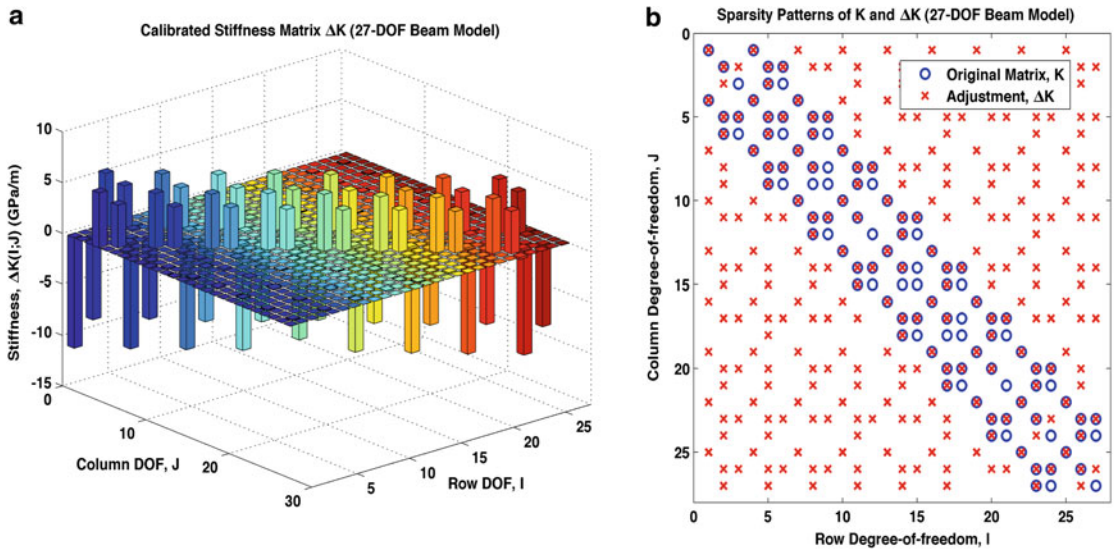

Fig. 6.4 Optimum matrix updating of a 27-DOF tapered wing beam model. (**a**) Global stiffness adjustment, ΔK. (**b**) Sparsity patterns of K and (K+ΔK)

Another characteristic of OMU methods is the manner in which the Kuhn–Tucker conditions [Eq. (6.18)] are solved. The emphasis, again, is to derive a closed-form solution that avoids computationally expensive solvers. A technique commonly encountered is to rely on the spectral decomposition properties of linear dynamics. For example, the updated stiffness can be decomposed as:

$$(\mathrm{K} + \Delta\mathrm{K}) = (\mathrm{M}\ \Psi)\ \Omega^2 \left(\Psi^{\mathrm{T}}\ \mathrm{M}\ \Psi\right)^{-1} (\mathrm{M}\ \Psi)^{\mathrm{T}} \tag{6.23a}$$

$$\Omega^2 = \left(\Psi^{\mathrm{T}}\ \mathrm{M}\ \Psi\right)^{-1} \Psi^{\mathrm{T}}\ (\mathrm{K} + \Delta\mathrm{K})\ \Psi \tag{6.23b}$$

With the spectral decomposition [Eq. (6.23b)], and assuming for simplicity that no mass correction is sought ($\Delta\mathrm{M} = 0$), it can be verified that the measured frequencies can be reproduced using:

$$\Omega^2 = \Omega^2_{\mathrm{Test}} \tag{6.24a}$$

$$\left(\Psi^{\mathrm{T}}\ \mathrm{M}\ \Psi\right)^{-1} \Psi^{\mathrm{T}}\ (\mathrm{K} + \Delta\mathrm{K})\ \Psi = \Omega^2_{\mathrm{Test}} \tag{6.24b}$$

$$\mathrm{K} + \Delta\mathrm{K} = (\mathrm{M}\ \Psi)\ \Omega^2_{\mathrm{Test}} \left(\Psi^{\mathrm{T}}\ \mathrm{M}\ \Psi\right)^{-1} (\mathrm{M}\ \Psi)^{\mathrm{T}} \tag{6.24c}$$

Equation (6.24c) defines the global stiffness correction ΔK without requiring an iterative solver. It corresponds to minimizing the cost function $\mathrm{J}(\Delta\mathrm{K}) = \|\Omega_{\mathrm{Test}}{}^2 - \Omega^2\|_2$, defined with the L^2 norm.

An illustration is given in Fig. 6.4 in the case of a 27-DOF beam model of cantilever wing with tapered cross-section. The first five measured eigenvalues $\Omega_{\mathrm{Test}}{}^2$ are specified in the right-hand side [Eq. (6.24c)], and the corresponding stiffness correction ΔK is shown in Fig. 6.4a. It is verified that the updated matrix representation (K + ΔK; M) reproduces the target frequencies exactly. Figure 6.4b compares the sparsity of the original and adjusted matrices. Non-zero entries of K are identified as blue circles, while those of (K + ΔK) are shown with red crosses. The tendency of the OMU solution to introduce fill-in is evident; it means that the updated matrix (K + ΔK) does not preserve the structural load path of the original FE representation.

Before closing this section on OMU, an important caveat must be evoked. Derivations assume that the FE representation is defined over the same set of DOF as the measurement locations. None of Eqs. (6.20)–(6.24) make sense if FE matrices (K; M), their corresponding mode shape vectors Ψ, and measured eigenvectors Ψ_{Test}, are defined at differing locations. This is referred to as spatial "incompatibility" in the remainder. Because this constraint is generally not satisfied, a pre-processing step must be implemented to define a common set of finite element DOF and measurement locations. FE matrices, for

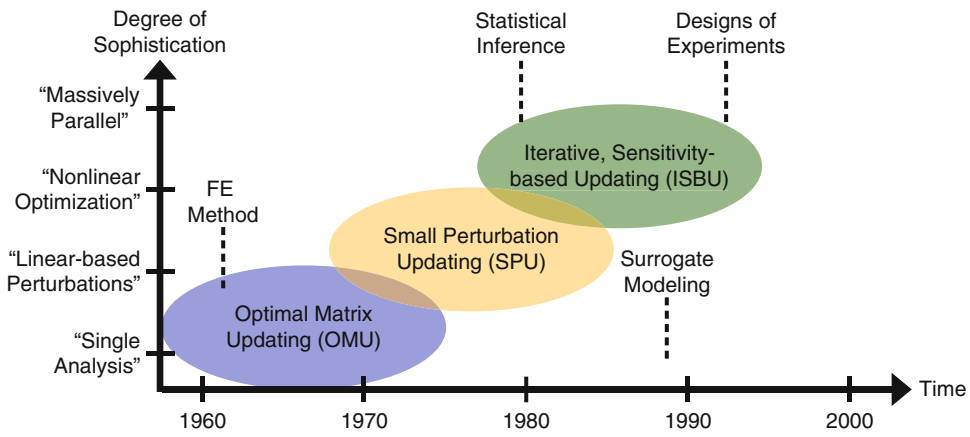

Fig. 6.5 Notional evolution of the FE model updating technology

example, can be converted to measurement locations using techniques inspired from Component Mode Synthesis (CMS). Such matrix reductions are used in [23–25]; Balmès [26] develops a coherent organization of CMS methods.

Kammer [27] proposes a unified framework for stiffness matrix correction, where the matrix adjustment is decomposed into normal and residual contributions, $\mathrm{K} + \Delta\mathrm{K} = \mathrm{K_N} + \mathrm{K_R}$. Different OMU schemes result from differing choices of $\mathrm{K_N}$ and $\mathrm{K_R}$, which are contributions to the stiffness matrix from measured normal modes and un-modeled residual dynamics, respectively. Another survey of OMU approaches can be found in [28].

6.3.4 *The Second Category: Small Perturbation Updating (SPU)*

The second category of updating methods is the Small Perturbation Updating (SPU). These methods are based on small perturbations of a quantity-of-interest or governing EOM. Hence, SPU borrows from the concept of re-design in engineering mechanics. Hoff et al. [29] and Chen and Lin [30] are two examples of application to linear dynamics.

The basic idea is to search for small perturbations of the mass and stiffness matrices, denoted by (δk; δm), that achieve small changes in predictions. These changes are sought such that the FE model predictions get closer to measurements. For example, adjusting a stiffness matrix, or some of its parameters, such that a resonant frequency is changed, can be written as:

$$\omega^2_{\mathrm{Test}} \approx \omega^2\,(\mathrm{k}) + \delta\omega^2\,(\mathrm{k} + \delta\mathrm{k}) \tag{6.25}$$

A condition such as Eq. (6.25) can then be expressed as an inverse problem that estimates the perturbation δk such that the adjusted prediction $\omega^2 + \delta\omega^2$ matches the measurement ${\omega_{\mathrm{Test}}}^2$. SPU methods are mostly based on small perturbations of the vibration equation. Garba and Wada [31] is an example of first-order Taylor series expansion that arrives at a linear system of equations to adjust parameters of the FE model. This strategy is discussed further in the following.

In many regards, SPU can be viewed as "stop-gap" technology that contributed to evolve model updating away from closed-form corrections of global matrices, and closer to sensitivity-based methods. The latter were out-of-reach in the late 1960s and early 1970s for lack of computing power and efficient software. As soon as these restrictions were lifted, methods were developed to achieve large adjustments by relying on either nonlinear optimization or iterative, sensitivity-based perturbations. Figure 6.5 expresses this view by mapping the three broad categories of FE model updating in a two-dimensional space, as a function of time and degree-of-sophistication.

To relate them to the OMU category discussed in Sect. 6.3.3, SPU methods can be thought of as a Taylor series expansion of the cost function that expresses the "distance" between physical measurements and FE model predictions:

$$\underset{\text{1-by-1}}{\mathrm{J}\,(\mathrm{k} + \delta\mathrm{k})} = \underset{\text{1-by-1}}{\mathrm{J}\,(\mathrm{k})} + \underset{\text{1-by-N}_\mathrm{P}}{\delta\mathrm{k}^\mathrm{T}}\ \underset{\text{N}_\mathrm{P}\text{-by-1}}{\nabla\mathrm{J}\,(\mathrm{k})} + \underset{\text{1-by-N}_\mathrm{P}}{\delta\mathrm{k}^\mathrm{T}}\ \underset{\text{N}_\mathrm{P}\text{-by-N}_\mathrm{P}}{\nabla^2\mathrm{J}\,(\mathrm{k})}\ \underset{\text{N}_\mathrm{P}\text{-by-1}}{\delta\mathrm{k}} + \mathrm{O}\left(\delta\mathrm{k}^2\right) \tag{6.26}$$

Equation (6.26) indicates dimensions (number of rows by number of columns) to emphasize that the equation is written for N_P calibration parameters, $k = (k_1; k_2; \ldots k_{NP})$. Limiting the expansion to first-order terms, and searching for an adjustment δk that minimizes the cost function, that is, $J(k + \delta k) = 0$, leads to a linearized system of equations:

$$\underset{1\text{-by-}N_P}{\nabla J(k)^T} \; \underset{N_P\text{-by-}1}{\delta k} = \underset{1\text{-by-}1}{-J(k)} \tag{6.27}$$

where the gradient vector of the dot-product collects partial derivatives of the cost function with respect to each parameter defined for calibration:

$$\underset{N_P\text{-by-}1}{\nabla J(k)} = \left[\frac{\partial J(k)}{\partial k_1} \; \frac{\partial J(k)}{\partial k_2} \cdots \frac{\partial J(k)}{\partial k_{NP}} \right]^T \tag{6.28}$$

Equation (6.27) can be solved in a single step, in which case the adjustment brought to FE model parameters is $k^{(\text{Updated})} = k^{(\text{Original})} + \delta k$. An alternative is to implement an iterative solver where the solution, at the $(n+1)^{\text{th}}$ iteration of the algorithm, is $k^{(n+1)} = k^{(n)} + \delta k^{(n+1)}$, with a least-squares solution obtained as:

$$\underset{N_P\text{-by-}1}{\delta k^{(n+1)}} = (1 - \lambda) \; \underset{N_P\text{-by-}1}{\delta k^{(n)}} - \lambda \left(\frac{J\left(k^{(n)}\right)}{\nabla J\left(k^{(n)}\right)^T \nabla J\left(k^{(n)}\right)} \right) \underset{N_P\text{-by-}1}{\nabla J\left(k^{(n)}\right)} \tag{6.29}$$

Equation (6.29) illustrates a predictor-corrector algorithm, where the scalar λ, $0 < \lambda \leq 1$, is a user-defined relaxation parameter that specifies the magnitude of the n^{th} correction step. The full step is given by $\lambda = 1$ (no iteration); a value $\lambda \leq 1$ defines a linear combination between the previous-iteration solution $\delta k^{(n)}$ and the current correction. Implementing iterations, such as suggested in Eq. (6.29), is a way to limit the FE adjustment to small perturbations. This matters greatly to ensure that the first-order approximation [Eq. (6.27)] remains valid; it is important in situations where the cost function J(k) is a nonlinear function of the correction parameters $(k_1; k_2; \ldots k_{NP})$.

The system of Eq. (6.27) is written for a single cost function; it can also be generalized to multiple cost functions. (Section 6.3.5 discusses this generalization.) Irrespective of this choice, the solution procedure involves linear matrix algebra. SPU methods, therefore, are somewhat more sophisticated than OMU that provides closed-form solutions for the calibration.

Many methods, based on the concept of small perturbation outlined in Eqs. (6.26)–(6.29), can be found in the literature. Chen and Garba [32] and Ojalvo [33] are two examples that use errors between measured and predicted resonant frequencies and mode shapes of a structure to define cost functions.

The analogy between SPU and design/shape optimization is noteworthy. Optimization methods attempt to optimize design parameters of a model to meet user-defined performance criteria that are often paired with design constraints. For example, the shape, mass and stiffness properties of an aircraft wing can be optimized to minimize weight, while avoiding a potential coupling of the bending and torsional modes to prevent aerodynamic flutter. Arora and Li [34] is an example of design optimization; Kikuchi et al. [35] is an application where the authors propose to optimize the topology of a FE mesh to achieve a performance requirement. It is interesting to note that the difference between SPU and design/shape optimization is that, in the latter, measurements are replaced by target requirements. The point made is that much about SPU can be learned by studying methods developed for design/shape optimization.

In the late 1980s and early 1990s, one witnesses an increasing integration of techniques developed for model updating (calibration), design and shape optimization, and the estimation of resonant frequency and mode shape derivatives. Similarly, CMS methods used to condense FE models and generate super-elements have demonstrated their maturity beyond the Guyan and Craig-Bampton reductions [25]. Such a "convergence" of the technology leads to the third category of sensitivity-based updating overviewed next.

6.3.5 The Third Category: Iterative Sensitivity-Based Updating (ISBU)

The third category of updating methods is the Iterative, Sensitivity-based Updating (ISBU). It represents the most versatile and powerful updating methods, which explains its popularity for a wide range of applications. ISBU methods seek to minimize a cost function that represents, as before, the error between measurements and model predictions. Spatial incompatibility between measurement locations and nodes of the FE discretization is treated with CMS-based reduction or expansion techniques [26], and numerical solvers are used to optimize the model parameters based on the error criterion defined.

Cost functions of ISBU do not need to be discussed in details, since they come from the same criteria of test-analysis correlation as those previously overviewed. The frequency differences of Eq. (6.19), as well as mode shape differences, are frequently encountered choices. Another common criterion is to minimize the norm of out-of-balance residual forces of Eq. (6.21):

$$J(p) = \left\| K\ \Psi_{Test} - M\ \Psi_{Test}\ \Omega^2_{Test} \right\| \tag{6.30}$$

Once the test-analysis correlation criterion is defined, and additional constraints are potentially added, an iterative method is implemented to solve the corresponding optimization problem, as illustrated in Eqs. (6.26)–(6.29). The main difficulties become implementation issues, such as the accuracy of cost function gradients, reaching a global optimum solution, and time-to-solution.

The first significant difference with SPU methods is that the definition of parameters optimized steps away from global corrections, such as "$p = \Delta K$," or matrix entries, such as "$p = K_{I,J}$." Instead, the parameters are defined at the element level. They can be corrections brought to the element-level mass and stiffness matrices, descriptors of the geometry (mass, length, thickness, etc.), or coefficients of a constitutive material model (modulus of elasticity, Poisson's ratio, etc.).

The second significant difference is that, generally, the system of equations defined to calculate the parameter correction, δp, is solved iteratively with successive re-evaluations of the gradient matrix. Iterative solvers are better suited to the estimation of parameter corrections that exercise a nonlinear effect on FE model predictions.

For illustration, we give an example of ISBU method where the expansion [Eq. (6.26)] is limited to first-order terms. The cost function is defined as the prediction error of resonant frequencies. Instead of writing a global error norm, such as shown in cost function [Eq. (6.19)], the linearization [Eq. (6.27)] can be written for each resonant frequency (ω_1; ω_2; ... ω_m). Collecting these "m" equations yields:

$$\begin{bmatrix} \frac{\partial \omega_1(p^{(n)})}{\partial p_1} & \frac{\partial \omega_1(p^{(n)})}{\partial p_2} & \cdots & \frac{\partial \omega_1(p^{(n)})}{\partial p_{N_P}} \\ \frac{\partial \omega_2(p^{(n)})}{\partial p_1} & \frac{\partial \omega_2(p^{(n)})}{\partial p_2} & \cdots & \frac{\partial \omega_2(p^{(n)})}{\partial p_{N_P}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \omega_m(p^{(n)})}{\partial p_1} & \frac{\partial \omega_m(p^{(n)})}{\partial p_2} & \cdots & \frac{\partial \omega_m(p^{(n)})}{\partial p_{N_P}} \end{bmatrix} \begin{Bmatrix} \delta p_1^{(n+1)} \\ \delta p_2^{(n+1)} \\ \vdots \\ \delta p_{N_P}^{(n+1)} \end{Bmatrix} = \begin{Bmatrix} \omega_1^{Test} - \omega_1(p^{(n)}) \\ \omega_2^{Test} - \omega_2(p^{(n)}) \\ \vdots \\ \omega_m^{Test} - \omega_m(p^{(n)}) \end{Bmatrix} \tag{6.31}$$

The system [Eq. (6.31)] can be solved in a least-squares sense if enough equations are available, that is, $m \geq N_P$. One difficulty is to evaluate entries $\partial\omega_p/\partial p_q$ of the gradient matrix. It can be verified that the partial derivative of the p^{th} frequency, with respect to the q^{th} parameter, is obtained as:

$$\frac{\partial \omega_p^2}{\partial p_q} = \frac{\Psi_p^T \frac{\partial K}{\partial p_q} \Psi_p}{\Psi_p^T M \Psi_p} - \frac{\Psi_p^T \frac{\partial M}{\partial p_q} \Psi_p}{\left(\Psi_p^T M \Psi_p\right)^2} \tag{6.32}$$

Equation (6.32) suggests that the difficulty is shifted away from estimating the gradients $\partial\omega_p/\partial p_q$, towards the partial derivatives of master matrices. This degree of difficulty depends on how the parameters are defined. A judicious choice is to select parameters that represent geometrical or material properties of individual elements, or groups of elements, such that the derivatives can be evaluated in closed-form. Optimizing, for example, the elastic modulus of a linear constitutive law, $\sigma = E \cdot \varepsilon$, gives an element-level sensitivity matrix that is simply $\partial k^{(e)}/\partial E = 1/E\ k^{(e)}$. The global sensitivity matrix, $\partial K/\partial E$, can then be assembled from the element-level sensitivities.

Derivations are more subtle for static deflection derivatives, $\partial U/\partial p_q$, or mode shape derivatives, $\partial\Psi_p/\partial p_q$. The evaluation of these sensitivities involves the computationally expensive procedure of inverting a master matrix. Nelson [36] and Hou and Kenny [37] propose schemes that can be implemented to obtain derivatives, such as $\partial\Psi_p/\partial p_q$, while bypassing the potential matrix singularities involved.

ISBU formulations based on out-of-balance residuals [Eq. (6.30)] are discussed in [38–40]. Ladevèze and Reynier [41] defines a dual approach where residuals are defined as virtual displacements as opposed to out-of-balance forces. An extension of this technique to damping errors is proposed in [42]. Piranda et al. [43] applies the sensitivity approach [Eqs. (6.26)–(6.29)] to resonant frequency and mode shape vector errors. The efficiency of many of these techniques has been assessed in benchmarks, such as the study reported in [44]. Link [45] offers a discussion of ISBU methods, and the advantages and limitations that this calibration technology offers.

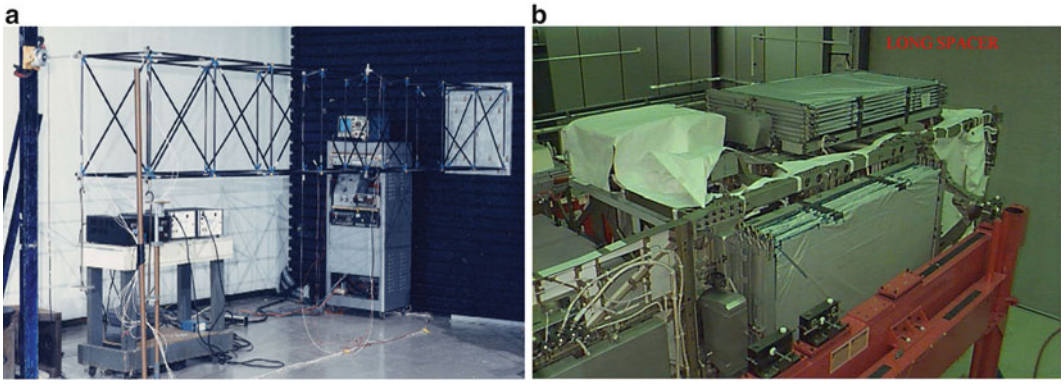

Fig. 6.6 Vibration testing of the ISS main truss. (**a**) NASA 8-bay truss testbed. (**b**) International Space Station main truss (courtesy: NASA [48])

Table 6.2 Correlation of the ISS main truss after calibration [48]

Mode number	Comparison of resonant frequencies			Mode shape correlation (MAC) (%)
	Measured (Hz)	Predicted (Hz)	Error (%)	
1	6.98	6.98	0.00	98.3
2	10.95	10.60	−3.19	97.5
3	12.42	12.40	−0.16	65.3
4	13.06	12.70	−2.76	89.8
5	13.87	13.67	−1.44	86.0

The versatility of ISBU has promoted application to many disciplines. One of them is Structural Health Monitoring (SHM), where model predictions augment physical measurements to locate, and assess the severity of, structural damage. Surveying the application of FE model calibration to SHM is beyond the scope of this discussion; a review can be found in [46]. One accomplishment, that we wish to highlight, is the work of Professor David Zimmerman, and his research team at the University of Houston, Texas. Their eigen-structure assignment technique [47, 48] was successful to calibrate models of the NASA International Space Station (ISS) main truss, illustrated in Fig. 6.6. Linear NASTRAN™ models with up to 66,000 DOFs were updated to achieve less than 3 % modal frequency error and at least 95 % mode shape correlation. These correlation requirements originate from recommended "best practices" [49]. Table 6.2 indicates a typical test-analysis agreement reached after FE model calibration for the ISS main truss.

6.4 A Brief Discussion of Challenges to Updating Methods

The overview of FE model calibration is concluded by briefly discussing several issues that present challenges. The first three are practical issues that, unfortunately, need to be dealt with: ill-conditioning, spatial incompatibility and error localization. Our view is that these issues do not offer fundamental challenges to FE model updating. The next four topics are more foundational: information-theoretic limitations of test-analysis correlation, experimental variability, truncation error and the application of FE model updating to nonlinear dynamics.

6.4.1 Numerical Ill-Conditioning of Inverse Problems

Inverse problems are, by definition, ill-posed. At best, it means that a parameter correction δp obtained by solving, for example, Eq. (6.27) or (6.31), is poor-quality due to contamination from numerical ill-conditioning. At worst, a calibration is ambiguous because an infinite number of solutions are available. Unless there are multiple solutions to choose from, ill-conditioning is not an issue that challenges FE model updating; it is an inconvenience to be dealt with.

Several approaches are available to mitigate the effects of ill-conditioning, such as discussed in [16]. Regularization can be applied. The well-known Tikhonov regularization adds a "minimum perturbation" term to the cost function, the effect of which is to orient the search towards solutions that do not deviate too much from the nominal values of model parameters. Calibration methods formulated with the Bayesian statistical framework introduce a "prior" term whose effect is analogous to regularization. Lastly, good practices of numerical analysis can be implemented, such as filtering out the null space of the gradient matrix inverted.

6.4.2 Mismatch Between Measurement Locations and FE Discretization

Spatial incompatibility between measurement locations and FE discretization nodes was evoked previously. (See, for example, the second-to-last paragraph of Sect. 6.3.3.) The fact is that a discretization is arbitrary; FE nodes have no reason to be co-located with measurement points. Spatial incompatibility, therefore, is another practical difficulty that must be dealt with.

There are two basic possibilities to handle spatial incompatibility. The first one is to expand the measurement locations to the FE spatial discretization. The second approach is to reduce the FE-based global matrices to measurement points. These two strategies are dual of each other. Methods, such as those presented in [26], can be applied to either expand the spatial measurements, reduce the FE matrices, or implement a hybrid approach that borrows from both strategies. The challenge is that both vector projection and matrix reduction rely on information derived from the FE model. Any modeling error "contaminates" the vector projection or matrix reduction step, hence, introducing an inextricable coupling between spatial incompatibility and the modeling error that one is attempting to correct through parameter calibration.

6.4.3 Localization of the Modeling Error

Calibration parameters are usually selected a priori. Sensitivity analysis can also be used to select them, keeping only those parameters that provide large derivatives of the cost function, $|\partial J/\partial p_k|$. A first issue is that the source and location of modeling error might be unknown, and the parameter selection might not allow to describe it accurately. Another issue is that modeling error does not necessarily coincide with high sensitivities, $|\partial J/\partial p_k|$. It implies that an adjustment is often brought to a FE parameter because this parameter exercises a large influence on the cost function, not necessarily because it is where the modeling error is located.

To address these uncertainties, it is often tempting to calibrate as many parameters as possible. Including large numbers of parameters, however, tends to increase ill-conditioning; it also tends to "spread" the corrections throughout the FE model instead of focusing them where they are really needed, that is, where modeling error is located. This dilemma goes back to a previously mentioned issue of FE modeling: there are limitations to inferring local model parameters from globally obtained, resonant frequency and mode shape information.

There are situations where error localization may not be such a serious issue. This is the case when the analyst knows the source of the modeling error, for example, at joints or connections between structural components. SHM is another application where damage scenarios might be known a priori. Deploying a posteriori error indicators, such as the residuals [Eq. (6.21)], can also offer a mapping of the modeling error on the computational mesh.

6.4.4 Information-Theoretic Limitations of Model Updating

Model updating can be viewed as a special case of the statistical inference of parameters given physical observations or measurements. In statistical sciences, the Cramér–Rao inequality provides a lower bound of the variance with which model parameters can be estimated [50]. The lower bound depends on the inverse of the Fisher information that represents the "curvature," or second derivative Hessian matrix, of the likelihood function defined for test-analysis correlation.

The information-theoretic limitation implies a lower bound on the accuracy of model parameters that can be estimated. This accuracy would depend on the formulation of the updating method, and amount and quality of physical measurements available for calibration. Even though it is not similar to the Cramér–Rao bound, one example of limit is derived in [51]:

$$\min_{1\le k\le m}\left|\omega^2_{\text{Test},k}-\omega^2_k\right| \le \frac{\|R\|_2}{\|\Psi_{\text{Test}}\|_2} \le \max_{1\le k\le m}\left|\omega^2_{\text{Test},k}-\omega^2_k\right| \tag{6.33}$$

where it is shown that the L^2 norm of out-of-balance residuals [Eq. (6.21)] cannot be made smaller than the minimum error in resonant frequency. The authors are not aware of investigations that would study lower bounds of parameter accuracy in the context of FE model updating. Nevertheless, it suggests that it makes no sense to calibrate model parameters beyond these theoretical limits. Doing so would be a manifestation of "over-fitting," which, unfortunately, is often encountered.

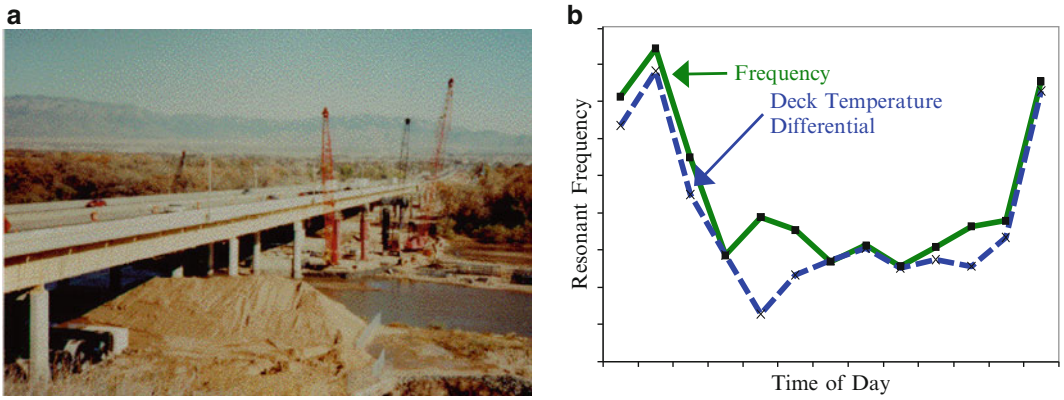

Fig. 6.7 Damage detection of the I-40 bridge in New Mexico. (**a**) I-40 bridge over the Rio Grande. (**b**) Variation of frequencies over time (credit: [53])

6.4.5 Accounting for Experimental Variability and Uncertainty

Another foundational challenge to FE model updating is experimental variability. As alluded to in Sect. 6.4.4, there is a theoretical limit to the accuracy of an inference. Likewise, the level of experimental variability, with which measurements are collected, should be taken into account as it sets a "threshold" beyond which calibrating the FE parameters becomes inefficient.

Farrar et al. [52, 53] were amongst the first ones to indicate the effect of experimental variability on the calibration of FE model parameters. Figure 6.7 illustrates this study that attempts to locate structural damage using a combination of vibration tests and FE predictions. Figure 6.7b plots the variation amplitude of the first resonant frequency over a 24-h period. It is observed that the variation is greater than the change induced by simulating structural damage with the model, leading to the conclusion that damage cannot be identified reliably in this application.

Many approaches inspired from Bayesian statistics have been developed in the mid-1990s to account for experimental variability in calibration. In Sect. 6.3.1, we have also evoked an early variant of Bayesian-like FE model updating [13], published in 1974. Rebba et al. [54, 55] offer a rigorous framework to account for experimental variability. With this and similar works, the focus of calibration shifts away from optimizing parameters, such that model predictions reproduce the measurements. Instead, one seeks to describe the parameter uncertainty with a probability law which, when sampled, yields a population of predictions that is consistent with the population of measurements. No single prediction matches any single measurement; test-analysis correlation is sought, instead, in the sense of ensemble statistics between predictions and measurements.

6.4.6 Accounting for Numerical Uncertainty Caused by Truncation

Similarly to experimental variability that can be thought of as defining bounds within which the "true-but-unknown" measurement is located relative to the data collected, truncation error defines bounds within which the "true-but-unknown" prediction is located relative to the values provided by simulations. Truncation error originates from discretizing the continuous EOM with a particular numerical method, and using a given level of mesh discretization. Using, for example, quadratic FE shape functions and low bulk viscosity does not produce the same error as a linear basis with high bulk viscosity. Truncation error is not known, except for a few code verification test problems, which means that it becomes an additional source of uncertainty in the problem.

Numerical uncertainty, that originates from truncation effects, should be accounted for during FE model updating, as it defines another contribution to the lower bound of prediction accuracy that can be achieved during calibration. Unfortunately, it is common in computational engineering to "run with a single mesh" and ignore truncation effects. Roache [56] and Hemez and Kamm [57] present techniques to quantify this truncation-based uncertainty, such that it can be accounted for. Applications can be found in the literature, for example, to the validation of wind turbine blade models [58, 59].

a

b

"Pride"	Assume that the code is correct.
"Sloth"	Only do a qualitative comparison (*e.g.*, the "viewgraph norm").
"Envy"	Use problem-specific settings.
"Wrath"	Use only code-to-code comparisons.
"Avarice"	Use only a single mesh.
"Gluttony"	Only show results that make the code look good.
"Lust"	Don't differentiate between accuracy and robustness-to-uncertainty.

Fig. 6.8 A metaphor of commonly encountered flaws in test-analysis correlation. (**a**) H. Bosch's "*Seven Deadly Sins*," 1485. (**b**) "Sins" of test-analysis correlation [credit: Prado Museum, Madrid, Spain (**a**) and [64] (**b**)]

6.4.7 Calibration of FE Representations for Nonlinear Dynamics

Model updating is typically applied to linear representations, using Fourier-based responses (resonant frequencies, mode shapes, etc.). When the response is significantly nonlinear and/or only very short time histories are available, assumptions underpinning the Fourier analysis can be violated. Sources of nonlinearity which can disrupt these assumptions are: nonlinear material response, contact, friction, other energy loss mechanisms and short-duration responses.

Nonlinearity, irrespective of its source, offers a foundational challenge to updating because the response can no longer be decoupled via modal analysis. Approaches have been proposed to expand the conventional space-time decoupling of the EOM to nonlinear systems [60, 61]. While these approaches have shown promise, applicability remains limited to "mild" nonlinearities that, for example, are isolated on the structure or do not perturb the (linear) response too severely. A non-exhaustive review of nonlinear model updating is proposed in [12].

Defining a general-purpose FE calibration technique, applicable to arbitrary nonlinear dynamics, seems an unattainable goal due to the vast diversity of sources and types of nonlinearities. One noticeable attempt is a formulation for nonlinear system identification, grounded in concepts of optimal control [62, 63]. These works highlight the significant computational burden of tracking a nonlinear time-domain response at multiple locations. To the best of the author's knowledge, this approach has not been applied to "real-life" problems. Fundamentally, a nonlinear response is described by three independent kinetic fields (displacement, velocity, acceleration) at every point of the structure. Experimentally, these three fields would have to be measured separately; computationally, they would have to be evolved independently. Neither experimental techniques nor computational methods are currently mature enough to meet these stringent requirements.

6.4.8 Closure

In closure, we re-iterate that technology for FE model updating has made great progress and achieved undeniable successes in three decades from the mid-1960s to the mid-1990s. This is especially the case for linear statics or linear dynamics. The technology, originally limited to closed-form corrections of master matrices, has evolved towards general-purpose sensitivity methods able to calibrate individual design parameters using diverse data (resonant frequency, mode shape, frequency response function, etc.). Current advances in modeling and analysis, pre- and post-processing software, and experimental methods for vibration testing, are likely to push forward the FE model updating technology even more.

Difficulties, some of which are discussed above, have also seriously hindered the transfer of FE model updating to industry. We prognosticate that some of these challenges will progressively disappear as simulation capabilities and experimental techniques keep improving. It should be the case for the quality of solutions (Sect. 6.4.1) and mismatch between measurement locations and FE discretization nodes (Sect. 6.4.2). Other difficulties, such as lower bounds on the value of information learned during calibration (Sect. 6.4.3) and nonlinear dynamics (Sect. 6.4.7), are foundational challenges to model updating. Another significant challenge is educational: "poor practices" of test-analysis comparison, illustrated in Fig. 6.8, are often encountered [64].

In the last 15 years, the computational physics and engineering communities have explored a change of paradigm towards Verification and Validation (V&V) [65]. We conclude by noting that V&V makes sense if the emphasis is on developing, and establishing the credibility of, a predictive capability. Verifying the quality of the analysis code, learning important sensitivities, and quantifying prediction uncertainty before starting to compare predictions to measurements, promotes "best practices" that mitigate short-comings, such as those illustrated in Fig. 6.8. The V&V technology offers another avenue of growth for FE model updating.

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