

A Control Scheme Towards Accurate Firing While Moving for a Mobile Robotic Weapon System with Delayed Resonators

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Abstract In the present paper a mobile robotic weapon system is considered. The system comprises of a ground vehicle equipped with a robotic manipulator carrying a gun. The goal is to perform accurate firing while moving and despite the vibrations due to uneven ground and reaction weapon forces. The vehicle is considered to be equipped with passive and active suspension systems. The active suspension involves delayed resonators feeding back the resonators' acceleration. Using the Euler-Lagrange approach, the model of the system is derived in the form of a nonlinear neutral time delay mathematical description. From the system design point of view, the goal is formulated as a command following problem with simultaneous disturbance attenuation, under appropriate constraints. To achieve these goals, an algebraic control scheme based on the linear approximant of the system's model is proposed. The controller is of the measurable output feedback dynamic type. Despite the complexity of the system's model, the derived controller is realizable in the sense that no predictors are required and is simple enough to be implemented to low level computer platforms. Thus the proposed controller offers itself to upgrade traditional armed ground vehicles. This upgrade appears to be of low cost. The good performance of the proposed controller is demonstrated through computational experiments upon the nonlinear model of the system.

Introduction

Vibration absorption is of great importance for several applications ranging from ride comfort improvement and passenger safety in conventional vehicles (see [1–6] and the references therein) and industrial applications (see [7–10] and the references

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therein) to military applications (see [11, 12] and the references therein). Besides the difficulty that arises from performance constraints, the designer has to take into account the actuator influence [13–17]. Of special interest, as far as military applications are concerned, are anti-mine robotic manipulators (see [18–20]). Also, such robotic systems are used for patrols around a military base or other government installation or for convoy support operations. Here we focus on such robotic systems where accurate firing is required while moving. Clearly, vibration minimization and compensation of the gun reaction force together with target following are required to be satisfied via an appropriate control scheme.

In the present paper, the mathematical description of a simplified robotic weapon system is presented. The system comprised of a ground vehicle equipped with a robotic manipulator carrying a gun. A two-stage control scheme towards performing accurate firing while moving and despite vibrations due to uneven ground and reaction weapon forces is designed. The control scheme is developed on the basis of the linear approximant of the system. The scheme is analyzed using an inner and an outer control loop. The inner loop controller achieves diagonalization for the transfer matrix mapping the external commands to the performance outputs as well as it satisfies a mixed disturbance rejection/disturbance attenuation criterion in order to reduce the influence of reaction weapon forces and road unevenness. The outer loop controller produces an appropriate command for the inner controller in order to follow fast and accurately a moving target. The performance of the proposed control scheme is demonstrated through simulations. The control scheme appears to contribute to force multiplication, expansion of the battle-space, extension of the warfighter's reach, and last but not least casualty reduction. Finally, it is noted that the present paper is a generalization of the results presented in [21].

Mathematical Description of a Mobile Robotic Weapon System

Consider the mobile robotic weapon presented in Fig. 1. Assuming that the vehicle moves on a straight path, the whole motion can be faced as a 2-D problem. The mobile robot consists of a platform carrying a two-joint robot which includes a prismatic and a revolute joint. In Fig. 1 the gun is represented by an abstractive arrow shape. The vehicle is assumed to move on a horizontal level due to a force that is always parallel to the road. Ground unevenness is modeled as a force disturbance. The vibrations of the vehicle are absorbed by four absorbers, two of which act as active absorbers and the rest two as passive absorbers. The passive absorbers (vehicle suspension) are two identical conventional spring-damper structures that connect the platform of the vehicle with the front and rear wheel, respectively. They are considered to be locked, i.e., no independent motion is allowed. Consequently, the platform cannot rotate. The active vibration absorbers are two identical mass-spring-damper trios that utilize acceleration feedback with controlled delay and are placed on top of the vehicle's platform at the position of the front and the rear

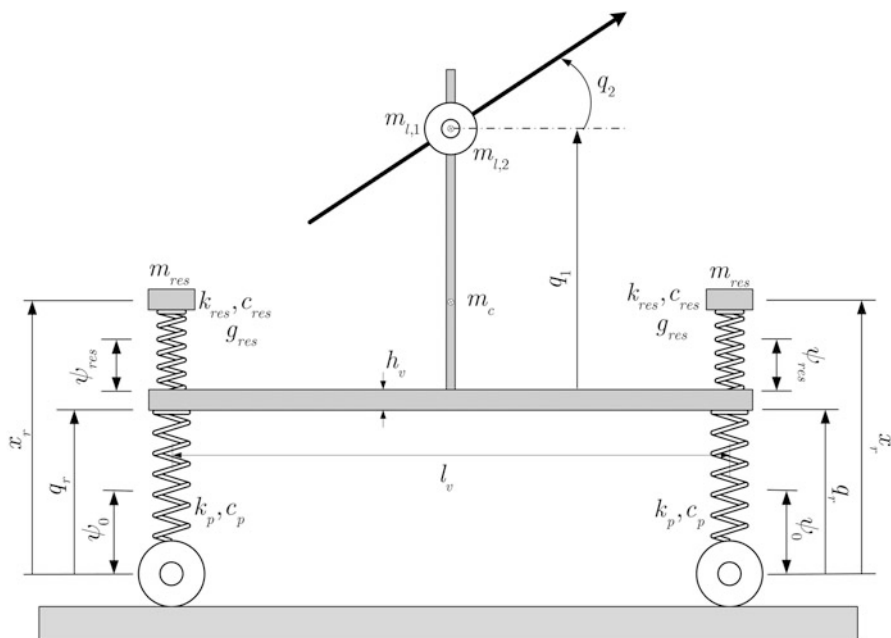


Fig. 1 A mobile robotic weapon system

wheel, respectively (see [7–9]). At the center of the platform, a robotic manipulator that carries a gun is considered. The “gripper” holds the gun from its center of mass. Three independent preinstalled controllers are considered to be used to regulate the forward velocity of the vehicle and the robot joints. These controllers are of the P/PD type with gravity compensation. The first controller feeds back the error between the respective commands and the forward velocity by appropriate gain $K_{p,1}$. The second and the third controller feed back the error between the respective commands and the joint variables as well as the derivatives of the joint variables, both multiplied by appropriate gains, $K_{p,i}$ and $K_{d,i}$ ($i = 2, 3$), respectively.

The mathematical model of the robotic system is in the following general neutral multi-delay form

$$E_0 \dot{x}(t) + E_1 \dot{x}(t - \tau) = A_0 x(t) + B_0 u(t) + D_0(x) \xi(t) + \Lambda_0 \quad (1)$$

where

$$x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8 \ x_9 \ x_{10}]^T$$

$$= [d_f \ q_r \ x_r \ q_1 \ q_2 \ \dot{d}_f \ \dot{q}_r \ \dot{x}_r \ \dot{q}_1 \ \dot{q}_2]^T \quad (2)$$

$$u = [u_1 \ u_2 \ u_3]^T \quad (3)$$

$$\xi = [\xi_1 \ \xi_2]^T = [f_g \ f_r]^T \quad (4)$$

and where d_8 is the horizontal distance of the center of mass of the cart from the point of origin, q_r is the passive (front or rear) suspension spring length, x_r is the distance of the (front or rear) active suspension mass from the (front or rear) wheel, respectively, q_1 is the prismatic joint variable, q_2 is the revolute joint variable, u_1 is the external command to the preinstalled P controller that regulates the forward velocity of the cart, u_2 is the external command to the preinstalled P-D controller that regulates the prismatic joint, u_3 is the external command to the preinstalled P-D controller that regulates the revolute joint, f_g is the gun reaction force, and f_r is the disturbance force generated from road unevenness. The matrices in (1) are in the forms $E_0 \in \mathbb{R}^{10 \times 10}$, $E_1 \in \mathbb{R}^{10 \times 10}$, $A_0 \in \mathbb{R}^{10 \times 10}$, $B_0 \in \mathbb{R}^{10 \times 3}$, $D_0 \in \mathbb{R}^{10 \times 2}$, $\Lambda \in \mathbb{R}^{10 \times 2}$, and their nonzero elements are the following: $(e_0)_{1,1} = 1$, $(e_0)_{2,2} = 1$, $(e_0)_{3,3} = 1$, $(e_0)_{4,4} = 1$, $(e_0)_{5,5} = 1$, $(e_0)_{6,6} = m_c + m_{l,1} + m_{l,2}$, $(e_0)_{7,7} = m_c + m_{l,1} + m_{l,2}$, $(e_0)_{8,7} = m_{l,1} + m_{l,2}$, $(e_0)_{8,9} = m_{l,1} + m_{l,2}$, $(e_0)_{7,9} = m_{l,1} + m_{l,2}$, $(e_0)_{9,10} = I_2$, $(e_0)_{10,8} = 1$, $(e_1)_{7,8} = -2g_{\text{res}}$, $(e_1)_{10,8} = g_{\text{res}}/m_{\text{res}}$, $(a_0)_{7,2} = -2(k_p + k_{\text{res}})$, $(a_0)_{10,2} = \frac{k_{\text{res}}}{m_{\text{res}}}$, $(a_0)_{7,3} = 2k_{\text{res}}$, $(a_0)_{10,3} = -\frac{k_{\text{res}}}{m_{\text{res}}}$, $(a_0)_{8,4} = -K_{p,2}$, $(a_0)_{9,5} = -K_{p,3}$, $(a_0)_{1,6} = 1$, $(a_0)_{6,6} = -K_{p,1}(m_c + m_{l,1} + m_{l,2})$, $(a_0)_{2,7} = 1$, $(a_0)_{7,7} = -2(c_p + c_{\text{res}})$, $(a_0)_{10,7} = \frac{c_{\text{res}}}{m_{\text{res}}}$, $(a_0)_{3,8} = 1$, $(a_0)_{7,8} = 2c_{\text{res}}$, $(a_0)_{10,8} = -\frac{c_{\text{res}}}{m_{\text{res}}}$, $(a_0)_{4,9} = 1$, $(a_0)_{8,9} = -K_{d,2}$, $(a_0)_{5,10} = 1$, $(a_0)_{9,10} = -K_{d,3}$, $(b_0)_{6,1} = K_{p,1}(m_c + m_{l,1} + m_{l,2})$, $(b_0)_{8,2} = K_{p,2}$, $(b_0)_{9,3} = K_{p,3}$, $(d_0)_{6,1} = \cos(x_5(t))$, $(d_0)_{7,1} = \sin(x_5(t))$, $(d_0)_{8,1} = \sin(x_5(t))$, $(d_0)_{6,2} = 1$, $(\lambda_0)_{7,1} = 2\psi_0 k_p - 2(h_v + \psi_{\text{res}})k_{\text{res}} - g(m_c + m_{l,1} + m_{l,2})$, $(\lambda_0)_{10,1} = -g + \frac{(h_v + \psi_{\text{res}})k_{\text{res}}}{m_{\text{res}}}$.

Note that m_c is the cart mass, $m_{l,1}$ is the first link mass, $m_{l,2}$ is the second link mass, m_{res} is the resonator mass, I_2 is the second link moment of inertia, h_v is the vehicle height, ψ_{res} is the resonator spring's free length, ψ_0 is the passive absorber's free length, k_{res} is the resonator spring constant, k_p is the suspension spring constant, c_{res} is the resonator damping factor, c_p is the suspension damping factor, g_{res} is the resonator feedback gain, τ is the resonator feedback delay, and g is the gravity acceleration. The performance outputs of the system description (1) are the forward velocity of the cart $y_1(t) = x_6(t)$, the distance of the center of mass (c.m.) of the gun from the ground $y_2(t) = h_v + r_0 + x_2(t) + x_4(t)$, and the inclination of the gun: $y_3(t) = x_5(t)$. Note that r_0 denotes the radius of the wheels. The performance output vector is given by the relation

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = Cx(t) + \begin{bmatrix} 0 \\ h_v + r_0 \\ 0 \end{bmatrix}$$

where

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

The measurable outputs of the system are denoted by

$$\psi(t) = \begin{bmatrix} \psi_1(t) \\ \psi_2(t) \\ \psi_3(t) \\ \psi_4(t) \end{bmatrix} = Lx(t)$$

where

$$L = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

To derive the linear approximant of the robotic system, first consider the following trim conditions

$$\bar{u}_1 = \bar{w}_1, \bar{u}_2 = \bar{w}_2, \bar{u}_3 = \bar{w}_3, \bar{\xi}_1 = 0, \bar{\xi}_2 = 0$$

$$\bar{x}_1 = v_n t, \bar{x}_2 = \psi_0 - \frac{g(m_c + m_{l,1} + m_{l,2} + 2m_{res})}{2k_p}$$

$$\bar{x}_3 = \psi_0 + \psi_{res} + h_v - \frac{g}{2k_p}(m_c + m_{l,1} + m_{l,2}) - \frac{g}{k_p k_{res}}(k_p + k_{res})m_{res}$$

$$\bar{x}_4 = \bar{u}_2, \bar{x}_5 = \bar{u}_3, \bar{x}_6 = v_n, \bar{x}_7 = 0, \bar{x}_8 = 0, \bar{x}_9 = 0, \bar{x}_{10} = 0$$

Second, consider the perturbations of the state and input variables

$$\delta y(t) = y(t) - \bar{y}, \delta x(t) = x(t) - \bar{x}, \delta u(t) = u(t) - \bar{u}$$

$$\delta \xi(t) = \xi(t) - \bar{\xi}, \delta \psi(t) = \psi(t) - \bar{\psi}$$

where $\bar{y} = C\bar{x}$ and $\bar{\psi} = L\bar{x}$. Thus, the linear approximant of the system is of the following generalized neutral form

$$E_0 \delta \dot{x}(t) + E_1 \delta \dot{x}(t - \tau) = A_0 \delta x(t) + B_0 \delta u(t) + D_0(\bar{x}) \delta \xi(t) \tag{7}$$

$$\delta y(t) = C \delta x(t) \tag{8}$$

$$\delta \psi(t) = L \delta x(t) \tag{9}$$

Controller Design

Here, the accurate performance of the system is analyzed in two design goals. The first is to achieve a desired closed loop transfer matrix while simultaneously achieving a mixed disturbance rejection/disturbance attenuation scheme and the second is accurate following of a target. The control scheme will be analyzed using an inner and an outer control loop. The inner control loop will satisfy the first design goal while the outer loop will satisfy the second design goal.

Inner Control Loop

The inner loop controller is considered to be of the dynamic multi-delay measurement output type:

$$\delta U(s) = K(s, z) \delta \Psi(s) + G(s, z) \delta R(s) \quad (10)$$

where $\delta U(s)$, $\delta \Psi(s)$, and $\delta R(s)$ are the Laplace transforms of the vector signals $\delta u(t)$, $\delta \psi(t)$, and $\delta r(t)$, respectively, while $\delta r(t) = [\delta r_1(t) \delta r_2(t) \delta r_3(t)]^T$ is the 3×1 vector of external inputs. Clearly, it holds that $\delta R(s) = [\delta R_1(s) \delta R_2(s) \delta R_3(s)]^T$. The Laplace transform of the perturbation of the measurement output vector is expressed by the relation $\delta \Psi(s) = L \delta X(s)$, where $\delta X(s)$ is the Laplace transform of the perturbation of the state vector $\delta x(t)$. The elements of the controller matrices are rational functions of s with coefficients being rational functions of $z = e^{-s\tau}$, i.e., $k_{i,j}(s, z)$, $g_{i,j}(s, z) \in \mathbb{R}(s, z)$, where $\mathbb{R}(s, z)$ is the field of rational functions of s with coefficients being rational functions of z . It is important to mention that for the implementation of the controller (10) the elements of the controller matrices should be realizable (see [22–24]). After substituting the controller (10) to system described by (7) to (9) the forced response of the closed loop system is given by the relation

$$\begin{aligned} \delta X(s) &= [I_{10} - H_u(s, z) K(s, z) L]^{-1} H_u(s, z) G(s, z) \delta R(s) \\ &+ [I_{10} - H_u(s, z) K(s, z) L]^{-1} H_\xi(s, z) \delta \Xi(s) \end{aligned} \quad (11)$$

where

$$H_u(s, z) = [s(E_0 + zE_1) - A_0]^{-1} B \quad (12)$$

$$H_d(s, z) = [s(E_0 + zE_1) - A_0]^{-1} D_0(\bar{x}) \quad (13)$$

The desired closed loop matrix should have the following three characteristics: First, the closed loop transfer matrix relating the external inputs to the performance

outputs is diagonal and invertible. This is the I/O decoupling design requirement. This design requirement is formally expressed as follows:

$$\begin{aligned}
 & C [I_{10} - H_u(s, z) K(s, z) L]^{-1} H_u(s, z) G(s, z) \\
 &= \begin{bmatrix} h_{m,1}(s, z) & 0 & 0 \\ 0 & h_{m,1}(s, z) & 0 \\ 0 & 0 & h_{m,1}(s, z) \end{bmatrix} \tag{14}
 \end{aligned}$$

where the rational functions $h_{m,1}(s, z)$, $h_{m,2}(s, z)$, and $h_{m,3}(s, z)$ are different than zero and they belong to $\mathbb{R}(s, z)$.

Second, the second row–first column element and the third row elements of the closed loop transfer matrix relating the disturbances to the performance outputs are equal to zero. This is a partial disturbance rejection design requirement. This design requirement is formally expressed as follows:

$$\begin{aligned}
 & C [I_{10} - H_u(s, z) K(s, z) L]^{-1} H_d(s, z) \\
 &= \begin{bmatrix} (h_d)_{1,1}(s, z) & (h_d)_{1,2}(s, z) \\ 0 & (h_d)_{2,2}(s, z) \\ 0 & 0 \end{bmatrix} \tag{15}
 \end{aligned}$$

According to the above design requirement, the gun reaction force being the main disturbance does not influence the distance of the center of mass (c.m.) of the gun from the ground and the inclination of the gun. Also, the inclination of the gun, having the greater influence to the accuracy of the shot, is not influenced by the disturbance force generated from road unevenness.

Third, the norm of the nonzero elements of the closed loop transfer matrix relating the disturbances to the performance outputs is enough small, i.e.,

$$\max \{ \|(h_d)_{1,1}(s, z)\|_\infty, \|(h_d)_{1,2}(s, z)\|_\infty, \|(h_d)_{2,2}(s, z)\|_\infty \} < \varepsilon \tag{16}$$

where ε is an enough small positive real guaranteeing attenuation of the influence of the disturbances to the performance outputs.

In what follows the elements of the controller matrices in (10) will be selected to satisfy all three of the above design requirements. The solution of the precompensator to satisfy the I/O decoupling design requirement is

$$\begin{aligned}
 G(s, z) &= \left\{ C [I_{10} - H_u(s, z) K(s, z) L]^{-1} H_u(s, z) \right\}^{-1} \\
 &\times \begin{bmatrix} h_{m,1}(s, z) & 0 & 0 \\ 0 & h_{m,1}(s, z) & 0 \\ 0 & 0 & h_{m,1}(s, z) \end{bmatrix} \tag{17}
 \end{aligned}$$

The solution of the feedback matrix to satisfy the partial disturbance rejection problem in (15) is the following

$$\begin{aligned}
 k_{2,4}(s, z) = & \left((s(H_u)_{1,1}(s, z)k_{1,4}(s, z) - 1) \right. \\
 & \times \left\{ (H_d)_{2,1}(s, z) \left[1 + (H_u)_{4,2}(s, z)(k_{2,1}(s, z) \right. \right. \\
 & \left. \left. - k_{2,2}(s, z)) \right] + (H_d)_{4,1}(s, z) \left[1 + (H_u)_{2,2}(s, z)(k_{2,2}(s, z) - k_{2,1}(s, z)) \right] \right\} / \\
 & \left\{ s \left[(H_d)_{1,1}(s, z) \left((H_u)_{2,2}(s, z) + (H_u)_{4,2}(s, z) \right) \right. \right. \\
 & \left. \left. - (H_u)_{1,1}(s, z) \left((H_d)_{4,1}(s, z) (H)_{2,2}(s, z) \right. \right. \right. \\
 & \left. \left. \left. - (H_d)_{2,1}(s, z) (H_u)_{4,2}(s, z) \right) (k_{1,1}(s, z) - k_{1,2}(s, z)) \right] \right\} \quad (18)
 \end{aligned}$$

$$\begin{aligned}
 k_{3,2}(s, z) = & \left[(H_d)_{4,1}(s, z) (H_u)_{2,2}(s, z) k_{2,2}(s, z) \right. \\
 & \left. + (H_d)_{2,1}(s, z) (1 - (H_u)_{4,2}(s, z) k_{2,2}(s, z)) \right] \\
 & \times k_{3,1}(s, z) / \left[(H_d)_{4,1}(s, z) \left((H_u)_{2,2}(s, z) k_{2,1}(s, z) - 1 \right) \right. \\
 & \left. - (H_d)_{2,1}(s, z) (H_u)_{4,2}(s, z) k_{2,1}(s, z) \right] \quad (19)
 \end{aligned}$$

$$\begin{aligned}
 k_{3,4}(s, z) = & \left(\left((H_d)_{2,1}(s, z) (H_u)_{4,2}(s, z) \right. \right. \\
 & \left. \left. - (H_d)_{4,1}(s, z) (H_u)_{2,2}(s, z) \right) \right. \\
 & \times \left(s(H_u)_{1,1}(s, z)k_{1,4}(s, z) - 1 \right) \left\{ (H_d)_{2,1}(s, z) \right. \\
 & \times \left[1 + (H_u)_{4,2}(s, z)(k_{2,1}(s, z) - k_{2,2}(s, z)) \right] \\
 & \left. + (H_d)_{4,1}(s, z) \left[1 + (H_u)_{2,2}(s, z)(k_{2,2}(s, z) - k_{2,1}(s, z)) \right] \right\} k_{3,1}(s, z) / \\
 & \left\{ s \left[(H_d)_{1,1}(s, z) \left((H_u)_{2,2}(s, z) + (H_u)_{4,2}(s, z) \right) \right. \right. \\
 & \left. \left. - (H_u)_{1,1}(s, z) \left((H_d)_{4,1}(s, z) (H_u)_{2,2}(s, z) \right. \right. \right. \\
 & \left. \left. \left. (H_d)_{2,1}(s, z) (H_u)_{4,2}(s, z) \right) (k_{1,1}(s, z) - k_{1,2}(s, z)) \right] \right. \\
 & \left. \left[(H_d)_{2,1}(s, z) (H_u)_{4,2}(s, z) k_{2,1}(s, z) \right. \right. \\
 & \left. \left. + (H_d)_{4,1}(s, z) (1 - (H_u)_{2,2}(s, z) k_{2,1}(s, z)) \right] \right\} \quad (20)
 \end{aligned}$$

where $(H_u)_{i,j}(s, z)$, denotes the (i, j) element of $H_u(s, z)$ and $(H_d)_{i,j}(s, z)$ denotes the (i, j) element of $H_d(s, z)$.

To satisfy the requirement of disturbance attenuation, the rest of the elements of the feedback matrices is selected to be real, i.e.,

$$\begin{aligned}
 k_{1,1}(s, z) = k_{1,1} \in \mathbb{R}, k_{1,2}(s, z) = k_{1,2} \in \mathbb{R}, k_{1,3}(s, z) = k_{1,3} \in \mathbb{R} \\
 k_{1,4}(s, z) = k_{1,4} \in \mathbb{R}, k_{2,1}(s, z) = k_{2,1} \in \mathbb{R}, k_{2,2}(s, z) = k_{2,2} \in \mathbb{R}
 \end{aligned}$$

$$k_{2,3}(s, z) = k_{2,3} \in \mathbb{R}, k_{3,1}(s, z) = k_{3,1} \in \mathbb{R}, k_{3,3}(s, z) = k_{3,3} \in \mathbb{R}$$

For the determination of the above real parameters, a heuristic algorithm is proposed (see [25–27]).

It is important to mention that a controller selection as in (18) to (20) with the remaining elements being static can guarantee the solvability of the closed loop system, i.e.,

$$\det [s(E_0 + zE_1) - A_0 - B_0K(s, z)L] \neq 0 \tag{21}$$

It can be verified that the same selection guarantees the realizability of the feedback matrix. Furthermore, it can be observed that the rational matrix $C[I_{10} - H_u(s, z)K(s, z)L]^{-1}H_u(s, z)$ resulting after the above selection of the elements of the feedback matrix is birealisable. Hence, for the precompensator in (17) to be invertible and realizable, it suffices to choose the rational functions $h_{m,1}(s, z)$, $h_{m,2}(s, z)$, and $h_{m,3}(s, z)$ to be delayless and different than zero, i.e.,

$$h_{m,1}(s, z) = h_{m,1}(s)$$

$$h_{m,2}(s, z) = h_{m,2}(s)$$

$$h_{m,3}(s, z) = h_{m,3}(s)$$

Finally, note that for the precompensator to be at least strictly proper, a possible selection for the models' transfer functions is

$$h_{m,1}(s) = \frac{p_{1,0}}{s + p_{1,0}}$$

$$h_{m,2}(s) = \frac{p_{2,0}}{s^2 + p_{2,1}s + p_{2,0}}$$

$$h_{m,3}(s) = \frac{p_{3,0}}{s^2 + p_{3,1}s + p_{3,0}}$$

where $p_{i,j} > 0$. It is mentioned that the above selection of the decoupled closed loop transfer functions guarantees asymptotic command following.

Outer Control Loop

As already mentioned the design goal of the outer loop is to follow accurately a target. Let $\alpha(t)$ be the inclination of the gun with respect to the vertical axis, i.e., it holds that $\alpha(t) = x_5(t) - \frac{3\pi}{2}$ (see Fig. 2). Let $l(t)$ be the distance between the

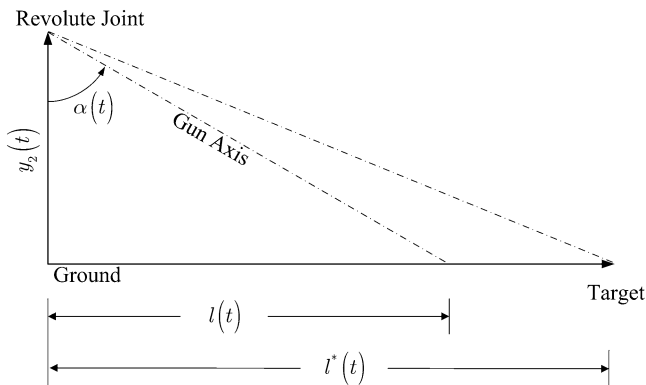


Fig. 2 A geometric interpretation of the distance between the aiming point and the target

projection of the revolute joint to the ground and the intersection between the gun axis and the ground level. Clearly it holds that $l(t) = y_2(t) \tan(\alpha(t))$. Finally, let $l^*(t)$ be the distance between the projection of the revolute joint to the ground and the target.

Let $\delta l(t) = l(t) - \bar{l}$ be the perturbation of $l(t)$, where

$$\bar{l} = \bar{y}_2 \tan(\bar{\alpha}) = (\bar{x}_2 + \bar{x}_4 + h_v + r_0) \tan\left(\bar{x}_5 - \frac{3\pi}{2}\right) \tag{22}$$

Thus, it holds that

$$\begin{aligned} \delta L(s) = & -\cot(\bar{u}_3) h_{m,2}(s, z) \delta R_2(s) + \\ & (h_v + r_0 + \bar{x}_2 + \bar{x}_4) \csc(\bar{u}_3)^2 h_{m,3}(s, z) \delta R_3(s) \end{aligned} \tag{23}$$

where $\delta L(s)$ denotes the Laplace transform of $\delta l(t)$. It is important to mention that after substituting the trim condition of the state variables presented in section “Mathematical Description of a Mobile Robotic Weapon System” to the relation (22) we get

$$\begin{aligned} \bar{l} = & -\left[h_v - 0.5gk_p^{-1} (m_c + m_{l,1} + m_{l,2} + 2m_{res}) \right. \\ & \left. + \psi_0 + r_0 + \bar{u}_2 \right] \cot(\bar{u}_3) \end{aligned} \tag{24}$$

Target following will be achieved by adjusting the distance of the revolute joint from the ground while preserving constant inclination of the gun, i.e., $\delta R_3(s) = 0$. Using $\delta R_3(s) = 0$ as well as (14) and (15) we observe (with respect to the linear model) that the inclination of the gun remains constant. Also, from (23) we observe (with respect to the linear model) that the influence of the disturbances to $\delta l(t)$ has been eliminated. With respect to the nonlinear model, it is significant to mention

that through the disturbance rejection controller the influence of the gun reaction to the distance of the center of mass of the gun from the ground and the inclination of the gun have been eliminated while the disturbance due to road unevenness has significantly been reduced.

Let $\delta l^*(t) = l^*(t) - \bar{l}$ be the perturbation of $l^*(t)$. Let $\delta L^*(s)$ be the Laplace transform of $\delta l^*(t)$. The outer loop controller is proposed to be of the following PID form

$$\delta R_2(s) = \left(f_p + \frac{1}{s} f_i + s f_d \right) (\delta L^*(s) - \delta L(s)) \tag{25}$$

Let $l(0-) = \bar{l}$. Hence using $\delta R_3(s) = 0$, (22) to (24) as well as the definition of the decoupled closed loop transfer function presented in section “Inner Control Loop,” the perturbation of the aiming point position is related to the perturbation of the target positions by the relation

$$\delta L(s) = H_l(s) \delta L^*(s) \tag{26}$$

where

$$\begin{aligned} H_l(s) = & - (f_d p_{2,0} \cot(\bar{u}_3) s^2 + f_p p_{2,0} \cot(\bar{u}_3) s \\ & + f_i p_{2,0} \cot(\bar{u}_3)) / [s^3 + (p_{2,1} - f_d p_{2,0} \cot(\bar{u}_3)) s^2 \\ & + p_{2,0} (1 - f_p \cot(\bar{u}_3)) s - f_i p_{2,0} \cot(\bar{u}_3)] \end{aligned} \tag{27}$$

The transfer function (27) is stable if and only if the PID controller parameters satisfy the following inequalities

$$f_p > \tan(\bar{w}_3) \tag{28}$$

$$f_i > 0 \tag{29}$$

$$f_d > \frac{f_i - f_p p_{2,1} + p_{2,1} \tan(\bar{u}_3)}{p_{2,0} - f_p p_{2,0} \cot(\bar{u}_3)} \tag{30}$$

To satisfy (28) to (30) the PID controller parameters are selected to be

$$f_p = \frac{[p_{2,0} - \lambda (1 + \lambda + \lambda^2) \rho^2] \tan(\bar{u}_3)}{p_{2,0}} \tag{31}$$

$$f_i = - \frac{\lambda^3 \rho^3 \tan(\bar{u}_3)}{p_{2,0}} \tag{32}$$

$$f_d = \frac{[p_{2,1} - (1 + \lambda + \lambda^2) \rho] \tan(\bar{u}_3)}{p_{2,0}} \tag{33}$$

where $\lambda > 0$, $\rho > 0$ are free parameters. Thus, (27) takes on the form

$$H_l(s) = \{[(1 + \lambda + \lambda^2)\rho - p_{2,1}]s^2 + [\lambda(1 + \lambda + \lambda^2)\rho^2 - p_{2,0}]s + \lambda^3\rho^3\} / [(s + \rho)(s + \lambda\rho)(s + \lambda^2\rho)] \quad (34)$$

To complete the determination of the outer control loop, it suffices to find appropriate $\lambda > 0$ and $\rho > 0$ such that the gun follows quickly and accurately any change to the distance of the target from the vehicle while simultaneously satisfying actuator constraints.

Step-Wise Supervisor

The above design scheme is based mainly upon adjustments of the distance of the revolute joint from the ground. To handle also the cases where the prismatic joint reaches its upper or lower bound, the following step-wise supervisor is proposed:

Case 1: If the distance of the target from the vehicle increases and the prismatic joint reaches its upper bound then

Step 1.1: lower the prismatic joint,

Step 1.2: increase the angle of the gun,

Step 1.3: reevaluate the linearized model and controller parameters and apply the design scheme presented in sections “Inner Control Loop” and “Outer Control Loop.”

Case 2: If the distance of the target from the vehicle decreases and the prismatic joint reaches its lower bound then

Step 2.1: lift the prismatic join,

Step 2.2: decrease the angle of the gun,

Step 2.3: reevaluate the linear model and controller parameters and apply the design scheme presented in sections “Inner Control Loop” and “Outer Control Loop.”

Simulation Results

The model parameters are considered to be:

$$m_c = 15.2(\text{kg}), m_{l,1} = 0.81(\text{kg}), m_{l,2} = 0.628(\text{kg})$$

$$m_{\text{res}} = 0.177(\text{kg}), k_p = 62,000(\text{N/m}), k_{\text{res}} = 3,490,000(\text{N/m})$$

$$c_p = 2,500(\text{kg/s}), c_{\text{res}} = 81.8(\text{kg/s}), g_{\text{res}} = 0.01833(\text{kg})$$

$$I_2 = 0.00081(\text{kg}\times\text{m}^2), \psi_0 = 0.3(\text{m}), r_0 = 0.02(\text{m})$$

$$h_v = 0.01(\text{m}), g = 9.81(\text{m/sc}^2), \tau = 0.000387(\text{s})$$

$$K_{p,2} = 100, K_{p,3} = 100, K_{d,2} = 10, K_{d,3} = 10$$

$$K_{p,1} = 2, v_n = 1(\text{m/s})$$

The nonzero trim conditions are:

$$\bar{u}_1 = 1(\text{m/s}), \bar{u}_2 = 0.25(\text{m}), \bar{u}_3 = 6.152(\text{rad})$$

$$\bar{x}_1 = t(\text{m}), \bar{x}_2 = 0.2987(\text{m}), \bar{x}_3 = 0.4087(\text{m})$$

$$\bar{x}_4 = 0.25(\text{m}), \bar{x}_5 = 6.152(\text{rad}), \bar{x}_6 = 1(\text{m/s})$$

The inner closed loop I/O transfer matrix parameters are selected to be:

$$p_{1,0} = 10, p_{2,0} = 50, p_{2,1} = 15, p_{3,0} = 50, p_{3,1} = 15$$

With respect to the feedback matrix, we select $\varepsilon = 5e - 5$. Then a set of controller parameters satisfying the inequality in (16) are:

$$k_{1,1} = 0, k_{1,2} = -466.015, k_{1,3} = -2.1622, k_{1,4} = -586.9273$$

$$k_{2,1} = -27.3172, k_{2,2} = -565.5233, k_{2,3} = -30.3138$$

$$k_{3,1} = -4.2357, k_{3,3} = -555.3803$$

With respect to the outer closed loop, it is observed that for the transfer function in (34) to have a maximum rise time of 0.575 s, it suffices to choose $\rho = 3$ and $\lambda = 2$ thus yielding $f_p = 0.407283$, $f_i = 1.15754$, and $f_d = 0.0321539$.

With respect to the gun reaction force (see Fig. 3) it will be assumed that it simulates the reaction force generated from a rifle shot (e.g., [28]). We will further assume that the force pattern is repeated every 2 s. With respect to the road disturbance, it is assumed that it is a random uniform noise in the form presented in Fig. 4. The target distance is considered to be $l^*(t) = (h_v + r_0 + \bar{x}_2 + \bar{x}_4) \tan(\bar{x}_5 - \frac{3\pi}{2}) + \sin(4t)$. The external commands are selected to be $\delta r_1(t) = 0$ and $\delta r_3(t) = 0$. The distances of the aiming point from the vehicle and the target point from the vehicle are presented in Fig. 5. Note that both responses are visually identical. The forward velocity is presented in Fig. 6. It is important to mention that the inclination of the gun is not affected neither by the disturbances nor variations of the other variables. The distance of the center of mass of the gun from the ground is presented in Fig. 7. The rest of the state variables of the system remains within acceptable limits.

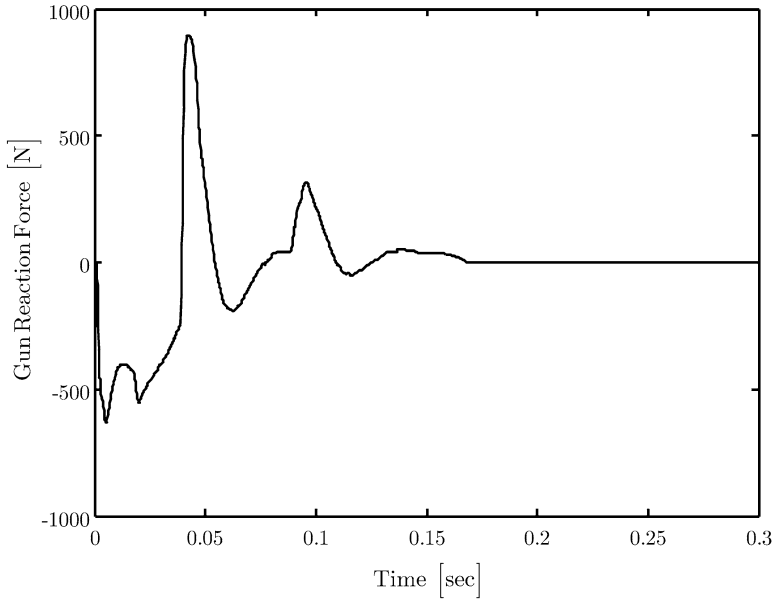


Fig. 3 Gun reaction force

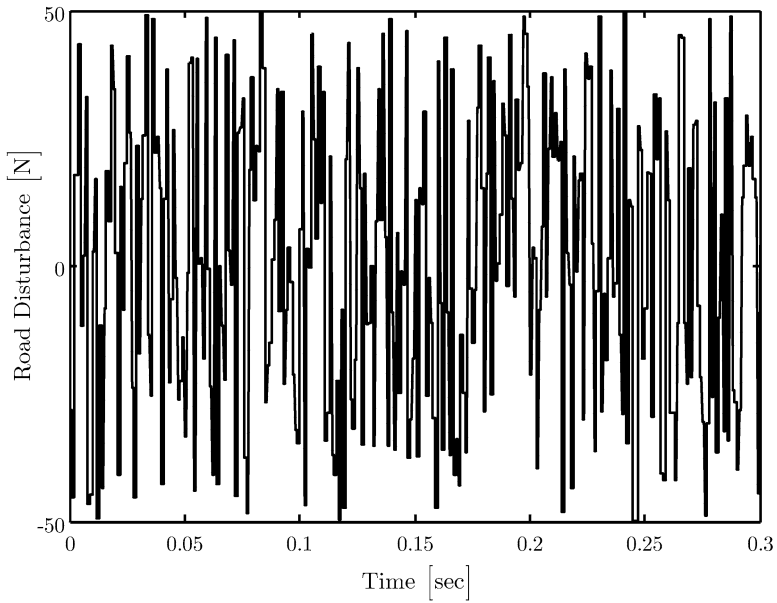


Fig. 4 Road disturbance force

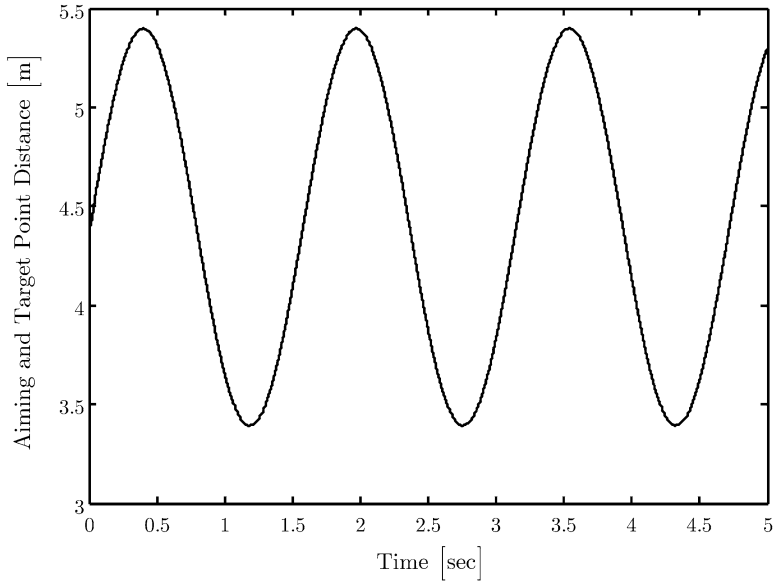


Fig. 5 Distance of the aiming and target points (cont. aiming, dotted target, visually identical)

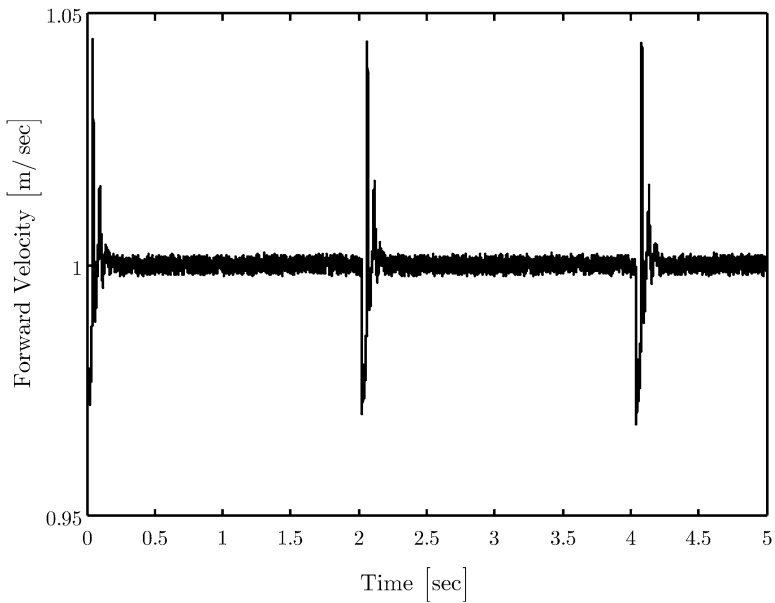


Fig. 6 Forward velocity of the cart

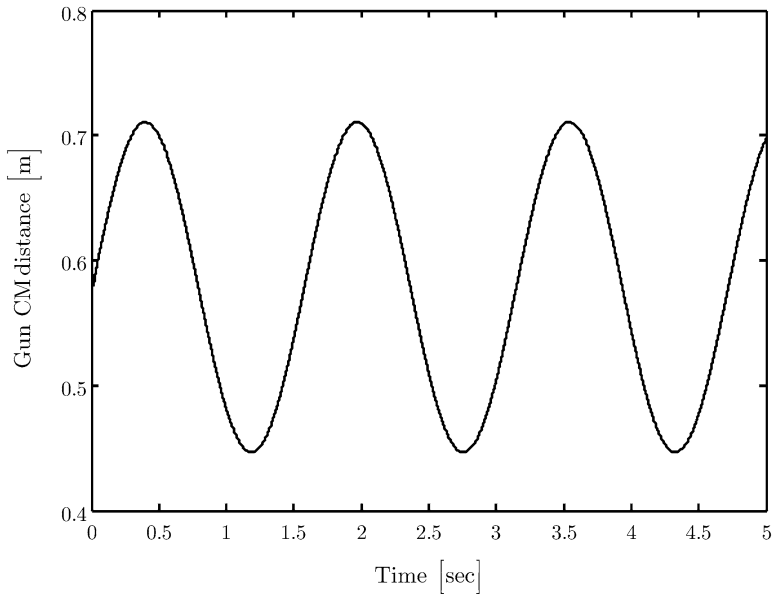


Fig. 7 Distance of the center of mass of the gun from the ground

Conclusions

The mathematical description of a simplified robotic weapon system has been presented. The system comprised of a ground vehicle equipped with a robotic manipulator carrying a gun. A two-stage control scheme towards performing accurate firing while moving and despite vibrations due to uneven ground and reaction weapon forces has been designed. The control scheme has been developed on the basis of the linear approximant of the system. The scheme has been analyzed using an inner and an outer loop. The inner loop controller achieved diagonalization for the transfer matrix mapping the external commands to the performance outputs, as well as a mixed disturbance rejection/disturbance attenuation scheme. The outer loop controller produced an appropriate command for the inner controller in order to follow fast and accurately the moving target. The performance of the proposed control scheme has been demonstrated through simulations. It has been observed that all design requirements have been satisfied. The gun aimed accurately at a moving target despite gun reaction and road unevenness, the influence of the disturbances to the forward velocity of the vehicle was small and the inclination of the gun remained unaffected.

References

1. C. Kim and P. Ro, 'A sliding mode controller for vehicle active suspension systems with non-linearities', *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 212, pp. 79–92, 1998.
2. D. Karnopp, 'Active damping in road vehicle suspension systems', *Vehicle System Dynamics*, vol. 12, pp. 291–311, 1983.
3. I. Fialho and G. J. Balas, 'Road adaptive active suspension design using linear parameter-varying gain-scheduling', *IEEE Transactions on Control Systems Technology*, vol. 10, pp. 43–54, 2002.
4. M. Rao and V. Prahlad, 'A tunable fuzzy logic controller for vehicle-active suspension systems', *Fuzzy sets and systems*, vol. 85, pp. 11–21, 1997.
5. T. Yoshimura, A. Kume, M. Kurimoto, and J. Hino, 'Construction of an active suspension system of a quarter car model using the concept of sliding mode control', *Journal of Sound and Vibration*, vol. 239, pp. 187–199, 2001.
6. F. N. Koumboulis and N. D. Kouvakas, 'A three term controller for ride comfort improvement', *Proceedings of the 19th Mediterranean Conference on Control & Automation (MED 2011)*, June 20–23, 2011, Corfu, Greece.
7. M. P. Tzamtzi, F. N. Koumboulis, N. D. Kouvakas and M. G. Skarpetis, 'Robustness in Liquid Transfer Vehicles with Delayed Resonators', *WSEAS Transactions on Systems*, vol. 7, no. 11, pp. 1359–1370, 2008
8. M. P. Tzamtzi, F. N. Koumboulis, N. D. Kouvakas and G. E. Panagiotakis, 'A Simulated Annealing Controller for Sloshing Suppression in Liquid Transfer with Delayed Resonators', *Proceedings of the 14th IEEE Mediterranean Conference on Control and Automation (MED 2006)*, June 28–30, 2006, Ancona, Italy.
9. M. P. Tzamtzi, F. N. Koumboulis and N. D. Kouvakas, 'A Two Stage Robot Control for Liquid Transfer', *Proceedings of the IEEE Conference on Emerging Technology and Factory Automation (ETFA 2007)*, September 26–28, 2007, Patras, Greece.
10. S. Futami, N. Kyura, and S. Hara, 'Vibration absorption control of industrial robots by acceleration feedback', *IEEE Transactions on Industrial Electronics*, vol. 30, no. 3, pp. 299–305, 1983.
11. S. R. Walker and F. J. Andrews, 'Shock and Vibration Isolators for Military Applications', *Journal of the American Defence Preparedness Association*, March 1986, pp. 2–5.
12. H. Wang, R. Li, H. Xue, H. Pan, 'The vibration identification of military vehicle seat based on wavelet-scale energy coefficients', *Proceedings of the 10th IEEE International Conference on Signal Processing (ICSP 2010)*, October 24–28, 2010, Beijing, China.
13. D. Hrovat, 'Optimal active suspension structures for quarter-car vehicle models', *Automatica*, vol. 26, no. 5, pp. 845–860, 1990
14. A. Hac and I. Youn, 'Optimal semi-active suspension with preview based on a quarter car model', *Proceedings of the American Control Conference*, pp. 433–438, Boston, USA, 26–28 June 1991
15. M. Ahmadian and C.A. Pare, 'A quarter-car experimental analysis of alternative semiactive control methods', *Journal of Intelligent Material Systems and Structures*, vol. 11, no. 8, pp. 604–612, 2000
16. G. Verros, S. Natsiavas and C. Papadimitriou, 'Design optimization of quarter-car models with passive and semi-active suspensions under random road excitation', *Journal of Vibration and Control*, vol. 11, no. 5, pp. 581–606, 2005
17. Y.M. Sam, J.H.S. Osman and M. Ghani, 'A class of proportional-integral sliding mode control with application to active suspension system', *System and Control Letters*, vol. 51, pp. 219–223, 2004
18. J. Trevelyan, 'Robots and landmines', *Industrial Robot: An International Journal*, vol. 24, pp. 114–125, 1997.

19. N. Andersson, C. P. da Sousa, and S. Paredes, 'Social cost of land mines in four countries: Afghanistan, Bosnia, Cambodia, and Mozambique', *BMJ: British Medical Journal*, vol. 311, p. 718, 1995.
20. M. A. Cameron, R. J. Lawson, and B. W. Tomlin, *To walk without fear: the global movement to ban landmines*, Cambridge Univ Press, 1998.
21. F.N. Koumboulis and N.D. Kouvakas, 'A Control Scheme Towards Accurate Firing While Moving for a Mobile Robotic Weapon System with Delayed Resonators', *2nd International Conference in Applications of Mathematics and Informatics in Military Science*, April 11–12, 2013, Vari, Athens, Greece.
22. F. N. Koumboulis, G. E. Panagiotakis and P. N. Paraskevopoulos, 'Exact Model Matching of Left Invertible Neutral Time Delay Systems', *International Journal of Modelling, Identification and Control*, Vol. 3, no. 4, pp. 376–384, 2008 / *Proceedings of the 13th Mediterranean Conf. on Control and Automation (2005 ISIC-MED)*, Limassol, Cyprus, pp. 1548–1555, 2005.
23. F.N. Koumboulis, G.E. Panagiotakis, 'Transfer Matrix Approach to the Triangular Decoupling of General Neutral Multi-Delay Systems', *Proceedings of the 17th IFAC World Congress*, Seoul, Corea, pp. 1279–1286, 2008.
24. F. N. Koumboulis, N. D. Kouvakas, and P. N. and Paraskevopoulos, 'Dynamic Disturbance Rejection Controllers for Neutral Time Delay Systems with Application to a Central Heating System', *Proc. International Conference on Modeling, Identification and Control*, Shanghai, China, June 29 – July 2, 2008 / *Science in China Series F: Information Sciences*, vol. 52, no. 7, pp. 1084–1094, 2009.
25. F.N. Koumboulis and M.P. Tzamtzi, 'A metaheuristic approach for controller design of multivariable processes', *12th IEEE International Conference on Emerging Technology on Factory Automation*, Rio-Patras, Greece, 2007, pp. 1429–143.
26. F. N. Koumboulis, N. D. Kouvakas, M. P. Tzamtzi and A. Stathaki, Metaheuristic control of substrate concentration for an activated sludge process, *International Journal of Modelling, Identification and Control*, vol. 10, no. 1, pp. 117–125, 2010.
27. F.N. Koumboulis and M.P. Tzamtzi, 'A generalised control metaheuristic framework for industrial processes', *International Journal of Automation and Control*, vol. 4, no. 2, pp. 218–233, 2010.
28. M. Pedzisz and M. Wojtyra, 'Dynamical Analysis of Semi-Automatic Sniper Rifle', *16th European ADAMS User Conference*, November 14–15, 2001, Berchtesgaden, Germany.