Chapter 85
Simple Stabilization Design for Perturbed Time-Delay Systems

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Abstract This paper addresses the stabilization design problem for the continuous perturbed systems subjected to a time delay. By using the Riccati equation approach associated with the upper bound of the solution of the Riccati equation, a new stabilizability criterion is proposed. This criterion is easy to be tested. According to the above criterion, a simple stabilization controller is developed. This controller is very simple and hence is easy to be implemented.

Keywords Stabilization · Time-delay · Perturbation · The Riccati equation · Upper solution bound

85.1 Introduction

It is known that time delay exist naturally in physical systems, engineering systems, and so on. Time delay can be considered as a of instability source of systems. On the other hand, perturbation is also a source of instability and must be integrated into system model. The control problem of systems with time delay(s) and/or perturbations then is complicate and hence has become an attractive research topic over past several decades. A number of research approaches have been proposed to solve control problems of systems with time delay(s) and/or perturbation(s) during the past decades [1–10]. In [2, 3, 5–10], stabilizability conditions have been developed and various feedback controllers have also been derived. It is

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seen that the proposed results often come to solving LMI. Since there are usually many free matrices in LMI, this might be a miscellaneous work. Therefore, to develop a simple stabilizability condition and a state feedback controller for perturbed time-delay systems is the objective of this paper. We first derive a simple upper matrix bound of the solution of the Riccati equation by choosing properly the positive definite matrix Q. Then, by using the Riccati equation approach associated with the proposed upper bound, a concise stabilizability criterion is presented. This criterion does not involve any Riccati equation and hence is easy to be tested. Furthermore, according to the obtained criterion, a simple stabilization controller is developed. This controller is very simple and hence is easy to be implemented. An algorithm is also proposed to construct the controller.

The following symbol conventions are used in this paper. Symbol $\mathbb{R}$ denotes the real number field. $A >\geq B$ means matrix $A - B$ is positive (semi)definite; $\lambda_1(A)$ denote the maximal eigenvalue of a symmetric matrix A. $\|A\|$ is the norm of matrix A. Furthermore, the identity matrix with appropriate dimensions is represented by I.

85.2 Main Results

Consider the time-delay systems with nonlinear perturbations

$$\dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) + f(x(t),t) + f_d(x(t-d),t) \tag{85.1}$$

where, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $d > 0$, respectively, denote the state, the input to be designed, and the delay duration, A, A_d, and B represent constant matrices with appropriate dimensions and A is a stable matrix, and $f(x(t), t)$ and $f_d(x(t-d), t)$ are nonlinear perturbations with the following properties:

$$\|f(x(t),t)\| \leq \delta\|x(t)\| \quad \text{and} \quad \|f_d(x(t-d),t)\| \leq \delta_d\|x(t-d)\| \tag{85.2}$$

where δ and δ_d are positive constants. It is assumed that the pair (A, B) is completely controllable. The objective of this paper is to derive simple stabilizability conditions and design a memoryless state feedback controller in the form of

$$u(t) = -Fx(t) \tag{85.3}$$

where F is the gain matrix such that the resulting closed-loop system is stable.

Before developing the main results, we first give the following useful result.

Lemma 1 *If there exists a positive constant η such that*

$$A^T + A - 2\eta BB^T + 2\left(\delta + \delta_d + \frac{\|A_d\|}{2}\right)I + \frac{A_d^T A_d}{\|A_d\|} < 0 \tag{85.4}$$

then the positive solution P of the Riccati equation

$$A^TP + PA - PBB^TP = -Q \tag{85.5}$$

has the upper bound

$$P < qI \tag{85.6}$$

where the $n \times n$ *real positive definite matrix* Q *is given as*

$$Q = q\left[(2\delta + \|A_d\| + 2\delta_d)I + \frac{A_d^TA_d}{\|A_d\|}\right] \tag{85.7}$$

and q *is a arbitrary positive constant.*

Proof Let a positive semi-definite matrix R be defined by

$$R \equiv (P - \eta I)BB^T(P - \eta I). \tag{85.8}$$

Then, we can rewrite the Riccati equation (85.5) as

$$\begin{aligned}&\left(A - \eta BB^T\right)^T(qI - P) + (qI - P)\left(A - \eta BB^T\right)\\ &\quad = -R + Q + \eta^2BB^T + q\left(A^T + A - 2\eta BB^T\right).\end{aligned} \tag{85.9}$$

In (85.9), we have

$$\begin{aligned}&Q + \eta^2BB^T + q\left(A^T + A - 2\eta BB^T\right)\\ &\quad = q\left[A^T + A - 2\eta BB^T + 2\left(\delta + \delta_d + \frac{\|A_d\|}{2}\right)I + \frac{A_d^TA_d}{\|A_d\|} + \frac{\eta^2}{q}BB^T\right].\end{aligned} \tag{85.10}$$

It is obvious that if the condition (85.4) is satisfied, then there must exist a constant $q \gg \eta^2\|B\|^2$ such that the right-hand side of (85.10) is a negative definite matrix. Furthermore, the condition (85.4) also implies that $A^T + A - 2\eta BB^T < 0$, one hence can conclude that the matrix $A - \eta BB^T$ is stable. Therefore, Eq. (85.9) is a Lyapunov equation and then its solution is positive definite. That is, the solution of the Riccati equation (85.5) has the upper bound (85.6). Thus, this completes the proof.

Then, by utilizing lemma 1 and some linear algebraic techniques, a stabilization controller for the system (85.1) is designed as follows.

Theorem 1 *If the stabilizability condition* (85.4) *holds, the perturbed time-delay system* (85.1) *can be stabilized by a memoryless state feedback controller in the form of* (85.3) *with*

$$F = 0.5B^TP \tag{85.11}$$

where the positive definite matrix P *satisfies the Riccati equation* (85.5) *and the positive definite matrix* Q *is given by* (85.7).

Proof Using the controller (85.3) with (85.11), the system (85.1) becomes

$$\dot{x}(t) = (A - 0.5BB^T P)x(t) + A_d x(t-d) + f(x(t), t) + f_d(x(t-d), t). \quad (85.12)$$

For this system, we construct a Lyapunov function as

$$V(x(t), t) = x^T(t)Px(t) + q\int_{t-d}^{t} x^T(\tau)\left(\frac{A_d^T A_d}{\|A_d\|} + \delta_d I\right)x(\tau)d\tau \quad (85.13)$$

where the positive definite matrix P satisfies (85.5). For convenience, we use symbols V, x, and x_d to replace $V(x(t), t)$, $x(t)$, and $x(t-d)$, respectively, in the following and later descriptions. Furthermore, $f(x(t), t)$ and $f_d(x(t-d), t)$ are also replaced by f and f_d, respectively. Now, taking the derivative along the trajectories of (85.1) gives

$$\begin{aligned}\dot{V} = x^T&\left[A^T P + PA - PBB^T P + q\left(\frac{A_d^T A_d}{\|A_d\|} + \delta_d I\right)\right]x - qx_d^T\left(\frac{A_d^T A_d}{\|A_d\|} + \delta_d I\right)x_d \\ &+ x_d^T A_d^T Px + x^T PA_d x_d + f^T Px + x^T Pf + f_d^T Px + x^T Pf_d.\end{aligned} \quad (85.14)$$

Since

$$\begin{aligned}x_d^T A_d^T Px + x^T PA_d x_d &\le \frac{1}{\|A_d\|}x_d^T A_d^T PA_d x_d + \|A_d\|x^T Px \\ &< q\left[\frac{1}{\|A_d\|}x_d^T A_d^T A_d x_d + \|A_d\|Ix^T x\right],\end{aligned} \quad (85.15)$$

$$f^T Px + x^T Pf \le \delta x^T Px + \frac{1}{\delta}f^T Pf < q\left[\delta x^T x + \frac{1}{\delta}f^T f\right] \le q2\delta x^T x, \quad (85.16)$$

and

$$\begin{aligned}f_d^T Px + x^T Pf_d &\le \frac{1}{\delta_d}f_d^T Pf_d + \delta_d x^T Px \\ &< q\left[\frac{1}{\delta_d}f_d^T f_d + \delta_d x^T x\right] \le q\delta_d\left[x_d^T x_d + x^T x\right],\end{aligned} \quad (85.17)$$

then

$$\begin{aligned}\dot{V} &< x^T\left[-Q + q\left(2\delta I + \|A_d\|I + \frac{A_d^T A_d}{\|A_d\|} + 2\delta_d I\right)\right]x \\ &= qx^T\left[-(2\delta + \|A_d\| + 2\delta_d)I - \frac{A_d^T A_d}{\|A_d\|} + 2\delta I + \|A_d\|I + \frac{A_d^T A_d}{\|A_d\|} + 2\delta_d I\right]x = 0\end{aligned} \quad (85.18)$$

where the upper bound (85.6) is used. Therefore, it is seen that if the condition (85.4) is satisfied, then the resulting closed-loop system (85.12) is asymptotically stable. Thus, the proof is completed.

Remark 1 An interesting consequence of this theorem is that the stabilizability condition (85.4) is independent of the Riccati equation (85.5). Furthermore, it is also independent of the free variable q.

Remark 2 Another benefit of the upper bound (85.6) is that we can use the bound qI to replace P in the memoryless feedback controller (85.3) to simplify the controller design. The result is given as follows.

Theorem 2 *If the stabilizability condition* (85.4) *is met, then the perturbed time-delay system* (85.1) *can be stabilized by making use of the feedback controller*

$$u(t) = -0.5B^T qIx(t) = -\eta B^T x(t) \tag{85.19}$$

where the positive constant η *defined by* $\eta \equiv 0.5q$ *is chosen by the designer.*

Proof From (85.19), the closed-loop system now becomes

$$\dot{x}(t) = (A - \eta BB^T)x(t) + A_d x(t-d) + f(x(t), t) + f_d(x(t-d), t). \tag{85.20}$$

Here, we choose the Lyapunov function as

$$V = x^T x + \int_{t-d}^{t} x^T(\tau)\left(\frac{A_d^T A_d}{\|A_d\|} + \delta_d I\right)x(\tau)d\tau. \tag{85.21}$$

This can lead to

$$\begin{aligned}\dot{V} &= x^T\left[A^T + A - 2\eta BB^T + \frac{A_d^T A_d}{\|A_d\|} + \delta_d I\right]x - x_d^T\left(\frac{A_d^T A_d}{\|A_d\|} + \delta_d I\right)x_d \\ &\quad + x_d^T A_d^T x + x^T A_d x_d + f^T x + x^T f + f_d^T x + x^T f_d \\ &\le x^T\left[A^T + A - 2\eta BB^T + 2\left(\delta + \delta_d + \frac{\|A_d\|}{2}\right)I + \frac{A_d^T A_d}{\|A_d\|}\right]x < 0.\end{aligned} \tag{85.22}$$

Therefore, it is seen that if the condition (85.4) holds, then the perturbed time-delay system (85.1) can be indeed stabilized by the controller (85.19). Thus, the proof is completed.

Note that the stabilization controller (85.19) is very simple. We also give the following algorithm for designing the positive constant η.

Algorithm 1

Step 1. Set $k = 0$. Give an initial value of $\eta_k = 0$.

Step 2. Substitute η_k into the stabilizability condition (85.4) and check it. If it is satisfied, then stop the algorithm and the controller is obtained. Otherwise, go to Step 3.

Step 3. Set

$$\eta_{k+1} = \eta_k + \varepsilon$$

where ε is an adequate positive constant. If $\eta_{k+1} > w$, then stop this algorithm and the stabilization controller can not be found where w is a default large value. Otherwise, go to Step 2.

85.3 Conclusions

The stabilization design problem of the continuous perturbed systems subjected to a time delay has been solved. A new stabilizability criterion is proposed to guarantee the existence of stabilization controller. This criterion does not involve any Riccati equation and hence is easy to be tested. Furthermore, a simple stabilization controller that is independent of the Riccati equation has also been developed. By the proposed algorithm, it is seen that this controller is easy to be implemented.

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