Chapter 25
Damage Detection Based on Wavelet Packet Transform and Information Entropy

S.A. Ravanfar, H. Abdul Razak, Z. Ismail, and S.J.S. Hakim

Abstract In this study, a new approach for damage detection in beam-like structures is presented. Damage feature such as relative wavelet packet entropy (RWPE) based on the decomposed component is used to identify the damage location and evaluate the damage severity. RWPE describes present information of relative wavelet energies correlated with different frequency ranges. It is should be noted that the acceleration measured in the all three directions should be used in computation of RWPE. Numerical and experimental results are used to verify the practicality of this method. Results show that the damage index technique has great potential in detection of location and depth of cut in a steel beam. The procedure can be applied for health monitoring of other complex structures.

Keywords Damage detection • Wavelet packet transform • Information entropy • Wavelet packet entropy • Beams

25.1 Introduction

To ensure structural integrity, safety and minimal maintenance, structural health monitoring (SHM) is a powerful tool to assess performance, by identifying and quantifying damage based on measurement by sensors and data analysis [1–4]. Vibration-based analysis has been reported as a promising method for SHM [5]. The premise of vibration-based SHM is that dynamic characteristics of a structure are a function of its physical properties. Therefore, changes in these physical properties, such as reduction in stiffness resulting from the localized structural damage will cause observable changes in the dynamic characteristics of the structure. The vibration-based damage detection methods have received great consideration in the past few decades, and several approaches have been suggested [6].

One subject matter of vibration-based damage detection techniques is to seek some damage features that are sensitive to damage of structure [7, 8]. The damage features that have been indicted with various degrees of achievement include mode shapes, natural frequencies, modal flexibility, mode shape curvatures, modal strain energy, etc. Doebling et al. [9] summarized the extensive development of damage evaluation methodologies based on these features as well as indicating their applicability and limitations. Most vibration-based structural damage detection techniques require the modal properties that are extracted from measured signals through system identification techniques. It is realistic to extract structural damage features directly from the measured vibration signals.

Recently, wavelet analysis has become a promising damage detection tool in the area of structural and machine health monitoring because of its shows potential features such as singularity detection, good handling of noisy data and being very informative about damage location/time and extent. For this reason, many damage detection studies focused on the wavelet transform (WT) scheme. Wavelet functions are included in the family of basis functions that are capable of depicting a

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signal in a localized frequency (or scale) and time (or space) domain. The main advantage obtained by using wavelets is the capability to execute local analysis of a signal, i.e. zooming on any interval of space or time. Wavelet analysis is capable of demonstrating some hidden features of the data that conventional Fourier analysis fails to detect. This characteristic is especially important for damage identification applications. An extensive literature survey on the subject is presented in the work of Kim and Melhem [10], with particular applications including damage detection of beam and mechanical gear and roller damage detection.

The wavelet packet transform (WPT) is a development of the WT, for recording the characteristic under every frequency band. In WT, a signal is split into an approximation and a detail. The approximation is then split into a second-level approximation and detail, and the process is repeated. In WPT, the details as well as the approximations are split, which provide complete decomposition of a signal. The WPT creates the same frequency bandwidths in every resolution. The wavelet packets are other bases formed by linear combinations of the common wavelet functions [11]. Sun and Chang [12] and Han et al. [13] have improved WPT-based component energy as damage sensitivity index for diagnosis of structural damage in a beam structure.

Damage will give arise to irregularity in the structural response signal. The entropy is a quantitative measure of the degree of disorder in measured signals. The wavelet entropy, which is a combination of entropy and wavelet, could take advantage of both methods to explain the characteristics of a signal, which are not directly visible in original space. In particular, Ren and Sun [14] proposed a damage-sensitive feature with the combination of discrete wavelet transform and information entropy to characterize the level of disorder in the measured signals to identify the occurrence and location of damage in beam structures.

This present study combines multi-resolution wavelet packet transform with information entropy to achieve higher as well as lower frequency signal components accuracy. Damage feature such as RWPE based on the decomposed component are used to identify the damage location and evaluate the damage severity through the vibration signals. RWPE is described to present information of relative wavelet energies correlated with different frequency ranges. Both numerically simulated and experimental data with different damage scenarios show that the proposed method has great potential in the field of damage detection of beam-like structures.

25.2 Damage Feature Extraction from Wavelet Packet Entropy

Wavelet packet transform (WPT) could be considered as an extension of the WT which provide a complete level-by-level time-frequency decomposition. In addition, it can give a rich structure to adapt a particular signal. WPT also enables multi resolution damage detection since it can localize multi-frequency bands in time domain. The wavelet packet function is defined as

$$\psi_{\mathrm{j,k}}^{\mathrm{i}}(\mathrm{t}) = 2^{j/2}\psi^{i}\left(2^{j}t - k\right) \quad i = 0, 1, 2, \ldots, 2^{j} - 1 \tag{25.1}$$

where a wavelet packet $\psi^{i}_{j,k}(\mathrm{t})$ is a function of three indices which integers i, j and k are the modulation, the scale and the translation parameter respectively. Moreover, $\psi^{0}(t) = \varphi(t)$ for $i = 0$ and $\psi^{1}(t) = \psi(t)$ for $i = 1$. The wavelet $\varphi(t)$ is called the scaling function and $\psi^{1}(t)$ called the mother wavelet function.

In this study, the measured structural dynamic response is decomposed into wavelet component functions. When the decomposition level is j, 2^{j} wavelet packet decomposed (WPD) components can be obtained. Figure 25.1 shows the decomposition process of a time-domain signal $f(\mathrm{t})$ up to the 3rd level. The original signal can be expressed as a summation of WPD components as,

$$f(t) = \sum_{i=1}^{2^{j}} f_{j}^{i}(t) \tag{25.2}$$

where t is time lag; $f_{j}^{i}(t)$ is the WPD component signal that can be represented by a linear combination of wavelet packet functions as follows:

$$f_{j}^{i}(t) = \sum_{k=-\infty}^{\infty} C_{j,k}^{i}\psi_{j,k}^{i}(t) \tag{25.3}$$

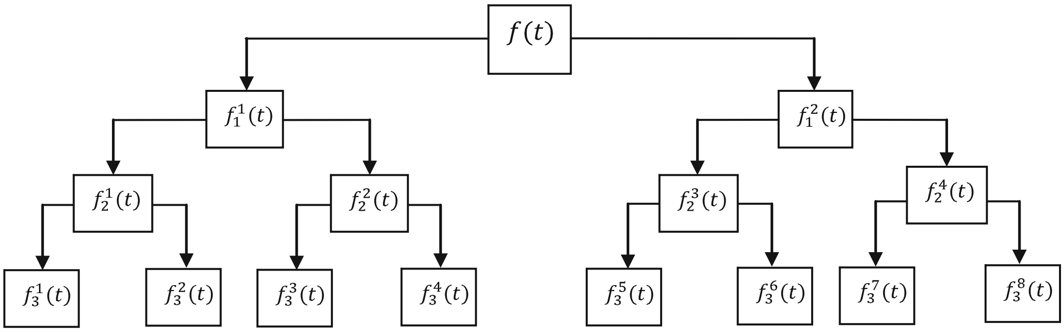

Fig. 25.1 Wavelet packet decomposition process

where $C_{j,k}^i$ are wavelet packet coefficient and can be calculated from

$$C_{j,k}^i = \int_{-\infty}^{\infty} f(t)\psi_{j,k}^i(t)dt \tag{25.4}$$

For WPT offers good time resolution in the high-frequency range of a signal and good frequency resolution in the low-frequency range of the signal. For application of WPT in SHM, there is no specific recommendation about the decomposition level. The level of wavelet packet decomposition is generally determined by trial and error sensitivity analysis, through which the decomposition level 6 is used in this study.

25.2.1 Wavelet Energy and Entropy

The wavelet packet component energy is a suitable tool to identify and characterize a specific phenomenon of signal in time-frequency domain. It has been shown in Yen and Lin's [11] study that the energy stored in a specific frequency band at a certain level of wavelet packet decomposition provides a more potential for signal feature than the coefficients alone. Sun and Chang [15] compared the sensitivity of four damage indices based on frequency change, mode shape change, flexibility change and wavelet packet energy change, and concluded that wavelet packet energy based index has the best ability to capture structural stiffness reduction. Ren et al. [16] has explored the practical application of wavelet packet energy change based damage detection method to the bridge shear connector monitoring. The wavelet packet energy E_f of a signal is defined as

$$E_f = \int_{-\infty}^{\infty} f^2(t)dt = \sum_{m=1}^{2^j}\sum_{n=1}^{2^j}\int_{-\infty}^{\infty} f_j^m(t) f_j^n(t)dt \tag{25.5}$$

where f_j^m and f_j^n are decomposed wavelet components. The total energy of signal can be extracted as the sum of wavelet packet components energies:

$$E_f = \sum_i^{2^j} E_{f_j^i} = \sum_{i=1}^{2^j}\int_{-\infty}^{\infty} f_j^i(t)^2 dt \tag{25.6}$$

Then, the normalized values for the j-th scale, which represents the relative wavelet packet energy is

$$p_{ij} = \frac{E_{f_j^i}}{E_f} \tag{25.7}$$

The p_{ij} values correspond to a ratio of the energy of a particular coefficient $E_{f_j^i}$ to the total energy. The p_{ij} value acts like a probability distribution of the energy. Therefore, the p_{ij} values sum to one.

The entropy of relative wavelet packet energy can characterize the level of order and disorder of a vibration signal, therefore it can supply valuable information about the dynamic process correlated with measured vibration signals. Ren and Sun [14] applied the concept of the wavelet entropy to structural damage detection problems. According to the Shannon entropy theory and wavelet energy ratio defined above, wavelet packet entropy is defined as

$$S_{WPE} = S_{WPE}(p) = -\sum\nolimits_j \sum\nolimits_i p_{ij}.lnp_{ij} \tag{25.8}$$

Damage to a structure causes a change in the entropy of wavelet. The damage detection problem can be formulated through the changes in the wavelet packet entropy before and after damage to characterize the location and quantification of damage. To identify the change in wavelet packet entropy of vibration signals from a structure, relative wavelet packet entropy (RWPE) is introduced as follows:

$$S^k_{RWPE}\left(p^k \middle| q^k\right) = \sum\nolimits_j \sum\nolimits_i \left| p^k_{ij} \ln\left(\frac{p^k_{ij}}{q^k_{ij}}\right)\right| \quad k = x, y, z \tag{25.9}$$

The RWPE value will be zero while the relative wavelet energy ratios p_{ij} and q_{ij} are identical. It is important that the measured accelerations in the same direction must be used in computations of RWPE. However, when the structure is damaged the values of p_{ij} and q_{ij} will become different, and as a consequence an increase in RWPE value.

25.3 Numerical Simulation

To verify the suitability of proposed damage identification method, numerical simulations were considered on the three I-section steel beams with a span length of 3 m and with several assumed damage elements, as shown in Fig. 25.2. A commercial finite element analysis package is employed to perform a modal dynamic analysis to provide the time history response of the beams.

Beam 0 is considered as the reference beam without damage while Beam 1 is the single damage scenario at the middle of the beam. Beam 2 is the single damage scenario with damage located at point 5. Dimension of all damages is 3 mm width with 3 mm depth. There was a total of 25 damages, and there was gradual increase of the damage depth of 3 mm for all beams at each step up to 75 mm.

A three dimensional solid element was used in the finite element modeling of beam geometry having Young's modulus of 210 G Pa and density of 7,850 kg/m^3 for the material properties. The analysis was carried out for different levels of damage severity as depicted in Fig. 25.2. The node acceleration responses of the beams under vibration test were obtained on sixteen locations on the top flange at the sampling frequency of 2,000 Hz and to find out the characteristics of the notch.

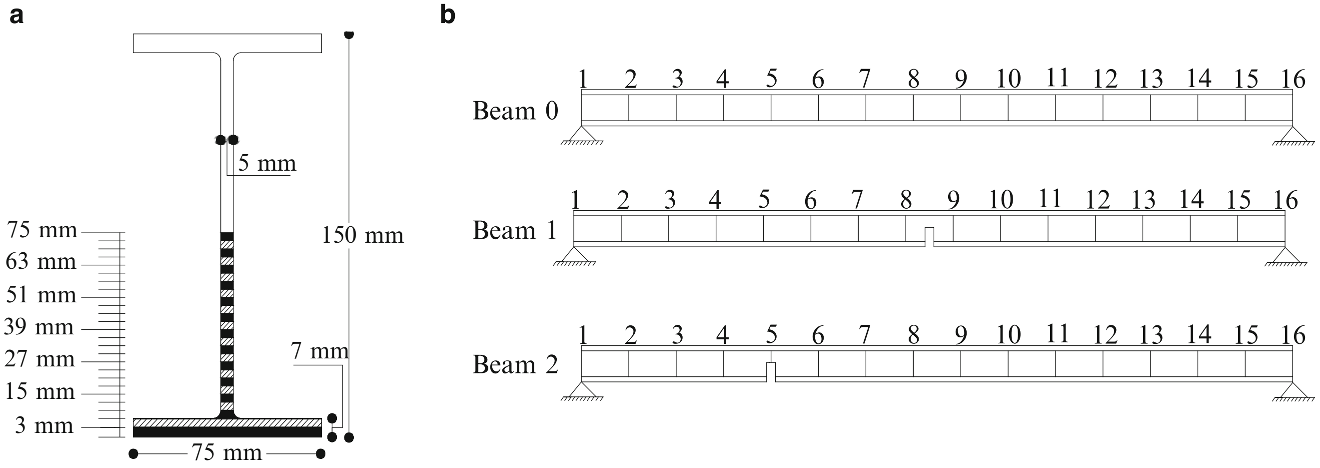

Fig. 25.2 I-beam damage scenario: (**a**) dimension and damage depth of beams; (**b**) damage location of beams

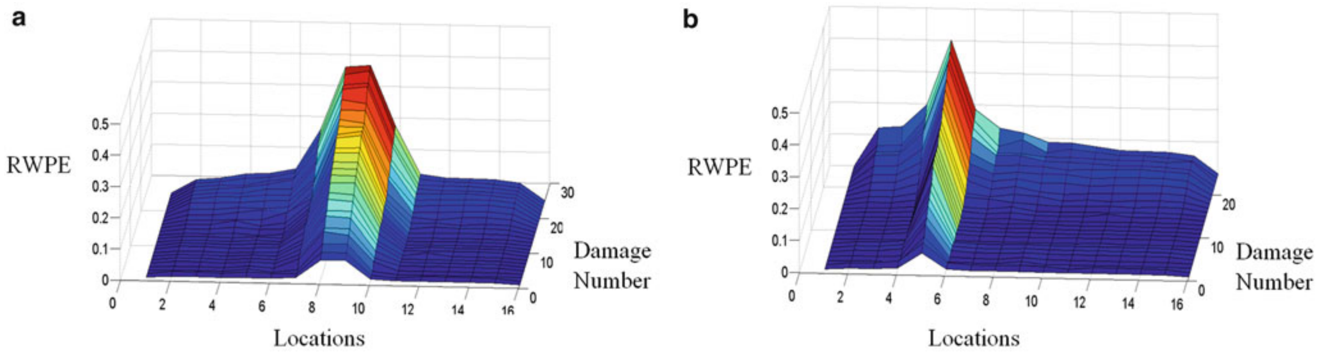

Fig. 25.3 Finite element model of the beams: (**a**) RWPE for damaged beam 1; (**b**) RWPE for damaged beam 2

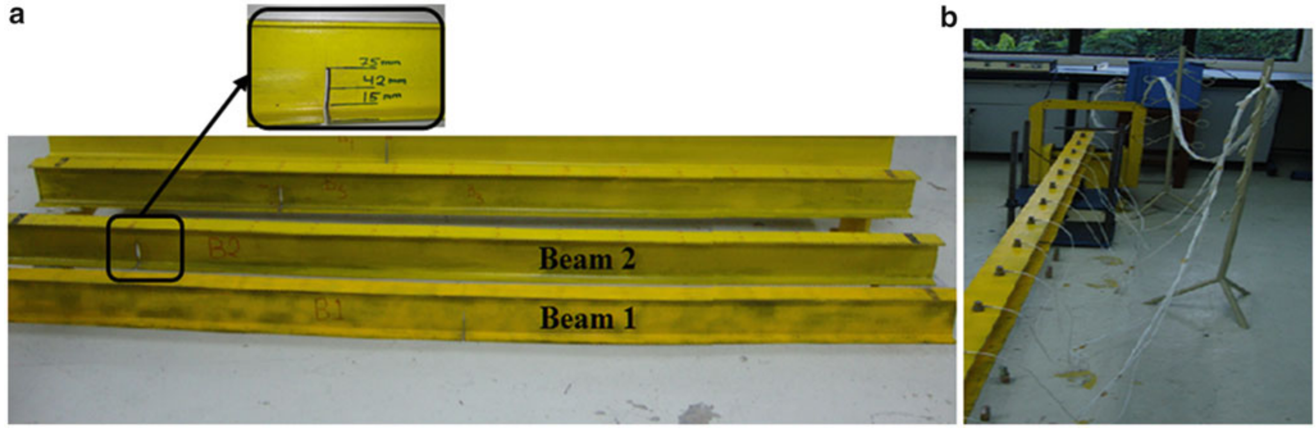

Fig. 25.4 Dynamic test in laboratory: (**a**) beams tested; (**b**) data acquisition

25.3.1 Damage Identification

Decomposition of wavelet packet resulted in 128 components by setting the decomposition level to 7 for the measured data obtained from the vibration tests. The RWPE at 16 locations calculated for each damaged scenario based on Eq. (25.9) gave the corresponding 3D contour plots of the RWPE of the single and multiple damage scenarios as shown in Fig. 25.3. According to these figures the value and distribution of RWPEs changed considerably after damage, which can be used as damage indicators.

Single damage scenarios indicate that the peak value of the RWPE in beam 1 occurred at the middle of the beam which is marginally greater than beam 2, which occurred at point 5. From the above observation, it may be construed that increase of damage depth in beams will influence the vibration response signal and subsequently RWPE values.

25.4 Experimental Verification

An experimental study on a test beam was conducted to validate the proposed procedure. The proposed damage identification technique has to be validated using real measurement data from vibration tests in the presence of measurement errors and noise. Vibration tests were carried out the noise and measurement error are present, as depicted in Fig. 25.4, under undamaged and several damage states.

Excitation is provided by a shaker at location 13. Sixteen accelerometers are placed along the beam to measure the time acceleration responses with a sampling frequency of 2,000 Hz. Figure 25.4 shows a typical acceleration response under white noise excitation. The level of decomposition is selected to be 7 resulting in a total of 128 component signals.

RWPE of acceleration responses of beam 1 and beam 2 are depicted in Fig. 25.5 demonstrates that the energy of structural vibration is increased with the depth of damage. Accordingly, the RWPE of every damage scenario indicates significant

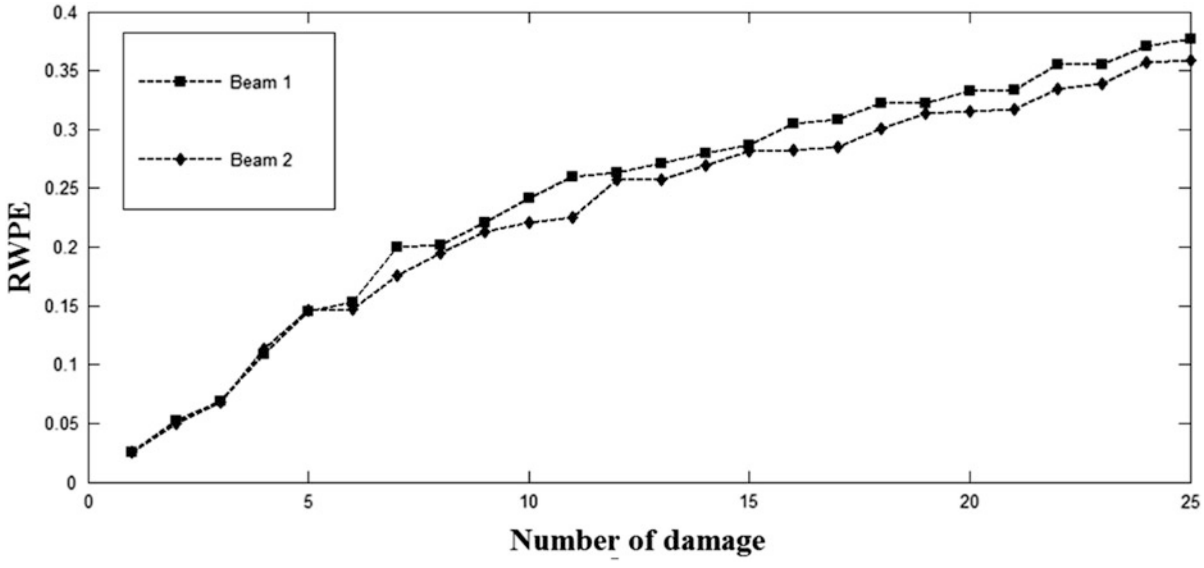

Fig. 25.5 RWPE for different damage depths in beam 1 and beam 2

difference of the signal distribution energy. Consequently, it is possible to identify the occurrence of structural damage based on the variation of RWPEs calculated from vibration signals of various damage scenarios. However, the extent of damage are recognized using RWPE.

All results demonstrated that the locations for all damage scenario as well as quantification of the extent of the damage can be successfully identified from the dynamic measurements.

25.5 Conclusion

In this paper, a new index RWPE based on the information entropy and wavelet packet energy through the vibration responses are proposed to identify the existence and quantify the extent of structural damage. To verify the viability and efficiency of the suggested method, both numerical simulation and experimental tests were carried out. It was shown that the proposed methods are sensitive to identify and also to quantify the damage using the vibration signals. The verification utilizes real measurement data when the signals are affected by experimental noise and measurement errors. Therefore, it can be deduced that the proposed indices are partially insensitive to noise of data collected.

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