Chapter 5
Prediction of Forced Response on Ancillary Subsystem Components Attached to Reduced Linear Systems

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Abstract Multi-component structural systems are commonly used in the modeling of dynamic systems. In order to simplify such complex mathematical models, peripheral/ancillary components are often times grouped as larger substructures of the total assembly. The dynamic response of the structural system will have the embedded characteristics of the appended ancillary components but the fidelity of the model will be highly dependent on the quality and resolution of the model. In particular, sufficient substructure information is needed for an accurate prediction of the response of the appendage and/or its coupling structure. This implies that proper characterization of the structure may require measurements at the subcomponent level or in the absence of sufficient data, a large and detailed finite element model.

In this work, analytical models of a multi-component beam system were created to investigate the prediction of the dynamic response of ancillary subcomponents. The ancillary structure will be assumed to be dynamically active but inaccessible/immeasurable. The models will be created first at full space as a reference and then reduction techniques will be used to determine the necessary information in order to accurately predict the force or displacement imparted to the appendages. The dynamic characteristics of the ancillary component will be extracted using the subcomponent information available from the system.

Keywords Forced linear response • Reduced order modeling

Nomenclature

Symbols

$\{X_n\}$	Full set displacement vector
$\{X_a\}$	Reduced set displacement vector
$\{X_d\}$	Deleted set displacement vector
$[M_a]$	Reduced mass matrix
$[M_n]$	Expanded mass matrix
$[K_a]$	Reduced stiffness matrix
$[K_n]$	Expanded stiffness matrix
$[U_a]$	Reduced set shape matrix
$[U_n]$	Full set shape matrix
$[U_a]^g$	Generalized inverse
$[T]$	Transformation matrix
$[T_U]$	SEREP transformation matrix
$\{p\}$	Modal displacement vector
$[M]$	Physical mass matrix

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[C] Physical damping matrix
[K] Physical stiffness matrix
{F} Physical force vector
$\{\ddot{x}\}$ Physical acceleration vector
$\{\dot{x}\}$ Physical velocity vector
{x} Physical displacement vector
α Parameter for Newmark integration
β Parameter for Newmark integration
Δt Time step
$[U_{12}]$ Mode contribution matrix

Acronyms

ADOF Reduced degrees of freedom
DOF Degrees of freedom
ERMT Equivalent reduced model technique
FEM Finite element model
MAC Modal assurance criterion
NDOF Full space degrees of freedom
SEREP System equivalent reduction expansion process
TRAC Time response assurance criterion

5.1 Introduction

During the analysis of complex structural models, these may be decomposed into simpler systems/components (or assemblies) that in turn can be further broken down to the subcomponent (or subsystem) level depending on the desired degree of resolution of the model. These subcomponents can be considered simple appendages or ancillary subcomponents of the assemblies. While the contribution to the dynamic characteristics of the system from the subsystem ancillary components can seem small, the accuracy of the prediction of the system level response may be compromised if a sufficiently detailed model is not used. In particular, the fidelity of the model can be drastically affected if these subcomponents are dynamically active and if the interaction with the coupling structure is nonlinear in nature. Therefore, there is significant motivation in developing a methodology for determining necessary model information for the accurate calculation of subsystem component response.

Recent developments in the computation of reduced order model response have allowed for the accurate calculation of system's time response while retaining all the highly refined and complex characteristics of full finite element models. Work by Thibault [1] and Marinone [2] showed that system level response can be accurately and efficiently calculated for highly reduced system models. Moreover, Pingle and Avitabile [3, 4, 5] demonstrated that the expansion of such systems can be used for the prediction of full field results such as stress and strain. The advantages of using reduced order models can be seen from a substantial reduction in computation time even when such systems involve nonlinear effects. Using these new efficient methodologies, this work aims to extend the application of reduced linear system modeling to the case in which the goal is not only the prediction of the dynamics of the full system but also the characterization of subsystem ancillary components.

In this paper, the common case of multi-component structural systems is addressed in the context of retaining embedded structural information of ancillary subcomponents for the calculation of reduced order model time response. A full space finite element model consisting of two systems, one of which contains a dynamically active ancillary subcomponent, will be reduced to a smaller set of degrees freedom and used for the prediction of the forced time response of the system. The reduced order model (with embedded ancillary subcomponent information) will then be used to expand back to the full space finite element model and to extract the predicted forced response of the ancillary subcomponent. This study will show advantages and drawbacks of common reduction/expansion methodologies (such as SEREP and Guyan reduction) in the characterization of the subsystem component from available reduced system information. Moreover, the selection of degrees of freedom during the reduction process will explore whether it is necessary to include the connecting degrees of freedom of the ancillary component and the larger coupling structure or if these can be omitted as long as the preserved modes of the reduced system span the space of all modes of interest of the system. This is of particular importance in real life experimentation, as often times, measurements cannot be made exactly at the connecting degrees of freedom ofmulti-component structures or highly detailed finite element models are necessary to approximate the behavior of the system at those locations.

5.2 Theory

The fundamentals of the study of the forced response of reduced linear systems spans a variety of theoretical topics briefly presented here. The summary starts with a description of linear multiple degree of freedom systems and continues with an overview of structural dynamic modification, analytical model reduction and expansion, model updating and forced time response computations. Further information can be found in the respective references.

5.2.1 Equations of Motion for Multiple Degree of Freedom System

The general equation of motion for a multiple degree of freedom system written in matrix form is

$$[M_1]\{\ddot{x}\} + [C_1]\{\dot{x}\} + [K_1]\{x\} = \{F(t)\} \tag{5.1}$$

Assuming proportional damping, the eigensolution is obtained from

$$[[K_1] - \lambda [M_1]]\{x\} = \{0\} \tag{5.2}$$

The eigensolution yields the eigenvalues (natural frequencies) and eigenvectors (mode shapes) of the system. The eigenvectors are arranged in column fashion to form the modal matrix $[U_1]$. Often times, only a subset of modes is included in the modal matrix to save on computation time. Exclusion of modes results in truncation error which can be serious if key modes are excluded. Truncation error will be discussed in further detail in the structural dynamic modification section.

The physical system can be transformed to modal space using the modal matrix as

$$[U_1]^T [M_1] [U_1] \{\ddot{p}_1\} + [U_1]^T [K_1] [U_1] \{p_1\} = [U_1]^T \{F(t)\} \tag{5.3}$$

Scaling to unit modal mass yields

$$[I_1]\{\ddot{p}_1\} + \left[{\Omega_1}^2\right]\{p_1\} = \left[U_1^n\right]^T \{F(t)\} \tag{5.4}$$

where $[I_1]$ is the identity matrix and $[{\Omega_1}^2]$ is the diagonal natural frequency matrix. More detailed information on the equation development is contained in Twenty Years of Structural Dynamic Modification – A Review [6].

5.2.2 Structural Dynamic Modification

Structural Dynamic Modification (SDM) is a technique that uses the original mode shapes and natural frequencies of a system to estimate the dynamic characteristics due to changes in the mass and/or stiffness of the system. First, the change of mass and stiffness are transformed to modal space as shown

$$\left[\Delta \overline{M}_{12}\right] = [U_1]^T [\Delta M_{12}] [U_1] \tag{5.5}$$

$$\left[\Delta \overline{K}_{12}\right] = [U_1]^T [\Delta K_{12}] [U_1] \tag{5.6}$$

The modal space mass and stiffness changes are added to the original modal space equations to obtain

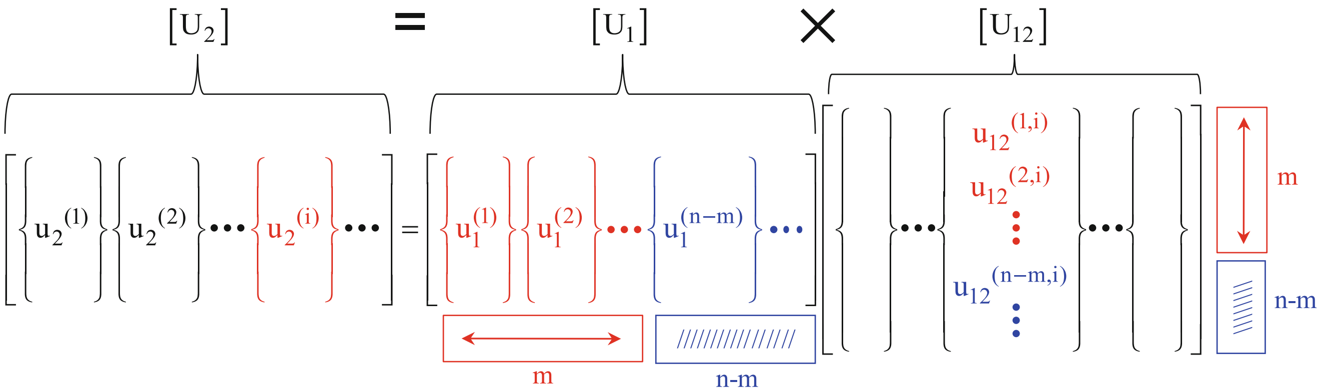


Fig. 5.1 Structural dynamic modification, mode contribution identified using U_{12} [7]

$$\left[\begin{bmatrix} \ddots & & \\ & \overline{M}_1 & \\ & & \ddots \end{bmatrix} + \left[\Delta\overline{M}_{12}\right]\right]\{\ddot{p}_1\} + \left[\begin{bmatrix} \ddots & & \\ & \overline{K}_1 & \\ & & \ddots \end{bmatrix} + \left[\Delta\overline{K}_{12}\right]\right]\{p_1\} = [0] \tag{5.7}$$

The eigensolution of the modified modal space model is computed and the resulting eigenvalues are the new frequencies of the system. The resulting eigenvector matrix is the $[U_{12}]$ matrix, which is used to transform the original modes to the new modes as indicated by

$$[U_2] = [U_1]\,[U_{12}] \tag{5.8}$$

The new mode shapes are $[U_2]$. The new mode shapes are formed from linear combinations of the original mode shapes. The $[U_{12}]$ matrix shows how much each of the $[U_1]$ modes contributes to forming the new modes. Figure 5.1 shows the formation of the new mode shapes as seen on Eq. (5.8). See [6] for additional information on SDM.

5.2.3 *General Reduction/Expansion Methodology and Model Updating*

Model reduction is a tool used to reduce the number of degrees of freedom (DOF) in order to diminish the required computation time of an analytical model, while attempting to preserve the full DOF dynamic characteristics. The relationship between the full space and reduced space model can be written as

$$\{x_n\} = \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = [T]\,\{x_a\} \tag{5.9}$$

where subscript 'N' signifies the full set of DOF (NDOF), 'a' signifies the reduced set of DOF (ADOF) and 'd' is the deleted DOF (those DOF not used during the reduced computation process). The transformation matrix [T] relates the full set of NDOF to the reduced set of ADOF. The transformation matrix is used to reduce the mass and stiffness matrices as

$$[M_a] = [T]^T\,[M_n]\,[T] \quad \text{and} \quad [K_a] = [T]^T\,[K_n]\,[T] \tag{5.10}$$

The eigensolution of these 'a' set mass and stiffness matrices are the modes of the reduced model. These modes can be expanded back to full space using the transformation matrix

$$[U_n] = [T]\,[U_a] \tag{5.11}$$

If an optimal 'a' set is not selected when using methods such as Guyan Condensation [8] or Improved Reduced System Technique [9], the reduced model may not perfectly preserve the dynamics of the full space model. If System Equivalent Reduction Expansion Process (SEREP) [10] is used, the dynamics of selected modes will be perfectly preserved regardless of the 'a' set selected.

5.2.3.1 Expansion of System Modes from Uncoupled Component Modes

Expansion is generally used for providing full N-space mode shape information extracted from limited a-space information. The expansion to full space in this paper is based on recent work by Nonis [11] showing that full N-space mode shape information for an assembled system model can be obtained using the expansion matrices from the uncoupled, unconnected, original component modes of each component. Figure 5.2 shows the entire expansion process schematically to further describe the overall procedure. Reference [11] further details the expansion process and considerations for modes included.

5.2.3.2 System Equivalent Reduction Expansion Process (SEREP)

The SEREP modal transformation relies on the partitioning of the modal equations representing the system DOFs relative to the modal DOFs [10]. The SEREP technique utilizes the mode shapes from a full finite element solution to map to the limited set of master DOF. SEREP is not performed to achieve efficiency in the solution but rather is intended to perform an accurate mapping matrix for the transformation. The SEREP transformation matrix is formed using a subset of modes at full space and reduced space as

$$[T_U] = [U_n]\,[U_a]^g \tag{5.12}$$

where $[U_a]^g$ is the generalized inverse and $[T_U]$ is the SEREP transformation matrix. When the SEREP transformation matrix is used for model reduction/expansion as outlined in the previous section, the reduced model perfectly preserves the full space dynamics of the modes in $[U_n]$ [10].

5.2.3.3 KM_AMI Reduction

A more recent technique has been developed that utilizes Guyan Reduction along with direct updating of the reduced system matrices with the full space modal vectors as targets for the updating process [12]. This reduction technique also overcomes some of the rank problems associated with SEREP and provides a reduced set of ADOF that retain all the eigenvalues and eigenvectors of the full system matrices. The Guyan reduced mass and stiffness matrices are updated using

$$[M_I] = [M_S] + [V]^T \left[\, [\,I] - \left[\overline{M}_S\right] \,\right] \,[V] \tag{5.13}$$

and

$$[\,K_I] = [K_S] + [V\,]^T \left[\, \left[\Omega^2_{REF}\right] + \left[\overline{K}_S\right] \,\right] \,[V] - [[K_S\,] \,\,[\,U_{REF}] \,\,[\,V]] - [[K_S] \,\,[\,U_{REF}] \,\,[V]]^T \tag{5.14}$$

with

$$[V] = \left[\overline{M}_S\right]^{-1} [U_{REF}]^T \,[M_S] \tag{5.15}$$

5.2.4 *System Forced Response Analysis*

The computation of the time response developed in this paper is based on the Equivalent Reduced Model Technique (ERMT), a technique developed by Avitabile and Thibault [1, 7]. This technique uses an exact reduced model representation for the calculation of the system response. Newmark integration technique [13] is used to perform the direct integration of the equations of motion for the ERMT solution process. From the known initial conditions for displacement and velocity, the initial acceleration vector is computed using the equation of motion and the applied forces as

$$\ddot{\vec{x}}_0 = [M]^{-1} \left(\vec{F}_0 - [C] \,\,\dot{\vec{x}}_0 - [K] \,\,\vec{x}_0 \right) \tag{5.16}$$

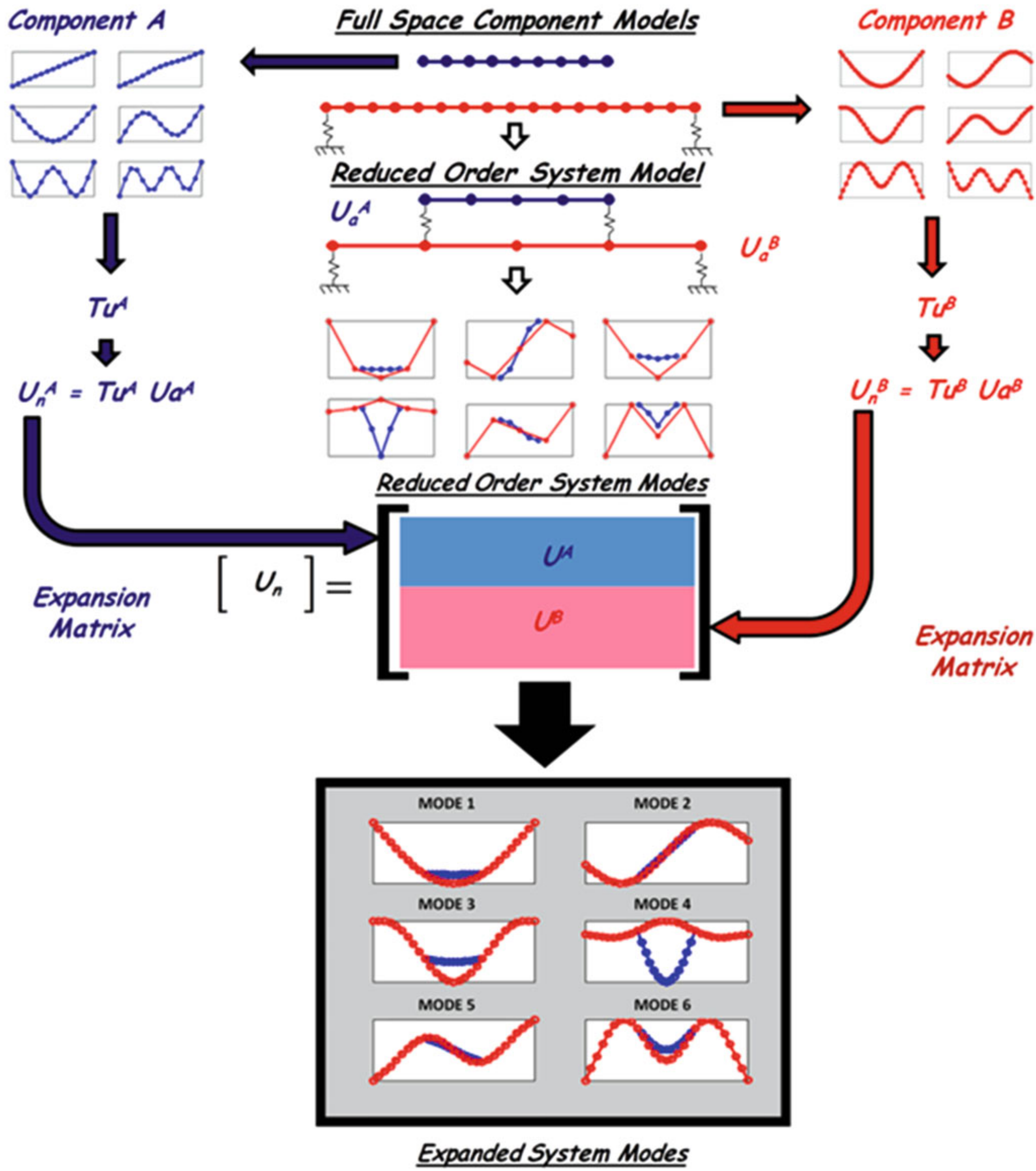


Fig. 5.2 Overall expansion process schematic using the transformation matrices from unconnected system components [11]

where

$\overrightarrow{\ddot{x}}_0 =$ Initial Acceleration Vector
$\overrightarrow{\dot{x}}_0 =$ Initial Velocity Vector
$x_0 =$ Initial Displacement Vector
$\overrightarrow{F}_0 =$ Initial Force Vector

Choosing an appropriate Δt, α, and β, the displacement vector is

$$\vec{x}_{i+1} = \left[\frac{1}{\alpha(\Delta t)^2}[M] + \frac{\beta}{\alpha(\Delta t)}[C] + [K]\right]^{-1} \left\{\vec{F}_{i+1} + [M]\left(\left(\frac{1}{\alpha(\Delta t)^2}\right)\vec{x}_i + \left(\frac{1}{\alpha(\Delta t)}\right)\dot{\vec{x}}_i + \left(\frac{1}{2\alpha} - 1\right)\ddot{\vec{x}}_i\right) + [C]\left(\left(\frac{\beta}{\alpha(\Delta t)}\right)\vec{x}_i + \left(\frac{\beta}{\alpha} - 1\right)\dot{\vec{x}}_i + \left(\frac{\beta}{\alpha} - 2\right)\frac{\Delta t}{2}\ddot{\vec{x}}_i\right)\right\} \tag{5.17}$$

The values chosen for α and β were ¼ and ½, respectively. This assumes constant acceleration and the integration process is unconditionally stable, where a reasonable solution will always be reached regardless of the time step used. However, the time step should be chosen such that the highest frequency involved in the system response can be characterized properly to avoid numerical damping in the solution.

Following the displacement vector calculation, the acceleration and velocity vectors are computed for the next time step using

$$\dot{\vec{x}}_{i+1} = \dot{\vec{x}}_i + (1 - \beta)\Delta t\,\ddot{\vec{x}}_i + \beta \Delta t\,\ddot{\vec{x}}_{i+1} \tag{5.18}$$

$$\ddot{\vec{x}}_{i+1} = \frac{1}{\alpha(\Delta t)^2}\left(\vec{x}_{i+1} - \vec{x}_i\right) - \frac{1}{\alpha \Delta t}\dot{\vec{x}}_i - \left(\frac{1}{2\alpha} - 1\right)\ddot{\vec{x}}_i \tag{5.19}$$

This process is repeated at each time step for the duration of the time response solution desired.

5.2.5 Time Response Correlation Tools

In order to quantitatively compare two different time solutions, two correlation tools were employed: The Modal Assurance Criterion (MAC) and the Time Response Assurance Criterion (TRAC).

5.2.5.1 Modal Assurance Criterion (MAC)

The Modal Assurance Criterion (MAC) [14] is widely used as a vector correlation tool. In this work, the MAC was used to correlate all DOF at a single instance in time. The MAC is written as

$$\mathrm{MAC}_{ij} = \frac{\left[\{X1_i\}^T\{X2_j\}\right]^2}{\left[\{X1_i\}^T\{X1_i\}\right]\left[\{X2_j\}^T\{X2_j\}\right]} \tag{5.20}$$

where X1 and X2 are displacement vectors. MAC values close to 1.0 indicate strong similarity between vectors, where values close to 0.0 indicate minimal or no similarity.

5.2.5.2 Time Response Assurance Criterion (TRAC)

The Time Response Assurance Criterion (TRAC) [15] quantifies the similarity between a single DOF across all instances in time. The TRAC is written as

$$\mathrm{TRAC}_{ji} = \frac{\left[\{X1_j\}^T\{X2_i\}\right]^2}{\left[\{X1_j\}^T\{X1_j\}\right]\left[\{X2_i\}^T\{X2_i\}\right]} \tag{5.21}$$

where X1 and X2 are displacement vectors. TRAC values close to 1.0 indicate strong similarity between vectors, where values close to 0.0 indicate minimal or no similarity.

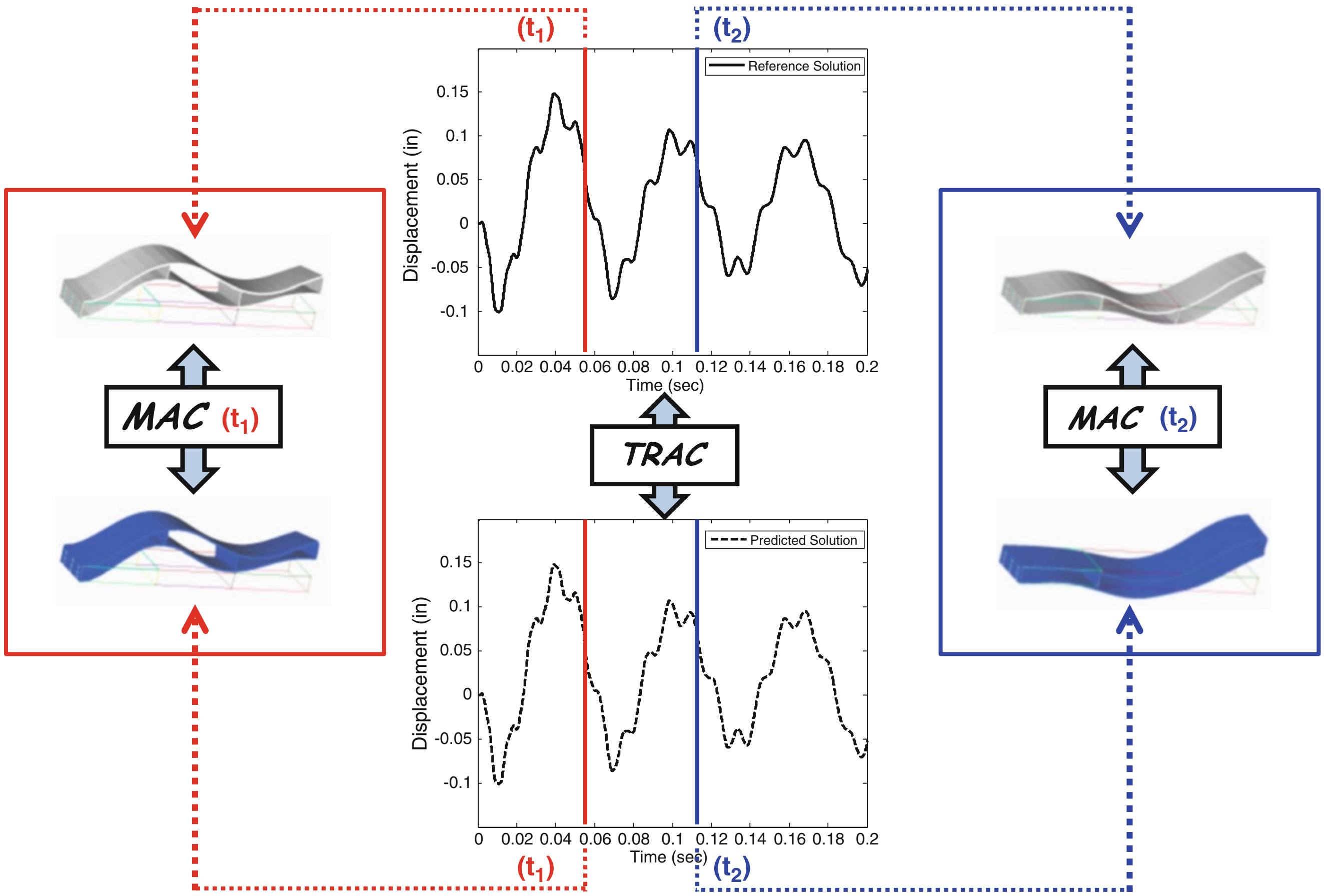


Fig. 5.3 Physical interpretation of MAC and TRAC

In this work, the MAC is calculated between the shapes of the full space reference solution and estimated solution obtained from the reduced order model at each time step. Similarly the TRAC is used to compare the time response from the reduced order model to the time response from the full space finite element solution at each degree of freedom. A diagram detailing the two comparison techniques is shown in Fig. 5.3.

5.3 Model Description

Analytical models of a multi-component beam system were created to investigate the prediction of the dynamic response of the system including subcomponents and ancillary attachments. The models consisted of three beams, as illustrated in Fig. 5.4, attached asymmetrically by linear springs and such that all components are dynamically active.

Planar element beam models of the three beams were generated using MAT_SAP [16], which is a finite element modeling (FEM) program developed for MATLAB [17] and forced response calculations were performed in MATLAB using Newmark integration scripts base on code originally written by Thibault [1]. The beam models were set to have dimensions and characteristics asdescribed in Figs. 5.5 and 5.6.

The 3 beam system was subjected to a double sided force pulse at the left end of the support beam (see Fig. 5.4) and this input force was set as to only excite the modes in the frequency band of approximately 1,000 Hz of the system as shown in Fig. 5.7.

With all 100 elements of the system (i.e. 206 DOF) the full N-space reference solution to the system was calculated and served as a point of comparison for all subsequent reduced order model calculations. The frequencies of the individual components and of the full assembled system with or without the ancillary subcomponent are shown in Table 5.1.

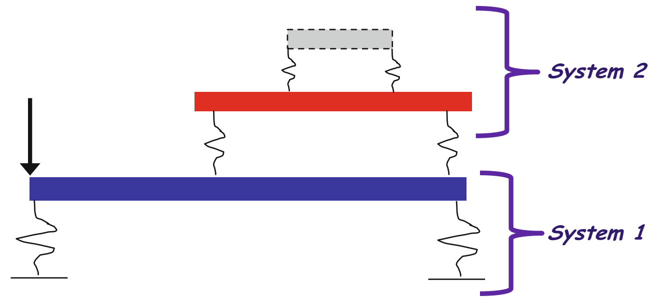
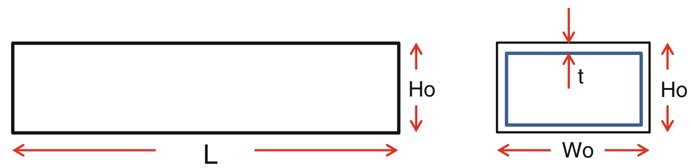
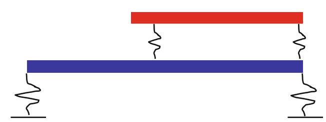


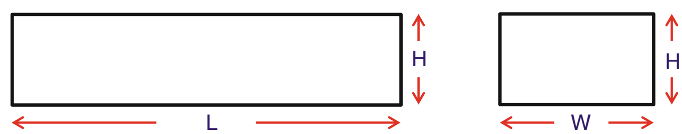
Fig. 5.4 Schematic of 3 beam analytical model and input force location (not to scale)

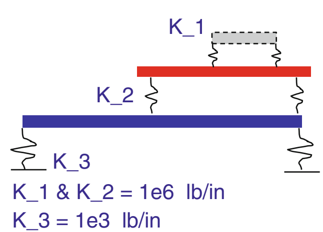




Property of Beam	Beam A	Beam B
Wo - Beam width (in)	2	3
Ho - Beam height (in)	1	4
t - Wall thickness (in)	3/8	1/8
L - Length (in)	50	140
E – Young's Modulus	10e6 psi	
ρ – Weight density	0.1 lb/in^3	
# of elements	20	70

Fig. 5.5 Dimensions of *top* (*red*) and support/base (*blue*) beams of the analytical beam system





Ancillary Beam Dimensions and Physical Properties	
L – Length	15 in
H – Height	0.25 in
W – Width	0.5 in
E – Young's Modulus	10e6 psi
ρ – Weight density	0.1 lb/in^3
# of Elements	10

Fig. 5.6 Dimensions and characteristics of ancillary subcomponent (*grey*) of 3 beam system

5.4 Cases Studied

The forced response of the full space reference model with 206 DOF was first calculated. Structural Dynamic Modification was used to calculate the U_{12} matrix and determine the necessary modes of the system to preserve the first five modes of the 3 beam system. Subsequently, reduction techniques were used to reduce the active DOF of the system to an 'a' set not including DOF on the ancillary beam. The forced response of the reduced ADOF linear system was calculated. The dynamic characteristics of the ancillary subcomponent were then extracted using the system information available from the reduction process. This is equivalent to assuming the subcomponent of the system (ancillary beam) is inaccessible/immeasurable and

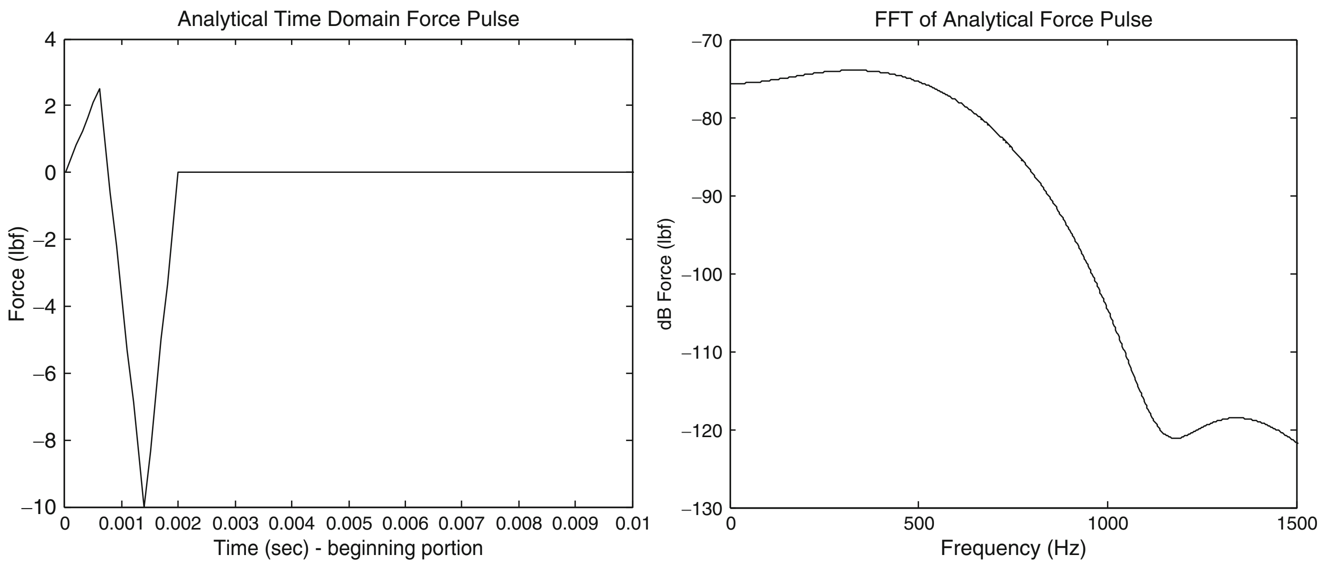
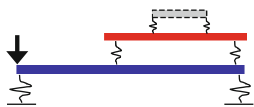


Fig. 5.7 Time (*left*) and frequency (*right*) domain plots of input analytical force pulse

Table 5.1 Frequencies of the first 15 modes of the 3 beam system and its components



Mode	Frequency (Hz)				
	Beam A	Beam B	Sub-Component	System w/o A.C.	System
1	0.0015	18.9	0.010	16.8	16.8
2	0.0024	45.9	0.014	37.6	37.5
3	87.6	80.9	224.5	68.2	68.1
4	241.4	161.4	619.0	86.9	84.6
5	473.2	299.7	1214.3	129.1	102.1
6	782.3	489.4	2010.4	210.5	129.1
7	1168.8	728.2	3011.1	282.8	210.0
8	1633.1	1015.5	4223.0	343.1	282.0
9	2175.4	1351.0	5653.7	477.1	343.0
10	2796.3	1734.7	7302.3	645.5	396.2
11	3496.7	2166.4	9071.4	716.4	477.3
12	4277.7	2646.2	12046.9	959.5	645.3
13	5140.7	3174.1	14493.9	1118.7	716.3
14	6087.4	3750.0	17502.7	1311.7	889.0
15	7120.0	4373.9	21038.3	1617.3	960.6

Yellow cells highlight similar frequencies after addition of ancillary subcomponent (AC) to the system

therefore predicting its response from the information of the other two components (the red and blue beam). The test cases presented here are intended to show the results when a proper set of modes are selected such that no information is lost in the reduction process and an inappropriate reduced model where the modes do not span the space of the system.

The cases presented here are summarized as:

Case 1—Reference Model
206 DOF Total; System 1/Beam B 142 DOF; System 2—Beam A 42 DOF and Ancillary 22 DOF
Case 2—Guyan Reduced Order Model
12 DOF Total; Beam B—ADOF 65, 117, 169, 199, and 205; Beam A—ADOF 23, 31, 33, 41, 43, 55 and 63

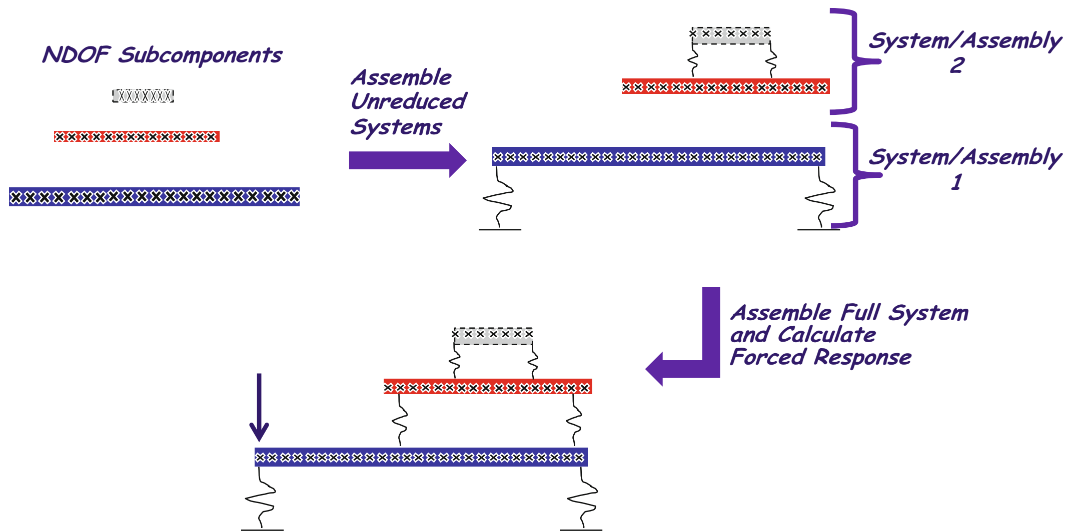


Fig. 5.8 Sequence for the development of assembled reference model

Case 3—SEREP Reduced Order Model
12 DOF Total; Beam B—ADOF 65, 117, 169, 199, and 205; Beam A—ADOF 23, 31, 33, 41, 43, 55 and 63
Case 4—KM_AMI Model Improvement from Guyan Reduced Order Model
12 DOF Total; Beam B—ADOF 65, 117, 169, 199, and 205; Beam A—ADOF 23, 31, 33, 41, 43, 55 and 63
Case 5—Considerations for Additional Modes in the Model Reduction
Case 6—Considerations for DOF Selection in the Model Reduction

5.4.1 *Case 1: Reference Model*

The NDOF unreduced subcomponents were tied together to form two systems, the support (System 1) and the top assembly (System 2). The frequencies and mode shapes of both untied and tied subcomponents were calculated for reference. The assembled 3 beam system consisting of the tied System 1 and System 2 was then tied together at full N-space and the linear forced response calculated using the analytical input force of Fig. 5.7. The forced response of this NDOF (206 DOF) served as the reference solution for the reduced cases. Figure 5.8 shows the sequence of the assembly of the system subcomponents used to create the assembled system reference solution.

The frequencies of the system and subcomponents are shown in Table 5.1 and the mode shapes of the 3 beam system can be seen in Fig. 5.9. The ancillary (grey) subcomponent can be observed to be dynamically active on the first 15 modes of the system and therefore sufficient component information needs to be preserved in the reduction process in order to not only accurately predict the force response of the ancillary subcomponent (during expansion) but also to properly capture the dynamics of the coupling red beam.

5.4.1.1 Component Mode Contribution—U_{12}

Calculation of the U_{12} matrix for the system response is of utmost importance to understand and mitigate the effects of truncation error in the reduction process. The modes from Systems 1 and System 2 required to preserve the first five modes of the assembled full 3 beam system were chosen using the resulting U_{12} shown in Fig. 5.10. As seen on the U_{12} contribution matrix, a total of 12 modes of the system components are required to preserve five modes of the assembled 3 beam system. System 1 (the blue support beam) contributes five modes to the reduced model while System 2 (the top red beam and its attached ancillary subcomponent) provides seven modes. The larger contribution from system 2 is already an indication of the strong influence of the dynamic ancillary subcomponent and emphasizes that a larger amount of information from this component (system 2) is needed to fully characterize the system level response of the 3 beam structure.

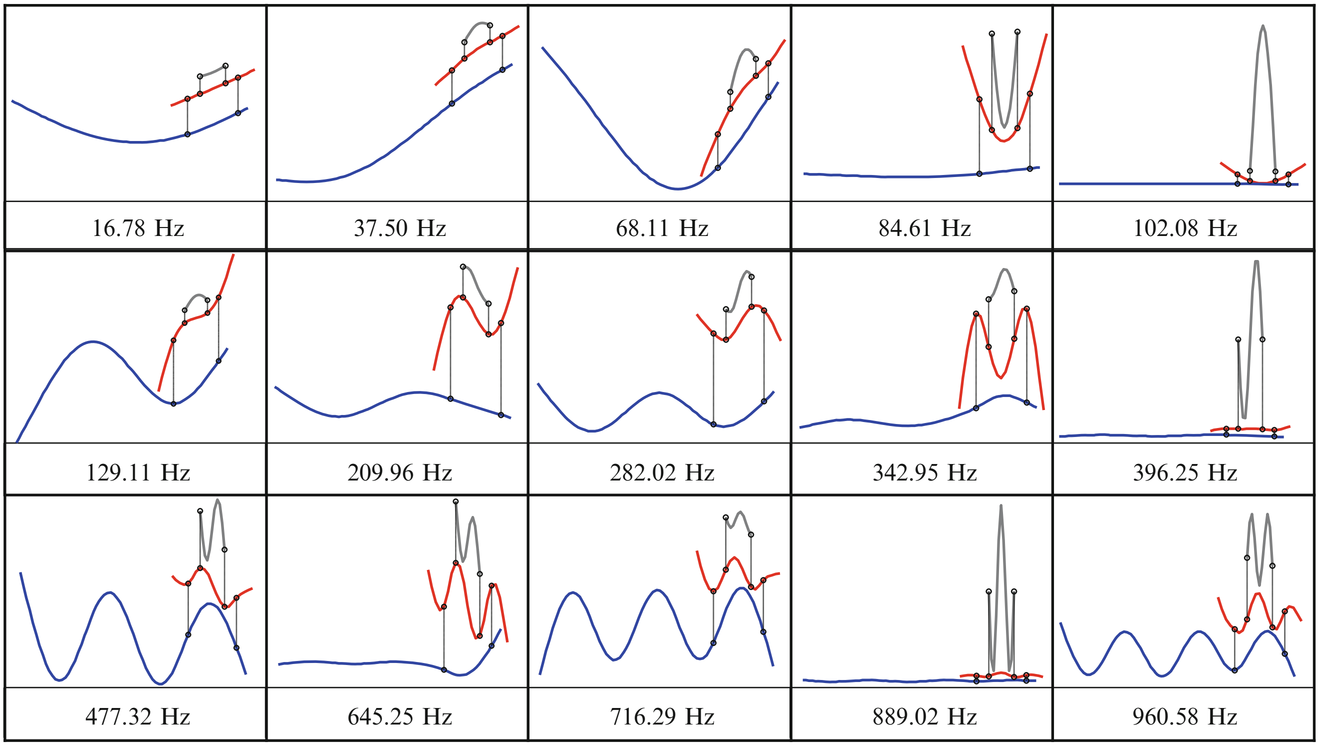


Fig. 5.9 Mode shapes of full 206 N-space 3 beam system used as reference solution

5.4.2 Overview of Reduced Models

Full N-space subcomponents were assembled into two systems (System 1 and System 2) as in the reference case. The two systems were reduced separately and then tied together. For System 2, containing the ancillary subcomponent, the reduction was made as to omit any active DOF in the ancillary beam. The idea is that the reduction process, if carried out successfully, should embed the necessary component information onto the reduced assembled system. Figure 5.11 shows the assembly sequence performed to reduce the system components and the extraction of the subcomponent response from the calculated reduced model forced response.

Three reduction methods were used to highlight particular advantages of each procedure. Guyan reduction, as explained in the theory section, does not fully preserve the dynamics of the system if an optimal ADOF set is not selected. On the other hand, SEREP will accurately preserve the selected modes of the system but issues may arise regarding full rank of the reduced mass and stiffness matrices of the system. Lastly, a model improvement technique, KM_AMI, is used to update the mass and stiffness matrices of the Guyan reduced model by seeding target frequencies and mode shapes and therefore preserving the exact model information while preserving the full rank advantages of a Guyan reduced model. These issues will be covered in the three cases described below.

5.4.3 Case 2: Guyan Reduced Model

As with the other reduced models covered in this study, the selected DOF for the reduction did not include DOFs of the ancillary subcomponent. As indicated by the U_{12} matrix in Figs. 5.10 and 5.12, there are 12 modes of the system's components necessary to accurately span the space of the first five modes of the assembled 3 beam system. Figure 5.13 shows the layout of connection DOF of the 3 beam system. Figure 5.14 shows the reduced order model frequencies compared to the reference N-space solution.

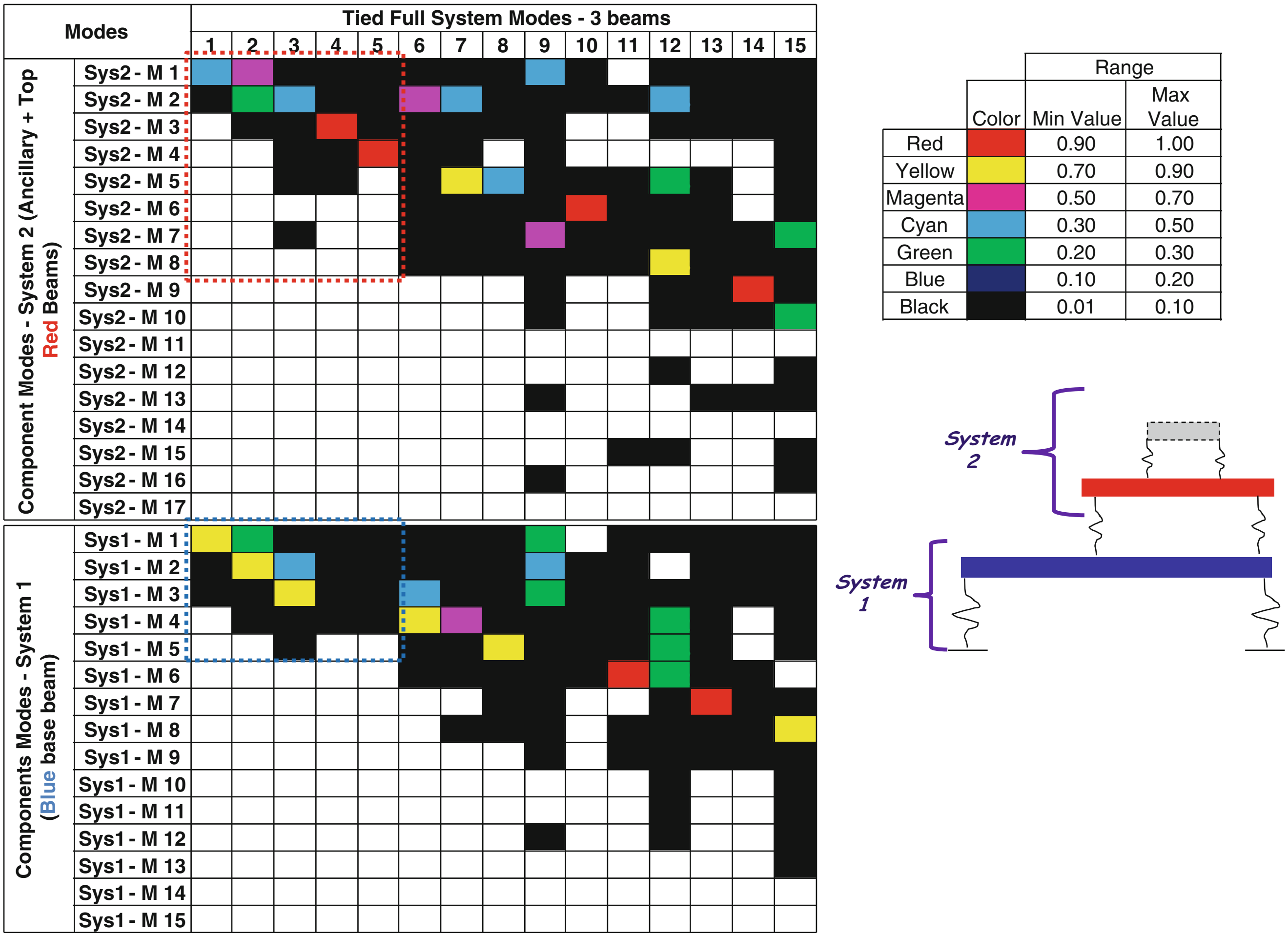


Fig. 5.10 U_{12} mode contribution matrix from system components to full 3 beam system

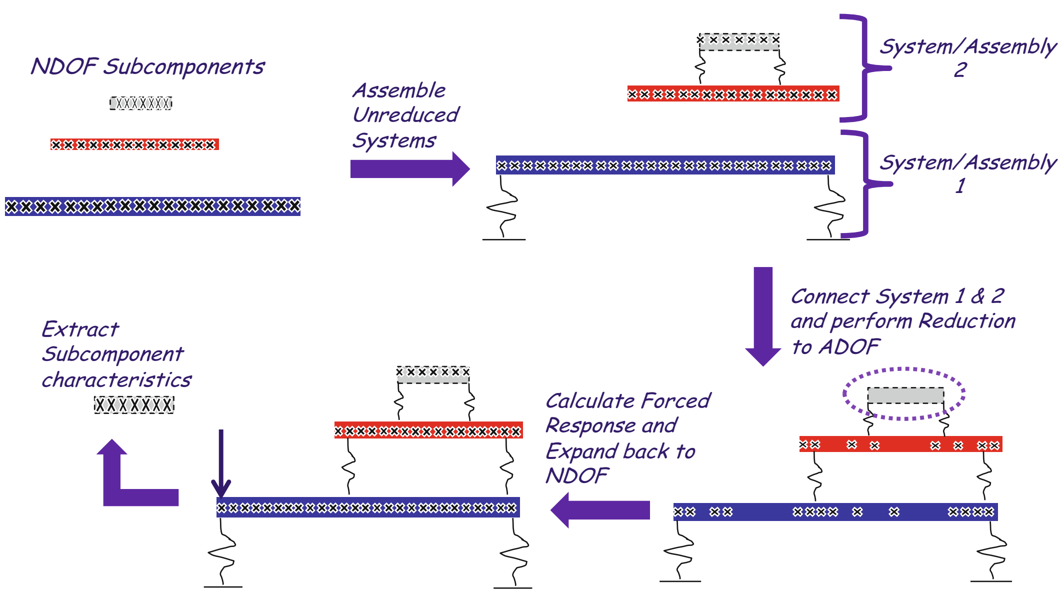
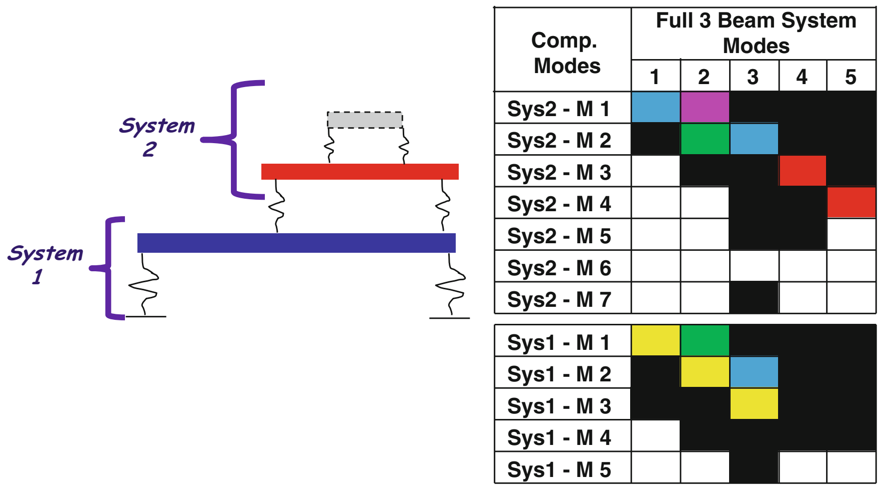


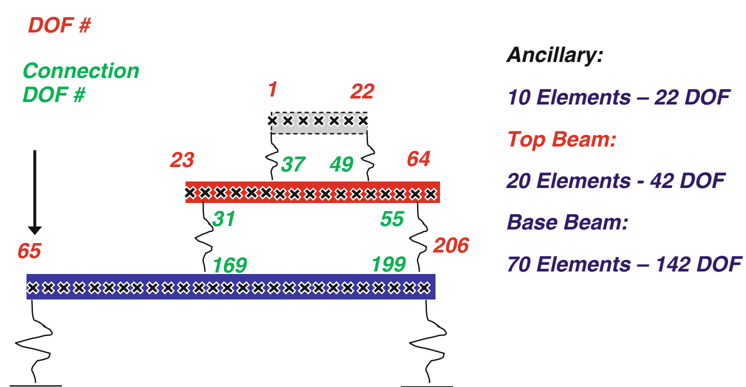
Fig. 5.11 Sequence for the development of reduced system response models

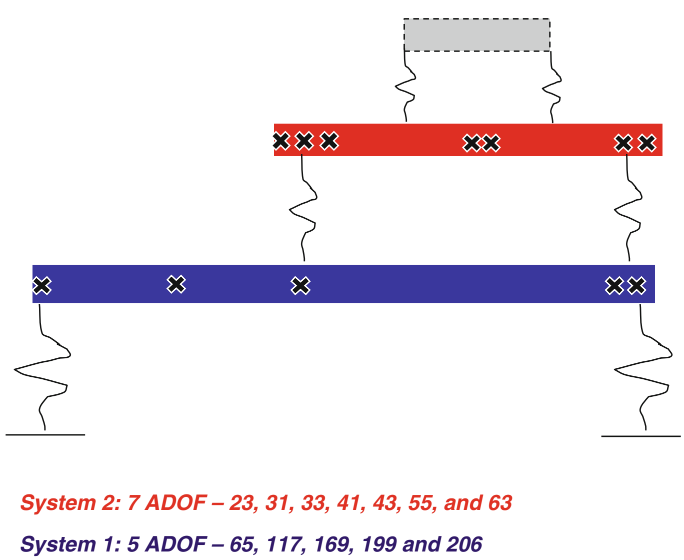


	Color	Range Min Value	Range Max Value
Red		0.90	1.00
Yellow		0.70	0.90
Magenta		0.50	0.70
Cyan		0.30	0.50
Green		0.20	0.30
Blue		0.10	0.20
Black		0.01	0.10

Fig. 5.12 Selected portion of U_{12} mode contribution matrix from system components to first five modes of full 3 beam system

Fig. 5.13 Summary of numbering of 3 beam system NDOF (206) and connecting DOF





	Reference System Freq. (Hz)	Guyan Reduced System Freq. (Hz)	% Difference
Mode 1	16.8	16.8	0%
Mode 2	37.5	37.7	0%
Mode 3	68.1	69.9	3%
Mode 4	84.6	86.4	2%
Mode 5	102.1	129.9	27%
Mode 6	129.1	220.6	71%
Mode 7	210.0	347.9	66%
Mode 8	282.0	663.8	135%
Mode 9	343.0	731.0	113%
Mode 10	396.2	1578.4	298%
Mode 11	477.3	3073.4	544%
Mode 12	645.3	3283.3	409%

Fig. 5.14 Comparison of Guyan reduced order model (12 DOF) frequencies with respect to (206 DOF) reference solution

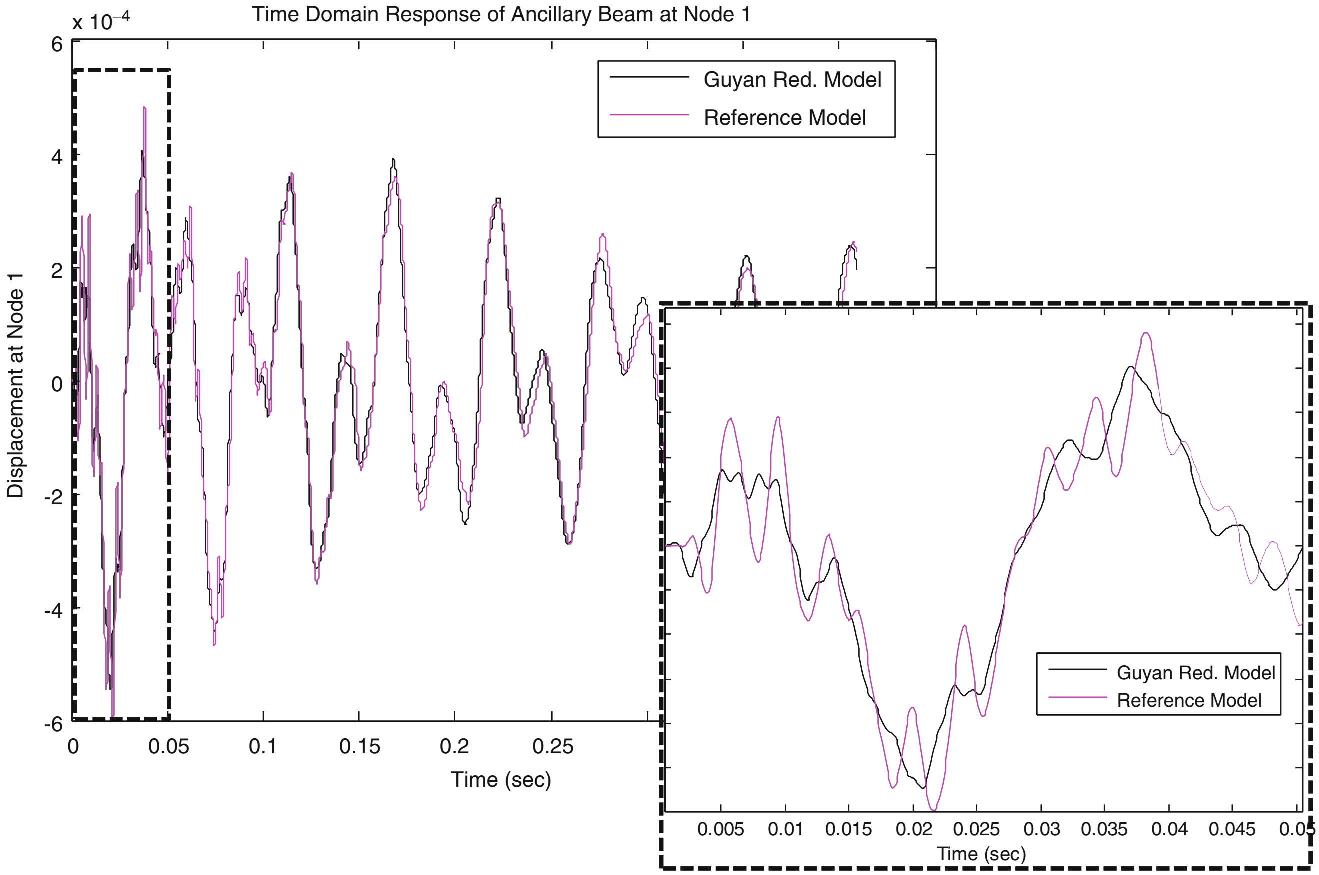


Fig. 5.15 Comparison of time response at node 1 of the ancillary subcomponent from the expansion of a 12 DOF Guyan reduced model versus the 206 DOF full space reference solution. Zoomed in region shows the response for the first 0.05 s

Note that there are no DOF from 1 to 22 since these are the ancillary subcomponent DOFs and moreover, DOF 37 and 49 connecting the top red beam to the ancillary beam were omitted as well. As discussed in the theory section, even with sufficient modes selected in the reduction process (according to the U_{12} contribution matrix) and a well placed selection of ADOF, Guyan reduced did not preserve the five modes of interest as seen in the table of Fig. 5.14. The error can be observed to increase quickly for modes higher than 3 and the predicted system response is not expected to yield accurate results. The forced response of the Guyan reduced order model was calculated and using the transformation matrix [T], the results were expanded back to full N-space (206 DOF). The ancillary subcomponent response was found from the expanded forced response of the system. Figure 5.15 shows a comparison of the displacement at the first node of the ancillary subcomponent with respect to the full space reference solution. The average MAC and TRAC were found to be 0.6 and 0.53 respectively. Figure 5.16 shows the MAC and TRAC for the expanded Guyan reduced model.

From Fig. 5.16 and from the calculation of time response correlation tools, the response of the expanded Guyan reduced model showed low correlation to the reference solution. Clearly mode truncation does not allowed for the accurate characterization of the reduced system, let alone the characterization of the ancillary subcomponent from embedded system information. While addition of extra DOF may diminish the effects of mode truncation, there is always an intrinsic risk of error in the Guyan reduction process and ADOF selection.

5.4.4 Case 3: SEREP Reduced Model

The goal of preserving the first five modes of the full 3 beam system requires the selection of five modes of System 1 and seven modes of System 2. The same ADOF selection from Case 2 was used during the SEREP reduction process. Figure 5.17 shows the resulting frequencies of the reduced system.

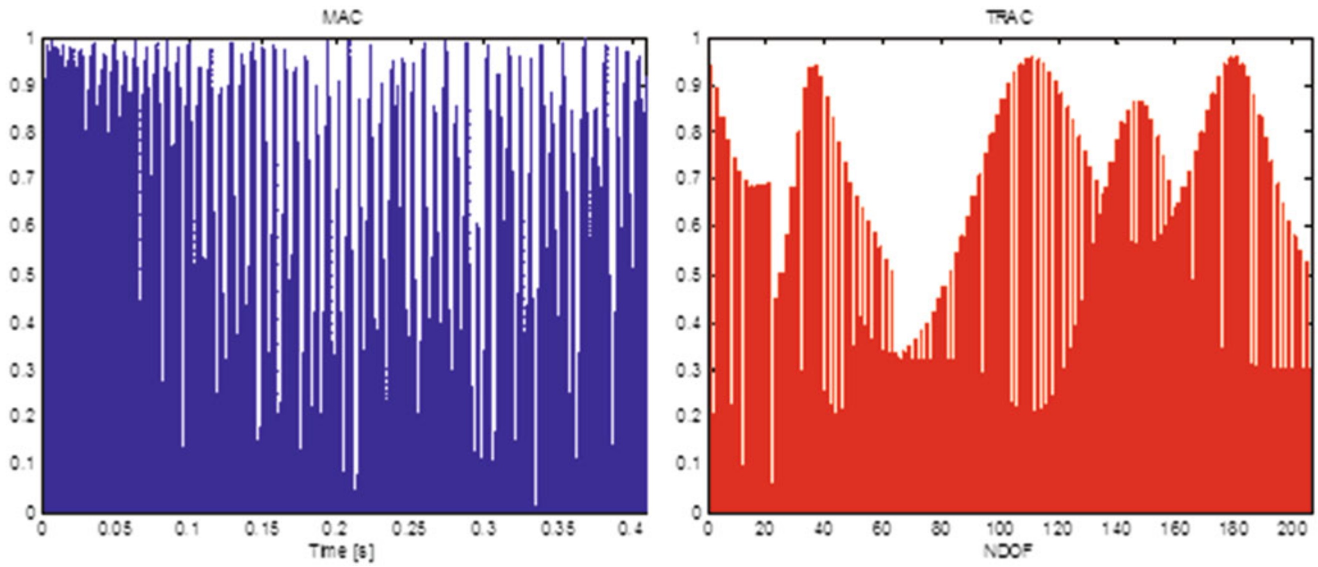


Fig. 5.16 MAC and TRAC bar plots showing the correlation of the expanded Guyan reduced model to the reference model

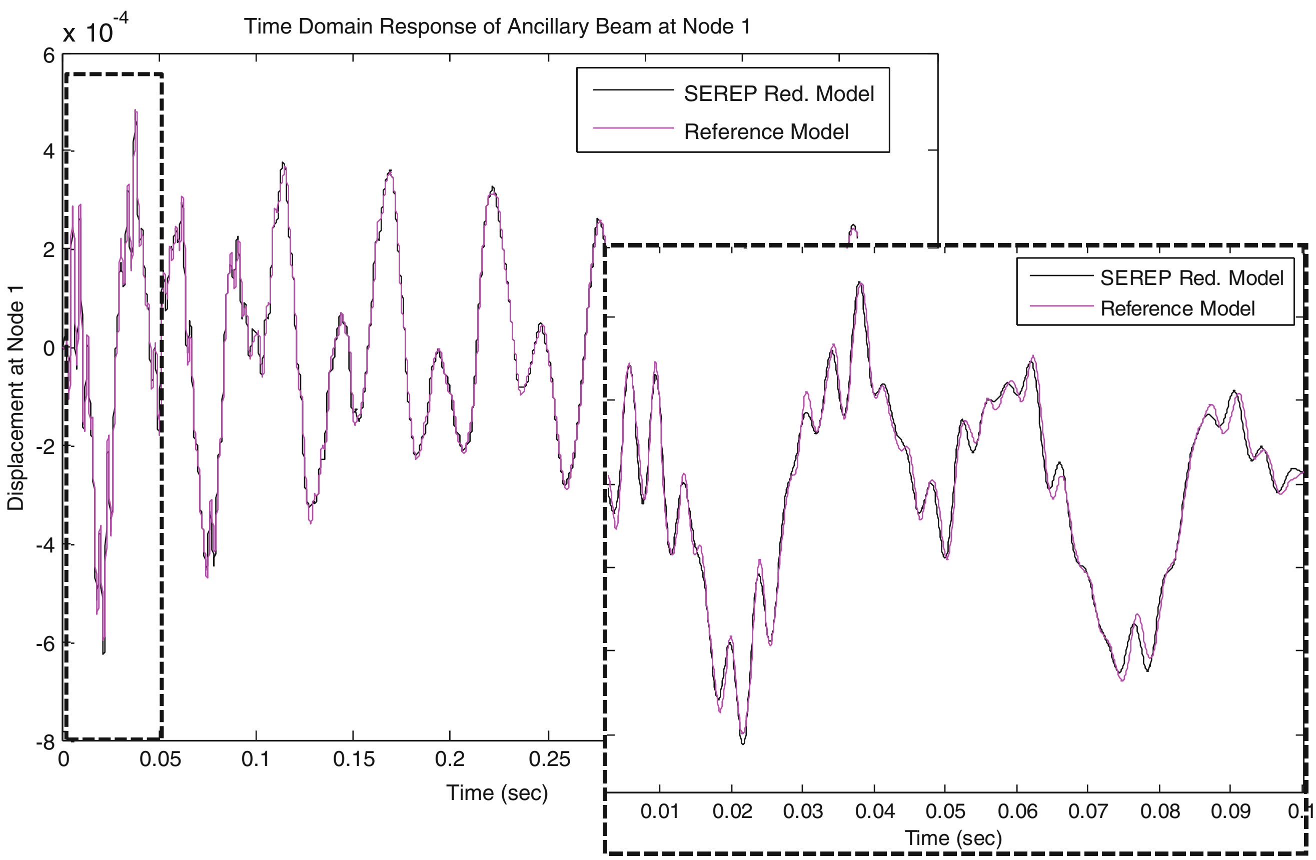
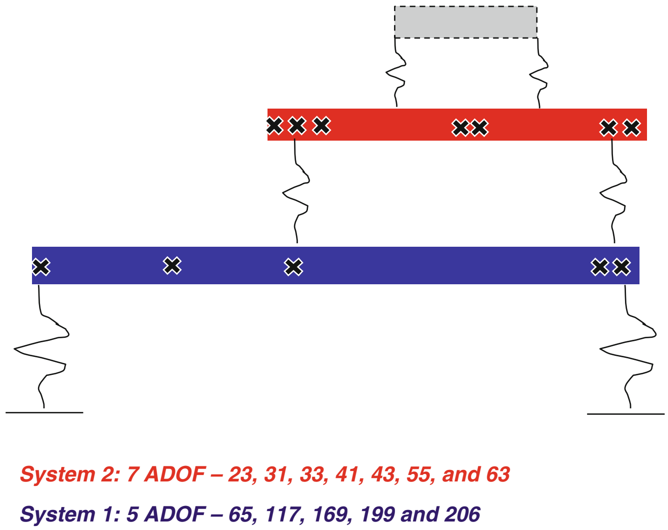


Fig. 5.17 Comparison of time response at node 1 of the ancillary subcomponent from the expansion of a 12 DOF SEREP reduced model versus the 206 DOF full space reference solution. *Zoomed in* region shows the response for the first 0.1 s

From Fig. 5.18 and upon comparison to Fig. 5.14 showed that the SEREP reduced order model preserved the five modes of interest and resulted on a smaller error for some of the remaining seven modes. Calculation of the time response was then performed for the ADOF reduced model. The resulting time response was then expanded to all 206 DOF using the $[T_U]$ SEREP transformation matrix. The response of the ancillary subcomponent was extracted from the expanded response of the



	Reference System Freq. (Hz)	SEREP Reduced System Freq. (Hz)	% Difference
Mode 1	16.8	16.8	0%
Mode 2	37.5	37.5	0%
Mode 3	68.1	68.2	0%
Mode 4	84.6	84.6	0%
Mode 5	102.1	102.1	0%
Mode 6	129.1	130.2	1%
Mode 7	210.0	211.5	1%
Mode 8	282.0	283.2	0%
Mode 9	343.0	351.2	2%
Mode 10	396.2	396.6	0%
Mode 11	477.3	2611.8	447%
Mode 12	645.3	3023.1	369%

Fig. 5.18 Comparison of SEREP reduced order model (12 DOF) frequencies with respect to (206 DOF) reference solution

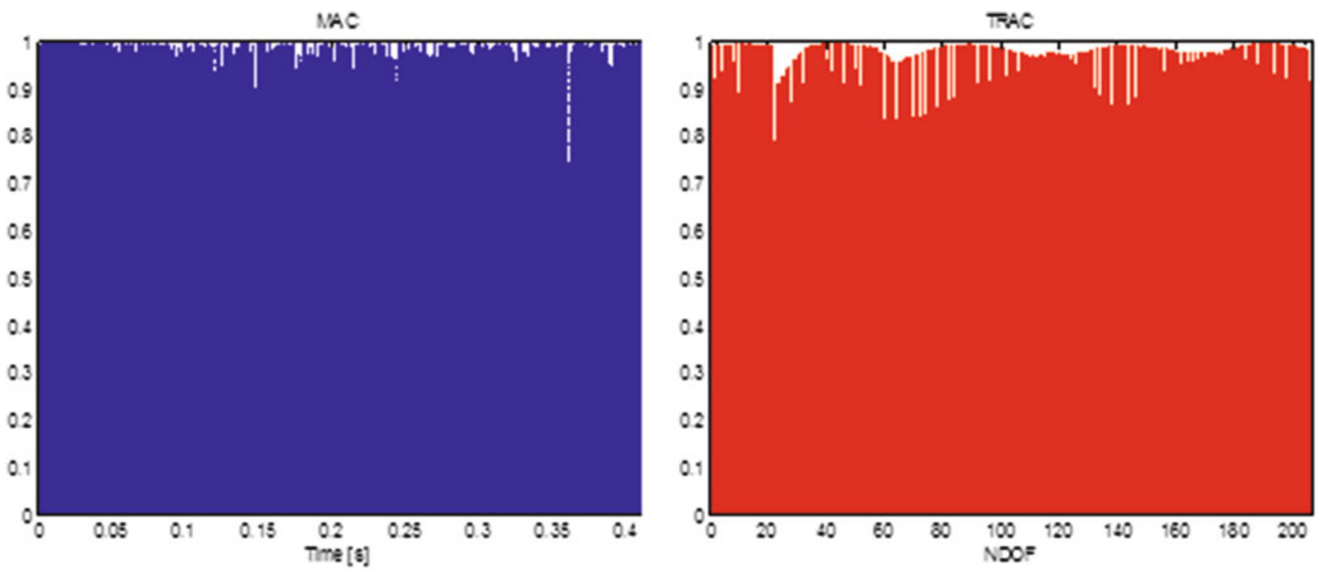


Fig. 5.19 MAC and TRAC bar plots showing the correlation of the expanded SEREP reduced model to the reference model

system. Figure 5.17 shows a comparison of time response at the first node of the ancillary beam (from the expanded SEREP reduced model) with respect to the reference solution; while only one node is shown for brevity, all of the nodes on the ancillary component had similar agreement. The average MAC and TRAC were found to be 0.97 and 0.95 respectively. Figure 5.19 shows the MAC and TRAC bar graphs correlating the expanded SEREP reduced model to the reference solution.

The SEREP reduction and expansion process resulted in high correlation using the same amount of DOF as the Guyan reduced model. Furthermore, the omission of the connecting DOF for the ancillary subcomponent did not yield additional error as will be shown in Case 6. Moreover, issues can only arise if the selected DOF do not yield full rank reduced mass and stiffness matrices but this issue was not encountered in the analytical models studied. Nevertheless, Case 4 discusses the KM_AMI Model Improvement which can alleviate any issues arising from the rank deficiency in the reduction process while preserving the accuracy of the SEREP reduction methodology.

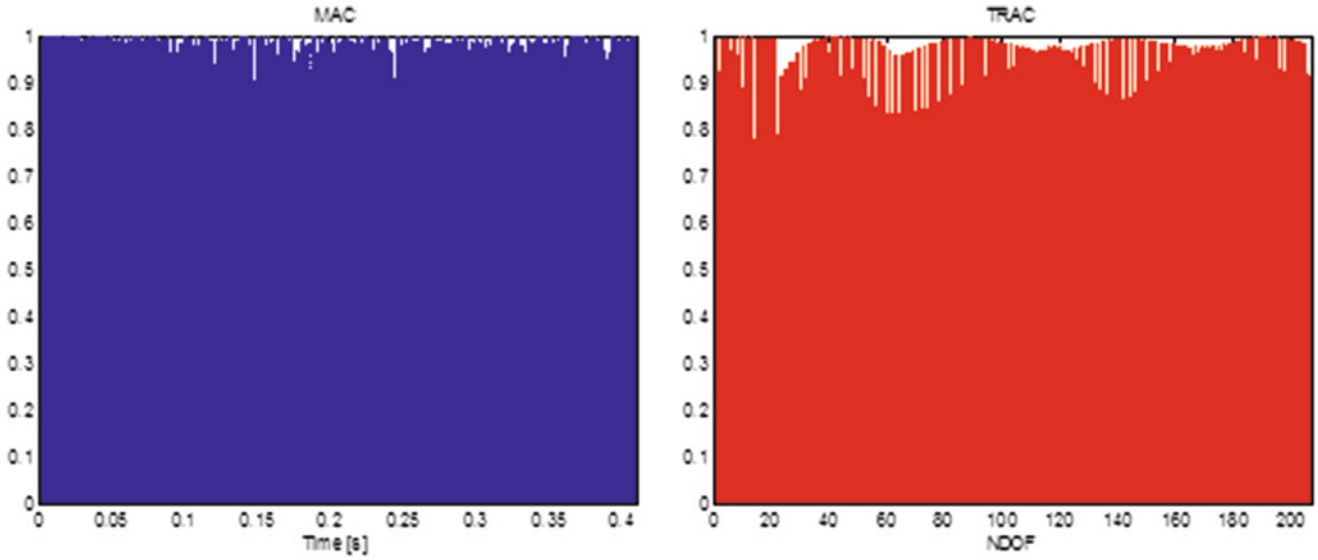


Fig. 5.20 MAC and TRAC bar plots showing the correlation of the expanded Guyan improved model to the reference model

5.4.5 *Case 4: KM_AMI Model Improvement from Guyan Reduced Model*

The Guyan reduced model of Case 2 was shown to be drastically affected by the selection of ADOF as well as the number of modes in the reduction. The need to use Guyan reduction may arise from limitations of currently available finite element software or from the need to have a fully ranked reduced mass and stiffness matrices. For such situations, this case explores using the exact frequencies and mode shapes of the system to update the reduced order model using the KM_AMI approach. The target frequencies and mode shapes (e.g. the first five modes of the fully assembled 3 beam system) are directly seeded to the Guyan reduced model of Case 2. The MAC and TRAC of the expanded KM-AMI updated Guyan model resulted in an average of 0.97 and 0.95 respectively just as the SEREP reduction model of Case 3. Figure 5.20 shows the MAC and TRAC bar plots for the improved Guyan reduced model using KM-AMI updating.

The improved reduced model using the KM_AMI approach showed significant gain in accuracy compared to the original Guyan reduced model. Not only where the five frequencies and mode shapes of the assembled model preserved exactly (as these were directly seeded to the reduced model) but also an accurate ancillary subcomponent response was extracted from the expanded reduced order model.

The cases discussed thus far have shown that the Guyan reduced model (Case 2) does not accurately preserve the embedded characteristics of the ancillary subcomponent during the reduction process while the SEREP and KM_AMI (Cases 3 and 4 respectively) model reductions produce remarkable accuracy in the prediction of the response of both components and the embedded ancillary subcomponent. Now, Cases 5 and 6 will discuss some considerations regarding the accuracy of the reduced model when either additional modes (modes beyond the ones specified by the U_{12} contribution matrix requirements) or DOF are used in the reduced order models.

5.4.6 *Case 5: Considerations for Additional Modes in the Model Reduction*

Addition of modes beyond the 12 modes indicated in the U_{12} matrix in the SEREP reduced model showed large improvement from the resulting expanded model response. Additional 5 modes were included, 2 from System 1 and 3 from System 2 for a total of 17 modes. The ADOF selected once again did not include the connecting DOF (37 and 49) of the ancillary subcomponent. The ADOF set selected was DOF 65, 100, 117, 169, 183, 199, and 205 from System 1/Beam B and ADOF 23, 27, 31, 33, 39, 41, 43, 51, 55 and 63 from Beam A of System 2. The expanded SEREP reduced model response resulted in an average MAC and TRAC of 0.998 and 0.997 respectively. Figure 5.21 shows the MAC and TRAC bar plots for this newly reduced 17 DOF model (Note the change in the y-axis of the MAC).

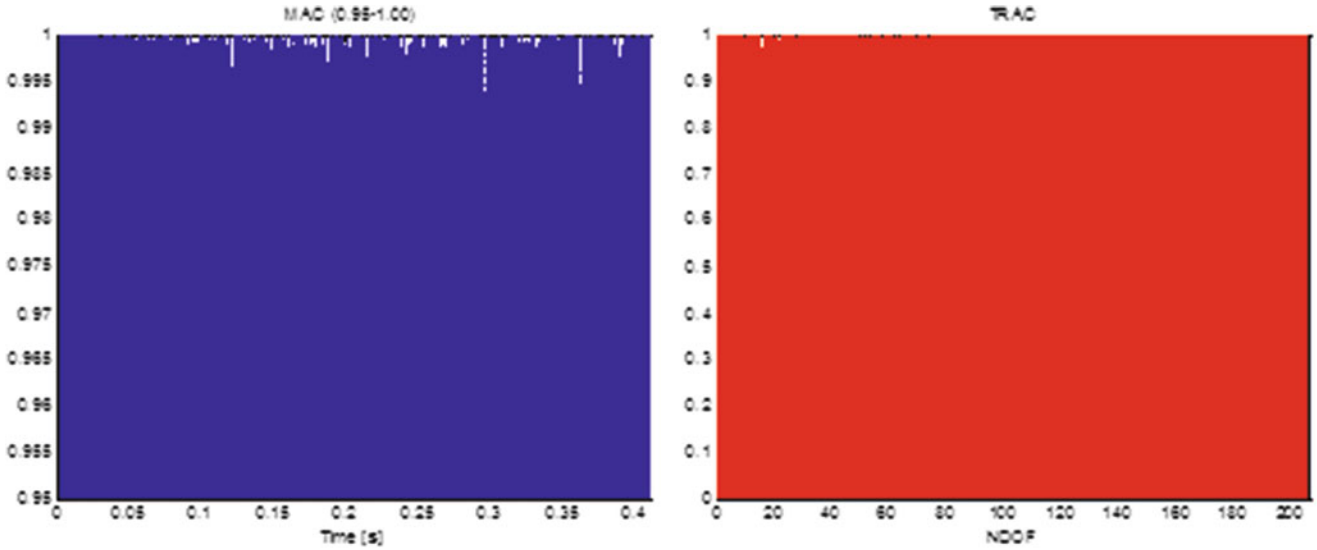


Fig. 5.21 MAC and TRAC bar plots showing correlation of the expanded 17 DOF SEREP reduced model to reference model. MAC y-axis is showing values from 0.95 to 1.0

When the reduction process is successful (as it was with the SEREP and KM_AMI models), the modes selected span the space of the system response. Modes beyond the 12 indicated by the U_{12} smooth the approximation of the system response and further addition of modes results in better results until the reduced 'a' space model approaches the full 'N' space solution and hence spans the whole space of the full assembled system response.

Up to this point, the selection of ADOF was almost arbitrary other than purposely avoiding the inclusion of the connection DOF in the reduction process. As explained in the theory section, Guyan reduced models are highly affected by DOF selection while this has no effect on the SEREP and KM_AMI methodologies. This independence from DOF selection is in fact a central point on why the connection DOFs can be omitted and how the subcomponent information is preserved during the SEREP reduction. Case 6 investigates the addition of the connection DOF in the model reduction.

5.4.7 *Case 6: Considerations for DOF Selection in the Model Reduction*

A reduced model using 12 DOF (as in Case 2, 3 and 4) was created using the connection DOF 37 and 49 from the ancillary subcomponent. The ADOF set selected was DOF 65, 117, 169, 199, and 205 from System 1/Beam B and ADOF 23, 31, 33, 37, 49, 55 and 63 from Beam A of System 2. Figure 5.22 shows a comparison of the SEREP reduction models carried out with and without use of the connection DOF of the ancillary subcomponent.

The 12 DOF reduced model utilizing the connecting DOF to the ancillary beam did not yield a higher average MAC or TRAC and no significant gain was observed from the selection of these DOF. This phenomenon is not surprising because SEREP reduction, as it has already been mentioned, is not dependent on the location of the selected DOF as long as the reduced set of modes span the space of the system. To further illustrates that omitted DOF are sufficiently recovered from the available system information, Fig. 5.23 shows a comparison of the time response at the connecting DOF of both the ancillary beam and the top red beam of System 2.

The connecting DOF of the subcomponents of System 2 are expected, and in fact, show the same time response. Therefore, the number of modes, as shown in Case 5, has a more significant contribution to the accuracy of the predicted response than the selection of ADOF in the reduced model.

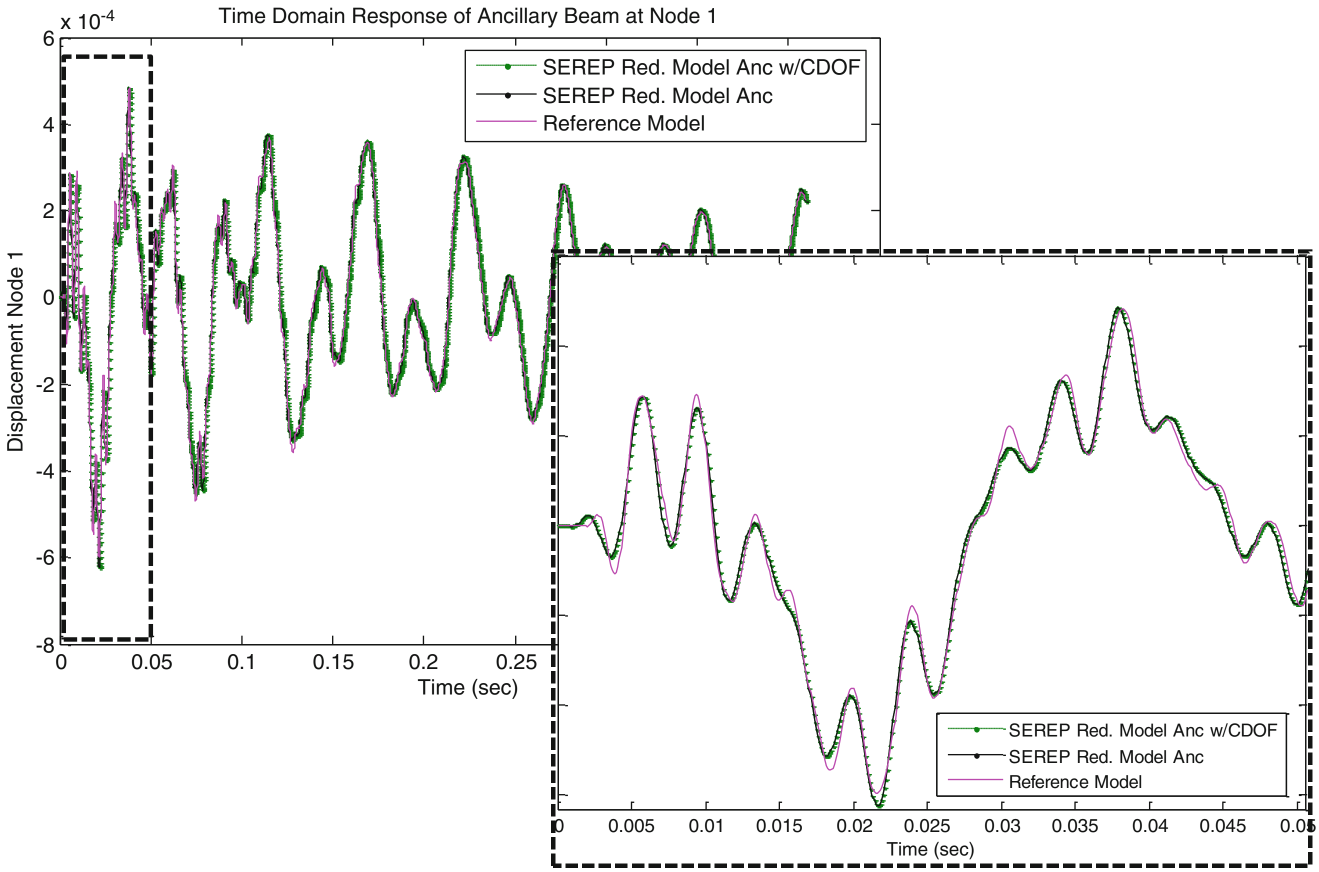


Fig. 5.22 Comparison of time response at node 1 of ancillary subcomponent from two expanded SEREP reduced models (Model in *green* uses the connecting DOF of the ancillary beam while the model in *black* uses the previously selected ADOF set)

5.4.8 Observations

The expanded SEREP reduced model of Case 3 and the KM_AMI model of Case 4 were shown to accurately preserve embedded structural component information. While there are slight differences in the time traces of the system response, these are not necessarily an artifact of the reduction process but more a result of the effect of higher order modes at the beginning of the response as well as the approximation of the damping in the system (damping was assumed to be 1 % for all modes of the system). Increasing the duration of the time pulse could reduce the number of higher modes excited and in turn yield a better representation without the need for more modes in the reduction process.

From Cases 5 and 6, selection of DOF was shown to be not as significant as the number of modes in the reduced model. Increasing the number of modes in the 'a' space resulted in a set of modes that better spans the space of the fully assembled reference model. Furthermore, when the 'a' space modes span the space of the system the selection of the connection DOF of the ancillary subcomponent brought no additional gain in accuracy in the predicted response of the system.

5.5 Conclusion

An efficient reduced order modeling technique for system forced response calculation (ERMT) was used to calculate the time response of multi-component structure. The reduced model was made of components with embedded subcomponent information. The transformation matrices from the reduction process were used to expand the time response from a selected ADOF set back to the full NDOF space. The selected ADOFs omitted the connecting DOF of the ancillary subcomponent and the expansion process was shown to correctly predict the response of the embedded component as long as the modes of the reduced system spanned the space of all the modes of interest.

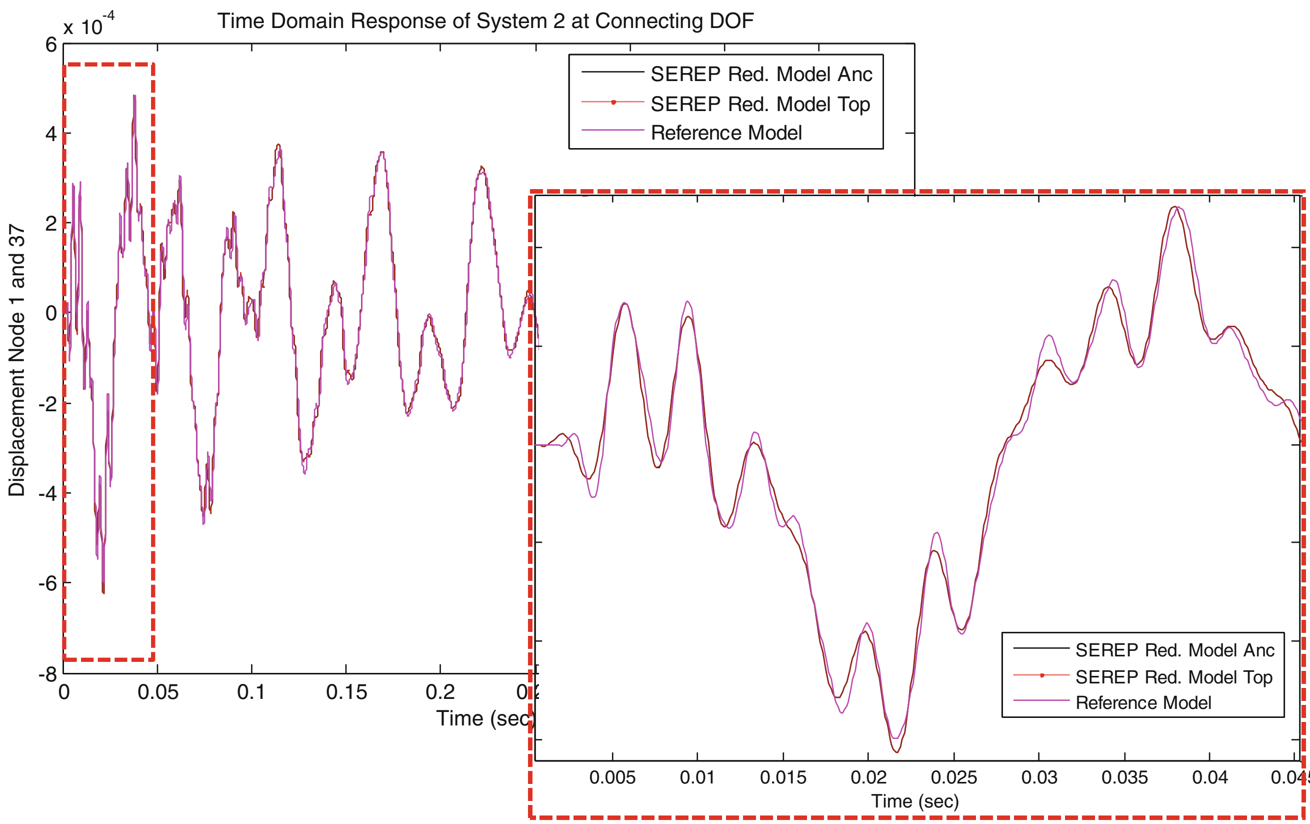


Fig. 5.23 Comparison of connecting DOF time response for both ancillary subcomponent and *top red beam* of System 2

Guyan and SEREP reduction were used to obtain the reduced order models but mode truncation in the Guyan reduced system resulted in poor correlation of the response of the system. SEREP reduced models and the expanded models were shown to accurately preserve the dynamics of the system as well as the dynamics of the embedded ancillary subcomponent; the reduced model did not contain any DOF associated with the ancillary connection to the subcomponent. No additional gain was found from the addition of the connecting DOF of the subcomponents as long as the modes selected were sufficient in accordance to the U_{12} contribution matrix.

KM_AMI updating of the Guyan reduced mass and stiffness matrices with target frequencies and mode shapes showed to mitigate the inherent errors in Guyan reduction process and ADOF selection. Addition of modes to the SEREP and KM-AMI models was shown to give significant gains in the correlation of the system to the full space reference solution.

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References

1. Thibault L (2012) Development of equivalent reduced model technique for linear modal components interconnected with nonlinear connection elements. Master’s Thesis, University of Massachusetts Lowell
2. Marinone T (2012) Efficient computational nonlinear dynamic analysis using modal modification response technique. Master’s Thesis, University of Massachusetts Lowell
3. Marinone T, Thibault L, Avitabile P (2013) Expansion of nonlinear system response using linear transformation matrices from reduced component model representations. Proceedings of the thirty-first international modal analysis conference, Garden Grove, CA, Feb 2013
4. Pingle P, Niezrecki C, Avitabile P (2011) Full field numerical stress-strain from dynamic experimental measured data. EURODYN 2011, Leuven, Belgium, July 2011

5. Avitabile P, Harvie J (2013) Reduced order system model nonlinear response and expansion for full field results. Eleventh international conference on recent advances in structural dynamics, Pisa, Italy, July 2013
6. Avitabile P (2003) Twenty years of structural dynamic modification – a review. Sound and Vibration Magazine, pp 14–27
7. Thibault L, Avitabile P, Foley J, Wolfson J (2012) Equivalent reduced model technique development for nonlinear system dynamic response. Proceedings of the thirtieth international modal analysis conference, Jacksonville, FL, Feb 2012
8. Guyan RJ (1965) Reduction of stiffness and mass matrices. AIAA J 3(2):380
9. O'Callahan JC (1989) A procedure for an improved reduced system (IRS) model. Proceedings of the seventh international modal analysis conference, Las Vegas, NV, Feb 1989
10. O'Callahan JC, Avitabile P, Riemer R (1989) System equivalent reduction expansion process. Proceedings of the seventh international modal analysis conference, Las Vegas, NV, Feb 1989
11. Nonis C, Avitabile P (2013) System model modes developed from expansion of uncoupled component dynamic data. Eleventh international conference on recent advances in structural dynamics, Pisa, Italy, July 2013
12. Marinone T, Butland A, Avitabile P (2012) A reduced model approximation approach using model updating methodologies. Conference proceedings of the society for experimental mechanics series 2012, Topics in modal analysis II, vol 6, pp 625–636
13. Newmark NM (1959) A method of computation for structural dynamics. J Eng Mech Div ASCE 85(3):67–94
14. Allemang RJ, Brown DL (2007) A correlation coefficient for modal vector analysis. Proceedings of the first international modal analysis conference, Orlando, FL, Feb 2007
15. Van Zandt T (2006) Development of efficient reduced models for multi-body dynamics simulations of helicopter wing missile configurations. Master's Thesis, University of Massachusetts Lowell
16. MAT_SAP/MATRIX (1986) A general linear algebra operation program for matrix analysis. Dr. John O'Callahan, University of Massachusetts Lowell
17. MATLAB R2010a. The MathWorks, Natick, MA 2010