Chapter 4
The New Role of Mathematical Risk Modeling and Its Importance for Society

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This book on risk and security is an example for the new role of mathematical modeling in science. In Newtonian times, mathematical models were mainly applied to physics and astronomy (e.g., planetary systems) as definitive mappings of reality. They aimed at explanations of past events and predictions of future events. Models and theories were empirically corroborated or falsified by observations, measurements and lab experiments. Mathematical predictions were reduced to uniquely determined solutions of equations and the strong belief in one model as mapping of reality. In probabilistic models, extreme events were underestimated as improbable risks according to normal distribution. The adjective "normal" indicates the problematic assumption that the Gaussian curve indicates a kind of "natural" distribution of risks ignoring the fat tails of extreme events. The remaining risks are trivialized. The last financial crisis as well as the nuclear disaster in Japan are examples of extreme events which need new approaches of modeling.

Mathematical models are interdisciplinary tools used in natural and engineering sciences as well as in financial, economic and social sciences. Is there a universal methodology for turbulence and the emergence of risks in nature and financial markets? Risks which cannot be reduced to single causes, but emerge from complex interactions in the whole system, are called *systemic risk*. They play a dominant role in a globalized world. What is the difference between microscopic interactions of molecules and microeconomic behavior of people? Obviously, we cannot do experiments with people and markets in labs. Here, the new role of computer simulations and data mining comes in.

These models are mainly stochastic and probabilistic and can no longer be considered as definitive mappings of reality. The reason is that, for example, a financial crisis cannot be predicted like a planetary position. With this methodic misunderstanding, the political public blamed financial mathematics for failing anticipations. Actually, probabilistic models should serve as stress tests. *Model ambiguity* does not allow to distinguish a single model as definitive mapping of reality. We have to consider a whole class of possible stochastic models with different weights. In this way,

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we can overcome the old philosophical skepticism against mathematical predictions from David Hume to Nassim Taleb. They are right in their skepticism against classical axiomatization of human rationality. But they forget the extreme usefulness of robust stochastic tools if they are used with sensibility for the permanent model ambiguity. It is the task of philosophy of science to *evaluate risk modeling* and to consider their *interdisciplinary possibilities and limits*.

Keywords Risk modeling · Systemic risk · Model scepticism · Risk measuring · Rational behavior

The Facts

- Mathematical modeling is crucial for understanding the dynamics of natural and societal systems.
- The emergence of systemic risks can be explained in nonlinear models of systems science.
- Philosophy of science delivers criteria of good models and their application in risk modeling.
- In risky situations, model skepticism is a challenge of research.
- Risk modeling has its historical origin in financial and insurance mathematics.
- Securitized credit models, their increasing networks of risks, and the crisis of risk modeling lead to a new paradigm of risk measuring and rational behavior.
- We can no longer trust in a single risk model, but we must consider a class of more or less appropriate models, supplemented by experimental behavioral case studies.

1 Introduction

Mathematical models are mathematical descriptions of systems in different sciences. They refer in particular to natural systems in astronomy (e.g., planetary systems), physics (e.g., atomic systems), chemistry (e.g., molecular bonds), and biology (e.g., cellular networks), but also to social systems in economics (e.g., financial markets), sociology (e.g., social networks) and political science (e.g., administrative organizations). When engineers analyze a technical system to be controlled or optimized, they also use a mathematical model. In mathematical analysis, engineers can build a model of the system as a hypothesis of how the system should work, or try to estimate how an unforeseeable event could affect the system. Examples are extreme events and risks emerging in complex systems. Similarly, in control of a system, engineers can try out different control approaches in simulations. Simulations are often represented by computer programs and tested on computers (Bungartz et al. [1]). In the natural sciences, the validity of models is tested by derived explanations or predictions which are confirmed or falsified by observations, measurements and experiments. A hypothetical model is a more or less appropriate mapping of reality.

A *mathematical model* usually describes a system by a set of variables and a set of equations that establish relationships between the variables (cf. Gershenfeld [5], Weidlich [13], Yang [14]). A *dynamical system* is characterized by its elements and the time-depending development of their states. The *states* can refer to moving planets, molecules in a gas, gene expressions of proteins in cells, excitation of neurons in a neural net, nutrition of populations in an ecological system, or products in a market system. The *dynamics* of a system, i.e. the change of system states depending on time, can mathematically be described by, e.g., time-depending differential equations. In physics, a *conservative system*, e.g. an ideal pendulum, is determined by the reversibility of time direction and conservation of energy. *Dissipative systems*, e.g., a real pendulum with friction, are irreversible. In a more intuitive way, a conservative system is "closed" with respect to external influences and only determined by its intrinsic dynamics. A dissipative system can be considered to be "open" to external influences, e.g., air or other material friction forces of the pendulum. Models of conservative and dissipative systems can also be applied in ecology and economics.

Case Study (Conservative and Dissipative Systems in Ecology) At the beginning of the 20th century, fishermen in the Adriatic Sea observed a periodic change of numbers in fish populations. These oscillations are caused by the interaction between predator and prey fish. If the predators eat too many prey fish, the number of prey fish and then the number of predators decreases. The result is that the number of prey fish increases, which then leads to an increase in the number of predators. Thus, a cyclic change of both populations occurs. In 1925, the Italian mathematicians Lotka [36] and Volterra suggested a dynamical model to describe the prey and predator system. Each *state* of the model is determined by the numbers of prey fish and the number of predator fish. So the *state space* of the model is represented by a two-dimensional Euclidean plane with a coordinate for prey fish and a coordinate for predator fish. The observations, over time, of the two populations describe a dotted line in the plane. Births and deaths change the coordinates by integers, a few at a time. To apply continuous dynamics, the dotted lines must be idealized into continuous curves. Obviously, the *Lotka-Volterra model* is closed to other external influences of, e.g., temperature or pollution of the sea. If these external forces of "ecological friction" were added to the model, its dynamics would change the cyclic behaviour.

Case Study (Conservative and Dissipative Systems in Economy) In 1967, the economist Goodwin proposed a conservative dynamical model to make the 19th-century idea of class struggle in a society mathematically precise (cf. Goodwin [26], Mainzer [7]). He considered an economy consisting of workers and capitalists. Workers spend all their income on consumption, while capitalists save all their income. Goodwin used a somewhat modified predator-prey model of Lotka and

Volterra. This *conservative model* supports the idea that a capitalist economy is permanently oscillating. Obviously it is superficial, because it does not refer directly to the functional income shares of capitalists and workers or to their population size. But it is mainly its conservative character that makes Goodwin's model seem economically unrealistic. Thus, the model has been made more realistic by the assumption of "*economic friction*". In reality, an economic system cannot be considered as isolated from other dynamical systems. An economic model of coupled oscillatory systems is provided by international trade. In other cases, economic systems are influenced by political interventions. We will come back to these examples later on.

Mathematical models can be classified in several ways (Mainzer [8, 9]). In classical physics, dynamics of a system is considered a *continuous process*. But, continuity is only a mathematical idealization. Actually, a scientist has single observations or measurements at discrete-time points which are chosen equidistant or defined by other measurement devices. In discrete processes, there are finite differences between the measured states and no infinitely small differences (differentials) which are assumed in a continuous process. Thus, *discrete processes* are mathematically described by difference equations.

Random events (e.g., Brownian motion in a fluid, mutation in evolution, innovations in economy) are represented by additional fluctuation terms. *Classical stochastic processes*, e.g. the billions of unknown molecular states in a fluid, are defined by time-depending differential equations with distribution functions of probabilistic states. In *quantum systems* of elementary particles, the dynamics of quantum states is defined by Schrödinger's equation with observables (e.g., position and momentum of a particle) depending on Heisenberg's principle of uncertainty which only allows probabilistic forecasts of future states.

2 Emerging Risks in Complex Dynamical Systems

2.1 Linear and Nonlinear Models

Historically, during the centuries of classical physics, the universe was considered a deterministic and conservative system. We say that a system is *deterministic* when future events are causally set by past events. A finite-difference equation like $x_{t+1} = f(x_t)$ is deterministic as long as $f(x_t)$ has only one value for each possible value of x_t. Given the past value x_t, the function f determines the future value x_{t+1}. The astronomer and mathematician P.S. Laplace (1814) assumed the total computability and predictability of nature if all natural laws and initial states of celestial bodies are well known. The Laplacean spirit expressed the belief of philosophers in determinism and computability of the world during the 18th and 19th century.

Laplace was right about linear and conservative dynamical systems. In general, a *linear relatio*n means that the rate of change in a system is proportional to its cause:

Small changes cause small effects while large changes cause large effects. Changes of a dynamical system can be modeled in one dimension by time series with changing values of a time-depending quantity along the time axis. Mathematically, linear equations are completely solvable. This is the deeper reason for Laplace's philosophical assumption to be right for linear and conservative systems.

In systems theory (Mainzer [8, 9, 39]), the complete information about a dynamical system at a certain time is determined by its state at that time. The *state of a complex system* is determined by more than two quantities. Then, a *higher dimensional state space* is needed to study the dynamics of a system. From a methodological point of view, time series and phase spaces are important instruments to study systems dynamics. The state space of a system contains the complete information of its past, present and future behavior.

Case Study (State Space in Ecology) Let us consider the state space of a Lotka-Volterra system of predator and prey fishes. The vector field on the *two-dimensional state space* can roughly be described in terms of four regions (Fig. 1a). In region A, both populations are relatively low. When both populations are low, predator fish decreases for lack of prey fish while prey fish increase because of less predation. The interpretation of this habitual tendency as a bound velocity vector is drawn as an arrow. In region B, there are many prey fish, but relatively few predators. But when there are many prey fish and few predator fish, both populations increase. This is interpreted by the vector in region B. In region C, both populations are relatively large. The predator fish are well fed and multiply, while the prey fish population declines. This tendency is shown by the vector in region C. In region D, there are few prey fish but many predator fish. Both populations decline. This tendency is shown by the vector in region D. The *phase portrait* of this system can be visualized by a closed trajectory, because the flow tends to circulate.

In Fig. 1b, the *phase portrait* is a nest of closed trajectories, around a central equilibrium point. As dynamical systems theory tells what to expect in the long run, the phase portrait enables the ecologist to know what happens to the two populations in the long run. Each initial population of predator and prey fish will recur periodically.

If some kind of ecological friction were added to the model, the center would become a point attractor. This would be a model for an ecological system in static equilibrium (Fig. 1c). A different but perhaps more realistic modification of the model results in a *phase portrait* like Fig. 1d, with only one periodic trajectory.

At the end of the 19th century, H. Poincaré (1892) discovered that celestial mechanics is not a completely computable clockwork, even if it is considered a deterministic and conservative system. The mutual gravitational interactions of more than two celestial bodies ('*Many-bodies-problem*') can be illustrated by causal feedback loops analytically represented by nonlinear and non-integrable equations with instabilities and irregularities. In a strict dynamical sense, the degree of complexity depends on the degree of nonlinearity of a dynamical system. According to the Laplacean view, similar causes effectively determine similar effects. Thus, in the

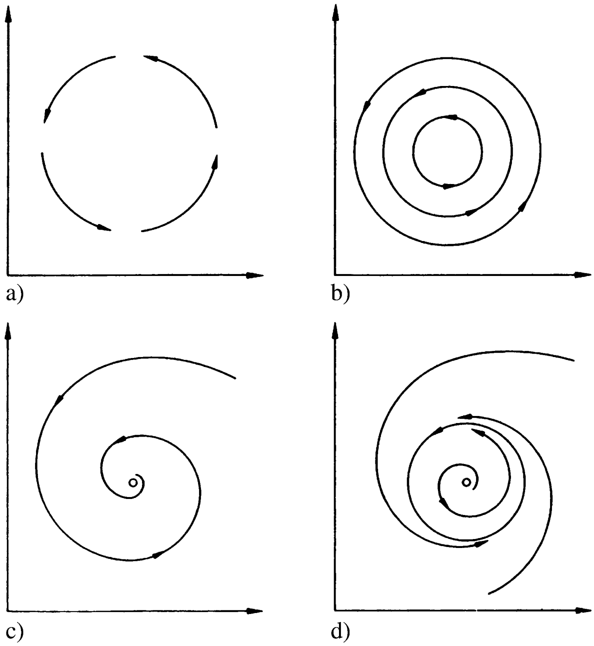

Fig. 1 Phase portraits of an ecological system with a prey and predator population (Lotka-Volterra): (**a**) a closed trajectory, (**b**) a nest of closed trajectories, (**c**) a point attractor, (**d**) a periodic trajectory [7, p. 114]

state space, trajectories that start close to each other also remain close to each other during time evolution. Dynamical systems with deterministic chaos exhibit an exponential dependence on initial conditions for bounded orbits: the separation of trajectories with close initial states increases exponentially.

Important Consequence for Risk Analysis (Butterfly Effect of Chaotic Dynamics) Consider two trajectories starting from nearly the same initial data. In chaotic dynamics only a tiny difference in the initial conditions can result in the two trajectories diverging exponentially quickly in the state space after a short period of time (Fig. 2). In this case, it is difficult to calculate long-term forecasts, because the initial data can only be determined with a finite degree of precision. Tiny deviations in digits behind the decimal point of measurement data may lead to completely different forecasts. This is the reason why attempts to forecast weather fail in an unstable and chaotic situation. In principle, the wing of a butterfly may cause a global change of development. This "*butterfly effect*" can be measured by the so-called *Lyapunov exponent*. A trajectory $\mathbf{x}(t)$ starts with an initial state $\mathbf{x}(0)$. If it develops exponentially fast, then it is approximately given by $|\mathbf{x}(t)| \sim |\mathbf{x}(0)|e^{\Lambda t}$. The exponent Λ is smaller than zero if the trajectory is attracted by attractors, such as stable points or orbits. It is larger than zero if it is divergent and sensitive to very small perturbations of the initial state.

Thus, tiny deviations of initial data lead to exponentially increasing computational efforts for future data limiting long-term predictions, although the dynamics is in principle uniquely determined. According to the famous *KAM-Theorem* of A.N. Kolmogorov (1954), V.I. Arnold (1963), and J.K. Moser (1967), trajectories in the

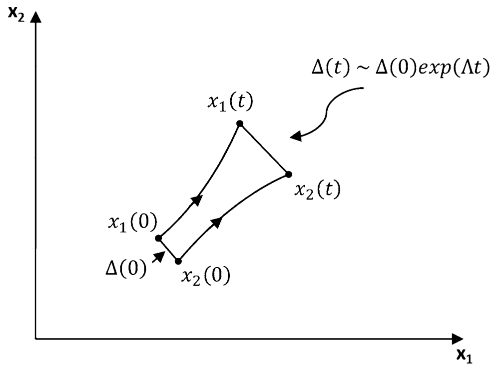

Fig. 2 Exponential dependence on initial conditions measured by Lyapunov exponent Λ [7, p. 83]

phase space of classical mechanics are neither completely regular, nor completely irregular, but depend sensitively on the chosen initial conditions.

Models of dynamical systems can be classified on the basis of the effects of the dynamics on a region of the state space (Weidlich [13]). A conservative system is defined by the fact that, during time evolution, the volume of a region remains constant, although its shape may be transformed. In a dissipative system, dynamics causes a volume contraction.

An *attractor* is a region of a state space into which all trajectories departing from an adjacent region, the so-called basin of attraction, tend to converge. There are different kinds of attractors (Lorenz [35]). The simplest class of attractors contains the *fixed points*. In this case, all trajectories of adjacent regions converge to a point. An example is a dissipative harmonic oscillator with friction: the oscillating system is gradually slowed down by frictional forces and finally come to a rest in an equilibrium point.

Conservative harmonic oscillators without friction belong to the second class of attractors with *limit cycles*, which can be classified as being periodic or quasi-periodic. A periodic orbit is a closed trajectory into which all trajectories departing from an adjacent region converge. For a simple dynamical system with only two degrees of freedom and continuous time, the only possible attractors are fixed points or periodic limit cycles. An example is a Van der Pol oscillator modeling a simple vacuum-tube oscillator circuit.

In continuous systems with a state space of dimension $n > 2$, more complex attractors are possible. Dynamical systems with *quasi-periodic limit cycles* show a time evolution which can be decomposed into different periodic parts without a unique periodic regime. The corresponding time series consist of periodic parts of oscillation without a common structure. Nevertheless, closely starting trajectories remain close to each other during time evolution. The third class contains dynamical systems with *chaotic attractors* which are non-periodic, with an exponential dependence on initial conditions for bounded orbits. A famous example is the chaotic attractor of a Lorenz system simulating the chaotic development of weather caused by local events, which cannot be forecast in the long run (*butterfly effect*).

2.2 Linear and Nonlinear Time Series Analysis

In the previous chapter we have analyzed dynamical systems and their types of behavior with fixed points, limit cycles, and chaos. Modeling means that these mathematical systems are applied to physical, biological or social systems of interest. The Lotka-Volterra equations, for example, constitute a mathematical system modeling the interaction of prey and predators in zoology. Modeling in this way is a top down procedure from mathematical equations to applications by appropriate interpretations of variables. In a bottom up approach, we start with a sequence of measurements and ask what the data themselves can tell us about the laws of dynamics. Sequences of data are called *times series*. Time series analysis is used to find types of appropriate equations fitting the data, or to compare the predictions of mathematical models to measurements made in the field of research.

In an ideal case, *time-series analysis* delivers a computer program providing a mathematical model fitting the measured data. But these data-generated models have a severe shortcoming, because they work without any understanding of the physical system. In practice, model building is combined with times-series analysis. Model building is based on knowledge of a physical system, while time-series analysis can be used to detect features of a system, inspiring model building.

Dynamical systems are governed by difference equations of the form $x_{t+1} = f(x_t)$ or differential equations of the form $dx/dt = g(x, y)$ and $dy/dt = h(x, y)$ with time-depending variables $x(t)$ and $y(t)$. In a *top-down approach* of modeling, the functions f, g, and h are given and the dynamical behavior with, e.g., fixed points, limit cycles, and chaos attractor is derived by mathematical analysis. In a *bottom-up approach*, we can only measure a limited set of quantities with limited precision. In our example of prey and predator dynamics, we might be able to measure the population of the predator only, although predator and prey are correlated and important for the dynamics of the whole prey and predator system.

For a mathematical model of observed data, we need an equation relating the measurements to the corresponding dynamical variables. The measurements approximate the dynamical variables with a difference which is called the *measurement error*. The measurement error depends on several factors like systematic bias, measurement noise, and dynamical noise. Systematic bias means a deficiency in the measurement process. Measurement noise results from random fluctuations in measurements. Dynamical noise is affected by outside influence, because dynamical systems are not isolated. A prey and predator system, for example, does not only depend on the two variables of prey and predator, but also on the environment with climate, nutrition, temperature et al.

Case Study (Linear Model of Dynamics) The dynamics of a finite-difference equation $x_{t+1} = A + \rho x_t$ has a steady state at $x_t = A/(1 - \rho) = M$ which is stable if $|\rho| < 1$. The solution to the finite-difference equation is exponential decay to the steady state. After the transient passes, there is steady-state behavior $x_t = M$. A direct measurement of the dynamical variable x_t is assumed. But, with respect to *measurement noise*, the measurement data at time t is $D_t = x_t + W_t$, where W_t is a

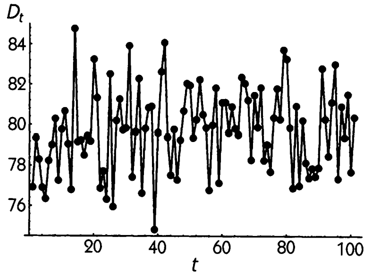

Fig. 3 Data dynamics of a linear model [6, p. 286]

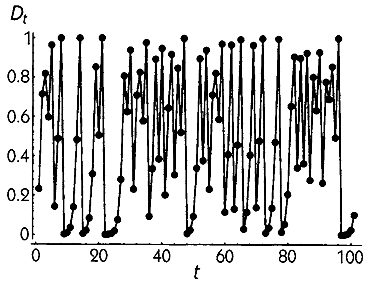

Fig. 4 Data dynamics of a nonlinear model [6, p. 302]

random number independently at each t in a Gaussian probability distribution with a mean of zero and standard deviation σ. Figure 3 shows data D_t generated by this model with $A = 4$, $\rho = 0.95$, and $M = 80$. P is Gaussian white measurement noise with a standard deviation of $\sigma = 2$.

The model describes a system maintained at a steady level (e.g., a population level or amount of prices at a market) without outside perturbations. For the interpretation of measured data, the model leads to following questions:

– What is the value of the steady state in the data?
– What is the level of measurement noise in the data?
– Is there evidence that there really is a steady state?
– Is there evidence that there is only measurement noise and no outside perturbations to the state x_t?

Case Study (Nonlinear Model of Dynamics) The previous model has *linear dynamics* and the stable fixed point is approached asymptotically in the absence of dynamical noise. *Nonlinear models* can have non fixed asymptotic behavior. For example, the quadratic map $x_{t+1} = \mu x_t(1 - x_t)$ can show a variety of behavior from stable fixed points to stable periodic cycles and chaos. The equation indicates no dynamical noise. Further on, there is no measurement noise, $D_t = x_t$ (Fig. 4). Thus, the model is completely deterministic. In this case all future data can be calculated for given initial conditions. In the case of chaos, there are practical limitations with respect to the sensitive dependence of the chaotic dynamics on initial data.

For a nonlinear model, the following questions may arise:

– What evidence is there that the data are generated by a deterministic process?

– What evidence is there for a nonlinear process?
– How large is the sensitive dependence on initial data in the case of chaos?

The mean of the data in Fig. 4 is $M_{est} = 0.471$. The fluctuations about the mean $V_t = D_t - M_{est}$ can be used to calculate the correlation coefficient between V_{t+1} and V_t. This is $\rho_{est} = 0.054$, close to zero. In fact, the autocorrelation function for the data of the nonlinear model is very similar to that for the data of the linear model. This suggests that the data from the nonlinear model are white noise, apparently contradicting the fact that the data are from a deterministic model. This paradox is solved by the fact that the correlation coefficient and the autocorrelation function measure linear correlations in the data. A scatter plot of V_{t+1} and V_t shows a very strong relationship, but actually the relationship is nonlinear and hence not accurately represented by the correlation coefficient and autocorrelation function.

Obviously, statistics of correlation coefficient and autocorrelation function cannot distinguish between the data in linear and nonlinear models. *Nonlinear time series analysis* helps to reconstruct nonlinear dynamics of a system from measured data. The idea of using a scatter plot to display the relationship between successive measurements is fundamental to the analysis of data from nonlinear systems. They are also called *return plot*, *Poincaré map*, or *return map*. In many cases, data are collected from a continuous-time dynamical system defined by differential equations rather than finite-difference equations. In these cases, it is appropriate to use the phase-plane or embedding reconstruction procedure to find the laws of dynamics from measured data.

Case Study (Harmonic Oscillator and Nonlinear Time Series Analysis) As an example, we consider a second-order differential equation describing a harmonic oscillator which is often used to model natural or economic systems with oscillating behavior (cf. Kaplan [6] p. 306): $d^2x/dt^2 = -bx$. In order to illustrate the flow of dynamics in a harmonic oscillator, this equation is rewritten with two first-order differential equations $dx/dt = y$ and $dy/dt = -bx$ for the variables x and y as coordinates of the phase plane of the system. In a *bottom-up approach*, we start with measuring a *time-series* $D(t) = x(t)$. In a next step, we must reconstruct the *state plane* and the flow on it from the measured data. At any instant, the position on the state plane is given by the coordinates (x, y) representing the state of the dynamical system at that instant. We can also measure $y(t)$ from $D(t)$ by noticing that $y = dx/dt = dD/dt$. If we plot dD/dt versus D, the trajectory in the state plane describes the flow based on the measured data.

But the harmonic oscillator is only a special case because dx/dt provides y. In general, dynamics on the state plane are given by a pair of coupled differential equations $dx/dt = f(x, y)$ and $dy/dt = g(x, y)$. Again, the question arises how to calculate the values of y if only $x(t)$ is measured. Measuring $x(t)$ and calculating dx/dt provide a direct measurement of x and a calculated value of $f(x, y)$. Some information about y is contained in the value of $f(x, y)$, and sometimes this information helps to an idea of the *whole dynamics* of the system.

Example (Chaotic Behavior and Weather Forecasting) Two-dimensional dynamics in a state plane cannot represent *chaotic behavior*. A continuous-time system generating chaos must consist of, at least, three equations. As an example, the Lorenz system of (simplified) weather forecasting is modeled by the three equations $dx/dt = 10(y - x)$, $dy/dt = 28x - y - xy$, and $dz/dt = 28xy - 8z/3$. If the values of $x(t)$, $y(t)$, and $z(t)$ can be measured simultaneously, it is easy to reconstruct the dynamics in a three-dimensional phase space. But if only one of the variables, e.g., $D(t) = x(t)$, can be measured, one must use heuristic procedures to reconstruct a model from measured data faithful to the geometry of the original.

2.3 Deterministic and Stochastic Models

Measurements are often contaminated by unwanted noise which must be separated from the signals of specific interest. Further on, in order to forecast the behavior of a system, the development of its future states must be reconstructed in a corresponding state space from a finite sequence of measurements. Thus, *time-series analysis* is an immense challenge in different fields of research from, e.g., climatic data in meteorology, ECG-signals in cardiology, and EEG-data in brain research to economic data of economics and finance. Beyond the patterns of dynamical attractors, randomness of data must be classified by statistical distribution functions.

Typical phenomena of our world, such as weather, climate, the economy and daily life, are much too complex for a simple deterministic description to exist. Even if there is no doubt about the deterministic evolution of, e.g., the atmosphere, the current state whose knowledge would be needed for a deterministic prediction contains too many variables in order to be measurable with sufficient accuracy. Hence, our knowledge does not usually suffice for a deterministic model. Instead, very often a stochastic approach is more situated. Ignoring the unobservable details of a complex system, we accept a *lack of knowledge*. Depending on the unobserved details, the observable part may evolve in different ways. However, if we assume a given probability distribution for the unobserved details, then the different evolutions of the observables also appear with specific probabilities. Thus, the lack of knowledge about the system prevents us from deterministic predictions, but allows us to assign probabilities to the different possible future states. It is the task of a time series analysis to extract the necessary information from past data.

Complex models contain nonlinear feedback, and the solutions to these are usually obtained by numerical methods (Bungartz et al. [1]). Statistical complex models are data driven and try to fit a given set of data using various distribution functions. There are also hybrids, coupling dynamic and statistical aspects, including deterministic and stochastic elements. Simulations are often based on computer programs, connecting input and output in nonlinear ways. In this case, models are calibrated by training the programs, in order to minimize the error between output and given test data.

Example (Power Laws and Risks) In the simplest case of statistical distribution functions, a *Gaussian distribution* has exponential tails situated symmetrically to the far left and right of the peak value. Extreme events (e.g., disasters, tsunamis, pandemics, worst case of nuclear power plants) occur in the tails of the probability distributions (Embrechts et al. [2]). Contrary to the Gaussian distribution, probabilistic functions $p(x)$ of heavy tails with extreme fluctuations are mathematically characterized by *power laws*, e.g., $p(x) \sim x^{-\alpha}$ with $\alpha > 0$. Power laws possess scale invariance corresponding to the (at least statistical) self-similarity of their time series of data. Mathematically, this property can be expressed as $p(bx) = b^{-\alpha} p(x)$ meaning that the change of variable x to bx results in a scaling factor independent of x while the shape of distribution p is conserved. So, power laws represent *scale-free* complex systems. The Gutenberg-Richter size distribution of earthquakes is a typical example of natural sciences. Historically, Pareto's distribution law of wealth was the first power law in the social sciences with a fraction of people presumably several times wealthier than the mass of a nation (Mainzer [8]).

3 Criteria of Risk Modeling in Philosophy of Science

3.1 What is a Good Model?

Mathematical modeling problems are often classified into black-box or white-box models, according to how much a priori information is available of the system. A *black-box model* is a system of which there is no a priori information available. A *white-box model* is a system where all necessary information is available. Practically all systems are somewhere between the black-box and white-box models, so this concept only works as an intuitive guide for approach.

Usually it is preferable to use as much a priori information as possible to make a model more accurate (cf. Gershenfeld [5]). Therefore the white-box models are usually considered easier, because if one has used the information correctly, then the model will behave correctly. Often the a priori information comes in forms of knowing the type of functions relating different variables. For example, if we make a model of how a climate model works in an ecological environment, we know that usually the amount of data is a varying function. Thus we are still left with several *unknown parameters*: how rapidly does pollution increase, and what is the initial state of the system? This example is therefore not a completely white-box model. These parameters have to be estimated through some means before one can use the model.

In black-box models one tries to estimate both the functional form of relations between variables and the numerical parameters in those functions. Using a priori information we could end up, for example, with a set of functions that probably could describe the system adequately. If there is no a priori information we would try to use functions as general as possible to cover all different models. The problem with using a large set of functions to describe a system is that estimating the parameters becomes increasingly difficult when the amount of parameters (and different types of functions) increases.

Another basic issue is the *complexity of a model*. If we were, for example, modeling the route of a railway plane, we could embed each mechanical part of the train into our model and would thus acquire an almost white-box model of the system. However, the computational cost of adding such a huge amount of detail would effectively inhibit the usage of such a model. Additionally, the uncertainty would increase due to a complex system, because each separate part induces some amount of variance into the model. It is therefore usually appropriate to make some approximations to reduce the model to a sensible size. Engineers often can accept some approximations in order to get a more robust and simple model. For example Newton's classical mechanics is an approximated model of the real world. Still, Newton's model is quite sufficient for most ordinary-life situations, that is, as long as particle speeds are well below the speed of light, and we study macro-particles only with respect to Einstein's theory of relativity and to quantum physics.

An important part of the modeling process is the *evaluation of an acquired model*. How do we know whether a mathematical model describes the system well? This is not an easy question to answer. Usually the engineer has a set of measurements from the system which are used in creating the model. Then, if the model was built well, the model will adequately show the relations between system variables for the measurements at hand. The question then becomes: how do we know that the measurement data is a representative set of possible values? Does the model describe well the properties of the system between the measurement data (interpolation)? Does the model describe well events outside the measurement data (extrapolation)?

Extrapolations are a challenge with increasing complexity of models. How well does this model describe events outside the measured data? Is it an adequate mapping of reality? Let us consider Newtonian classical mechanics-model, again. Newton made his measurements without advanced equipment, so he could not measure properties of particles travelling at speeds close to the speed of light. Likewise, he did not measure the movements of molecules and other small particles, but macro particles only. It is then not surprising that his model does not extrapolate well into these domains, even though his model is quite sufficient for ordinary life physics.

3.2 Model Skepticism—From David Hume to Nassim Taleb

Since Newton's century, there have been deep doubts in causality and the reliability of model-based predictions. An important progress of this criticism was the British philosopher David Hume (1711–1776) who was—like Adam Smith—one of the most important figures of Scottish Enlightenment. From a methodological point of view, Hume's critical analysis of human reason was a milestone in the history of philosophy. Kant mentioned that it was Hume waking him up from his "dogmatic slumbers". The problem concerns the question of how we are able to make inductive inferences. *Inductive inference* is reasoning from the observed behavior of objects to their behavior when unobserved. As Hume said, it is a question of how things behave when they go beyond the present test by our senses, and the records of our memory. He noticed that we tend to believe that things behave in a regular manner, i.e., that patterns in the behavior of objects will persist into the future, and throughout the unobserved present.

Hume's argument is that we cannot rationally justify the claim that nature will continue to be uniform, as justification only allows two arguments, and both of these are inadequate. According to Hume, the two sorts are: (1) *demonstrative reasoning*, and (2) *probable reasoning*. With regard to (1), Hume argues that the regularity of nature cannot be demonstrated, as, without logical contradiction, we can assume that nature might stop being regular. Considering (2), Hume argues that we cannot hold that nature will continue to be uniform because it has been in the past, as this is using the very sort of reasoning (induction) that is under question: it would be circular reasoning. Thus no form of justification will rationally warrant our inductive inferences.

Hume's solution to this skeptical problem is to argue that, rather than reason, it is natural *instinct* that explains our ability to make inductive inferences. He asserts that "All inferences from experience, therefore, are effects of custom, not of reasoning". (Hume [31]).

On the same line, the Lebanese philosophical essayist and practitioner of finance Nassim Taleb has argued in front of the recent financial crisis (Taleb [12]). His argument centers on the idea that predictive models are based on axiomatic "*Platonism*" (cf. Popper [47]), gravitating towards mathematical purity and failing to take some key ideas into account, such as: complete information is impossible, small unknown variations in the data could have a huge impact, and flawed models are based on empirical data without considering events that have not taken place but could have taken place. These rare and risky events are symbolized as "*black swans*" against the general belief that all swans are white. From a methodological point of view, Taleb follows Sir Karl Popper's philosophy of falsification (Popper [11]).

Logical Excursion (Falsification and Black Swans) In more details, Popper argues in the following way. A *general hypothesis* like "All swans are white" has the logical form "For all objects x is assumed: if x is a swan, then x is white". This general statement is especially true for a special object x_o, i.e. "If x_o is a swan, then x_o is white". Let the condition of this conclusion be true for a special object x_o, i.e. "x_o is a swan" is true. Then, our hypothesis *predicts* for the special swan x_o that it is white. This prediction follows by a logical direct conclusion (modus ponens): let A and B be propositions which can be either true or false. The *direct conclusion* (*modus ponens*) claims if A is true and conclusion $A \rightarrow B$ ("if A, then B") is true, then B is true. If we observe that the prediction is true, i.e. the observed swan x_o is actually white, then the general hypothesis is only *corroborated* by the example x_o, but not *verified* for *all* possible cases. In general, it is not possible to verify a general statement of empirical sciences for all possible objects, locations, and points of time. Only in mathematics, we can verify a general proposition on all natural numbers by a proof of complete induction. Therefore, according to Popper, a general hypothesis in empirical sciences can logically only be *falsified*, but not verified: Again, let the condition A ("x_o is a swan") be true. By observation, the swan is not white, but black, i.e. B is false. Then, the conclusion $A \rightarrow B$ must be false by logical reasons. In this case, the general hypothesis "All swans are white" is said to be falsified by the example x_o of a black swan.

The occurrence of black swans may be rare, but we must take black swan events into account. Therefore, according to Taleb, the foundations of quantitative economics are faulty and highly self-referential. He states that statistics is fundamentally incomplete as a field, as it cannot predict the risk of rare events, a problem that is acute in proportion to the rarity of these events. Taleb sees his main challenge as mapping his ideas of "*robustification*" and "*anti-fragility*", that is, how to live and act in a world we do not understand, and build robustness to black swan events. He advocates what he calls a "black swan robust" society, meaning a society that can withstand difficult-to-predict events. Like Hume he argues that, rather than mathematical modeling, it is natural instinct that explains our ability to make inductive inferences. He favors "*stochastic tinkering*" as a method of scientific discovery, by which he means experimentation and fact-collecting instead of top-down directed modeling.

3.3 Human Instinct, Probabilistic Thinking, and the Brain

Most of Taleb's critique could only be detected by sophisticated mathematical analysis. Thus, the question arises how Hume's and Taleb's confidence in *human instinct* can be sufficient in front of a world with increasing complexity. Traditionally, philosophy of science defended the belief in human rationality and the possibility of logical reasoning. Therefore, in the 20th century, logical empirism argued for scientific rules of inductive reasoning.

Logical Excursion (Inductive Logic) Since Isaac Newton, induction was proclaimed a fundamental method to derive a general natural law or hypothesis from observational data and measurements. Although there is no logical justification to derive a general proposition for all cases of a domain from some confirmed examples, logicians and philosophers of science suggested formal rules to handle the problem of induction. Rudolf Carnap (1891–1970) suggested a probabilistic calculus of hypotheses. The probability of a hypothesis h is defined as degree of belief in h with respect to given data of experience e. The task of inductive logic is the definition of a function of confirmation $c(h, e) = r$, which correlates an inductive resp. a priori-probability r to the proposition h. Carnap's c-function was defined on elementary propositions, complex propositions of logically connected elementary propositions, and general propositions for infinite many cases (e.g., all space-time points). But his axioms were too weak for practical applications. Thus, he did not longer rely in one unique inductive method, but suggested a class of different confirmation functions. Anyway, in modern philosophy of science, probabilistic arguments and the meaning of probability play a crucial role. To evaluate the probability of a hypothesis, the concept of *Bayesianism* assumes some prior probability, which is then updated with respect to new data (Hacking [28]).

Carnap also initiated a logical theory of rational decisions under risk. The degree of belief of a person at time T is defined by a belief function Cr which is interpreted as betting quotient. Obviously, this approach makes no sense in the natural sciences, because natural laws do not depend on betting. In social sciences, decisions under risks depend on personal degrees of belief which Carnap assumed to be measurable. But, in modern brain research and cognitive science, gut feeling is no longer only a source of irrationality. New insights in human intuition and unconscious experience lead to behavioral skills which are even useful in management. The philosopher of science Michael Polyani (1891–1976) introduced the term "*tacit knowledge*", in order to describe these unconscious abilities. Polyani argued that we sometimes cannot only more than we can express by language, but that all kind of knowledge is based on tacit knowledge (Polyani [46]). Daily activities like car driving or the routines of our jobs are rooted in unconscious abilities which were trained and learnt in earlier time. These schemes of behavior let us react under stress and risk. Without trust in these abilities, we would not be able to act under risk. Modern brain research and cognitive science are extremely interested to understand these mechanisms. Therefore, *experimental* and *behavior-oriented economics* as well as *neuroeconomics* provide important tools to complement mathematical risk modeling (Fehr [23]).

4 Classical Risk Modeling in Financial and Insurance Mathematics

In economics as well as in financial theory uncertainty and information incompleteness prevent exact predictions. A widely accepted belief in financial theory is that time series of asset prices are unpredictable. Chaos theory has shown that unpredictable time series can arise from deterministic nonlinear systems. The results obtained in the study of physical, chemical, and biological systems raise the question whether the time evolution of asset prices in financial markets might be due to underlying nonlinear deterministic dynamics of a finite number of variables. If we analyze financial markets with the tools of nonlinear dynamics, we may be interested in the reconstruction of an attractor. In time series analysis, it is rather difficult to reconstruct an underlying attractor and its dimension. For chaotic systems, it is a challenge to distinguish between a chaotic time evolution and a random process, especially if the underlying deterministic dynamics are unknown. From an empirical point of view, the discrimination between *randomness* and *chaos* is often impossible. Time evolution of an asset price depends on all the information affecting the investigated asset. It seems unlikely that all this information can easily be described by a limited number of nonlinear deterministic equations.

4.1 Beginning of Insurance Mathematics: Poisson Distribution of Risks

Mathematical modeling in finance and insurance can be traced back for centuries. Insurance of risks against the chances of life is an old topic of mankind (cf. Mainzer [8]). *Commercial insurance* dates back to Renaissance, when great cities of trading introduced bets on safe routes of ships. In the 17th century, the great British insurance company Lloyd arose from this system of bookmakers. The philosopher and mathematician Gottfried Wilhelm Leibniz (1646–1716) already suggested a health insurance in which people should pay with respect to their income. In Germany, the ingenious idea of Leibniz was realized not earlier than in the 19th century by Bismarck. In the time of Leibniz, life insurances were first applications of probability calculations.

Historical Excursion (Huygens and Insurances in the 17th Century) The Dutch physicist Christiaan Huygens (1629–1695) applied the law of large numbers to calculations of insurance rates. In his approach, an insurance is considered as a game between the insurer and clients. The insurer diminishes his risk by adapting the premium payed by a client. Let $c_1, \dots, c_n$ be the costs of the insurer and $p_1, \dots, p_n$ the probabilities that the damages happen. The expected damage of the insurer is assumed to be $p_1 c_1 + \cdots + p_n c_n$. The average gain is equal to the premium Q paid by the clients. His *risk* is zero for a premium $Q = p_1 c_1 + \cdots + p_n c_n$. The risk of clients is also zero, their loss Q and the expected gain $p_1 c_1 + \cdots + p_n c_n$. In this case, Q is called a fair premium to be paid by clients. It is assumed that the probabilities $p_1, \dots, p_n$ can be estimated according to the law of large numbers. But this assumption was the flaw of Huygens' approach. The law of large numbers cannot be applied in cases of rare damages with extreme costs.

In 1898 the Russian economist and statistician Ladislaus Josephovich Bortkiewicz (1868–1931) published a book about the Poisson distribution, titled *The Law of Small Numbers*. In this book he first noted that events with low frequency in a large population follow a Poisson distribution even when the probabilities of the events varied. Modern insurance mathematics started with the thesis of the Swedish mathematician Filip Lundberg (1876–1965). He introduced the *collective risk m*odel for insurance claim data. Lundberg showed that the homogeneous Poisson process, after a suitable time transformation, is the key model for insurance liability data. Risk theory deals with the modeling of claims that arrive in an insurance business and which gives advice on how much premium has to be charged in order to avoid ruin of the insurance company. Lundberg started with a simple model describing the basic dynamics of a *homogeneous insurance portfolio*.

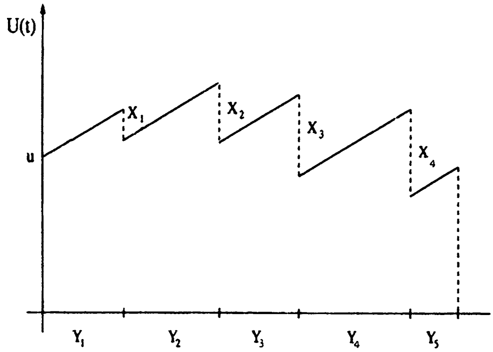

Fig. 5 A realization of Lundberg's risk process [2, p. 9]

Lundberg's Model of a Homogeneous Insurance Portfolio This means a portfolio of contracts for similar risks (e.g., car or household insurance) under three assumptions:

- Claims happen at time T_i satisfying $0 \leq T_1 \leq T_2 \leq T_3 \leq \cdots$ which are called claim arrivals.
- The ith claim arriving at time T_i causes the claim size. The latencies between the claim arrivals T_i are iid (exponential) distributed.
- The claim size process (X_i) and the claim arrival process (T_i) are mutually independent.

According to Lundberg's model, the *risk process* $U(t)$ of an insurance company is determined by the *initial capital* u, the *loaded premium rate* c and the *total claim amount* $S(t)$ of claims X_i with $U(t) = u + ct - S(t)$ and $S(t) = \sum_{i=1}^{N(t)} X_i (t \geq 0)$. $N(t)$ is the *number of the claims* that occur until time t. Lundberg assumed that $N(t)$ is a *homogeneous Poisson process*, independent of (X_i). Figure 5 illustrate a realization of the risk process $U(t)$.

Lundberg's model is fine for *small claims*. But the question arises how the global behaviour of $U(t)$ is influenced by individual extreme events with *large claims*. Under Lundberg's condition of small claims, Harald Cramér estimated bounds for the *ruin probability* of an insurance company which are exponential in the initial capital u. Actually, claims are mostly modeled by *heavy-tailed distributions* like, e.g., Pareto which are much heavier than exponential.

4.2 *Beginning of Financial Mathematics: Gaussian Distribution of Risks*

With the up-coming stock markets during the period of industrialization, people became more and more interested in their risky dynamics. Asserts price dynamics are assumed to be stochastic processes. An early key-concept to understand stochastic processes was the random walk. The first theoretical description of a *random walk* in the natural sciences was performed in 1905 by Einstein's analysis of molecular interactions. But the first mathematization of a random walk was not realized in physics, but in social sciences by the French mathematician Louis Jean Bachelier (1870–1946). In 1900 he published his doctoral thesis with the title "*Théorie de la Spéculation*" [17]. During that time, most market analysis looked at stock and bond prices in a causal way: something happens as cause and prices react as effect. In complex markets with thousands of actions and reactions, a causal analysis is even difficult to work out afterwards, but impossible to forecast beforehand. One can never know everything. Instead, Bachelier tried to estimate the odds that prices will move. He was inspired by an analogy between the diffusion of heat through a substance and how a bond price wanders up and down. In his view, both are processes that cannot be forecast precisely. At the level of particles in matter or of individuals in markets, the details are too complicated. One can never analyze exactly how every relevant factor interrelate to spread energy or to energize spreads. But in both fields, the broad pattern of probability describing the whole system can be seen.

Bachelier introduced a stochastic model by looking at the bond market as a *fair game*. In tossing a coin, each time one tosses the coin the odds of heads or tails remain 1:2, regardless of what happened on the prior toss. In that sense, tossing coins is said to have no memory. Even during long runs of heads or tails, at each toss the run is as likely to end as to continue. In the thick of the trading, price changes can certainly look that way. Bachelier assumed that the market had already taken account of all relevant information, and that prices were in equilibrium with supply matched to demand, and seller paired with buyer. Unless some new information came along to change that balance, one would have no reason to expect any change in price. The next move would as likely be up as down.

Actually, prices follow a *random walk*. Imagine a blind drunk staggering across an open field. How far will he have gotten after some time? He could go one step left, two steps right, three backwards, and so on in an aimless path. On average, just as in tossing coins, he gets nowhere. On the average, his random walk will be forever stuck at his starting point. In the same way, the prices on markets can go up or down, by big increments or small. With no new information to push a price in one direction or another, a price on average will fluctuate around its starting point. In that case, the best forecast is the price today. Each variation in price is unrelated to the last. In a stochastic model, the price-changes form a sequence of *independent* and *identically distributed random variables*. In that case, a chart of changes in price from moment to moment illustrates a more or less *uniform distribution over time*. The size of most price changes varies within a narrow range. There are also bigger fluctuations. But they barely stand up from the bulk of changes, as some outliers of grass rise above the average height of an unmown lawn, in that most of the blades of grass fall within a narrow range of heights, while a minority rise above this range (Mainzer [8], Mandelbrot and Hudson [10]).

In order to illustrate this smooth distribution, Bachelier plotted all of a bond's price-changes over a month or year onto a graph. In the case of independent and identically distributed price-changes, they spread out in the well-known bell-curve shape of a *normal* ("*Gaussian*") *distribution*: the many small changes clustered in the center of the bell, and the few big changes at the edges. Bachelier assumed that price changes behave like the random walk of molecules in a Brownian motion. Long before Bachelier and Einstein, the Scottish botanist Robert Brown had studied the way that tiny pollen grains jiggled about in a sample of water. Einstein explained it by molecular interactions and developed equations very similar to Bachelier's equation of bond-price probability, although Einstein never knew that. In 1923 (*Journal of Mathematical Physics* 2, 131–174), Norbert Wiener proved the existence of Brownian motion and considered advanced related mathematical theories. Therefore, Brownian motion is also called a Wiener process. It is a remarkable *interdisciplinary coincidence* that the *movement of security prices*, the *motion of molecules*, and the *diffusion of heat* are described by mathematically analogous models.

Bachelier's Hypotheses of Price Changes In short, Bachelier's model depends on the three hypotheses of (1) *statistic independence* (i.e., each change in price appears independently from the last), (2) *statistic stationarity* of price changes, and (3) *normal distribution* (i.e., price changes follow the proportions of the Gaussian bell curve).

4.3 Models of Efficient Markets and Computable Risks

But it took a long time that economists recognized the practical virtues of describing markets by the laws of chance and Brownian motion (Mainzer [8], Mandelbrot and Hudson [10]). In 1956, Bachelier's idea of a fair game was used by Paul A. Samuelson and his school to formulate the *Efficient Markets Hypothesis*. They argued that in an ideal market, security prices fully reflect all relevant information. A financial market is a fair game in which buyer balances seller. By reading price charts, analyzing public information, and acting on inside information, the market quickly discounts the new information that results. Prices rise or fall to reach a new equilibrium of buyer and seller. The next price change is, once again, as likely to be up as down. So, one can expect to win half the time and loose half the time. If one has special insights into a stock, one could profit from being the first in the market to act on it. But one cannot be sure to be right or first, because there are many clever people in a market as intelligent as oneself.

Since Samuelson Bachelier's theory was not only elaborated into a mature theory of how prices vary and how markets work. It was more important for the financial world that the theory has been translated into practical tools of finance. In the 1950s, Markowitz [43] was inspired by Bachelier to introduce *Modern Portfolio Theory* (MPT) as a method for selecting investments. In the early 1960s, Sharpe [51] devised a method of valuing an asset, called Capital Asset Pricing Method (CAPM). A third tool is the *Black-Scholes formula* for valuing options contracts and assessing risk. Its inventors were Black and Scholes [18] in the early 1970s. These three innovations, CAPM, MPT, and Black-Scholes, are still the fundamental tools of classical financial theory until today, resting on Bachelier's hypotheses of financial markets.

Black-Scholes Conditions of Financial Markets The Black-Scholes formula tries to implement risk-free portfolios. Black and Scholes assumed several conditions of financial markets: (1) The change of price $Y(t)$ at each step t can be described by the stochastic differential equation of a *geometric Brownian motion*. This assumption implies that the changes in the (logarithm of) price are Gaussian distributed. (2) *Security trading* is continuous. (3) *Selling of securities* is possible at any time. (4) There are no *transaction costs*. (5) The *market interest rate r* is constant. (6) There are no *dividends* between $t = 0$ and $t = T$ (maturity).

(7) There are no *arbitrage opportunities*. Arbitrage is a key concept for the understanding of markets. It means the purchase and sale of the same or equivalent security in order to profit from price discrepancies. A stock may be traded in two different stock exchanges in two different countries with different currencies. By buying several shares of the stock in New York and selling them in Frankfurt, the arbitrager makes a profit apart from the transaction costs. Traders looking for arbitrage opportunities contribute to a market's ability to evolve the most rational price for a good. The reason is obvious: if someone has discovered an arbitrage opportunity and succeeded in making a profit, he will repeat the same action. After carrying out this action repeatedly and systemically for several opportunities, the prices will be adapted and no longer provide arbitrage opportunities. In short: New arbitrage opportunities continually appear in markets. But as soon as they are discovered, the market moves in a direction to eliminate them gradually (Mandelbrot and Hudson [10]).

Now, in the absence of arbitrage opportunities, the change in the value of a portfolio must equal the (expected) gain obtained by investing the same amount of money in a riskless security providing a return per unit of time. The assumed dynamics of prices allows to derive the *Black-Scholes partial differential equation* which is valid for both call and put European options. Under some boundary conditions and substitutions the Black-Scholes partial differential equation becomes formally equivalent to the heat-transfer equation of physics which is analytically solvable.

Assumptions of Classical Economic Models These financial tools are deeply rooted in assumptions of classical economic models, but refuted by observables of real human behavior (Mandelbrot and Hudson [10]):

1. Assumption: People are rational in the sense of Adam Smith's *homo oeconomicus*. Consequently, when presented with all the relevant information about a stock or bond, investors will make the obvious rational choice leading to the greatest possible wealth and happiness. Their preferences can be expressed in mathematical formulas of utility functions which can be maximized. By that, rational investors make a rational model of an efficient market. Actually, people do not

only think in terms of mathematical utility functions, and are not always rational and self-interested. They are driven by emotions distorting their decisions. Sometimes, they miscalculate probabilities and feel differently about loss than gain.

2. Assumption: *All investors are alike*. Consequently, people have the same investment goals and react and behave in the same manner. In short: they are like the molecules in an idealized gas of physics. An equation that describes one such molecule or investor can be replaced to describe all of them. Actually, people are not alike. If one drops the assumption of homogeneity, one gets a more complex model of the market. For example, there are at least two different types of investors: a fundamentalist believes that each stock has its own value and will eventually sell for that value. On the other side, a chartist ignores the fundamentals and only watchs the price trends in order to jump on or off band waggons. Their interactions can lead to price bubbles and spontaneously arising crashes. The market switches from a well-balance linear system in which one factor adds predictably to the next, to a chaotic nonlinear system in which factors interact with the emergence of synergetic and unanticipated effects.
3. Assumption: *Price change is practically continuous*. Consequently, stock quotes or exchange rates do not jump up or down, but move smoothly from one value to the next. In this way, continuity has been assumed in classical physics, according to the motto of Leibniz "natura non facit saltum" (nature does not make leaps) which was repeated by Alfred Marshall in his text book "Principles of Economics" (1890) for economic systems. From a methodological point of view, the belief in a continuous behavior of nature and economy opens the possibility to apply continuous functions and differential equations, in order to solve physical or economic problems analytically. But actually, prices in economy and quantum states in quantum physics do jump, and discontinuity, far from being an anomily, characterizes the reality. Contrary to Einstein's famous objection against quantum physics: god plays with dice—in nature and society.
4. Assumption: *Price changes follow* a *Brownian motion*. The Brownian motion is also a famous model of physics applied to financial markets by Bachelier. In more details, it implies three assumptions: first, each change in price is believed to appear independently from the last (statistical independence). Second, the process generating price changes stays the same over time (statistical stationarity). Third, price changes follow the proportions of the Gaussian bell curve (normal distribution). Financial data clearly contradict to a smooth normal distribution of changing prices. The analysis of the real distribution patterns is a challenge of stochastic mathematics and systems theory and opens new avenues to the complexity of modern society.

4.4 Securitized Credit Model and Increasing Networks of Risks

Nevertheless, the demand for profit and security has initiated a wave of financial innovation, based on these classical assumptions. They are focused on the origination, packaging, trading and distribution of *securitised credit instruments*. Simple forms of securitised credit have existed for almost as long as modern banking. But from the mid-1990s the system entered explosive growth in both scale and complexity. We observe a huge growth in the value of the total stock of credit securities, an explosion in the complexity of the securities sold, with the growth of structured credit products, and with the related explosion of the volume of credit derivatives, enabling investors and traders to hedge underlying credit exposures, or to create synthetic credit exposures.

This financial innovation sought to satisfy the demand for yield uplift. It was predicated on the belief that by slicing, structuring and hedging, it was possible to create value, offering investors combinations of risk, return, and liquidity which were more attractive than those available from the direct purchase of the underlying credit exposures. It resulted not only in massive growth in the importance of securitised credit, but also in a profound change in the nature of the securitised credit model. As securitisation grew in importance from the 1980s on, its development was praised as a means to reduce banking system risks and to cut the total costs of credit intermediation, with credit risk passed through to end investors, reducing the need for unnecessary and expensive bank capital. Credit losses would be less likely to produce banking system failure (Turner [54]).

But there is no "*free lunch*" or financial "*perpetuum mobile*". When the crisis broke, it became apparent that this diversification of risk holding had not actually been achieved. Instead most of the holdings of the securitised credit, and the vast majority of the losses which arose, were not in the books of end investors intending to hold the assets to maturity, but on the books of highly leveraged banks and bank-like institutions. This reflected an evolution of the securitised credit model away from the initial descriptions. To an increasing extent, credit securitised and taken off one bank's balance sheet, was not simply sold through to an end investor, but bought by the propriety trading desk of another bank, sold by the first bank but with part of the risk retained via the use of credit derivatives, resecuritised into increasingly complex instruments (e.g. CDOs and CDO squareds) or used as collateral to raise short-term liquidity (International Monetary Fund [32]).

The financial innovations of structured credit resulted in the creation of products, e.g. the lower credit tranches of CDOs or even more so of CDO-squareds, which had very high and imperfectly understood embedded leverage, creating positions in the trading books of banks which were hugely vulnerable to shifts in confidence and liquidity. This process created a complex chain of multiple relationships between multiple institutions, each performing a different small slice of the credit intermediation and maturity transformation process, and each with a leveraged balance sheet requiring a small slice of capital to support that function (Sinn [52]). A *complex network of dependences* has emerged in a hidden and intransparent world of financial shadows. The new model left most of the risk still somewhere on the balance sheets of banks and bank-like institutions but in a much more complex and less transparent way.

The evolution of the *securitised credit model* was accompanied by a growth in the relative size of financial services within economy, with activities internal to the banking system growing far more rapidly than end services to the real economy. The growing size of the financial sector was accompanied by an increase in total system leverage. But this process also drived the boom and created vulnerabilities of the whole financial network that have increased the severity of the crisis. According to the Turner Report [54], from about 2003 onwards, there were significant increases in the measured on-balance sheet leverage of many commercial and investment banks, driven in some cases by dramatic increases in gross assets and derivative positions. This was despite the fact that measures of leverage (e.g. Value at Risk (VaR) relative to equity) showed no such rise. This divergence reflected the fact that VaR measures of the risk involved in taking propriety trading positions, in general suggested that risk relative to the gross market value of positions had declined. It is clear in retrospect that the VaR measures of risk were faulty (Stutz [53]).

4.5 The Risk of Value at Risk (VaR)

The increasing complexity of the securitised credit market was obvious to some participants, regulators and academic observers (Greenspan [27]). But the predominant assumption was that increased complexity had been matched by the evolution of mathematically sophisticated and effective techniques for measuring and managing the resulting risks (Colander et al. [19]). Central to many of the techniques was the concept of *Value-at-Risk* (VaR), enabling inferences about forward-looking risk to be drawn from the observation of past patterns of price movement. The *risk-forecasting models* of value-at-risk (VaR) are based on the assumption that forecasting credit risk is an activity not unlike that of forecasting weather. It is assumed that one's own action, based on past volatility, does not affect future volatility itself just like forecasting weather does not influence future weather.

This technique, developed in the early 1990s, was not only accepted as standard across the industry, but adopted by regulators as the basis for calculating trading risk and required capital. Therefore, VaR was incorporated within the European Capital Adequacy Directive (Danielsson et al. [21]). In financial mathematics and financial risk management, Value at Risk (VaR) is a widely used risk measure of the risk of loss on a specific portfolio of financial assets. For a given portfolio, probability and time horizon, VaR is defined as a *threshold value* such that the probability that the mark-to-market loss on the portfolio over the given time horizon exceeds this value in the given probability level. VaR has five main uses in finance: risk management, risk measurement, financial control, financial reporting and computing regulatory capital (Kleeberg and Schlenger [33]). VaR is sometimes used in non-financial applications as well. Important related ideas are economic capital, backtesting, stress testing and expected shortfall.

Mathematical Definition of VaR Mathematically (Föllmer and Schied [3, 24]; compare also Chap. 5 of Biagini et al.), the uncertainty in the future of a portfolio is usually described by a function $X : \Omega \to R$, where Ω is a fixed set of scenarios.

For example, X can be the *value of a portfolio*. The goal is to determine a number $\rho(X)$ that quantifies the risk and can serve as a capital requirement or the minimal amount of capital which, if added to the position and invested in a risk-free manner, makes the position acceptable. Given some *confidence level* $\alpha \in (0, 1)$, the *Value at Risk* (VaR) of the portfolio value X at the confidence level α is given by the smallest number $m \in R$ such that the probability of a loss is not larger than the confidence level α:

$$VaR_\alpha(X) = \inf\bigl\{m \in R \,|\, P(X + m < 0) \le \alpha\bigr\}.$$

Obviously, value at risk (VaR) only pays attention that the boundary of the confidence level is not exceeded. But, it does not consider the degree of loss. Further on, it assumes that the probability distribution of losses is well-known because of historical data. Only in this case value at risk (VaR) can forecast credit risk like weather, which means that future volatility can be derived from past volatility.

There are, however, fundamental questions about the validity of VaR as a measure of risk. The use of VaR measures based on relatively short periods of historical observation (e.g. 12 months) introduced dangerous procyclicality into the assessment of trading book risk (Turner [51]). Short-term observation periods and the assumption of normal distribution can lead to large *underestimation of probability of extreme loss events*. Interconnected market events in complex networks can produce *self-reinforcing cycles* which models do not capture. *Systemic risk* may be highest when measured risk is lowest, since low measured risk encourages behavior which creates increased systemic risks.

This kind of mathematics, used to measure and manage risk by VaR, was not very well understood with all its conditions and restrictions by top management and boards to assess and exercise judgement over the risks being taken. Mathematical sophistication ended up not containing risk, but providing false assurance that other indicators of increasing risk (e.g. rapid credit extension and balance sheet growth) could be safely ignored.

The global financial system, combining with macroeconomic imbalances, created an *unsustainable credit boom* and *asset price inflation*. Those consequences of the financial crisis transmitted financial system problems into real economy effects. The shock to the banking system has been so great that its impaired ability to extend credit to the real economy has played a major role in enforcing the economic downturn, which in turn undermines banking system strength in a self-reinforcing feedback loop.

From a historical point of view, it is remarkable that the academic professionals were well aware of the methodological weakness of VaR measures. In an "Academic Response to Basel II" [21], the methodology of value-at-risk (VaR) was criticized to be insufficient: (1) VaR risk models treat risk as a fixed *exogenous* process, but its *endogeneity* may matter enormously in times of crisis. (2) VaR is a misleading risk measure when the returns are not *normally distributed*, as in the case with credit, market, and operational risk. It does not measure the distribution of risk in the tail, but only provides an estimate of a particular region in the distribution. Thus, VaR models generate imprecise and widely fluctuating forecasts.

5 New Paradigm of Risk Modeling and Rational Behavior

The development of an expanded financial sector and the rapid growth and increased complexity of the securitised model of credit intermediation was accompanied by the development of increasingly sophisticated mathematical techniques for the measurement and management of position taking risks. The techniques entailed numerous variants to cope with, for instance, different categories of option. Their application required significant computing power to capture relationships between different market prices, the complex nature of structured credit instruments, and the effects of diversification across correlated markets. But the underlying methodological assumption was the old idea that analysis of past price movement patterns could deliver statistically robust inferences relating to the probability of price movements in future.

5.1 Crisis of Risk Modeling

The financial crisis has revealed, however, severe problems with these techniques. They suggest the need for significant changes in the way that VaR-based methodologies have been applied. But, the most fundamental question concerns our ability in principle to infer *future risk* from *past observed patterns*. Can financial models still be considered true mappings of an external world in order to derive predictions of future events like in the natural sciences? (Lux and Westerhoff [37].)

Models in the tradition of Bachelier assume that the distribution of possible events, from which the observed price movements are assumed to be a random sample, is normal with the shape of a Gaussian bell curve. But there is no clearly robust justification for this assumption. Actually, the financial market movements are inherently characterized by *fat-tail distributions*. This implies that any use of VaR models needs to be analyzed by the application of stress test techniques which consider the impact of extreme movements beyond those which the model suggests are at all probable.

One explanation of fat-tail distributions may lie in the complex networks of financial dependences. VaR models implicitly assume that the actions of the individual firm, reacting to market price movements, are both sufficiently small in scale as not themselves to affect the market equilibriums, and independent of the actions of other firms. But this is a deeply misleading assumption if it is possible that developments in markets will induce similar and simultaneous behavior by numerous players. If this is the case, which it certainly was in the financial crisis, VaR measures of risk may not only fail adequately to warn of rising risk, but may convey the message that risk is low and falling at the precise time when systemic risk is high and rising.

For example, according to VaR measures, risk was low in spring 2007. Actually, the system was overwhelmed with huge *systemic risk*. This suggests that *stress tests* are needed to consider the impact of second order effects, for example, the impact on one bank of another bank's likely reaction to the common systemic stress.

5.2 A New Paradigm of Risk Modeling

The most fundamental insight is, however, philosophical: it is important to realize that the assumption that past distribution patterns carry robust inferences for the probability of future patterns is methodologically insecure. It involves applying to the world of social and economic relationships a technique drawn from the world of physics, in which a random sample of a definitively existing universe of possible events is used to determine the probability characteristics which govern future random samples. But it is doubtful when applied to economic and social decisions with inherent uncertainty. Economists sometimes refer to it as "*Knightian*" uncertainty which is a reference to the classical distinction between risk and uncertainty in Frank Knights' Ph.D. "Risk, Uncertainty, and Profit" [34] from 1921. But it would also suggest that no system of regulation could ever guard against all risks and uncertainties.

Analysis of the causes of the crisis suggests that there is a limit to the extent to which risks can be identified and offset at the level of the individual firm. We explained how the origins of the crisis lay in systemic developments: the crucial shift required in regulatory philosophy is towards one which focuses on macro-analysis, *systemic risks* and judgements about business model sustainability, and away from the assumption that all risks can be identified and managed at a firm specific level. As a result most of the changes we propose relate to the redesign of global regulation combined with a major shift in methodology (Colander et al. [19]).

But improvements in the effectiveness of *internal risk management* and *firm governance* are also essential. While some of the problems could not be identified at firm specific level, and while some well run banks were affected by systemic developments over which they had no influence, there were also many cases where internal risk management was ineffective and where boards failed adequately to identify and constrain excessive risk taking. Achieving high standards of risk management and governance in all banks is therefore essential. Detailed proposals are necessary to support an FSA (Financial Service Authority) in all countries.

The origins of the past crisis entailed the development of a complex, highly leveraged and therefore risky variant of the securitised model of credit intermediation. Large losses on structured credit and credit derivatives, arising in the trading books of banks and investment banks, directly impaired the capital position of individual banks, and because of uncertainty over the scale of the losses, created a *crisis of confidence* which produced *severe liquidity strains* across the entire system. As a result, a wide range of banking institutions suffered from an impaired ability to extend credit to the real economy, and have been recapitalized with large injections of taxpayer money.

The mathematical rigor and numerical precision of risk management and asset pricing tools has a tendency to conceal the weakness of models and their assumptions to those who have not developed them and do not know the potential weakness of the assumptions. *Models* are only approximations to the real world dynamics and partially built upon *idealized assumptions*. A typical example is the belief in normal distribution of asset price changes completely neglecting the importance of extreme events. Considerable progress has been made by moving to more sensitive models with *fat-tailed Lévy processes* (Mandelbrot [41]). Of course, such models better capture the intrinsic volatility of markets. But they might again contribute to enhancing the control illusion of the naïve user.

Therefore, market participants and regulators have to become more sensitive towards the potential weakness of risk management models. Since there is not only one true model, robustness should be a key concern. *Model uncertainty* should be taken into account by applying more than a single model. For example, one could rely on probabilistic procedures that cover a whole class of specific models. The theory of robust control provides a toolbox of techniques that could be applied for this purpose.

5.3 Convex Models of Risk

In the field of *financial economics* there are a number of ways that risk can be defined (Marrison [40]). To clarify the concept mathematicians have axiomatically described a number of properties that *a risk measure* might or might not have (Föllmer and Schied [24], York [55]).

Mathematical Definition of Coherent Risk Measure A *coherent risk measure* (Artzner et al. [16]) is a risk measure ρ that satisfies properties of *monotonicity*, *sub-additivity*, *homogeneity*, and *translational invariance*. Consider a random outcome X viewed as an element of a linear space L of measurable functions, defined on an appropriate probability space. A functional $\rho : L \to R$ is said to be a coherent risk measure for L if it satisfies the following properties:

Monotonicity: If $X_1, X_2 \in L$ and $X_1 \leq X_2$, then $\rho(X_1) \leq \rho(X_2)$.

That is, if portfolio X_2 always has better values than portfolio X_1 under all scenarios then the risk of X_2 should be less than the risk of X_1.

Sub-additivity: If $X_1, X_2 \in L$, then $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$.

Indeed, the risk of two portfolios together cannot get any worse than adding the two risks separately. This is the diversification principle.

Positive homogeneity: If $\alpha \geq 0$ and $X \in L$, then $\rho(\alpha X) = \alpha\rho(X)$.

Loosely speaking, if you double your portfolio then you double your risk.

Translation invariance: If $m \in R$ and $X \in L$, then $\rho(X + m) = \rho(X) - m$.

The value m is just adding cash to the portfolio X, which acts like an insurance. The risk of $X + m$ is less than the risk of X, and the difference is exactly the added cash m. Therefore, translational invariance is also called cash invariance. In particular, if $m = \rho(X)$ then $\rho(X + \rho(X)) = 0$.

The notion of coherence has been subsequently relaxed. Indeed, the notions of sub-additivity and positive homogeneity can be replaced by the notion of convexity:

Convexity: If $X_1, X_2 \in L$ and $0 \leq \lambda \leq 1$, then $\rho(\lambda X_1 + (1-\lambda)X_2) \leq \lambda\rho(X_1) + (1-\lambda)\rho(X_2)$.

Consider the collection of possible future outcomes that can be generated with the resources available to an investor. One investment strategy leads to X_1, while a second strategy leads to X_2. If one diversifies, spending only the fraction λ of the resources on the first possibility and using the remaining part for the second alternative, one obtains $\lambda X_1 + (1-\lambda)X_2$. Thus, the axiom of convexity gives a precise meaning to the idea that diversification should not increase the risk.

It is well known that *value at risk* (VaR) is positively homogeneous, but it is not in general a coherent risk measure as it does not respect the sub-additivity property. Hence, it is *not convex*. An immediate consequence is that *value at risk* might discourage diversification. *Value at risk* is, however, *coherent*, under the assumption of *normally distributed* losses when the portfolio value is a linear function of the asset prices. However, in this case the value at risk becomes equivalent to a mean-variance approach where the risk of a portfolio is measured by the variance of the portfolio's return. *Average value at risk at level* $\lambda \in (0, 1]$,

$$AVaR_\lambda = \frac{1}{\lambda}\int_0^\lambda VaR_\alpha(X)d\alpha$$

also called *conditional value at risk*, expected shortfall, or tail value at risk, is a coherent risk measure (Detlefsen and Scandolo [22], Riedel [48]).

We previously underlined that model uncertainty should be taken into account, since we do not know the distinguished true model of financial reality. Therefore, we should consider a whole class of possible probabilistic models with different penalty. In the dual representation theory of convex risk measures one aims at deriving their representation in a systematic manner. The class M contains possible *probabilistic models* Q which are taken more or less seriously according to the size of a *penalty function* $\pi(Q)$. In this way, we take the message of praxis seriously that we should not rely on one single model, but flexibly vary the models with respect to different contextual applications under special attention to the worst case.

Mathematical Definition of Convex Risk Measure A dual representation of a *convex risk measure* computes the *worst case expectation* taken over all *models* Q and *penalized* by $\pi(Q)$. The class M of possible probabilistic models is a set of probability measures such that the *expectation* $E_Q(X)$ is well defined for all models Q and *portfolios* X. According to Föllmer and Schied [21], the dual representation of a convex risk measure ρ has the form

$$\rho(X) = \sup_{Q\in M}\big(E_Q(-X) - \pi(Q)\big).$$

These models are no longer considered definitive mappings of reality. But they serve as stress tests. One does not rely on a fixed model, but chooses the sure side for every position and focuses on the corresponding worst case model. Thus, the model ambiguity is explicitly considered during the procedure.

5.4 Model Ambiguity and Rational Behavior

Model ambiguity is linked to the economic theory of rational behavior under uncertainty (Cont [20], Maccheroni [38]). Classical economic models are mainly built upon the two assumptions of rational expectations with well-known probabilities of utilities and a representative agent ("*homo oeconomicus*"). They imply a complete understanding of the economic laws governing the world. These models leave no place for *imperfect knowledge* discovered in empirical psychological studies of real humans (Frydman and Goldberg [4, 25]). Their behavior in financial markets is even strongly influenced by emotional and hormonal reactions. Thus, economic modeling has to take *bounded rationality* seriously. But, model ambiguity does not mean the collapse of mathematical modeling. Mathematically, a fixed probability measure of expected utilities should be replaced by a convex risk measure which simultaneously considers a whole class of possible stochastic models with different penalties. Financial praxis warned us not to rely on a fixed model, but to vary possible models in a flexible way and to pay attention to the worst case. This is also the mathematical meaning of a convex risk measure.

The differences between the overall system and its parts, macro- and microeconomics, remain incomprehensible from the viewpoint of classical rationality which assumes a *representative agent*. Since interaction depends on differences in information, motives, knowledge and capabilities, this implies heterogeneity of agents (Hayek [29, 30]). Only a sufficiently rich structure of connections between firms, households and a dispersed banking sector will allow insights in *systemic risks* and synergetic effects in the financial sector. The reductionism of the representative agent or "homo oeconomicus" has prevented economists from modeling these phenomena.

For natural scientists, the distinction between micro-level phenomena and those originating on a macro originated from the interaction of microscopic units is well-known. In those models, the current crisis would be seen as an *emergent phenomenon* of the macroeconomic activity (Aoki and Yoshikawa [15], Mainzer [40]). The reductionist paradigm blocks any understanding of the interplay between micro and macro level.

Models with interacting heterogeneous agents would also open the door to interdisciplinary research from different sciences. Complex networks of different agents or statistical physics of interacting agents can model dynamic economic systems (Mantegna and Stanley [38], McCauley [45]). *Self-organized criticality* is another area that seems to explain boom-and-bust cycles of the economic non-equilibrium dynamics (Scheinkman [49]).

6 Food for Thought

In macroeconomics, data mining is often driven by the pre-analytic belief in the validity of certain models which should justify *political* or *ideological opinions*. The political belief in deregulation of the 1990 years is a typical example. Rather than misusing statistics as a means to illustrate these beliefs, the goal should be to put theoretical models to scientific tests like in the natural sciences. We should follow the line of a more data-driven methodology.

A chain of specification tests and estimated statistical models for simultaneous systems would provide a benchmark for the tests of models based on economic behavior. Significant and robust relations within a simultaneous system would provide empirical regularities that one would attempt to explain, while the quality of fit of the statistical benchmark would offer a confidence for more ambitious models. Models that do not reproduce (even) approximately the quality of the fit of statistical models would have to be rejected. This methodological criterion also has an aspect of *ethical responsibility* of researchers: economic policy models should be theoretically and empirically sound. Economists should avoid giving policy recommendations on the base of models with a weak empirical grounding and should, to the extent possible, make clear to the public how strong the support of the data is for their models and the conclusions drawn from them.

A neglected area of methodology is the degree of connectivity and its interplay with the *stability of the complex system*. It will be necessary for supervision to analyze the network aspects of the financial system, collect appropriate data, define measures of connectivity and perform macro stress testing at the system level. In this way, new measures of financial fragility would be obtained. This would also require a new area of accompanying academic research that looks at agent-based models of the financial system, performs scenario analyses and develops aggregate risk measures. Network theory and the theory of self-organized criticality of highly connected systems would be appropriate starting points (Scheinkman and Woodford [50], Mainzer [7]).

Such scientific analysis must be supported by more practical consequences. The hedge fund market is still widely unregulated. The interplay between *connectivity*, *leverage* and *system risks* needs to be investigated at the whole level. It is highly likely that extreme leverage levels of interconnected institutions impose dangerous social risks on the public.

On the macroeconomic level, it would be desirable to develop *early warning schemes* that indicate the formation of bubbles. Combinations of indicators with time series techniques could be helpful in detecting deviations of financial or other prices from their long-run averages. Indication of structural change would be a sign of changes of the behavior of market participants of a bubble-type nature (McCauley [45]).

Obviously, there is no single causal model as definitive mapping of reality. In this sense, David Hume and his followers were right in their skepticism against classical axiomatization of rationality in the world. But that does not mean a complete deny of mathematical tools and models. We have to consider whole *classes of possible* *stochastic models* with different weights. They must be combined with a *data-driven methodology* and insights in the factual human behavior and its diversity. Therefore, *psychological* and *sociological case studies* of human behavior under risk conditions (e.g., stakeholders at stock markets) are necessary. In experimental economics, decision behavior is already simulated under laboratorial conditions. Even *philosophical ethics* can no longer only argue with arm-chaired considerations and a priori principles, but must relate to empirical observations of factual decision behavior. That is done in the new approaches of experimental ethics. We argue for this kind of *interdisciplinary methodology* which opens new avenues for mathematical modeling in science. In this case, robust stochastic tools are useful, because they are used under restricted conditions and with sensibility for the permanent model ambiguity.

7 Summary

In a globalized world, risks are mainly *systemic* and cannot be reduced to single causes. They emerge from *complex interactions* in natural, technical, economic, and social systems. Examples are complex information and communication networks, power ("smart") grids as well as cellular interactions in organisms or transactions in financial markets. Therefore, *systems theory* with *linear* and *nonlinear dynamics*, *stochastic* and *statistic modeling*, and *computer models* are important methodologies in RISE. We must consider their explanatory power as well as their limitations. Then, they can supplement themselves mutually.

But, formal models are not sufficient. *Risk-awareness* even of experts is often *subjective* and depends on individual experience, societal and cultural contexts. Remember the extremely different reactions of the public to the Fukushima disaster in Japan and Germany. Therefore, formal risk-models must be complemented by sociological and cultural studies. Psychic behavior in decision situations must also be taken into account. Therefore, experimental economics and ethics relate to observations of factual behavior of people, e.g., at stock markets. Behavioral studies under experimental lab conditions are even useful for social philosophy and ethics.

The past crises might be characterized as example of final stages of well-known boom-and-bust patterns that have been repeated so many times in the course of economic history. But, there are several new aspects leading to a shift of methodological paradigm: the preceding boom had its origin in the development of new financial products with increasing complexity which seemed to promise diminishing risks. The financial market detaches itself from the real market. Profit seems to be possible by clever financial innovations loosing their connection to real economy. But, like in nature, there is no "*free lunch*" or "*perpetuum mobile*" of profit in finance. Further on, the past crises were due to the increasing complexity of interconnected financial networks. These aspects have been largely ignored by traditional economic models.

Therefore, we cannot trust in a single risk model, but must consider a *class of more or less appropriate models*, supplemented by *experimental behavioral case* *studies*. The lack of methodological understanding of models and the lack of ethical responsibility to warn the public against the limitations of models were the main reasons of the past economic crises. It is the task of *philosophy of science* to evaluate scientific modeling and the ethical responsibility of scientists. During booming periods we should better prepare the next crisis in a countercyclical manner.

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