Assessing the Status of the Common Cause Principle

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1 Introductory Comments

Since Kant's awakening from his dogmatic slumber it has been known that general metaphysical claims cannot be verified. Nor can they be conclusively falsified empirically; especially not, as Popper has taught us, if they are pure existential claims: one cannot be sure that a certain entity does *not* exist or that a certain state of affairs is absent from the world – simply because it is impossible to check empirically the whole universe, past, present and future, to make sure the entity in question does not exist or that a particular condition never obtains.

Given this well-known lesson from history of philosophy one should be very careful when it comes to assessing the metaphysical claim about the causal structure of the world known as the Common Cause Principle. The Common Cause Principle states that if two events A and B are probabilistically correlated, then either there is a direct causal link between A and B that is responsible for the correlation, or there exists a third event C, a common cause, that brings about the correlation. The Common Cause Principle is clearly a pure existential claim about causal connections lurking behind correlations and if it could be shown to be false, then one would have falsified not only the Common Cause Principle but even the metaprinciple that metaphysical principles stating existence are non-falsifiable. This would be interesting; however, the meta-principle pronouncing the non-falsifiability of general existence claims does not seem to have been falsified by the Common Cause Principle: the aim of this paper is to argue that assessing the status of the Common Cause Principle is a very subtle problem, and that at present we do not have strictly empirical evidence that it does not hold.

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The complete argument that does justice to the complexity of the issue requires a full book (Hofer-Szabó et al. 2013). In this short paper just the main ideas and the most important concepts and claims are presented in a sketchy manner, typically without giving precise definitions of all the concepts involved. Specifically, standard mathematical notions from measure theory and operator algebra theory are used without explanation. (The Appendix in Hofer-Szabó et al. (2013) collects the most crucial mathematical facts involved.)

2 The Common Cause Principle

Given a classical probability measure space (X, S, p) with the set X of elementary events, Boolean algebra S of some subset of X and probability measure p, the events $A, B \in S$ are called positively correlated if

$$p(A \cap B) > p(A)p(B) \tag{1}$$

Reichenbach's Common Cause Principle: If A, B are correlated events then either the events A and B stand in a direct causal relation responsible for the correlation, or, if A and B are causally independent, $R_{ind}(A, B)$, then there exists a third event C causally affecting both A and B, and it is this third event, the so-called (*Reichenbachian*) common cause, which brings about the correlation by being related to A and B in a specific way spelled out in the following definition:

Definition 1. *C* is a *common cause* of the correlation (1) if the following (independent) conditions hold:

$$p(A \cap B|C) = p(A|C)p(B|C)$$
⁽²⁾

$$p(A \cap B|C^{\perp}) = p(A|C^{\perp})p(B|C^{\perp})$$
(3)

$$p(A|C) > p(A|C^{\perp}) \tag{4}$$

$$p(B|C) > p(B|C^{\perp}) \tag{5}$$

where

$$p(X|Y) \doteq \frac{p(X \cap Y)}{p(Y)}$$

denotes the conditional probability of X on condition Y, and it is assumed that none of the probabilities p(X) ($X = A, B, C, C^{\perp}$) is equal to zero.

The above notion of common cause is due to Reichenbach (1956), the Common Cause Principle was articulated especially by Salmon (see e.g. Salmon 1984) and it has been discussed extensively in the literature. The bibliography in Hofer-Szabó

et al. (2013) contains an extensive list of papers on the topic, papers published in the last 10 years include Butterfield (2007), Cartwright (2007), Henson (2005), Hofer-Szabó (2008), Hofer-Szabó (2011), Hofer-Szabó et al. (1999), Hofer-Szabó et al. (2002), Hoover (2003), Mazzola (2012a), Mazzola (2012b), Portmann and Wüthrich (2007), Rédei and San Pedro (2012), Sober (2008), Sober and Steel (2012), San Pedro (2008), Wrónski (2010), Wrónski and Marczyk (2010), Wrónski and Marczyk (2013) and Wüthrich (2004).

3 How to Assess the Status of the Common Cause Principle?

Unless one takes the very problematic position that the truth of non-analytic statements can be decided by a priori reasoning without having a look at the world, the way to assess the status of the Common Cause Principle is to turn to our best descriptions of the world, namely to our confirmed scientific theories, to see if they do comply with the Common Cause Principle by being causally complete in the sense of providing a causal explanation of all the correlations they predict – either in terms of causal connections between the correlated entities or by displaying a common cause of the correlations. Doing so one can encounter the following cases in principle:

1. Our probabilistic theories are causally complete.

This is confirming evidence for the Common Cause Principle – but not *proof* of its truth because our theories can be incomplete: not describing all the correlations that exist, and some of those they do not describe may not have a causal explanation.

- 2. *Our probabilistic theories are causally incomplete.* There are two subcases of this latter situation:
 - a. A theory is causally incomplete but causally completable. These theories do not provide disconfirming evidence for the Common Cause Principle.
 - A theory is causally incomplete and causally incompletable.
 Only such theories provide disconfirming evidence for the Common Cause Principle.

By causal completion of a probabilistic theory *T* that models the world in terms of a classical probability space (X, S, p) is meant a causally complete theory *T'* that describes the probabilistic aspect of the world by another probability measure space (X', S', p') that is an extension of the probability space (X, S, p) in the sense that there exists a Boolean algebra homomorphism $h: S \to S'$ that preserves the measure: p'(h(A)) = p(A) for all $A \in S$.

The essential message of this paper is that probabilistic theories are either causally complete, or, if they are not, then they *are* causally completable; furthermore, if we require reasonable, well-motivated conditions on the common cause *in*

addition to the four conditions in the definition of common cause, which obviously make causal completability of probabilistic theories more difficult, then it is an open problem whether good candidates for causally incomplete theories (quantum theory) are in fact causally incomplete, and if so, whether they are causally incompletable.

In view of the above it is therefore of interest to find out if probabilistic theories are causally complete. A particularly strong form of causal completeness is *common cause completeness*: a probability space (X, S, p) is common cause complete by definition if it contains a common cause of every correlation it predicts and it is called *common cause completable* with respect to a pair (A, B) of correlated events A, B in it if there exists an extension (X', S', p') of (X, S, p) such that (X', S', p')contains a common cause of the correlation between A and B. Call a probability space *strongly* common cause completable with respect to a pair (A, B) of correlated events A, B in it if, given any *type* of a common cause, there exists an extension (X', S', p') of (X, S, p) such that (X', S', p') contains a common cause of the given type of the correlation between A and B. "Type" of the common cause refers here to some additional probabilistic constraints one can in principle impose on the common cause (see Hofer-Szabó et al. 1999 for details).

It is not difficult to see that probability spaces with a finite number of elementary events are typically *not* common cause complete, not even causally complete with respect a causal independence relation R_{ind} satisfying certain plausible conditions (Gyenis and Rédei 2004, 2011a,b). "Large" probability spaces (probability spaces with an uncountably infinite Boolean algebra) are however common cause complete:

Proposition 1 (Gyenis and Rédei 2011).

- 1. If a classical probability space is purely nonatomic as a measure space, then it is common cause complete.
- 2. Every classical probability measure space has an extension that is purely nonatomic; hence:
- 3. Every classical probability measure space is common cause completable with respect to any set of correlated events.

Note that (3) in Proposition 1 does *not* say that every probability space is *strongly* common cause completable; whether this is true, is not known. The conjecture is that measure theoretically purely nonatomic probability spaces are *strongly* common cause complete: containing a common cause of *every* admissible type of *every* correlation they predict – by Proposition 1 this would entail that any probability space is strongly common cause completable with respect to every correlation. We only have a weaker result so far¹:

¹After this paper was completed, the author was informed that the conjecture has been proved by Marczyk and Wrónski (2013). See also their review paper in this volume: M. Marczyk and L. Wrónski: "A Note on Strong Causal Closedness and Completability of Classical Probability Spaces".

Proposition 2 (Hofer-Szabó et al. 1999). Every classical probability measure space is strongly common cause completable with respect to any finite set of pairs of correlated events.

The significance of the above two propositions is that they entail that Reichenbach's Common Cause Principle can always be defended against attempts of falsification by claiming that a correlation predicted by a probability theory is due to a possibly hidden common cause – hidden in the sense of not being part of the original theory predicting the correlation.

Thus one has to be epistemically modest when assessing the truth status of the Common Cause Principle: one only can aim at finding out whether scientific theories provide confirming or disconfirming evidence for the Principle, and the propositions above also entail how one can in principle display disconfirming evidence for the Common Cause Principle:

- 1. Impose extra conditions on the common cause that are not part of Reichenbach's definition.
- 2. Argue that the additional conditions are justified.
- 3. Display a theory *T* that predicts empirically testable correlations common causes of which should satisfy the extra conditions.
- 4. Show that theory *T* is causally incomplete.
- 5. Prove that theory T is not extendable into a richer theory that contains common causes satisfying the extra conditions.

Good candidates for the above are quantum correlations, common causes of which should be *local* in some sense; it will be argued however in the next section that (4) and (5) have *not* been shown yet for quantum theory. In case of quantum correlations one has to be careful however what one means by a common cause: as we have seen, Reichenbach's definition of common cause was formulated originally within the framework of classical, Kolmogorovian probability measure spaces, and quantum theory can be viewed as non-classical probability theory.

A general non-classical probability theory is a pair (\mathcal{L}, ϕ) where \mathcal{L} is an orthocomplemented, orthomodular but non-distributive lattice (with respect to lattice operations \land, \lor, \bot) replacing the Boolean algebra of events and ϕ is a countably additive map (generalized probability) measure from \mathcal{L} into the interval [0, 1]. Special cases of generalized probability spaces are the quantum probability spaces $(\mathcal{P}(\mathcal{N}), \phi)$ with $\mathcal{P}(\mathcal{N})$ being the lattice of projections of a von Neumann algebra \mathcal{N} and ϕ being a normal state on \mathcal{N} . Standard Hilbert space quantum theory is an even more special case where $\mathcal{N} = \mathcal{B}(\mathcal{H})$ is the set of all bounded operators on a Hilbert space \mathcal{H} .

Given a general probability measure space (\mathcal{L}, ϕ) , two *compatible* elements $A, B \in \mathcal{L}$ are called correlated in ϕ if

$$\phi(A \wedge B) - \phi(A)\phi(B) > 0 \tag{6}$$

One can define then the common cause in \mathcal{L} of the correlation (6) by reformulating the conditions (2)–(5) without modification if one requires the common cause to commute with both A and B:

Definition 2. $C \in \mathcal{L}$ is called a common cause of the correlation (6) if *C* is compatible with both *A* and *B* and the following conditions hold.

$$\phi(A \wedge B|C) = \phi(A|C)\phi(B|C) \tag{7}$$

$$\phi(A \wedge B|C^{\perp}) = \phi(A|C^{\perp})\phi(B|C^{\perp})$$
(8)

$$\phi(A|C) > \phi(A|C^{\perp}) \tag{9}$$

$$\phi(B|C) > \phi(B|C^{\perp}) \tag{10}$$

where

$$\phi(X|Y) \doteq \frac{\phi(X \wedge Y)}{\phi(Y)} \tag{11}$$

Notions such as type of the common cause, (strong) causal and common cause completeness, and (strong) causal and (strong) common cause completeness can now be defined in non-classical probability measure spaces in complete analogy with the classical definitions, and one is then led to a number of problems, some of which are still open. It is important however that the non-commutative analogue of Proposition 1 holds:

Proposition 3 (Kitajima 2008; Gyenis and Rédei 2013).

- 1. If a non-classical probability space is purely nonatomic as a measure space, then it is common cause complete.
- 2. If a \mathcal{L} is a σ -complete non-atomic lattice and ϕ is a faithful general probability measure on \mathcal{L} , then the probability space (\mathcal{L}, ϕ) is measure theoretically purely non-atomic.

Note that a probability space (both classical and generalized) *can* be common cause closed and *not* purely non-atomic as a measure space, measure theoretic non-atomicity is just sufficient but not necessary for the theory to be common cause complete: if the space has at most one measure theoretic atom, then, and only then it is common cause complete (Gyenis and Rédei 2011, 2013). Also note that it is not known if the quantum analogues of (2) and (3) of Proposition 1 hold, i.e. it is not known whether every general probability space is extendable into a purely non-atomic one; it is conjectured that this is true.

4 Quantum Correlations and the Common Cause Principle

It is known that (relativistic) quantum field theory (QFT) predicts correlations between observables associated with spacelike separated hence causally independent spacetime regions such as spacelike separated double cones D_1 and D_2 : this is a consequence of violation of Bell's inequality in quantum field theory (Rédei and Summers (2002) reviews the most relevant facts about spacelike correlations in QFT). Causal completeness of QFT would therefore mean that the spacelike correlations have a common cause which is local in the sense that it is an observable (a projection) that belongs to an algebra of observables that is localized in the *intersection* of the backward light cones of the regions D_1 and D_2 . "Common cause" in the previous sentence refers to the concept of common cause in the sense of Definition 2.

If the correlation is predicted by a faithful state, then there exist common cause projections in the local von Neumann algebra associated with a double cone region $D \supset D_1 \cup D_2$: this is entailed by Proposition 3 and the highly nontrivial feature of quantum field theory that the double cone algebras are type III von Neumann algebras and that the projection lattice of type III von Neumann algebras are atomless - so faithful states on type III von Neumann algebras define a measure theoretically purely nonatomic quantum probability space $(\mathcal{P}(\mathcal{N}), \phi)$, which are common cause complete by Proposition 3. It is not known however whether there exists common causes of the spacelike correlations that are localized in the *intersection* of D_1 and D_2 . This problem has remained open since it was first formulated (Rédei 1997) (also see Chap. 12 in Rédei 1998). We only know that if a quantum field theory satisfies the Local Primitive Causality condition, then common causes exist that are localized in the union of the backward light cones of D_1 and D_2 (Rédei and Summers 2002, 2007, see also Chap. 8 in Hofer-Szabó et al. 2013). QFT is thus weakly causally complete and definitely cannot be considered as disconfirming evidence for the Common Cause Principle at this time.

It should be noted that *lattice* quantum field theory behaves differently from the perspective of causal completeness: since in lattice quantum field theory the local algebras of observables are finite dimensional matrix algebras and the quantum probability spaces that they determine are thus discrete (not purely nonatomic in the measure theoretic sense), lattice quantum field theory in (1 + 1) dimension contains spacelike correlations that do not have common causes *at all* – local or no local (Hofer-Szabó and Vecsernyés 2012). This result motivates weakening of definition notion of common cause (Definition 2) by allowing the common cause to *not* commute with the commuting correlated projections. It seems that this weakening compensates for the measure theoretic discreteness of the non-classical probability spaces in lattice quantum field theory: lattice quantum field theory in 1 + 1 dimension causes (see Chap. 8 in Hofer-Szabó et al. 2013 for the details).

The notorious EPR correlations are typically considered as disconfirming evidence for the Common Cause Principle. The standard argument is that assumption of common causes of EPR correlations entails Bell's inequality, hence in view of violation of Bell's inequality one concludes that common causes of those correlations cannot exist. A careful look at the problem reveals however that problem is very subtle and that the standard arguments are too quick.

The first relevant observation is that before even raising the issue of possibility of common causes of EPR correlations, one has to set up a probabilistic model in which the EPR correlations are described – probability statements, such as events being correlated or independent, are meaningful only within a fixed probabilistic model, not in general. It is non-trivial but provable that the EPR correlations can be represented by a classical probability measure space if the correlations are properly interpreted, i.e. if the values of quantum probabilities are considered as classical conditional probabilities, where the conditioning events are the measurement setups (see Szabó 2001; Rédei 2010; Hofer-Szabó et al. 2013). Once the probability models describing the EPR correlations have been set up, one can invoke the common cause extendability results presented in the previous section and conclude that the EPR correlations *can* in principle have common causes.

The second observation is therefore that one has to impose further conditions on the common cause models of the EPR correlations, conditions that express, in probabilistic terms, the causal relations that hold among the random events (including the random events representing the hypothetical common causes) by virtue of the fact that they have particular spatio-temporal locations. These are the "locality conditions". There are two types of locality conditions: those that are empirically testable because they involve observable random events (such as the outcome events and the events setting up the measuring device in a correlation experiment) – these are called "surface locality" conditions – and those that are *not* observed in the correlation experiment (such as the hypothetical common cause events and their occurrence in combination with observed events) – these are called "hidden locality conditions".

The third observation is that it turns out that in order to be able to derive a Bell-type inequality from the assumption of local common causes of EPR correlations, it is not sufficient to impose locality conditions: one also has to require "no conspiracy conditions". These latter requirements are statistical independence conditions expressing independence of the hypothetical common causes from the events that set up the measuring device in a fixed direction in the EPR correlation experiments. To complicate matters further, the "no conspiracy" conditions come in two forms: weak and strong. The difference is related to the fact that in an EPR correlation experiment one has more than one pair of correlations to explain in terms of common causes and each of the correlations can in principle have its own, distinct common cause that need not have anything to do with the common causes of other correlations. The strong no conspiracy condition demands then that any combination of the hypothetical common causes of the different pairs of correlated events occurring in an EPR correlation experiment is probabilistically independent of any combination of measurement setups measuring the correlations in question. Under these conditions one can prove the following.

Proposition 4 (Proposition 9.16, Hofer-Szabó et al. 2013). There exist no hidden local, strongly non conspiratorial common cause models of all the EPR correlations: there exist four directions in the left, and four directions in the right wings of the EPR measurement setup such that the 16 correlations arising from measuring spin components in the 4×4 possible pairs of direction in the left and right wings, respectively do not all have a hidden local and strongly non conspiratorial common cause model.

The proof of the above proposition is based on deriving Bell-type inequalities from the assumptions and showing that the inequality is violated by the 16 correlations (Hofer-Szabó et al. 2013, p. 170).

It must be emphasized that the assumptions in Proposition 4 *cannot* be weakened in the following sense: one can show that there *do* exist local (as opposed to *hidden* local), non-conspiratorial (as opposed to *strongly* non-conspiratorial) common cause explanations of EPR correlations (see Hofer-Szabó et al. (2013) and the references therein). The significance of this is that, since the hidden locality conditions involve non-observed events, the hidden locality conditions (Bell's inequality) that are the basis on which one claims that the EPR correlations are disconfirming evidence for the metaphysical Common Cause Principle, this is in harmony with the spirit of the opening remark in this paper: metaphysical principles cannot be conclusively (dis)proved empirically – when assessing metaphysical principles, a large amount of epistemological modesty is required.

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