Chapter 5 Truth-Makers and Convention T

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Abstract This chapter discusses the place, if any, of Convention **T** (the condition of material adequacy of the proper definition of truth formulated by Alfred Tarski) in the truth-makers account offered by Kevin Mulligan, Peter Simons, and Barry Smith. It is argued that although Tarski's requirement seems entirely acceptable in the frameworks of truth-maker theories at first sight, several doubts arise under a closer inspection. In particular, **T**-biconditionals have no clear meaning as sentences about truth-makers. Thus, the truth-maker theory cannot be considered as the semantic theory of truth enriched by metaphysical (ontological) data. The problem of truth-makers for sentences about future events is discussed at the end of this chapter.

Keywords Bivalence · Logic · Metalanguage · Metalogic · T-biconditionals

Since Convention **T** is a very important ingredient of the semantic theory of truth, every comparison of Tarski's construction with other approaches to the concept of truth must, sooner or later, discuss the equivalence:

(1) S is true if and only if A^* ,

where A is a sentence in an interpreted (this qualification is important, because it dispenses us with worries whether propositions or sentences function as bearers of truth) language \mathbf{L} , S is a name of this sentence and the symbol A^* refers to embedding, for example, via translation, of A into a metalanguage \mathbf{ML} . Convention \mathbf{T} requires that any materially correct truth-definition \mathbf{Df} logically entails every instance of (1), that is, the specialization of this scheme for an arbitrary sentence of \mathbf{L} ; such concrete equivalences are called \mathbf{T} -sentences, \mathbf{T} -biconditionals, or \mathbf{T} -equivalences. Note that \mathbf{T} -sentences are something more than usual material equivalences, because we have $\mathbf{Df} \models B$, for any B being an instance of (1) (see Woleński 2008 for a discussion of this problem). According to Tarski, (1) does not constitute a truth-definition, although it can be considered as a partial one. Take the content of the sentence A as the set of all its consequences, formally $\mathbf{Cont}(A) = Cn(\{A\})$. Clearly, $\mathbf{Cont}(\mathbf{Df}) > B$. In fact, the content of \mathbf{Df} exceeds the collection (rather the content) of all instantiations of (1), because truth-definitions usually contain elements

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(expressions) which do not occur in **T**-sentences, for instance, terms borrowed from set theory, like "the empty set" or "sequence" as in the case of the semantic definition of truth. Another argument that (1) cannot serve as an adequate truth-definition is that it holds for falsehoods as well.

Kevin Mulligan, Peter Simons, and Barry Smith (MSS for brevity) claim in their seminal paper (see Mulligan et al. 1984, 10–11; page-reference to 2007 reprint) as follows:

Putnam [...] has argued that Tarski's theory of truth, through its very innocuousness, its eschewal of "undesirable" notions, fails to determine the concept it was intended to capture, since the formal characterization still fits if we reinterpret "true" to mean, for instance, "warrantedly assertable" and adjust our interpretation of the logical constants accordingly. Putnam's conclusion [...] is that if we want to *account* for truth, Tarski's work needs supplementing with a philosophically non-neutral correspondence theory. If we are right that the Tarskian account neglects precisely the atomic sentences, then its indeterminacy is not surprising. [...]. If as we suggest, the nature of truth is underdetermined by theories like that of Tarski, then an adequate account of truth must include considerations which are other than purely semantic in the normally accepted sense. Our suggestion here—a suggestion which is formulated in a realist spirit—is that the way to such a theory lies through direct examination of the link between truth-bearers, the material of logic, and truth-makers, that in the world in virtue of which sentences or propositions are true.

Although MSS modestly call their proposal a mere suggestion, the actual task of introducing the concept of truth-makers seems to be much more ambitious, namely offering at least an outline of a full-blooded theory of truth, in particular, metaphysically grounded. In fact, their paper inaugurated a considerable and hot discussion (see Armstrong 2004, and the papers in Beebee and Dodd 2005, Monnoyer 2007, Loewe and Rami 2009).

Disregarding the details of Putnam's criticism of Tarski (however, see Woleński 2001 for a defense of the semantic definition of truth against Putnam's arguments), I will investigate how MSS's account of truth is related to that of Tarski. More precisely, I will discuss an application, if any, of Convention T when truth-makers are used in an explanation of the concept of truth. Two interpretations of the quoted passage are possible. Firstly, the concept of truth-maker supplements the vocabulary of Tarski's theory. Such a reading seems to assume that although Tarski's account correctly captures very general properties of truth, it requires additional conceptual resources in order for the indeterminacy noted by MSS to disappear. If so, Convention T should be fully preserved. Secondly, introducing truth-makers as a notional device leads to a different truth-definition which partially or even entirely is at odds with the semantic account. In this case, however, Convention T can be rejected, modified, or preserved in its full form. Each of these three possibilities should be excluded in advance; Tarski himself did not claim that his definition of truth satisfies only the requirement of material adequacy established by Convention T. MSS's literal formulations tend to be closer to the first option, because they explicitly postulate supplementing the typical semantic machinery by "a philosophically nonneutral correspondence theory" in order to achieve a determination of truth theory. This could suggest that formal semantics cum metaphysics (realist in the version of MSS) represented by the concept of truth-makers provides a fully determinate, that is, ontologically involved, theory of truth. Yet, and this circumstance suggests the second interpretation as also possible, MSS characterize "an adequate account of truth" as a philosophical construction that "must include considerations [...] other than semantic in the normally accepted sense." The meaning of the context "considerations which are other than purely semantic in the normally accepted sense" can be understood as referring to a theory radically opposed to that offered by Tarski. I will argue that MSS offer a theory which cannot be regarded as the semantic account of truth supplemented by a certain amount of metaphysics.

Alfred Rami (see Rami 2009, 3) proposed the following general characterization of truth-maker theories (I deliberately disregard all appeals to the truth-making relation as a necessary connection). All assume the so-called truth-maker principle in the following form:

(2) For every x, x is true if and only if there is a y such that y is a truth-maker for x.

This statement implies:

- (3) For every x, if x is true, then x has a truth-maker;
- (4) For every x, if x has a truth-maker, then x is true.

Implication (2) expresses truth-maker maximalism, but (4) is the principle of truth-maker purism. If we combine (3) and (4), we obtain

(5) For every x, x is true if and only if x has a truth-maker,

which is a more convenient formulation of (2), at least for my considerations in this chapter. In order to neutralize semantic antinomies, (5) should be rewritten as the scheme:

(6) S is true if and only if a sentence named by S has a truth-maker.

This equivalence can be regarded as generating formulas very close to T-sentences. Perhaps we can introduce the name "TM-biconditionals" as a label for instances of (6). Consider the sentence (i) "snow is white." Assume that English supplemented by a simple mathematical notation serves as a metalanguage. The related T-equivalence for the sentence in question can be written as

(7) The sentence "snow is white" is true if and only if **snow** \in **WHITE**,

where the word **WHITE** refers to the set of white entities. The right side of (7) translates the sentence (i) into the chosen metalanguage; this is the language of very elementary algebra of classes supplemented by logical constants and syntactic devices allowing us to form names of sentences belonging to the object language. This translation can even be interpreted as pointing out a truth-maker for the sentence in question. Thus, the affinity between **T**-equivalences and **TM**-biconditionals is striking (see Smith and Simon 2007, 80–81 for an opposite view).

Rami observes then that a truth-maker theory does not need to accept both statements (3) and (4). Speaking more precisely, he argues that it is fairly possible to accept truth-maker purism without being committed to truth-maker maximalism. On the other hand, Rami qualifies (4) as an analytic truth. His argument makes use of a *reductio ad absurdum*. Assume that A has a truth-maker, but it is untrue. If A has a truth-maker A is not true. Thus, A is true and untrue, which is impossible. In fact, MSS reject (3) in its

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full generality (see also Mulligan 2007, Smith and Simon 2007) and replace it by a restricted principle

(8) A
$$\vdash \exists tm(tm \Vdash A)$$
,

which can be read "that A is true entails that there is a truth-maker **tm** making A true." The principle (4) has the form:

(9)
$$\operatorname{tm} \Vdash A \vdash A$$
,

and its meaning is captured by the statement "what is made true, is true." Although (8) and (9) implicitly use (1), this fact is not essential, because one can replace A by "A is true" or "it is true that A" without making any appeal to T-biconditionals.

If someone accepts the maximalist truth-maker theory as David Armstrong does (see Armstrong 2004), that is, with (3), introducing a surrogate of Convention T (note that Armstrong does not take this step) no major problem arises. Let the symbol TMT refer to such a truth-maker theory. We can claim that TMT is materially adequate if and only if it entails every instance of (2) or (6), that is, a TM biconditional for any sentence. The issue looks differently in the case of the MSS account. Denote their theory by TMT'. First of all, (8) restricts the set of TM-equivalences to atomic sentences and some other cases, for instance, conjunctions of sentences which are simultaneously made true by the same objects as truth-makers. The restricted TM-scheme is expressed by

(10)
$$A(\text{is true}) \Leftrightarrow \exists \mathbf{tm}(\mathbf{tm} \Vdash A).$$

Unfortunately, we have no simple way to formulate a counterpart of Convention T for TMT'. The problem is that the limitations of (1) are extralogical in character and depend on a tension between the truth-functional (extensional) and the mereological character of truth-makers. Hence, the scope of the existential quantifier in (10) is not precisely established in advance. The situation is even much worse, because we do not know which elements of TMT' imply the correct TM-equivalences. Perhaps, this situation motivated Barry Smith and Jonathan Simon (see Smith and Simon 2007, 97) to their diagnosis that we should not define truth via truth-makers, because this task is simply unrealizable. Although TMT' justifies some, mostly very simple or elementary, T-conditionals, no generally formulated condition of its material adequacy, similar to Convention T, is available. Hence, TMT' cannot be regarded as a metaphysically improved semantic theory of truth. It should be considered as an alternative to Tarski's account.

Finally, I would like to make some remarks about the status of (3) and (4). There is a simple argument that the latter is analytic or even a theorem of (meta)logic, but the former is not. In order to make the argument easier, let me rewrite both formulas as

(11)
$$TA \Rightarrow A$$
.

(12)
$$A \Rightarrow TA$$
.

Formula (11) (see, for example, Turner 1990) is frequently adopted as one of the axioms of the logic of truth, when truth operates as a modality, but (12) is either rejected for its role in generating the Liar paradox or suitably modified as in (1). However, another motivation for rejecting (12) as universally valid can be given. This motivation is completely independent of the problem of semantic antinomies. Suppose that a three-valued logic, for example, Łukasiewicz's logic, functions as the basic system. Take a valuation v such that $v(A) = \frac{1}{2}$. Clearly, the metalogical statement ' $\mathbf{v}(A) = \frac{1}{2}$ ' is true, but TA is false. This observation shows that the implication $A \Rightarrow TA$ cannot be considered as a theorem of metalogic, although the formula $TA \Rightarrow A$ still holds in many-valued logic and its metatheory. We have here a simple analogy with alethic modal logic. The operator T behaves quite analogously to the operator \Rightarrow expressing the concept of necessity. Any modal logic admits the formula $A \Rightarrow A$ and rejects the formula $A \Rightarrow A$ as a logical truth. If we accept the implication $TA \Rightarrow A$ as tautological, but reject the reverse conditional $A \Rightarrow TA$ as logically valid, the formula $TA \Leftrightarrow A$ shares the fate of the latter and cannot be considered as a logical theorem.

Nevertheless, we have a way to justify the biconditional $TA \Leftrightarrow A$. Suppose that we accept the equivalence

(13)
$$\mathbf{F}A \Leftrightarrow \neg \mathbf{T}A \Leftrightarrow \mathbf{T} \neg A$$
,

which postulates that the falsity, non-truth of A, and the truth of not-A are equipollent, (12) becomes acceptable. Otherwise speaking, introducing bivalence legitimizes the full T-scheme as a good theorem of metalogic, provided that devices blocking semantic paradoxes are blocked. Thus, the principle of bivalence is a very important ingredient of the semantic theory of truth. It is quite unclear how TM' is related to bivalence and whether if a many-valued logic were analyzed by the conceptual machinery of truth-makers, the intermediate logical values would have their own makers or not; the same question concerns falsity-makers (see Armstrong 2004 for a discussion of falsity-makers). Consider the sentence (ii) "Tomorrow there will be a sea battle." Certainly, (ii) has no truth-maker at the present moment, but it will or will not have one tomorrow. Some authors (see Nef 2007) propose abstract truthmakers, but this way out seems to be very expensive (too expensive in my opinion) from the metaphysical point of view; the same concerns Josh Parsons's (see Parsons 2005) ideas connecting truth-makers for statements about past and future events with the realism/antirealism controversy. If we are not radical indeterminists, assertions about the future can have something like possibility-makers before they become realized or not. Even without introducing many-valued logic, the assertion "(ii) has a possibility-maker" is true, but the statement "(ii) has a truth-maker" is false. I guess that TMT' or any other non-maximalist truth-maker theory must be supplemented in order to be able to cope with statements about future. No metaphysically grounded theory of truth can ignore this issue, although purely semantic 84 J. Woleński

(model-theoretic) constructions do not need discuss this question. I am inclined to think, unlike MSS and most authors dealing with truth-makers, that semantics should be seen as autonomous in principle with respect to ontology or metaphysics. As a corollary, we have that the semantic definition of truth as such does not require any metaphysical or ontological enrichment. Thus, Convention T suffices as the condition of material adequacy as far as the issue concerns the very general properties of truth. On the other hand, nothing prevents making syntheses, realist or not, of semantics and ontology. Truth-maker theories go in this direction.

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