Braess-Type Paradox in Self-optimizing Wireless Networks

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Abstract. The Braess's paradox, also called Braess paradox, in transportation networks states that adding extra capacity to a network, when moving entities with incomplete information selfishly choose their routes, can in some cases *reduce* the overall network performance. In this paper, we observe a similar phenomenon in wireless networks. More specifically, we consider a single-cell system with two different types of access points, one of them with a fixed rate and one of them with a variable rate, i.e., a rate that depends on the number of users connected to that access point. We observe that, under certain conditions, the intersystem connection between these two types of access points does not necessarily improve the overall system performance. In other words, after the interconnection, the individual rates of users as well as their sum rate might get worse. This is similar to the original Braess paradox where adding a new route does not necessarily improve the overall traffic throughput. We develop a general model that describes under which conditions and for which families of variable rate functions this paradox happens. *abstract* environment.

Keywords: Braess paradox, game theory, wireless networks, information theory, intersystem interconnection.

1 Introduction

In traffic networks, Braess paradox [1] states the following: Suppose a road network where cars go from a starting point to a destination through various routes. The delay incurred on each route depends on the number of users on that specific route. Hence, whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. Under these conditions, one wishes to estimate the total time of travel at the equilibrium state. If each driver takes the path that looks most favorable for him, the resulting running times need not be minimal. It turns out that [1] an extension of the road network with a new road may cause a redistribution of the traffic that results in *longer* individual running times (which will not happen if a centralized approach

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is used). This is known as the Braess paradox. Key reasons for Braess paradox are the greedy behavior of individual users and the lack of global information and coordination.

In the case of wireless networks, similar phenomena may occur if users are allowed to behave selfishly. The decentralized and selfish behavior of users have recently appeared in wireless networks as a solution to cope with the excess protocol signaling in highly mobile scenarios. Indeed, centralized approaches require feedback protocols and complex optimization, which turns out to be incompatible with the coherence time of mobile flexible networks, where terminals must decide on their own, on different resource allocation criteria [2, 3]. Decentralization appears also as a first step towards self-optimization networks, which are at the heart of future networks enabling to reduce monitoring and managing costs. Consequently, game theory turned out to be a natural mathematical framework to analyze the outcome of intelligent devices which make decisions based on their local information. As a result, many game theoretical approaches [4–6] have been proposed recently to let the users autonomously optimize their resources.

The general idea is to understand to what extent local optimization provides a global optimization of the network, which is difficult to achieve for scaling networks. Unfortunately, the distributed approaches often suffer from similar paradoxes as the original Baress's paradox, due to greedy and selfish behavior and lack of global information. In particular, in many cases and for a fixed number of devices, one can show that the equilibria states with respect to a given utility are Nash equilibria that are very often far from the Pareto optimal performance. Most solutions proposed in the literature are either to introduce some pricing mechanisms or to change the network architecture.

The goal of this paper is to show that the change in the network architecture, by interconnecting two different technologies (e.g., a technology with a fixed rate and a technology with a variable rate) in order to improve the network traffic has to be done carefully as it may worsen the utilities of different users and lead to a paradoxical situation in which network performs worse than the original one. Here we analyze to what extent the intersystem connection does not improve the network performance.

The rest of this paper is organized as follows: In Section 2, we describe three possible models of wireless networks, in which intersystem connection does not improve the downlink rate of the users. The general functions for the access points and conditions under which the Braess paradox happens is considered in Section 3. Finally, Section 4 concludes the paper.

2 System Models

We consider a single-cell network with two access points (APs). A number of mobile users (terminals) are connected (Fig. 1) to these two access points.

Assume that the first access point, denoted in Fig. 1 by AP_1, offers a fixed rate r_F and the rate offered by the second access point, denoted by AP_2, offers a per user rate that depends on the number of users connected to it $r_V(n)$. An

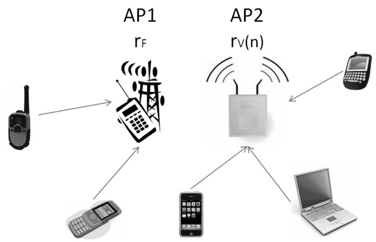

Fig. 1. A single cell with two different access points - one offering a fixed rate r_F and the other offering a rate that depends on the number n of users connected to it $r_V(n)$

example of such a system in practice could be a network that has two types of access points. The first type is an access point that uses an orthogonal multiple access scheme where each user gets a *fixed rate*. The second type is an access point that offers a *variable rate* depending on the number of users connected to it. An example of such an access point could be based on a system which offers a rate that depends on the number of users connected to it. In this system the more users are connected to it, the bigger the interference is. This leads to a lower signal-to-interference-and-noise ratio (SINR) and, therefore, to a lower rate. In the following text, we demonstrate that if one tries to improve the system by allowing an *intersystem connection*, i.e., connection between the two types of APs, this may lead to a performance that is worse than in the original system. In this section we present three different models of the variable rate access point that may lead to a Braess type of paradox in wireless networks.

2.1 Model 1

Assume that AP_2 in Fig. 1 offers the following per user rate as a function of the number of users connected to it

$$r_V(n) = \frac{r_0}{n+k}, \tag{1}$$

where n is the number of users connected to AP_2, $r_0 > 0$ is the total available rate and $k \geq 0$ is some fixed penalty that has to be paid when connected to AP_2. In other words, the total rate of the users connected to AP_2 is never equal to r_0. For instance, if we have ℓ users connected to AP_2, their sum rate will be $\ell r_0/(\ell + k)$ and is always smaller or equal to r_0.

At equilibrium, the total rate through both base stations will be the same and no user at that point will have an incentive to deviate to another access point. In that case if there are N users in the cell, we have

$$m \cdot \frac{r_0}{m+k} = (N-m)r_F, \tag{2}$$

where m is the number of users that are connected to AP_2 and the rest $(N-m)$ users are connected to AP_1. Solving (2) we get that in equilibrium:

$$m = \frac{N - k - \frac{r_0}{r_F}}{2} + \frac{\sqrt{(N - k - \frac{r_0}{r_F})^2 + 4kN}}{2}. \tag{3}$$

Note that each user connected to AP_1 has a rate of $r_1 = r_F$ and each user connected to AP_2 has a rate of $r_2 = r_0/(m+k)$. The number of users m that in equilibrium are connected to AP_2 as a function of r_0/r_f for different k is shown in Fig. 2. Note that for a given r_0/r_F, m is larger for larger k.

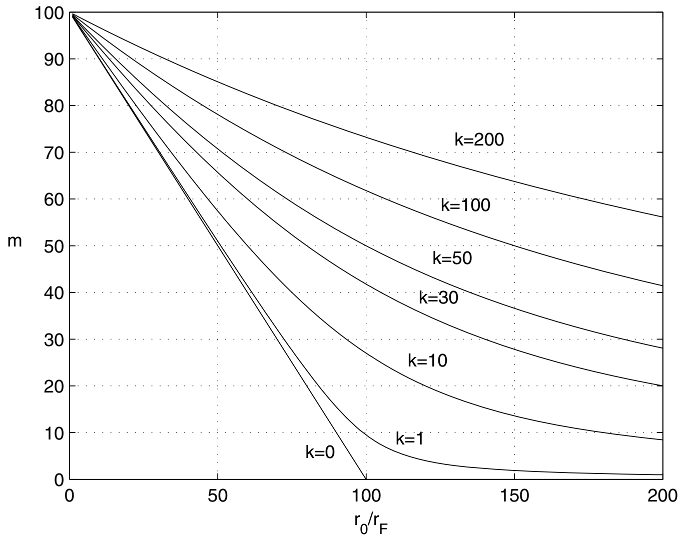

Fig. 2. The number of users m connected to AP_2 in equilibrium for the model (1), for $N = 100$ and various k

Assume now that the system provider decides to interconnect the two different technologies (and provide also terminals with the possibility to be connected on the two different technologies at the same time to split their packets), namely the access points with fixed and variable rates. With this new system, the network is no longer in equilibrium with the original rate flow since a new configuration has appeared. Users can, therefore, use a combination of the two links to send their packets. In particular, the individual rate of each user after interconnection is

$$\tilde{r}_2 = \frac{r_0}{N+k} > r_F.$$

Hence, all users would prefer to be connected to AP_2 since they get better rate than connecting to AP_1. In other words, no user has an incentive to change its access point, i.e., to connect back to AP_1.

Here, we consider two types of *paradox*. A paradox may happen with respect to the individual rate of users connected to AP_2, i.e., $\tilde{r}_2 \le r_2$. For this model, this paradox happens since each user that was connected to AP_2 gets a smaller individual rate, since after interconnection

$$\tilde{r}_2 = \frac{r_0}{N+k} < \frac{r_0}{m+k} = r_2,$$

where m is given by (3). Note that the paradox happens if $1 \leq m < N$. In the case $m = N$ noting changes after the interconnection, i.e., this case is trivial. In the case $m = 0$, the paradox does not happen, since all users previously connected to AP_1 get better rate by switching to AP_2.

More interesting paradox for the system designer is the paradox that happens with respect to the *total sum rate*. In other words, after interconnection is done, the total sum rate gets worse than before. The total sum rate in equilibrium, before interconnection is

$$r_T = (N-m)r_F + \frac{mr_0}{m+k} = 2(N-m)r_F.$$

Since after interconnection, the total sum rate is

$$\tilde{r}_T = \frac{Nr_0}{N+k},$$

the paradox happens with respect to the sum rate if

$$r_T = 2(N-m)r_F > \frac{Nr_0}{N+k} = \tilde{r}_T.$$

Example 1. Consider a system in which $N = 100$, $k = 10$, $r_0 = 120$ Mbps, and $r_F = 1$ Mbps. In this case, from (3), we get that $m = 20$. That means, in equilibrium, 20 users will be connected to AP_2 with a per user rate of $r_2 = r_0/(20+10) = 4$ Mbps, and the other 80 users will be connected to AP_1 with a fixed rate of $r_1 = r_F = 1$ Mbps. The sum rate is

$$r_T = (80 \cdot 1 + 20 \cdot 4) \text{ Mbps} = 160 \text{ Mbps}.$$

If an intersystem connection is applied, then all users will connect to AP_2 since in that case they get

$$\tilde{r}_2 = r_0/(N+k) = 120/110 \text{ Mbps} = 1.091 \text{ Mbps},$$

a rate larger than $r_F = 1$ Mbps. The paradox happens since the sum rate after interconnection is

$$\tilde{r}_T = N\tilde{r}_2 = \frac{Nr_0}{N+k} = 109.1 \text{ Mbps},$$

which is smaller than the sum rate $r_T = 160$ Mbps of the original system.

2.2 Model 2

Assume that AP_2 in Fig. 1 offers the following per user rate as a function of the number of connected users

$$r_V(n) = \frac{r_0}{n} + r_1. \tag{4}$$

Here n is the number of users connected to AP_2, $r_0 > 0$ is the total available rate and $r_1 \geq 0$ is some fixed rate that is guaranteed no matter how many users are connected to AP_2. In that case, since there are a total of N users in the cell, in equilibrium, if the access points are not interconnected we have the same amount of traffic through both access points, that is

$$m \cdot \left(\frac{r_0}{m} + r_1\right) = (N - m) r_F. \tag{5}$$

Here, m is the number of users connected to AP_2, while the rest $(N - m)$ users are connected to AP_1. Solving (5) we get that at the equilibrium state:

$$m = \frac{N r_F - r_0}{r_F + r_1} = \frac{N - r_0/r_F}{1 + r_1/r_F}. \tag{6}$$

Note that each user connected to AP_1 has a rate of $r_1 = r_F$ and each user connected to AP_2 has a rate of $r_2 = r_0/m + r_1$. The number of users m that in equilibrium are connected to AP_2 is shown in Fig. 3 for different r_1/r_F. Note that for a given r_0/r_F, m is larger for larger r_1/r_F.

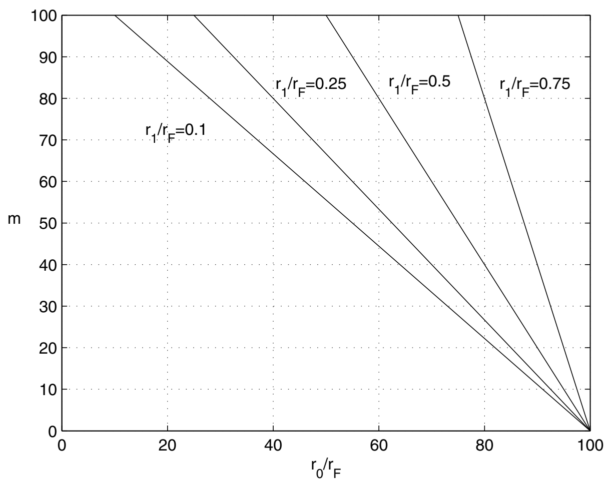

Fig. 3. The number of users m connected to AP_2 in equilibrium for the model (4), for $N = 100$ and various ratios r_1/r_0

Assume now that the system provider decides to interconnect the two access points with fixed and variable rates. In that case, if we assume that

$$\tilde{r}_2 = r_0/N + r_1 > r_F,$$

then all users prefer to be connected to AP_2 since they get a better rate if they are connected to AP_2 than to AP_1. Here again, no user has an incentive to connect to AP_1. In this case, the paradox happens since each user that was connected to AP_2 gets a smaller individual rate than in the previous case, since

$$\tilde{r}_2 = \frac{r_0}{N} + r_1 < \frac{r_0}{m} + r_1 = r_2,$$

where m is given by (6). The paradox happens if $1 \leq m < N$.

The sum rate before interconnection is

$$r_T = 2r_F(N - m) = 2m(r_0/m + r_1,)$$

while after interconnection it is

$$\tilde{r}_T = N(r_0/N + r_1).$$

Hence, the paradox happens with respect to the sum rate if

$$r_T = 2r_F(N - m) > r_0 + Nr_1 = \tilde{r}_T.$$

Example 2. Consider a system in which $N = 100$, $r_0 = 80$, $r_1 = 0.25$ Mbps, and $r_F = 1$ Mbps. In that case from (6), we get $m = 16$. This means that, in equilibrium, 16 users will be connected to AP_2 with a rate of $r_2 = 5.25$ Mbps and the other 84 users will be connected to AP_1 with a rate of $r_1 = r_F = 1$ Mbps. The sum rate is

$$r_T = (84 \cdot 1 + 16 \cdot 5.25) \text{ Mbps} = 168 \text{ Mbps}.$$

If an intersystem connection is done then all users will connect to AP_2, since in that case they get

$$\tilde{r}_2 = r_0/N + r_1 = 1.05 \text{ Mbps},$$

a rate that is larger than $r_F = 1$ Mbps. The paradox happens since $\tilde{r}_2 < r_2$ although the interconnection is applied. Moreover, the sum rate is

$$\tilde{r}_T = r_0 + Nr_1 = 105 \text{ Mbps}.$$

This is smaller than the sum rate of $r_T = 168$ Mbps in the original system.

2.3 Model 3

Assume that AP_2 in Fig. 1 offers the following rate as a function of the number of users connected to it

$$r_V(n) = \log_2\left(1 + \frac{P}{\sigma^2 + (n-1)P}\right), \tag{7}$$

where n is the number of users connected to AP_2, $P > 0$ is the received power of a given user, and σ^2 is the variance of the additive Gaussian noise. This model is used for CDMA based systems where if in the uplink, n users are connected to the access point. Here we assume an AWGN channel, however the extension to fading channels is straightforward. Since power equalization is applied, each user has the same received signal-to-interference-and-noise (SINR) ratio given by $P/(\sigma^2 + (n-1)P)$. In that case, since there are N users in the cell, we have that, in equilibrium,

$$m \cdot \log_2\left(1 + \frac{P}{\sigma^2 + (m-1)P}\right) = (N-m)r_F. \tag{8}$$

Here again, m is the number of users connected to AP_2 and the rest $N-m$ is connected to AP_1. Equation (8) is a transcendental equation and can be solved only numerically. Fig. 4 depicts the number of users m that in equilibrium are connected to AP_2 as a function of r_F for different SNRs P/σ^2. Note that for low SNR= P/σ^2, using the approximation $\log_2(1+x) \approx x/\ln 2$, (8) simplifies to

$$\frac{mP}{\sigma^2 + (m-1)P} \cdot \frac{1}{\ln 2} = (N-m)r_F.$$

For high SNR= P/σ^2, (8) simplifies to

$$m \cdot \log_2\left(1 + \frac{1}{m-1}\right) = (N-m)r_F,$$

and does not depend on P/σ^2. Assuming that the solution m is large enough, using again $\log_2(1+x) \approx x/\ln 2$ we get

$$\frac{m}{m-1} = (N-m)r_F \cdot \ln 2$$

Assume now that the system provider decides to interconnect the two access points with fixed and variable rates. In that case, if we assume that

$$\tilde{r}_2 = \log_2\left(1 + \frac{P}{\sigma^2 + (N-1)P}\right) > r_F,$$

then all users prefer to connect to AP_2 since they get better rate if all are connected to AP_2 than to AP_1. The system is in equilibrium since no user wants to connect back to AP_1. The paradox happens since each user that was connected to AP_2 gets a smaller individual rate than in the previous case, since

$$\begin{aligned}\tilde{r}_2 &= \log_2\left(1 + \frac{P}{\sigma^2 + (N-1)P}\right) \\ &< \log_2\left(1 + \frac{P}{\sigma^2 + (m-1)P}\right) = r_2,\end{aligned}$$

where m is given by the solution of (8). Note that the paradox happens only if $0 \leq m < N$.

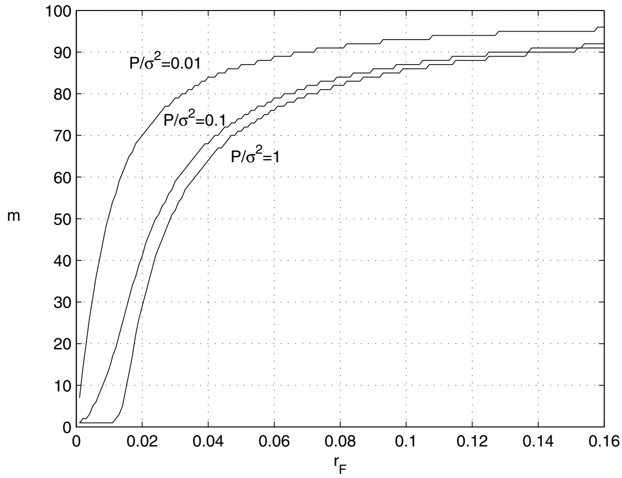

Fig. 4. The number of users m connected to AP_2 in equilibrium for the model (7), for $N = 100$ and various SNRs P/σ^2

The sum rate before interconnection is

$$r_T = 2r_F(N - m),$$

while after interconnection it is

$$\tilde{r}_T = N \log_2 \left(1 + \frac{P}{\sigma^2 + (N-1)P}\right) = N\tilde{r}_2.$$

The paradox happens for the sum rate if

$$\begin{aligned} r_T &= 2r_F(N - m) \\ &> N \log_2 \left(1 + \frac{P}{\sigma^2 + (N-1)P}\right) = \tilde{r}_T. \end{aligned}$$

Example 3. Consider a system in which $N = 100$, $P/\sigma^2 = 0.1$, and $r_F = 12.5$ kbps. In that case solving numerically (8), we get that in equilibrium $m = 21$. This means that, in equilibrium, 21 users will be connected to AP_2 with a rate of $r_2 = 47.3$ kbps and the other 79 users will be connected to AP_1 with a fixed rate of $r_1 = r_F = 12.5$ kbps. The sum rate is $r_T \approx 1.981$ Mbps. By interconnecting the two access points, all users will connect to AP_2 since in that case they get

$$\tilde{r}_2 = \log_2 \left(1 + \frac{P}{\sigma^2 + (N-1)P}\right) \approx 13.18 \text{ kbps},$$

a rate larger than $r_F = 12.5$ kbps. The paradox happens since $\tilde{r}_2 < r_2$ although the interconnection is done. Moreover, the sum rate is $\tilde{r}_T = 1.318$ Mbps and it is smaller than the sum rate of r_T =1.981 Mbps in the previous case.

3 General Model

In this section, we describe in general for what type of functions $r_V(n)$ for AP_2 in Fig. 1, the paradox happens, by interconnecting the fixed and variable rate access points. We describe the conditions under which paradox happens for both the individual and the sum rate of users.

From the previous analysis we see that $r_V(n)$ has to be a decreasing function of n, namely,

$$\begin{aligned} &r_V : \mathbb{N} \mapsto \mathbb{R}_+ \\ &r_V(n+1) - r_V(n) \leq 0. \end{aligned}$$

The paradox happens only if a certain number of users in the original system are connected to both access points. Mathematically, this means that the solution of

$$m \cdot r_V(m) = (N-m)r_F \tag{9}$$

gives a number between 1 and $N-1$. In order to have an equilibrium in the new system and the paradox to happen for the individual rate, the following has to be satisfied

$$r_F < r_V(N) = \tilde{r}_2.$$

Then, the paradox with respect to the individual rate of users connected to the access point with variable rate happens since $r_V(N) < r_V(m)$. An example of such a variable rate function $r_V(n)$ and a fixed rate r_F is shown in Fig. 5.

It is of greater interest to consider under which conditions the paradox happens with respect to the sum rate. From the previous analysis we see that it happens if the following condition is satisfied

$$N \cdot r_V(N) < 2(N-m)r_F = 2m \cdot r_V(m)$$

where m is an integer solution to (9) and $1 \leq m < N$.

In summary, we give the conditions that the function $r_V(n)$ has to satisfy such that the performance gets worse after the interconnection of the two access points. Again we present the conditions for the two types of paradox.

In the first case the following conditions have to be satisfied such that the paradox happens with respect to the individual rate:

(1) $r_V(n+1) \leq r_V(n)$, for $1 \leq n < N$,
(2) $\tilde{r}_2 = r_V(N) > r_F$, and
(3) the solution m of $m \cdot r_V(m) = (N-m)r_F$ has to satisfy $1 \leq m < N$.

Note that the paradox follows directly from (1), since $r_2 = r_V(m) > \tilde{r}_2$.

In the second case the following conditions have to be satisfied such that the paradox happens with respect to the sum rate:

(1) $r_V(n+1) \leq r_V(n)$, for $1 \leq n < N$,
(2) $\tilde{r}_2 = r_V(N) > r_F$,
(3) the solution m of $m \cdot r_V(m) = (N-m)r_F$ has to satisfy $1 \leq m < N$, and
(4) $N \cdot r_V(N) < 2(N-m)r_F = 2m \cdot r_V(m)$.

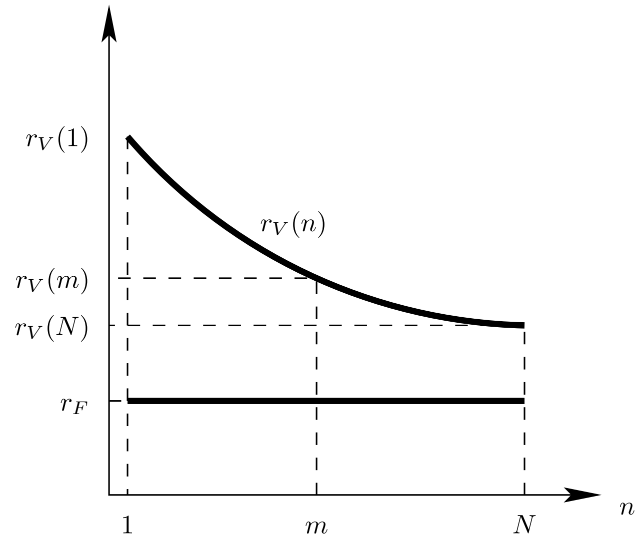

Fig. 5. An example of a network with $r_V(n)$ in which a Braess-type paradox happens

4 Conclusions

Based on the observation of Braess paradox in transportation, we investigate the impacts of greedy behaviors and lack of global information. From our analysis, we conclude that interconnecting two different access points (with the ability of the users to use these two access points at the same time, i.e., cross-system diversity) may not always improve the performance of the network. Under certain conditions, we showed that the rate of users connected to the access point with variable rate decreases after interconnection. We also identified the conditions under which the sum rate does not improve if a system interconnection is performed. Our analysis is based on game theory, and it suggests what are the conditions under which the Braess-type paradox happens. A future work will consist in designing a mechanism that encourages user cooperation in order to avoid the paradox under any scenario. In this case, users may also not cooperate, however, the design of a certain utility will have the same effect as in the case of cooperation.

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