Chapter 15
Mathematical Modeling in Problems of Vibration Acoustics of Shells

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Small oscillations of shells in the ideal compressible unlimited linear acoustic medium (fluid) under the influence of harmonic loads are studied. In such a statement the acoustics of vibrating constructions are modeled. Different versions of simulation of an environment influence are discussed. In strict statement the boundary-value problem of hydroelasticity on joint oscillations of a fluid and an elastic shell construction contacts with the liquid is considered. Here the integro-differential problem with using of the integral formula of Helmholtz with integration on contact surface of the construction and the medium is solved. More effective method applying the formulae for a dynamic pressure of the fluid, obtained in simpler problems, is offered also. It is discussed application of some iterative processes for obtaining numerical results.

15.1 Equations of Linear Acoustics

The equations of linear acoustics follow from the main hydrodynamic equations including equations of continuity, motion and adiabatic state that leads to wave equation: $\Delta\Phi = c_f^2 \Phi_{,tt}$. Here $\Delta = \nabla^2$ is the operator of Laplace, ∇ is the operator of Hamilton, Φ is the velocity potential of medium points: $v_f = -\mathrm{grad}\Phi = -\nabla\Phi$, c_f is the sound speed in the medium. Acoustic pressure, p, and relative change of a liquid density also satisfy the wave equation. The pressure p and potential Φ are coupled by dependence: $p = \rho_f \Phi_{,t}$, where ρ_f is the liquid density in the static state. This dependence fulfills inside and on field boundary. If the field boundary represents an impervious shell, we have on the contact surface the condition: $w_{,t} = \Phi_{,n_o}$, where $w_{,t}$ is the displacement velocity of shell, W is the displacement on normal $\mathbf{n}_o$ (exterior to the shell), $\Phi_{,n_o} = n_o \cdot \nabla\Phi$ is the normal derivative.

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In stationary (harmonic) version of oscillation process, it is fulfilled a time dependence: $\Phi = \Phi_a \cdot \exp(-s_0 i\omega t)$, where $s_0 = -1, +1$, ω is the circular frequency. Then the wave equation reduces to the equation of Helmholtz relatively of amplitudes:

$$\Delta\Phi_a + k_o^2\Phi_a = 0, \quad k_o^2 = \omega/c_f. \tag{15.1}$$

Below, for simplification, we shall omit the lower index "a", pointing amplitude, keeping for amplitudes the denotations of considered functions. In this case, the relations (for amplitudes) accept the following forms: $p = -s_0 i\omega\rho_f\Phi$, $\Phi_{,n_o}|\ S = -s_0 i\omega w$.

At statement of the problem external to the closed surface for obtainment of the unique solution (taking into account that the waves decrease at infinity), we use Somerfield conditions of radiation in the form: $\lim\limits_{R\to\infty}(\Phi_{,R} - s_0 ik_o\Phi) = 0$, $\lim\limits_{R\to\infty}\Phi = 0$, where R is the radius of spherical coordinate system with the origin of coordinates in region, bounded by closed surface (shell) O_o. These conditions are equivalent to the requirement of transformation of the waves radiated from a surface into spherical dispersing damping waves at infinite propagation along any ray starting in the restricted field.

Particular solutions of tasks for the Helmholtz equations on propagation of oscillations into fluid from shells (solids) with the given velocity distribution on the contact surface are sufficiently simply determined in spherical and cylindrical coordinate systems for sphere and infinite circular cylinder. More common solutions are constructed in series on effectively computed special functions (of Legendre, Bessel, Neumann, Hankel). Some approximate solutions are obtained in spheroid coordinates for the prolate ellipsoids and slightly oblate spheroids.

Complexity of solution of the radiation and dispersion problem increases in the case of joint oscillations of fluid and shells of sufficiently common form (the shells of revolution which are distinct from canonical forms, with complex connections and others). Here, we develop the corresponding methods based on integral equations and formulae.

Let the surface O_o divides space into the internal finite area O_+ and external one O_-. The following fundamental solution satisfies Helmholtz equation and radiation conditions:

$$\lambda(x,\xi) = \exp(s_0 ik_o\bar{R})/(4\pi\bar{R}),\ \bar{R} = |x-\xi| = \sum_{j=1}^{3}(x_j-\xi_j)^2, \tag{15.2}$$

where $\bar{R}$ is the distance between two points of space. By using solution (15.2) and Green's formula the integrated formula for pressure is deduced [1]:

$$\iint\limits_{O_o}[p(\xi)\lambda(x,\xi)_{,n_o} - \lambda(x,\xi)p(\xi)_{,n_o}]\,dS = K_p p. \tag{15.3}$$

Here $\xi \in O_o$, $(..)_{,n_o} = \partial(..)/\partial n_o$ is the derivative on normal to the surface. For point x in exterior field $K_p = 1$ and integral (15.3) is usually named by an integral of Helmholtz. It can be applied to sound field calculation by using known pressure profile and gradient of pressure (displacements, velocities) on surface O_o. At $x \in O_o$, coefficient $K_p = 0.5$ and the relationship (15.3) becomes an integral equation of Fredholm of the second kind respectively of pressure p. When $x \in O_o$, coefficient $K_p = 0$. Thus in strict statement, the task of calculation of shell oscillations and sound field in the medium is integro-differential (ID) one.

15.2 Equations for Vibrating Shells

The main equations of shell oscillations could be found in [2, 3] and other references. In the case of shell with main surface in the form of closed rotationally symmetric form, the effective algorithms are methods of solution of the boundary-value problem into framework of the Cauchy problem. In this case, trigonometric Fourier series on the peripheral coordinate are applied to two-dimensional equation set. As a result, the prototype set of the two-dimensional equations reduces to one-dimensional one, depending on the remained longitudinal coordinate and number of the peripheral mode as on parameter. Then the equations are led to a normal form of ordinary differential equations sets of the first order, each of which respectively of the mixed group of solving functions. One half of these functions represent power components, other half define the kinematic ones. These components are included into natural boundary conditions following from variational principles of Lagrange (in statics) or Hamilton (in dynamics). From these principles, the matching conditions on lines of breaking meridian and reinforcing discrete ring ridges also are derived. The boundary-value problems for number of resolving sets of equations, including matching and boundary conditions, are solved by using the method of differential sweep with orthogonalization on Godunov at the intermediate points of integrating range. Variability of mechanical characteristics of shells along the meridian is thus admitted. The shells can be multi-connected and preliminary statically stress-strained. In principle, the equations can lead to base functions as displacement components. For compactness of further presentation, it will be assumed.

Symbolically the equations of simple harmonic motions of the reinforced shells we will write respectively of displacement amplitudes by using the compact operator form:

$$C(\mathrm{U}) + \omega^2 A(\mathrm{U}) + \mathrm{q} - \mathrm{p} = 0, \tag{15.4}$$

where C, A are the elastic and inertial operators of the shell reinforced by ribs, $U = \{u, v, w\}$ is the displacement amplitude vector, $q = \{q_1, q_2, q_3\}$ is the amplitude of loading, $p = \{0, 0, p\}$ is the amplitude of dynamic pressure of liquid. We consider, that the operator equation (15.1) includes matching conditions at discrete ribs and boundary conditions through area of definition of the operators C and A.

Further we shall use the equations with dimensionless quantities, pass to which is conducted by using the following formulae:

$$\begin{gathered} \{\Phi\}_L = \{\Phi\}_D/(\omega R_*^2);\ \{\Delta\}_L = \{\Delta\}_D R_*^2;\ \{\nabla\}_L = \{\nabla\}_D R_*; \\ \{n_o,\ R\}_L = \{n_o,\ R\}_D/R_*;\ \{u,\ v,\ w\}_L = \{u,\ v,\ w\}_D/h_*; \\ \{p,\ q_j\}_L = \{..\}_D R_*^2 \bar{v}_*/(E_* h_*^2);\ \{\rho_f\}_L = \{\rho_f\}_D/\rho_*; \\ \{c_f\}_L = \{c_c\}_D/c_*;\ c_* = [E_*/(\rho_* \bar{v}_*)]^{1/2};\ \bar{v}_* = 1 - v_*^2; \\ \varepsilon_1 = h_*/R_*;\ \Omega = \omega R_*/c_*. \end{gathered} \tag{15.5}$$

Here the subscript "L" corresponds to dimensionless quantities; the index "D" defines the dimensional ones; h_*, R_*, ρ_*, c_* are the proper small and large scales, material density, and sound velocity in the shell; ε_1, Ω are the parameters of thin-wallness and frequency, respectively.

Further in Sect. 15.3, we shall work with dimensionless quantities, but we shall omit subscript "L" with aim of simplification. Then in the dimensionless form above relationships reduce to the following ones:

$$\begin{aligned} &\Delta\Phi + \bar{k}^2\Phi = 0; \quad \bar{k} = \Omega/c_f;\ p = -s_0 i \rho_c (\Omega/\varepsilon_1)^2 \Phi; \\ &n_o \cdot \nabla\Phi|_{S_o} = s_0 i \varepsilon_1 w;\ \lim_{R\to\infty} R(\Phi_{,R} - s_0 i \bar{k}\Phi) = 0; \quad \lim_{R\to\infty} \Phi = 0; \\ &p(r_2) = (2\pi)^{-1} \iint\limits_{O_o} [p(r_1) f(R)_{,n_o} - f(R) p(r_1)_{,n_o}]\, dr_1; \end{aligned} \tag{15.6}$$

$$C(\mathrm{U}) + \Omega^2 A(\mathrm{U}) + \mathrm{q} - \mathrm{p} = 0. \tag{15.7}$$

The integral (15.6) is written for the surface of contact, $f(R) = \exp(i\bar{k}R)/R$, where $\mathbf{r_1}, \mathbf{r_2}$ are the radius-vectors of points on the shell surface, $R = |r_1 - r_2|$ is the distance between them.

If necessary to take into account intrinsic losses in fluid, its sound velocity is given as a complex variable [4]. For simulation of intrinsic losses of vibrational energy in construction, the method of complex elastic moduli (or complex stiffnesses) based on Sorokin's hypothesis [5] is used.

15.3 Method of Eigenforms

Method of eigenforms (EF-method) uses integral equation (15.6) and differential equation (15.7) (ID-formulation) and in the case of rotationally symmetric shells contains three stages [6, 7]: (i) calculation of base functions which are eigenforms of shell oscillations in vacuum, (ii) obtainment of pressure and displacement distributions (forms of forced oscillations) on the shell-medium interface, (i) calculation of acoustic field in the medium.

Basic functions are defined in the process of the solution of eigenvalue problem for the shell equations in which the circumferential coordinate is separated by application of trigonometrical Fourier serieses. Further factors for shell displacements and pressure in liquids on the contact surface further are represented in the form of series on longitudinal eigen-oscillation modes of shell in vacuum. By using the eigenmodes for normal displacement of shell, the set of orthogonal basic functions is found on the base of Gramm-Shmidt's process of orthogonalization. With its help, it is possible to express Fourier factors of pressure through factors of displacements. On the base of boundary integral equation and application of Bubnov-Galerkin's procedure for each n-th circumferential mode, the set of the linear algebraic equations of M-order is formed, where M is the number of the retained longitudinal modes. By this, the orthogonality of vectors of eigenforms is used in power spaces of elastic and inertial operators of the equations for shell oscillations. The equation set solution gives distribution of pressure and pressure gradient at the boundary-contact surface.

Demonstrating a correctness in the solution of the coupled problem for "shell—liquid" system, EF-method is difficult for applications. It also does not possess flexibility to changes of parametres of construction. At multiple calculations, it is necessary to recalculate eigenfunctions (which are required some tens) for ensuring good convergence. A practical applicability of the EF-method is limited by sufficiently average frequencies since natural obstacle for expension EF-method to higher frequencies is the dense spectrum of eigenfrequencies mainly of bending normal forms with zerous circumferential mode of the shell (after frequency of the first radial resonance).

15.4 Method of Local Impedance Modeling

Due to above reason the method of local impedance modelling (LIM-method) is more effective, when a *priori* the dependence of the dynamic pressure on velocity (or amplitudes of displacements) is given on the shell surface. Attractiveness of this method consists in that the fluid account only insignificantly complicates algorithm for "dry" shells with intrinsic vibrational energy dissipation.

The solution of the acoustics equations in cylindrical coordinate system may be deduced by method of variables separation. After that, it is possible to obtain for n-th circumferential mode the impedance linking amplitudes of dynamic pressure and velocities of partticles of the ideal infinite liquid on surface of infinitely long cylindrical shell of radius R_c [4]:

$$p_{n(s)} = Z_{cn} c_{n(s)}; \quad Z_{cn} = \left[s_o i \rho_f c_f \gamma_o H_n^{(s_o)}(\gamma)\right] \Big/ \left[\gamma H_n^{(s_o)'}(\gamma)\right], \tag{15.8}$$

where $\gamma_o = kR_y;\ \gamma = k_r R_y;\ k_r = \sqrt{k^2 - k_z^2};\ k_o = \omega/c_f;\ k_z = m\pi/L; s_o = \pm 1;\ \ k_o,\ \ k_r,\ \ k_z$ are the wave numbers; $H_n^{(s_0)},\ \ H_n^{(s_0)'}$ are Hankel functions and their derivatives on argument.

For a plane problem when the solution does not depend from z, $k_z = 0$; $k_r = k_o$; $\gamma = \gamma_o$. Hankel functions of the first kind correspond to value of $s_0 = 1$, and Hankel functions of the second kind relate to $s_0 = -1$:

$$H_n^{(s_o)}(\gamma) = J_n(\gamma) + s_o i \mathrm{Y}_n(\gamma);\ H_n^{(s_o)'}(\gamma) = nH_n^{(s_o)}(\gamma)/\gamma - H_{n+1}^{(s_o)}(\gamma).$$

Here J_n, Y_n are Bessel functions of the first and second kind, respectively.

By passing in (15.8) from velocities $c_{n(s)}$ to displacements $w_{n(s)}$, taking into account $c_{n(s)} = -s_o i\omega w_{n(s)}$we obtain

$$p_{n(s)} = Z_{wn} w_{n(s)};\quad Z_{wn} = [\omega\rho_c c_c \gamma_o H_n^{(S_o)}(\gamma)]\Big/[\gamma H_n^{(s_o)'}(\gamma)], \tag{15.9}$$

where Z_{wn} has a sense of the mechanical stiffness coefficient of medium.

After transition to non-dimensional quantities by means of the relationships:

$$\begin{aligned}
&\tilde{p}_{n(s)} = p_{n(s)} v_*/(E_* \varepsilon_1^2);\ \ \tilde{\rho}_f = \rho_f/\rho_*;\ \ \tilde{w}_{n(s)} = w_{n(s)}/h_*;\\
&\tilde{c}_c = c_c/c_*;\ \ \Omega = \omega R_*/c_*;\ \ \Omega_1 = \Omega/(\tilde{k}_2 \tilde{c}_2) = \gamma_o;\\
&\tilde{R}_y = R/R_*,\quad \tilde{L} = L/R_*,\quad \tilde{k}_2 = 1/\tilde{R};\\
&\varepsilon_1 = h_*/R_*,\ \ \bar{v}_* = 1 - v_*^2,\ \ c_* = \sqrt{E_*/(\rho_* \bar{v}_*)};\\
&\gamma = \Omega_1 \kappa,\ \ \kappa = \sqrt{1 - \kappa_1^2},\ \kappa_1 = m\pi c_c/(\omega L) = m\pi \tilde{c}_c/(\Omega \tilde{L});\\
&c_* = [E_*/(\rho_* \bar{v}_*)]^{1/2},\ \bar{v}_* = 1 - v_*^2,\ \varepsilon_1 = h_*/R_*,\ \Omega = \omega R_*/c_*
\end{aligned}$$

the expression of dynamic rigidity takes the form:

$$\tilde{p}_{n(s)} = \tilde{Z}_{wn} \tilde{w}_{n(s)},\ \ \tilde{Z}_{wn} = [\Omega \tilde{\rho}_c \tilde{c}_c H_n^{(S_o)}(\Omega_1 \kappa)]\Big/[\varepsilon_1 \kappa H_n^{(S_o)'}(\Omega_1 \kappa)]. \tag{15.10}$$

For $\kappa_1 > 1$, arguments of the Hankel functions becomes purely imaginary, and they can be presented by McDonald functions, or modified Bessel functions of first ($s_0 = 1$) and second ($s_0 = -1$) kind. In this case $\tilde{Z}_{wn}$ is represented as

$$\tilde{Z}_{wn} = [\Omega \tilde{\rho}_c \tilde{c}_c K_n^{(s_o)}(\Omega_1 |\kappa|)]\Big/[\varepsilon_1 |\kappa| K_n^{(s_o)'}(\Omega_1 |\kappa|)] \equiv \tilde{Z}_n^-, \tag{15.11}$$

where $K_n^{(s_o)}$ are McDonald functions, real and positive at the real and positive argument, $K_n^{(s_o)'} = nK_n^{(s_o)}(x)\Big/x - K_{n+1}^{(s_o)}(x)$.

At $n \neq 0$ and small argument $x = \Omega_1 |\kappa| \to 0$, we obtain

$$K_n^{(s_o)}(x) \to [(n-1)!/2](2/x)^n,\ \ K_n^{(s_o)'}(x) \to [n!/4](2/x)^{n+1},$$
$$\tilde{Z}_n^- \to -(\tilde{\rho}_c\Omega^2)\big/(\varepsilon_1 n\tilde{k}_2).$$

At $n = 0$ and $x \to 0$, we obtain

$$K_0^{(s_o)}(x) \to 0.11593 - \ln x,\ \ K_0^{(s_o)'}(x) \to -K_1^{(s_o)}(x) \to 1/x,$$
$$\tilde{Z}_0^- \to -[\tilde{\rho}_c\Omega^2/(\varepsilon_1\tilde{k}_2)](\ln x - 0.11593).$$

The presented formulae are obtained for cylinder. At their usage for shells of revolution there are alternative versions. In one of them, it is possible to substitute $\tilde{R}_c$ on $\tilde{r}$ being polar radius of shell at the given point on generatrix, and then $\tilde{k}_2 = 1/\tilde{R}_c$. In other version, it is possible to treat $\tilde{k}_2$ as curvature of normal cross-section in circumferential direction, and then $\tilde{k}_2 = (\sin\beta)/\tilde{r}$, where β is the angle of slope of the generatrix normal to rotation axis.

When edges of the shell of revolution are enclosed by rigid plates of radius $\tilde{r}_p$, then at longitudinal oscillations on the zero mode ($n = 0$), the medium response at the edges could be simulated through the impedance of the rigid piston without screen [8]. In the dimensionless form

$$\tilde{p}_0^{(n)} = \tilde{Z}_{w_0}^{(n)}\tilde{w}_0^{(n)}, \quad \tilde{Z}_{w_0}^{(n)} = -\varepsilon_1^{-1}\tilde{\rho}_c\tilde{r}_n\Omega^2(\Omega_1\kappa)(2/\pi + s_o i\tilde{r}_n/(2\tilde{c}_c)). \qquad (15.12)$$

Calculation of dynamic rigidity of environment is conducted in the form of complex procedure with logic branching. Different formulae are realized on branching in dependence on that where pressure is calculated on the shell or plate. We take into account the mode number and behavior of argument $x = \Omega_1|\kappa|$.

After obtainment of the solution on the shell surface, the field pressure in fluid is determined by Helmholtz integral (15.3) for exterior area O_-.

For shells of revolution, numerical experiments on selection of the longitudinal wave numbers m depending on vibration mode of shell with finite size were executed. Comparison shows possibility to use the plane problem version ($m = 0$, $\kappa = 1$) in sufficiently broad range of frequencies at construction of external field characteristics. Acceptability of such model confirms by comparison with EF-method.

15.5 Iterative Processes

The further development of ID-formulation and LIM-method are iterative methods where given impedance could be used as initial approach of iterative processes (IPs) at the solution of the original equations. For simply connected shells of axisymmetrical geometry (without branching) some variants of IPs, having different areas of convergence, are realized in [9].

15.6 External Field Calculation

In the briefly presented methods (EF, LIM, itterative ones), the exterior acoustic field is calculated at the last stage by using Helmholtz integral (15.3), where $x \in O_-$. The field gives a distribution of dynamic pressure amplitudes (sound) in the fluid. Obsrvation point coordinates **x** (radius-vector) are convenient to give in spherical coordinate system $r, \;\; \varphi = \alpha_2, \;\; \theta = \theta_1 + \gamma_1$. Here r is the radius, $\varphi \in [0, 2\pi]$ is the circumferential coordinate (longitude), $\theta_1 \in [0, \pi]$ is the coordinate calculated along meridian (latitude). The origin of spherical coordinate system is usually placed at the center of area O_+ (see Fig. 15.1).

Common procedure of calculation allow us to construct the near and far sound field by using repeated integration of the double integral:

$$p(r) = (4\pi)^{-1} \iint\limits_{O_o} [p(r_1) f(R)_{,n_o} - f(R) p(r_1)_{,n_o}]\, dr_1 \tag{15.13}$$

on surface O_o with repeated application of Simpson method. Here $\mathbf{r}, \mathbf{r_1}$ are the radius-vectors of points into medium and on the shell surface, respectively, $\mathrm{R} = \sqrt{\mathrm{r}^2 + \mathrm{r}_1^2 - 2\mathrm{r}_1 \mathrm{r} \cdot \cos(\gamma_1)}$.

For recalculation of levels $|p|$ in decibels two versions of normalizations were used:

(i) in relation to a threshold level of audibility $p_1 = 2 \cdot 10^{-5}$ H/M^2, or $\tilde{p}_1 = 8.84 \cdot 10^{-13}$, where tilde marks non-dimensional value;
(ii) in relation to a maximum level of the field from the force radiating in the liquid half-space (dipole):

$$|p_2| = \Omega Q/(2\pi R c_f), \; L_{|p|}^{(k)} = 20 \ln(|\tilde{p}|/|\tilde{p}_k|), \; k = 1, \; 2. \tag{15.14}$$

For constructions with sections such as in Fig. 15.2, some of the eigenmodes are shown in Fig. 15.3. Note that the three-section shell consisted of conical and two cylindrical parts. The shell sections were divided into plates and rigid rings. The last scheme well works and allows us to simplify algorithm. So, instead of using the superelement scheme, it is possible to solve two-dimensional boundary-value problem. Comparison of the applied methods was executed on multi-section long shells with additional masses. The amplitude-frequency characteristics of the field levels in the "illuminated" zones for the same observation points (for these methods) are shown in Fig. 15.4.

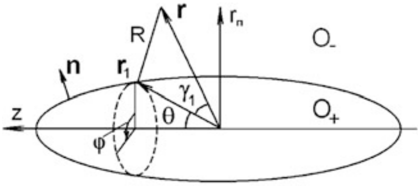

Fig. 15.1 Coordinate system for external field calculation

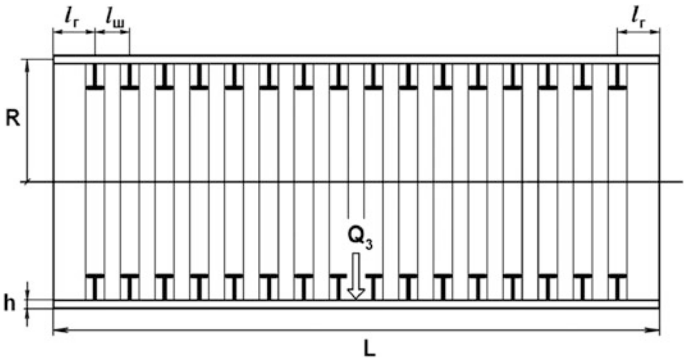

Fig. 15.2 Typical section of shell construction

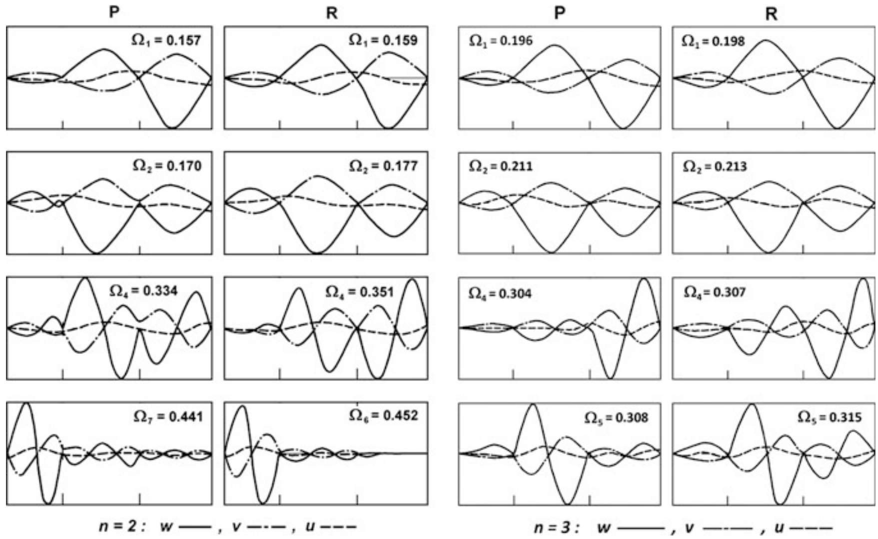

Fig. 15.3 Eigenform of oscillations for three-section shell, divided into plates (*P*) and rings (*R*)

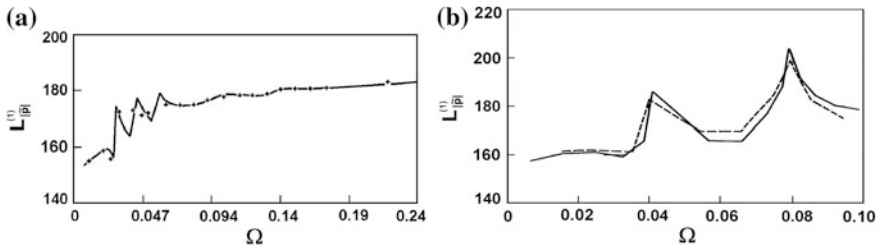

Fig. 15.4 Comparison of methods of eigenforms and local impedance modeling for **a** transverse (n = 1) and **b** longitudinal (n = 0) oscillations

15.7 Conclusions

Application of the EF-method and methods of iterative processes meets significant challenges for the implementation in algorithms. Therefore, it is expedient to use the LIM-method. On the basis of the LIM-method, there were analyzed the shells with no axisymmetrical rigidities and masses [10, 11], multicoupled [12], coaxial [13, 14] ones, and also the shells with discrete rings of variable stifness [15], incircular cylindrical shells [16] and others.

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