Investigation of OWA Operator Weights for Estimating Fuzzy Validity of Geometric Shapes

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Abstract. The estimation of fuzzy validity (*f*-validity) of complex fuzzy objects (*f*-objects) by using fuzzy geometry (f-geometry) may be a useful tool in revealing unknown links or patterns e.g. finger prints, shoe print, face sketch of a criminal etc. at crime site. The Extended Fuzzy Logic (FLe) is a combination of Fuzzy Logic (FL) and Unprecisiated Fuzzy Logic (FLu). Whenever a precise solution of any problem is either impossible or bit costlier, then we opt for the concept of FLe. The f-geometry is an example of Unprecisiated Fuzzy Logic (FLu). The *f*-geometry has different *f*-objects like *f*-point, *f*-line, *f*-circle, *f*-triangle, etc. The aggregation models can be used for aggregating the component of *f*-objects. The Minimizing Distance from extreme Point (MDP), which is a nonlinear ordered weighted averaging (OWA) objective model, is used to estimate *f*-validity of fuzzy objects. The results generated by the MDP model are found closer to degree of OR-ness. The objective of this paper is to lay the foundation and encourage further discussion on the development of` methods for defining as well as estimating *f*-validity of some more complex *f*-objects for forensic investigation services.

1 Introduction

Sometime forensic experts have to make decision on the basis of clues collected by crime site team e.g. face sketch of criminal drawn by expert on the basis of onlooker's statement. Onlooker's statement is based on perception. Practically, exact interpretation of these statements into a face is either impossible or bit costlier. Because face sketch is drawn by free hand without use of ruler and compass. That may result in unclear face of criminal. This may help criminal to escape from law. Computational forensics is an emerging research area focusing on the solution of forensic problems using algorithmic and software methods. Its primary goal is the discovery and advancement of forensic knowledge involving modeling, computer simulation, computer based analysis and recognition [1]. Computer methods enable forensic professionals to reveal and identify previously unknown patterns in an objective and reproducible manners [2]. In forensic research field f-geometry may play a vital role. The f-geometry is a counterpart of Euclidian geometry in crisp theory. In *f*-geometry, objects are drawn by liberated hand without the use of geometric instruments. In FLu,

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there is a concept of perception based f-valid reasoning. The *f*-geometry is an example of FLu. L.A.Zadeh has given the concept of Extended Fuzzy Logic (FL). FLe is a combination of Fuzzy Logic and Unprecisiated Fuzzy Logic. Whenever a perfect solution cannot be given or a process falls excessively costly then the role of the FLe comes into play [3,4,6,7]. The FLe provides the f-valid solution. The Ordered Weighted Averaging (OWA) provides a unified decision making platform under the uncertainty [5]. The f-geometry is a tool that is useful in sketching the different types of geometric shapes on the basis of perceptions i.e. the natural language statements. In this direction Sketching With Words is an emerging research area. In [6,7] authors have applied Sketching With Word technique for the estimation of perceptions in geometric shapes by using triangular membership function.

The f-objects may be the components of the different clues. The estimation of f-objects may be a useful tool in revealing unknown links or patterns e.g. finger prints, shoe print, face sketch of a criminal etc. at crime site. It is clearly shown in Fig.1 the skeleton of a face is made of different f-geometric objects e.g. f-circle, f-triangle, f-quadrilateral, and many more. In f-geometry different f-objects like f-point, f-line, f-circle, f-triangle, f-rectangle, f-square, and f-parallelogram have been defined in [4,6,7]. The f-rhombus is a complex f-object. The definition of f-rhombus as well as a method for estimation of f-validity is explained in this paper. The f-geometry may provide a framework for estimating the components of the face on the basis of the onlooker's statements. This may provide a scientific basis for human face recognition system by simulating the forensic sketch expert in the discipline of computational forensic. The Minimizing Distance from extreme Point (MDP) is most recent model and results generated by MDP aggregation model are closer to degree of OR-ness. So we have used MDP model for estimating the f-validity of f-rhombus. The estimation of f-rhombus may be a useful model for defining more complex f-objects. The objective of this paper is to lay the foundation and encourage further discussion on the development of` methods for defining as well as estimating f-validity of some more complex f-objects for forensic investigation services.

Fig. 1. Skeleton of Face

In this way f-geometry along with OWA operator weights may provide a vital link in catching the criminal in the field of computational forensics. In the literature some nonlinear Ordered Weighted Aggregation objective models like Maximum Entropy Model (MEM), Minimal Variability Model (MVM), Chi Square Model (CSM), Least Square Deviation Model (LSM) and Minimizing Distance from extreme Point Model (MDP) have been reported [9]. In [10] authors have analyzed the capabilities of memetic algorithms for the 3D image reconstruction of forensic objects.

The proposed work may be considered as a first step in aggregating the component of complex f-objects. Further we can aggregate the different parts of complete shape e.g. face sketch. In this paper the methodology of Computing With Words is not taken into account. That is out of scope for this paper. Only f-line and f-similar angle are used in this work. That is why we are going to explain only these two components. Please refer [4,6,7,8] in order to have the details of the rest of the components of f-geometry.

This paper is organized as follows. In Section 2, we have briefly looked into the f-geometry. The section 3 incorporates proposed f-theorem. In section 4, we have reviewed OWA techniques for estimating f-validity of f-objects. In section 5, we have estimated the f-validity by using f-theorem and OWA weights with experimental work. The final section comprises of conclusion and future directions.

2 Fuzzy Geometry

The Unprecisiated Fuzzy Logic introduces the concept of f-geometry. In Euclidean geometry crisp concept C corresponds to a fuzzy concept, in f-geometry. In this section we have discussed fuzzy line (f-line), fuzzy similar angle (f- similar angle), fuzzy theorem(f- theorem), fuzzy similarity(f-similarity), fuzzy validity, and fuzzy proof (f-proof). More detail can be seen in [4, 6, 7, 8].

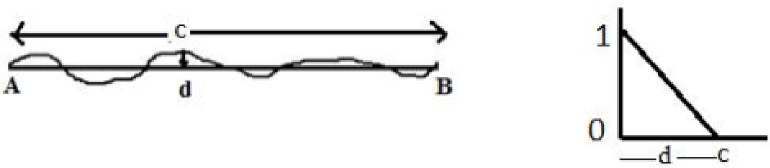

Fig. 2. (a) f-Line (b) Membership Function

2.1 *f*uzzy-Line

Let us consider an f-line which is like a curve that passes through a straight line *AB*, such that the distance between any point on the curve and the straight line *AB* is very small or negligible. With reference to Fig. 2(a), this implies that we are assigning a small value to the distance d [4].

$$\mu(\text{f- line}) = \begin{cases} \dfrac{c-d}{c} & \text{if} \quad 0 \leq d \leq c \\ 0 & \text{if} \quad c \leq d \end{cases} \tag{1}$$

In equation (1) d << D, where d is the largest difference between the f-line and straight line. D is the length of the crisp line AB. Here c is some real number.

From Fig. 2(b) we can conclude that the membership function increases with the decrease in the value of d. If the value of d is equal to 0 then we can conclude that given f-line is a straight line with validity index 1.

2.2 *f*uzzy Triangle

In f-geometry a closed shape is said to be f-triangle if its membership value is closer to the membership value of triangles. As shown in Fig.3.

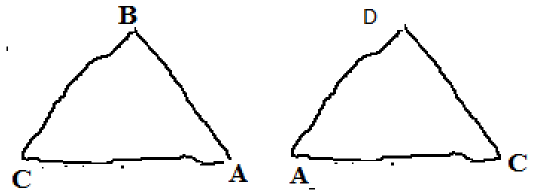

Fig. 3. *f*- Triangles

2.3 *f*uzzy-Similarity and Fuzzy-Validity

In f-geometry any two f-objects are said to be f-similar, if both of them have same shape. Very specifically, by uniform scaling one must be congruent to other. Conversely, f-similar polygons may be of same f-angles and scaling of f-sides may be proportionate. In section 2.4 we illustrate the concept of f-similarity. On the other hand f-validity is a measure of the degree of belongingness of any f-objects against the exact geometric object.

2.4 *f*uzzy-Theorem

The f-theorem in f-geometry is f-transform of a theorem in Euclidean geometry. In f-theorem, we try to formalize the f-concept in f-geometry, generally in the form of membership functions, e.g. by application of any rules. Assuming a formal illustration of the concept of the f-theorem, let us consider the Fig.3. The f- triangles ABC and ADC constituted with three fuzzy lines and three fuzzy angles.

2.4.1 Side Angle Side Postulates of Similar Triangle (SAS)

In f-geometry, two triangles are said to be *f*-similar if their membership function has high validity index for the property of similar triangles (SAS). The membership value of similar triangles decreases as difference in corresponding angle and difference in the proportion of two corresponding sides' increases. Membership function is given by (2).

$$\mu(\text{f-similar})=\mu_{S1}*\mu_{A2}*\mu_{S3} \tag{2}$$

Where μ_{S1} and μ_{S3} are membership functions of *f*- proportions of *f*-sides. The μ_{A2} is membership function of *f*-similar angle.

Equation (3) shows *f*- proportions of *f*-sides and (4) shows *f*-similar angle of *f*-triangle. In case of SAS, we assume that AB/DC' *=BC/AD' *= k (A constant) i.e. corresponding sides of the two triangles are in the same ratio as in geometry. Here AB/DC' and BC/AD' are in the same ratio as in geometry. Here AB/DC' and BC/AD' have taken the fuzzy proportion values $k_1$and k_2 respectively. The point to be noted here is AB/DC *= BC/AD means AB/DC' is approximately equals to BC/AD' [6,7]. The membership function of the difference in proportion is computed by (3). Where the value of j is given by $k-k_1$ and $k-k_2$ for AB/DC' and BC/AD' respectively.

$$\mu(f\text{-similar side})=\begin{cases}\frac{c-j}{c-b} & \text{if } b\leq j\leq c\\ 0 & \text{if } c\leq j\end{cases} \tag{3}$$

In the following equation membership function of difference in θ_1 and θ_2 angle is given .

$$\mu(f\text{-similar angle})=\begin{cases}\frac{c-h}{c-b} & \text{if } b\leq h\leq c\\ 0 & \text{if } c< h\end{cases} \tag{4}$$

Here, $h=\theta_1 - \theta_2$ is the difference between the angles as shown in Fig. 4.

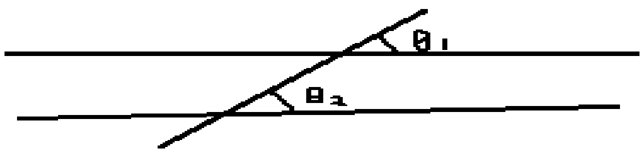

Fig. 4. *f*-Angle

2.5 *f*uzzy-Proof

In f-geometry, f-proof may be either pragmatic or logical. The pragmatic f-proof involves experiments while the logical f-proof is the f-transform of their counterpart of crisp geometry [1,4,5].

3 Proposed *f*uzzy-Theorem

This section introduces the definition of the f-rhombus. Further we have discussed f-theorem and their f-proof by using validation principle. Afterwards we have estimated f-validity of f-rhombus with MDP OWA weights technique. Similar concept can be used for defining more complex f-objects.

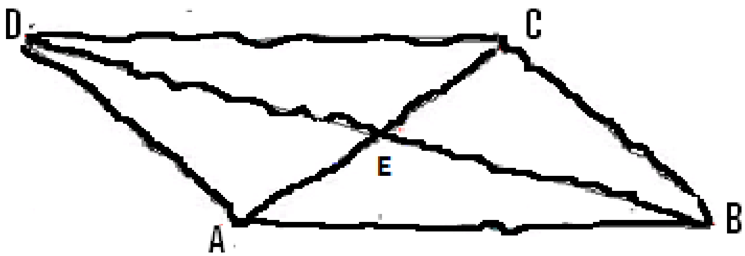

Fig. 5. *f*-Rhombus

3.1 *f*-Rhombus

In a four sided closed shape if all of its *f*-sides are *f*-similar such that the difference between all four *f*-sides are very small or negligible, then it is called f-rhombus as shown in Fig. 5. The membership function of f-rhombus is given below .

$$\mu(f\text{-rhombus}) = \mu_{s1} * \mu_{s2} * \mu_{s3} * \mu_{s4} \quad (5)$$

Where $\mu_{s1}, \mu_{s2}, \mu_{s3},$ and μ_{s4} are given by (3).

f-theorem1: If diagonals (f-lines) of f-parallelogram are perpendicular to each other then f-parallelogram will be a f-rhombus.

f-proof :“ if the difference between interior angle made by the diagonals from right angle is either small or negligible ” then f-parallelogram ABCD has a higher degree of validity index of f-rhombus. The validity index is given by (6).

$$\mu(\text{f-validity}) = \mu_{IA1} * \mu_{IA2} * \mu_{d1} * \mu_{d2} * \mu_{D} \quad (6)$$

The μ_{IA1},and μ_{IA2} are given by (4).

The μ_D is given by (7)

$$\mu_D = \mu_{d3} * \mu_{d4} * \mu_{d5} * \mu_{d6} \quad (7)$$

4 OWA Operator Weights Methods

4.1 Minimizing Distances from Extreme Points Method

This method is introduced by Byeong Seok Ahn. The MDP method has three steps, first step is identification of extreme points on the basis of degree of OR-ness (β) is given by the user, second is generation of weights , and final step is an aggregation of MDP weights with inputs [10].

4.1.1 Identification of Extreme Points

Any weighting vector w ϵK can be represented by a convex combination of the extreme points E= (e_1, e2…e_m) where e_j is a unit vector with 1 in the j^{th} position and 0 elsewhere. A weight set K^{A-C} can be constructed by combining attitudinal character constraint (β) with set K.

$$K^{A-C} = \{\mathbf{w}: Aw = b, w \geq 0\} \quad (8)$$

Where b= (1,β)′ is a 2-D vector and A is a 2xm matrix.

$$A=(a_1,a_2,\ldots a_m) = \begin{pmatrix} 1 \ldots & 1 \ldots & 1 \\ \frac{m-1}{m-1} & \frac{m-i}{m-1} & 0 \end{pmatrix}$$

$$A_{ji} = (a_j, a_i) \; j < i, j=1\ldots m$$

$$A_{ij}^{-1} = \frac{m-1}{j-i} \begin{pmatrix} m - j/m - 1 & -1 \\ i - m/m - 1 & 1 \end{pmatrix} \quad (9)$$

In certain cases, however, a weighting vector **w** is negative due to wrong choice of index j and i. That is avoid deriving a legitimate index set such as

$$j \leq m(1-\beta)+\beta,$$

$$i \geq m(1-\beta)+\beta$$

On the basis of legitimate index, extreme points are given below.

$$ext_j^k = (\beta*(m-1)-(m-i))/(i-j) \quad (10.1)$$

$$ext_i^k = ((m-i)-\beta*(m-1))/(i-j) \quad (10.2)$$

Here β is a level of OR-ness, k denotes the number of extreme points, and m denotes the number of criteria.

4.1.2 Generation of Weights

The coordinates wise averaging of the extreme points results in the MDP OWA operator weights.

$$w_j = \frac{\sum_{j=1}^{k} ext_j^i}{k} \qquad j=1,\ldots m \tag{11}$$

4.1.3 Aggregation of Weights and Inputs

OWA determines the f-validity in f-objects by using (12). Where $X=(x_1, x_2, x_{3,\ldots} x_m)$ are input parameters with the multi-criteria of size m. The y_i is the i^{th} largest input parameter.

$$OWA(x_1,x_2,x_3\ldots\ldots x_m) = \sum w_j y_i \tag{12}$$

5 Estimation of f-Validity Using OWA Operator

In this section the f-validity of f-rhombus is estimated by using the MDP OWA method.

5.1 Experiments and Results

The computations of the f-validity are performed by applying the concept of f-theorem on f-rhombus. The f-theorem1 is illustrated in Example1. The sample images used in experimental work are shown in Fig. 6.

Example1: The f-rhombus shown in Fig.5 has f-transform distance for the f-lines AB, BC, CD and AD are 2, 25, 3, and 6 respectively. This results in μ_d as {0.97, 0.74, 0.9681, 0.9309} by using (1). The f-transformation distance of f-lines AC and DB are 5 and 6 respectively. The membership values of f-lines AC and DB are estimated by (1) is 0.95 and 0.93 respectively. The values of internal angles DEC and AEB are 102.254 and 94.90173 respectively. The membership values of f-similarity of these angles from right angle calculated by (4) are 0.38, and 0.75.

To compute the value of μ_D we have applied MDP OWA model. Here we consider four parameters as inputs i.e. m=4.The weight vector {0.816,0.1,0.05, 0.033} which is generated by (11) with membership values {0.97, 0.74, 0.9681, 0.9309} produces results by (12) is

$$\begin{aligned}\mu_D &= \mu_{d1}*w_1+\mu_{d2}*w_2+\mu_{d3}*w_3+\mu_{d4}*w_4\\ &=0.97*0.816+0.96*0.1+0.93*0.05+0.74*0.033\\ &= 0.95.\end{aligned}$$

Membership of f-rhombus is calculated by using MDP OWA methods. The weight vector for m =5 by (11) is {0.79, 0.1, 0.05, 0.0325, 0.025}. For the degree of ORness (β) 0.9 produces f-validity by (12) is

$$\mu_{(f\text{-rhombus})} = \mu_D * w_1 + \mu_{d5} * w_2 + \mu_{d6} * w_3 + \mu_{IA2} * w_4 + \mu_{IA1} * w_5$$
$$= 0.95*0.79 + 0.95*0.1 + 0.93*0.005 + 0.75*0.0325 + 0.38*0.025$$
$$= 0.92$$

The f-validity is computed by the MDP OWA method for different values of degree of the OR-ness (β) from 0.9 to 0.6 are shown in Fig.7. It can be clearly seen in Fig.7 the results are closer to degree of OR-ness.

Fig. 6. Sample Images

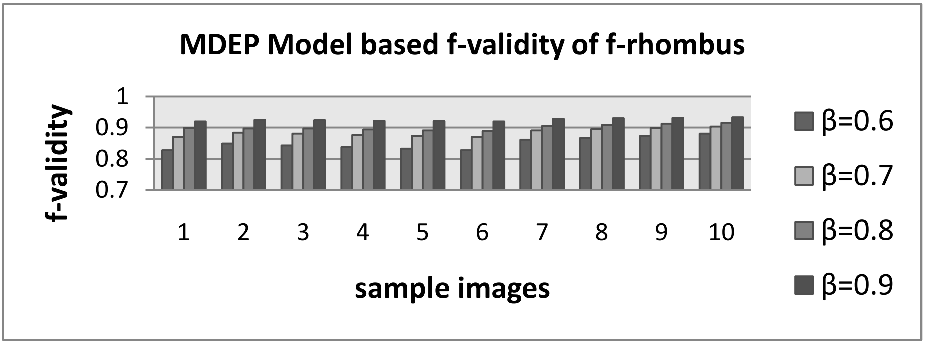

Fig. 7. *f*-validity generated MDP methods by *f*- theorem 1 for sample images of *f*-rhombus

6 Conclusion and Future Directions

In this paper we have proposed the definition of *f*-rhombus in *f*-geometry. Then we have estimated the *f*-validity of *f*-rhombus by transforming a crisp theorem to an *f*-theorem. We have reviewed and applied MDP aggregation model for estimating the degree of OR-ness of the *f*- rhombus. The results generated by MDP are closer to degree of the OR-ness. This characteristic of MDP weights may be very helpful in drawing shape of f-objects with desire degree of OR-ness. This work may open the door for formalizing more complex f-objects. The proposed work may be considered as a first step in aggregating the component of complex f-objects. Further we can aggregate the different parts of face to make a complete face. The concept of *f*-similarity can be used for estimating the similarity of different faces, which may be useful in matching the face of criminal with existing face sketches.

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