Chapter 14
Financial Networks

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14.1 Introduction

The financial system performs vital functions for the world economy. Very often one of more aspect of this system can be described by means of a complex graph. In this chapter under the generic name of *financial networks* we indicate several different systems all related to the world of finance. Such a coarse graining is justified by the fact that in all the various situations we always find similar behaviours. We shall present here a series of examples passing from the study of stock-price correlations to the study of the web of exposures between different companies, and finally to the lending of money between banks.

Indeed in every of the abovementioned systems we encounter similar mathematical structures. Furthermore we are interested in similar basic questions. More particularly we always find a scale-free architecture, a scale-free distribution of centrality and betweenness. At the same time, in all these cases we want to know which institutions is more important for the stability of the whole system, what is the global impact of a local bankruptcy, and finally how we can act on the system in order to change its properties or to recover the initial stability.

These questions have similar answers, with details changing from one case to another, but in any case related to the issue of centrality and controllability.

The possibility to provide to regulators a set of simple indicators that can be used as a thermometer of the financial situation in order to prevent crises is one of the most challenging perspectives. For this reason more and more often scientists and research groups involve regulators in the research activity. The recent crisis has spurred a profound debate about the role of policy and regulations in financial markets. The debate has drawn the attention of researchers from many areas of

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science as well as of the civil society at large to the needs for new approaches to policy modelling. Overall, it has emerged as a prominent societal issue the need to build a sustainable global financial system that serves the global policy goals. In particular, many observers share the view that the current financial crisis should be seen as an opportunity to strengthen climate finance and not as an excuse to postpone the environmental objectives that were previously put forward.

It is well known that in financial markets, while contracts are beneficial to the parties involved they can also entail unforeseen (negative/positive) externalities to other parties [21, 36]. In particular, incentives for parties to take excessive risk as individuals lead to systemic risk for the market as a whole. According to the direct contacts of our Consortium with regulators (e.g., Bank of England, Bank of Italy, Deutsche Bundesbank, DG-Markt) [6, 16], the problem that many regulators are facing today is that (1) it is not clear what externalities could arise from certain contracts and (2) what could be done to mitigate the negative externalities and strengthen the positive ones. The problem is even more acute when linkages with the environmental sphere are introduced via climate finance [28].

The lack of data is the immediate cause of this situation. A first step has been very recently moved in the direction of collecting systematic information for instance for OTC derivatives. In December 2012, the European Commission has adopted new technical standards, the so-called European Market Infrastructure Regulation (EMIR). However, there are more fundamental causes. The continuous injection of financial innovations as well as their inherent complexity makes it difficult to keep track of the possible externalities [28]. In fact, the interactions of market players across the globe through various instruments such as OTC derivatives, security lending and repurchase agreements make, today more than ever, the financial market a complex network [8, 20] with highly non-linear dynamics [6, 15, 22]. Our current understanding of what undesirable systemic effects may arise and how to cope with them is very limited. Progress in this direction is vital for the well-being of the economy and requires combined efforts and competences [35]. This is the topic of the first section of this chapter that focuses on reconstructing the missing links (Fig. 14.1).

Linkages among financial institutions can have ambiguous effects: on the one hand they increase individual profitability and reduce individual risk, but on the other hand they propagate contagion and distress, thus increasing systemic risk (see Fig. 14.2, for an illustration). On this topic a significant body of work has grown in the recent years. The issue of cascade of failures was initially investigated in simple four-node graphs [1] and in ring structures. The fact that too many linkages can be a factor of systemic risk is a result that has emerged in various works, from various underlying mechanisms. In some models, the balance between positive and negative effects of the network density depends on the level of market capitalization [31], in others on the persistence in the dynamics of financial fragility [5], and yet in others on the level of market liquidity [4]. More links in the network can result also in the so-called robust-yet-fragile property, namely the network resists well to most of the shocks but break down entirely in a non-negligible fraction of cases [15]. Moreover, the tendency towards complexity in the instruments and at the same time towards homogeneity in the risk models and investment strategies adopted by banks is per se

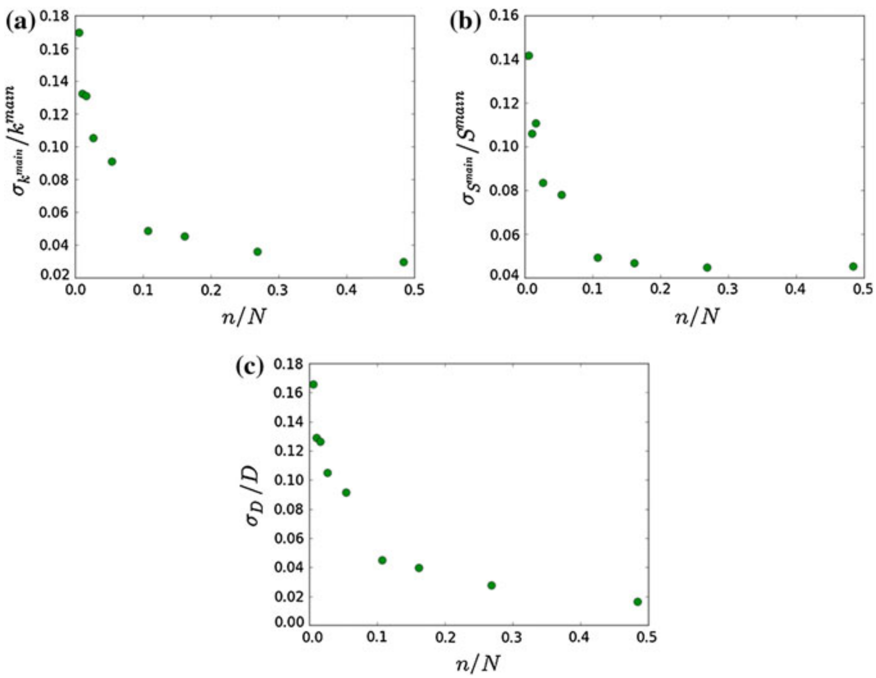

Fig. 14.1 The WTW network. The plots from *top left* represent respectively: **a** the relative error in the estimation of average degree of the main core $\sigma^k{}_{main}$ /k$_{main}$ computed with real WTW network **b** the same as in (**a**) but for the relative error in the size of main core, **c** same as in (**a**) but for the density of the links D. In all the 3 plots it is shown that the quality of the reconstruction of the WTW network increases with the number of nodes used to generate the network ensemble

a source of systemic risk [22]. Finally, indirect linkages can emerge from the fact that banks invest in overlapping portfolios, resulting in a tension between the incentive to reduce individual risk and the social cost of having systemic crises.

Following this perspective we present here results that are mostly based on the activity of the FET Open project "Forecasting Financial Crises" a consortium of several European institutions and the European Central Bank whose continuous feedback proved crucial in order to refine our research.

14.2 Reconstructing the Missing Links

One first issue in the study of financial networks is to deal with partial and/or incomplete information. While the reconstruction of the original network may be proven to be very difficult, it has to be noticed though, that in most of the application we need only the statistical properties of it. Addressing this issue has many concrete applications. Typically we can consider a system made of financial institutions as

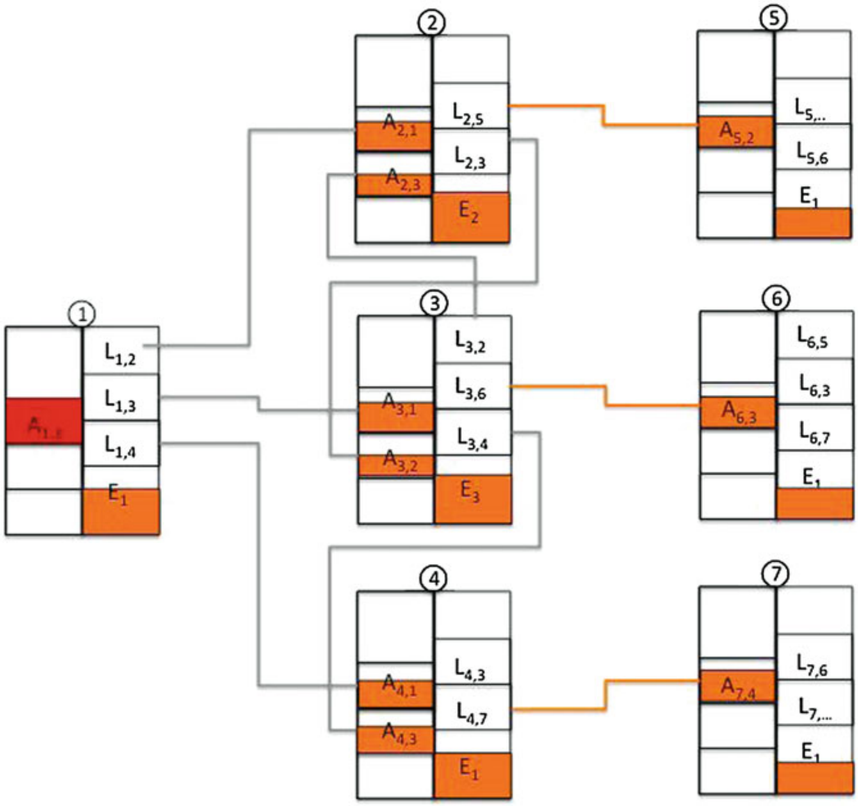

Fig. 14.2 Illustration of distress propagation across a network of banks connected via liabilities. Each block is a schematic balance-sheet of a bank

vertices and edges formed by various kinds of financial ties such as loans or derivative contracts. These ties result in dependencies among institutions and constitute the ground for the propagation of financial distress across the network. The resilience of the system to the default or the distress of one or more institutions depends on the topological structure of the whole network [4]. Unfortunately, the information that regulators are able to collect on the mutual exposures among institutions is very limited (since the confidentiality issues).

Various methods have been presented in order to reconstruct the network in the papers analysing systemic risk. One of the most successful is the so-called Maximum Entropy (ME) algorithm. This method assumes that the network is fully connected (for this reason this class of approaches is called "dense reconstruction methods"). Edges are weighted and these weights are obtained via a maximum homogeneity principle. This means that each node is assumed to bear a similar level of dependence from all other nodes. After that, the method proceeds by looking for the matrix that minimizes the distance from the uniform matrix (where every entry has the same value), while satisfying certain constraints (imposed in this case by the budget of the individual banks). Such a matrix is found by minimizing an objective mathematical function known as the Kullback-Leibler divergence.

However, the hypothesis of "graph completeness" strongly limits the ME algorithm, since empirical networks show instead a largely heterogeneous degree distribution. Moreover, such "dense reconstruction" leads to an underestimation of the systemic risk [29, 32]. To overcome these limitations a sparse reconstruction" algorithm has been proposed. The procedure is similar to the one for dense graph. Again we minimise the Kullback-Leibler divergence and we obtain a matrix with an arbitrary level of heterogeneity given certain constraints. The latter approach is more reliable but leaves open the question of what value of heterogeneity would be appropriate to choose. Finally the density of connections must be specified ex-ante and it is not recovered by the algorithm.

Recently a third approach wanted to overcome these problems. The new procedure is called Bootstrapping Method (BM) and it is a new general method to deal with incomplete information [30]. This method does not aim at reconstructing the original network but rather to estimate its global properties.

In more detail, among all the possible topological properties, the authors focused on those that in the literature have been shown to play an important role in contagion processes and in the propagation of distress, i.e., the network density and the k-core structure [25]. For the resilience, they focussed on a recently introduced notion, DebtRank [5], that allows to measure the systemic impact of an initial shock on one or more nodes, whenever the links in the network represent the financial dependencies among nodes. It is also possible to determine the accuracy of the estimation upon the size of the subset of nodes for which the information is available.

In this method, the allocation of the links among nodes is carried out using the fitness model ([9, 18]). Differently from other network generation models, the fitness model distinguishes amongst different vertices. In particular it generates a network structure starting from a non-topological variables (fitness) associated to the nodes. This approach has been used in the past to reproduce the topological properties of several empirical economical networks, including the network of equity investments in the stock market [19], the interbank market [12], and the WTW [18].

The validity of this method has been proved on both synthetic networks as well as examples of real economic systems. In these few cases, there is full information on the system. The whole adjacency matrix is available and therefore it is possible to evaluate the accuracy of the method by using only part of the information. The two empirical cases of study presented in [30] are

- the World Trade Web (WTW), i.e. the network in which nodes are countries and links are trade volumes (in US dollars) among them,
- the interbank loan network of the so-called e-mid interbank money market.

The result of this analysis is that information on the degree of a relatively small fraction of nodes is sufficient to estimate with good approximation the above mentioned topological properties, as long as the fitness of all nodes is known.

For instance, with only about 7 % of the nodes (10 out of 185) we have a relative error of about: 7 % on the density, 10 % on the average degree of the main core, 7 % on the size of the main core. Similarly, t with about 7 % of the nodes the resilience can be estimated with a relative error within 10 %.

14.3 Evaluating the Impact of Linkages Through Centrality Measures: DebtRank

During the period March 2008–March 2010 many US and international financial institutions received aid from the US Federal Reserve Bank (FED) through emergency loans programs, including the so-called "FED Discount Window". Recently this dataset has been released thereby providing a unique and important opportunity to study the distribution of debt across institutions and across time.

One of the papers based on the analysis of this dataset wanted to estimate the impact of a node on the others. This is done by means of a novel measure inspired by feedback centrality. Such quantity termed DebtRank [5] takes recursively into account the impact of the distress of an initial node across the whole network. More particularly DebtRank of vertex i, is a number (i.e. dollars or euros) measuring the fraction of the total economic value in the network potentially affected by the distress or the default of node i. This quantity can be used to construct a ranking, but it is not itself a particular rank of the node considered.

Its computation differs from the methods based on the default cascade dynamics [4, 11, 29] in which, below the threshold no impact is propagated to the neighbors. In this respect DebtRank is more similar to other feedback centrality measure that have found successful applications in many domains ranging from rankings in the world-wide-web (e.g. PageRank) to corporate control in economic networks.

Feedback centrality can be considered as the in-flow in a non-homogeneous diffusion process. Exactly in this spirit the presence of a cycle in the network represent a potential problem. In this case there is an infinite number of reverberations of the impact of a node to the others and back to itself, which leads to no simple and measurable economic interpretation. DebtRank overcomes this problem by only allowing for walks that do not visit the same edge twice.

Consider a directed network where the nodes represent institutions and the links represent financial dependencies.

- Denote the amount invested by i in the funding of j as A_{ij}. Thus, A is the weighted adjacency matrix of the investment network. The total value of the asset invested by i in funding activities is $A_i = \Sigma_j A_{ij}$.
- Denote by E_i the tier capital of i giving the buffer of i against shocks. If $E_i < c$ (where c is a positive threshold) the firm defaults.

If the node i defaults, the node j faces a loss of A_{ji} (in the first instance, any recovery is excluded). Similarly also the node j defaults if $A_{ji} > \mathrm{E}_j$. When the loss

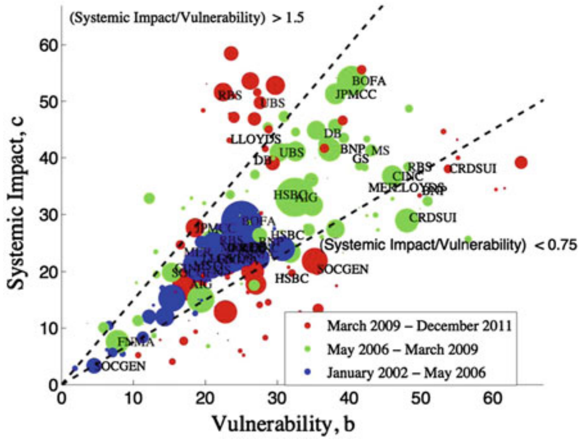

Fig. 14.3 DebtRank-like algorithms allow to monitor over time in a 2-dimensional plot those players that have at the same time high impact on the others and high vulnerability to other players' shocks (see [24] for more details on the calculations). As we can see, in the intermediate period (*green*) a number of players were at the same time highly vulnerable and systemically important

exceeds the capital the impact is fixed to 1, so that in general the impact of i on j can be defined as $W_{ij}=\min\{\,1, A_{ji}/E_j\,\}$. The total amount of the impact of institution i is $I_i=\Sigma_j\, v_j W_{ij}$.

The problem is to take into account the impact of i on its indirect successors, that is, the nodes that can be impacted from i at distance 2 or more.

Authors define an iterative equation of kind

$$I_i=\Sigma_j v_j W_{ij} + \beta \Sigma_j W_{ij} I_j$$

where the second term accounts for the indirect impact via the neighbours. The parameter β is a dampening factor.

In a cycle ($W_{ij} > 0$ and $W_{ji} > 0$), the impact of node i to j hits back on i and keeps cycling an infinite number of times (although with dampening). A single reverberation of the impact of i back to itself is realistic and mathematically acceptable. Further reverberations lead instead to an inconsistency because the impact could become larger than one. The reason is that if the impact is repeated several times through a cycle, then the impact of a node on another one is counted more than once. The same problem applies also to any cycle not involving i, but located downstream of i in the network. Removing the cycles altogether from the network and considering its corresponding acyclic graph would remove entirely the reverberation and cut many links, thus strongly underestimating the impact. In contrast, DebtRank is computed on the original network by excluding the walks in which one or more edges are repeated (Fig. 14.3).

14.4 Interbank Controllability

As shown above the network theory can provide some suggestions to improve the stability of financial systmes against crises. Especially in the credit sector, scientists tried to emphasize the systemic implications of distress in economic systems [1, 14]. Indeed, the fragility in specific countries, markets and financial institutions can propagate and damage the whole economy [2, 33]. For this reason an increasing research effort is being invested in the study of economic and financial networks [3, 4]. Interestingly, many (practically relevant) properties of these networks can be quantitatively investigated. For example, it has been widely recognized the role of Too-Big-To-Fail (TBTF) [35] hubs in determining the fragility of the system with respect to distress propagation or its resilience against link failures. The connectivity structure plays a fundamental role in it [1]. Against this background, Delpini et al. [13] investigated which (if any) policy could improve the stability of the network toward a less risky situation.

The basis of this work is to apply to the interbank money market the notion of *controllability structural* [26, 27] based on control theory concepts. The idea is that the whole network can be controlled by acting on specific drivers. An immediate application arises when a central bank must give credit to the banking system and therefore needs to know which actions (amongst many) are likely to affect the whole structure. The problem of finding the driver vertices of the system is completely solved by finding a maximum matching of the corresponding oriented graph [27].

Since the matching does not depend on the specific values of the weights (which are by the way largely unknown or affected by errors for the majority of real networks), the results hold in a variety of different situations explored with reference to the Italian case.

In particular it is possible to assess the controllability of interbank money markets empirically, focusing on the specific case of the Italian electronic trading system (e-MID), which is open to European banking players, and for which a time series of micro data is available [13]. Following the network evolution over time it is possible to detect the banks that are more relevant from a control perspective. For them, the authors considered the changes of the relevant topological and financial quantities, clarifying the role played by drivers in this system. Through this approach it is also possible to address the resilience of the network drivers, that is the correlation between driver sets at different times. At every aggregation scale and for every available network instance, it is possible to identify the set of driver nodes (Fig. 14.4a shows a daily network instance, with the driver nodes highlighted).

The number of maximum matchings for directed graphs of the size of the Italian interbank market is rather large. Searching for a maximum matching takes into account the graph edges only and enumerating all the possible control configurations is an intensive numerical task. Nevertheless, this can be simplified by assuming that some configurations may be more significant than others from a control perspective. Delpini et al. [13] selected the maximum matching with maximum weight (the weight being given by the sum of edge weights in the matching). The intuition behind

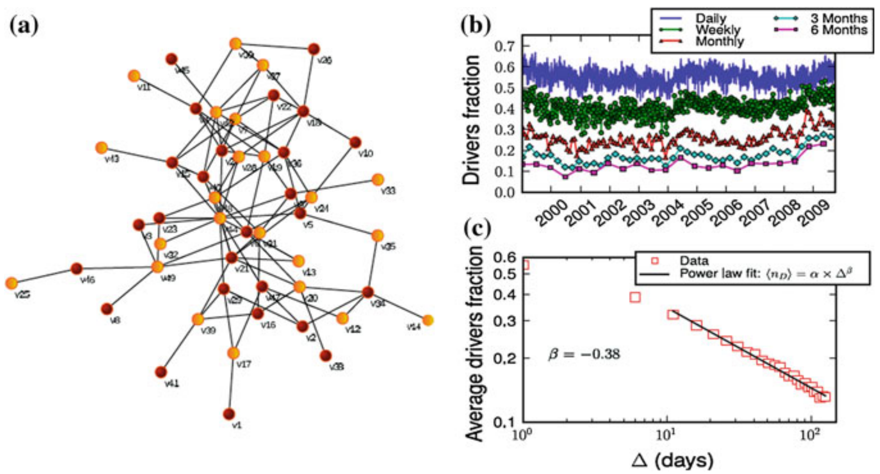

Fig. 14.4 **a** A sample snapshot of the daily interbank lending network. External inputs on the *yellow* nodes (drivers) allow to control the state of the whole system. **b** Time evolution of the fraction of drivers: at the monthly scale less than 40 % of the banks drive the system. **c** The average fraction of drivers decays with a neat power law scaling with the aggregation scale Δ

this choice is that cash flows are proxies for the strength of the influence relationship between two banks.

The main results are that there is no characteristic time scale for the fraction of drivers in the system. In other words it is not possible to select an optimal aggregation time. Rather, different levels of aggregation correspond to networks with different connectivity, which requires different control strategies. Different scales could serve different supervision purposes and policy makers could adopt the instruments that are more effective for the time horizon of interest for the control. The control set of drivers will change over time. However, inspection of the drivers resilience, shows also that the system is characterized by long-range memory. The survival function has a very slow, almost linear decay, and after 6 months nearly 60 % of drivers are still in the control set. Surprisingly, the level of the curve does not follow a monotonic trend with Δ: the control configuration is maximally stable at the monthly scale. This result supports the proposal of the Basel III Committee to introduce a 30-day liquidity coverage ratio, and suggests the monthly scale as a reasonable time window for observing the system.

This is important, since recent financial crisis has been forcing central banks to implement aggressive and creative policy actions. Radically new strategies have been proposed to cope with liquidity shocks within interbank markets. Traditionally, policies have been mainly based on liquidity injection through open market operations, but it has been proved that targeted intervention on individual banks could be more effective in guaranteeing and restoring the efficient allocation of credit. The above findings suggests the need for monitoring the system and keeping track of banks that are systemically relevant from a control perspective [13]. Since no characteristic scale exists in the decay of the fraction of drivers with the time resolution, this implies that no optimal timing for bank supervision can be selected based only on that. Nevertheless, other network statistics, such as the persistence of control configurations, indicate the monthly scale as natural for observing the system. Another result of this analysis is that the more relevant banks to the overall state of the credit network are neither the most connected nor the top lenders. This strongly suggests the necessity to rethink the policies based exclusively on the TBTF specification of a systemically important institution. According to recent regulation proposals, the ECB has recognized that allowing recapitalization interventions directly on individual banks is a necessary procedure.

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