Chapter 4 Problems of the Aesthetic Induction

In Chap. 2 we surveyed McAllister's study of beauty in science. That study is the most sophisticated available on the topic, and its claim that evaluations play a role in the development of science is supported by compelling evidence. McAllister's work is insightful not only in the sense that it accounts for phenomena like scientific revolutions and the role of aesthetic evaluations in science, but also in that it shows the advantages of having an explicit aesthetic theory to make sense of aesthetic phenomena in an methodologically sound way. In this respect, McAllister's idea of the aesthetic evaluations compatible with the rationalist image of science. However, I believe that there is room for improving McAllister's ideas and that further insight can be gained by addressing the weakest points in those ideas. In this chapter, I shall identify some problems with McAllister's approach. In the next chapter, I shall address those problems by introducing a more accurate model of aesthetic evaluations in science.

4.1 Two Kinds of Problems

Since McAllister's work is the first attempt to formulate an articulated model of the role of beauty in science and it has very ambitious goals, it is no surprise that some problems can be identified in it. Here, we shall concentrate on issues that directly concern our goal—the formulation of a consistent theory of mathematical beauty. In this respect, I have identified two types of problems with McAllister's approach: explanatory anomalies and theoretical tensions.

4.1.1 Anomalies

McAllister's central idea, the aesthetic induction, has a significant relevance to our discussion not only because it allows us to capture the dynamic character of beauty,

but also because it allows us to explain episodes of theory choice and even make predictions such as McAllister conjecture on computer-assisted proofs. As we saw in the previous chapter, predictions like the fate of the aesthetic merit of computerassisted proofs can be contested if we extend our approach to consider the role of our inner experience. But this is not the problem I would like to address here—after all, only time can tell whether or not a conjecture is correct; the problem I have in mind has to do with the explanations of historical episodes offered by the aesthetic induction. The problem is that there exist a whole class of patterns of evolution of aesthetic preferences that cannot be explained by the aesthetic induction; that is, the aesthetic induction has significant explanatory anomalies.

The aesthetic induction cannot account for the patterns of evolution of what I call historical constants (especially, of negative historical constants). I elaborate: in the aesthetic induction, the track record of experiences with certain property determines the intensity of the preference for that property, at least in principle. The aesthetic induction, as presented by McAllister, is equally valid for all properties. It does not differentiate between, for example, the property of simplicity and its opposite, complexity. But in actuality, as we shall see below, the aesthetic induction seems to affect different properties in different ways. Properties of theories such as being an abstract theory (in the sense of relying on abstract mathematical models), being a visualizing theory (in the sense of not relying on abstract mathematical models, but rather on offering a visualization of phenomena), or being tractable by mechanistic analogy, according to McAllister's own illustrations, seem to evolve in great accord with the aesthetic induction: they have exhibited varying degrees of preference in different historical periods, in close association with their empirical success. How these properties fare historically in terms of preference is a contingent matter. I label this type of properties *historical contingencies*.¹ In contrast, properties such as harmony, symmetry or simplicity seem to consistently exhibit high degrees of preference throughout history. McAllister himself recognizes that this feature may even mislead us into thinking that the beauty associated with those properties is an objective property-let us remember that McAllister endorses projectivism-or even that it might have some metaphysical basis [62, Chaps. 3 and 7].² The nature of such properties is a fascinating topic in itself, but one beyond the scope of this book. Here, our concern is that these properties exist, and that their degrees of preference remain constant over time. I label these properties historical constants.

¹As a matter of fact, from today's perspective, it is difficult to see how properties such as tractability by mechanistic analogy, abstractness or being visualizing can be regarded as *aesthetic* qualities of theories. This is precisely because the appreciation of such properties is determined by contingent historical circumstances. Different historical contexts influence what properties are seen as aesthetically appealing by a scientist living in such contexts. Our contemporary context is one in which tractability by mechanistic analogy or being visualizing seem simply deprived of any aesthetic appeal. This fact supports my labelling them *contingencies*.

 $^{^{2}}$ McAllister claims that if there is any relation between beauty and truth such a relation must be established empirically.

McAllister's work shows that the aesthetic induction can account for the evolution of historical contingencies such as the property of being visualizing: this property increased its degree of preference as theories that relied on visualizing phenomena accumulated a track record of empirical success. However, it is more difficult to account for the pattern of evolution of historical constants. Consider, for example, the properties of simplicity and complexity. Already in Ancient Greece, simple theories were preferred over complicated ones. A similar situation can be found throughout history and among contemporary scientists, and this is perhaps even more evident in mathematics: The Elements of Euclid, which set the standards of rigour and logical structure that characterize mathematics, shows great commitment to proving theorems in the simplest possible way. This commitment to simplicity is appreciated even today, as it is testified by the fact that Euclid's proof of the infinity of primes is the very first item in *Proofs From The Book* [2, p. 3], a contemporary compilation of the most beautiful mathematical proofs.³ It is not only in particular proofs, but also in Euclid's general style where we find a preference for simplicity:

To prove a good theorem with the weakest possible tools is rather like landing a large trout on an old and beloved silk line. It does not make for speed or brevity, but it has an undeniable charm. Euclid is not always given to swiftness, but he is rather devoted to the task of getting as much as he can with as little as he can get away with [1, p. 57].

In the second century, astronomers also showed an explicit preference for simplicity:

The classical and Alexandrian astronomers not only constructed theories but fully realized that these theories were not the true design but just descriptions that fit the observations. Ptolemy says in the Almagest that in astronomy one ought to seek as simple as possible a mathematical model [45, p. 159].

Newton saw simplicity as a prominent precept in the investigation of nature: it appears as the first of the "rules of reasoning in philosophy" in his *Principia* [71, p. 3]. Contemporary scientists still value simplicity. Stephen Weinberg makes this evaluation: "Einstein's general theory of relativity [...] Newton's theory of gravity [...] are equally beautiful" [23, p. 107]. He argues that the simplicity of Einstein's general relativity makes it as beautiful as Newton's gravity, even if they exhibit different kinds of simplicity [23, pp. 107–108]. Philosophers of science are also aware of the importance of simplicity, as Donald Hillman remarks: "Principles of simplicity have been abundant, from Occam's Razor in fourteenth century philosophy all the way down to various twentieth-century attempts to interpret simplicity in its scientific connection" [34, p. 226].

Simplicity has enjoyed a high degree of preference throughout history. McAllister does not see this as problem with the aesthetic induction. After all, the evolution of the preference for simplicity does not directly contradict the

³The compilation was inspired by the mathematician Paul Erdos, who used to say that God had a book that contained all the most beautiful mathematical proofs. Erdos used to exclaim "This is a proof from the Book" whenever he found a proof he considered extraordinarily beautiful.

mechanism of the aesthetic induction, since simple theories do have a track record of empirical success to explain their high degree of preference. What is peculiar about simplicity is that, although preferences change constantly over time, the preference for simplicity seems to remain unchanged, even across scientific revolutions. The aesthetic induction may be consistent with the evolution of the preference for simplicity, but it cannot account for the fact that such preference was already present at the very beginnings of the study of nature, nor that the preference never seems to lose momentum, despite the ever changing historical contexts. McAllister himself seems to recognize that there is something anomalous in properties like simplicity, since he devotes an entire chapter [62, Chap.7] to discuss simplicity: he concludes that simplicity plays a complex role that involves empirical and nonempirical criteria for theory choice. We shall not address the nature of simplicity here, but it is worth mentioning that simplicity certainly plays a diversity of roles in scientific practice. The simplicity of a theory, of an explanation or of a mathematical formalism has epistemological, pragmatic and methodological advantages. For example, Karl Popper relates the degree of simplicity of a theory to its degree of falsifiability, arguing that simple statements are highly prized "because they tell us more: because their empirical content is greater; and because they are better testable" [75, p. 128]. Pragmatically and methodologically, a simple mathematical formulation, for instance, enables quicker and more accurate calculations, as well as further formal development. In addition, some authors interpret simplicity as an indicator of empirical adequacy and, thus, as a valid empirical criterion for theory choice [34, pp. 225–226]. The intricate nature of simplicity might somehow explain why the aesthetic induction seems to play a marginal role in its evolution. But if we focus on the instances of simplicity that have an aesthetic character, its evolution still poses questions for the aesthetic induction.

Now, the pattern of evolution of the preference for simplicity may pose questions for the aesthetic induction, but at least it is consistent with it. Much more problematic are properties that exhibit patterns of evolution inconsistent with the aesthetic induction, as we shall see now. First of all, recall that in McAllister's model, the aesthetic canon includes all possible aesthetic properties [62, pp. 78– 79]. This means that the aesthetic canon involves aesthetic properties that evoke positive, negative or neutral (indifferent) responses. Thus, we can classify our aesthetic properties not only as historically contingent or constant, but also, as positive, negative or neutral. Since the aesthetic induction does not differentiate properties, let us consider simplicity's opposite: *complexity*. The history of the preference for complexity is, of course, the mirror image of the history of simplicity: the unappealing character of complexity remains unchanged throughout history; even across scientific revolutions. Now, a significant fact about the preferences for simplicity and complexity is that they are often overlooked to achieve empirical and epistemic success. This means that in the history of science there are not only simple theories with a track record of success, but also complicated theories with a track record of success. This would not be a problem if complexity were a historical contingency, but it is a constant. Complexity in mathematics provides us with clear examples of this. We know that Greek mathematicians had a strong predilection for simplicity. However, in order to further advance the discipline, they had to sacrifice their aesthetic prejudices. Morris Kline comments:

By insisting on a unity, a completeness, and a simplicity for their geometry, and by separating speculative thought from utility, classical Greek geometry became a limited accomplishment. It narrowed people's vision and closed their minds to new thoughts and methods. It carried within itself the seeds of its own death. The narrowness of its field of action, the exclusiveness of its point of view, and the aesthetic demands on it might have arrested its development, had not the influences of the Alexandrian civilization broadened the outlook of Greek mathematicians [45, p. 175].

In general, mathematicians are well aware that complicated theories or methods are necessary to achieve epistemic success. However, the aesthetic merit of complicated mathematics has not increased over time. The different methods of proof illustrate this. There are several methods of proof that mathematicians utilize on a regular basis. There are simple, beautiful methods such as *reductio ad absurdum*, which we discussed earlier. But there are also complicated, brute force methods of proof. Complicated methods, despite their undeniable epistemic soundness, were not found appealing by Greek mathematicians, and that is still the case today. An example of this are proofs by cases.⁴ G.H. Hardy comments on the complexity of proofs by cases:

We do not want many 'variations' in the proof of a mathematical theorem: 'enumeration of cases', indeed, is one of the duller forms of mathematical argument. A mathematical proof should resemble a simple and clear-cut constellation, not a scattered cluster in the Milky Way [33, p. 113].

This type of proof is even the target of harsh words. A page later, Hardy comments on the soundness of proofs by cases:

[...] All this is quite genuine mathematics, and has its merits; but it is just that 'proof by enumeration of cases' [...] which a real mathematician tends to despise [33, p. 114].

It must be emphasized that the different methods of proof are equally sound: a result proved by *reductio ad absurdum* is just as true as one proved by cases. The point is that mathematicians abhor proofs by cases despite the fact that *they are a sound method of proof*. Furthermore, throughout history, prominent mathematicians achieved many relevant results by proofs by cases or by methods involving proving special cases: proofs by cases appear already in *The Elements* of Euclid (about 300 B.C.). Cardano (1501–1576), who presented for the first time in history a method for solving cubic equations, justified his method by treating separately the many cases of cubic and quadratic auxiliary equations involved in the solution [45, pp. 265–266]. Leibniz (1646–1716) conceived the solution to the problem of orthogonal trajectories by tackling special cases of it. Jacob (1654–1705) and Johann Bernoulli (1667–1748) devised solutions to special cases of isoperimetric problems, which

⁴In a proof by cases one divides the statement to be proven into a finite number of mutually exclusive cases, and then shows and documents independently that in each case the statement holds.

eventually led to a general solution [47, pp. 575–577]. For many years Fermat's last theorem, $x^n + y^n = z^n$, was approached by attempting to prove it for special cases of *n*; by Euler (1707–1783), Lagrange (1736–1813), Legendre (1752–1833), Gauss (1777–1855), and Dirichlet (1805–1859) among others [46, 47]. More recently proofs by cases have become conspicuous by achieving spectacular results and arising heated controversies. As we have seen, Appel and Haken's 1976 computer-assisted proof of the four-color theorem is a proof by cases, with almost 2000 cases, which posed all kinds of questions about its validity. Unflattering adjectives are still regularly applied to this proof:

[...] this particular "proof" is almost always what mathematicians think of when asked "What is an example of ugly mathematics?" [70]

Proofs by cases have a very long history of success. But mathematicians' preference for them does not seem to increase. This contradicts the aesthetic induction. Furthermore, Gian-Carlo Rota, as we have seen, suggests that ugly proofs play a significant role in the development of mathematics as incentives to look for alternative proofs [80, pp. 9–10].

Complicated yet successful theories can be found not only in mathematics. In physics, the Standard Model, one of the greatest achievements of science, is not necessarily regarded as a paradigm of beauty despite its great success:

At present, there has been no experimental deviation from the Standard Model. Thus, it is perhaps the most successful theory ever proposed in the history of science. However, most physicists find the Standard Model unappealing because it is exceptionally ugly and asymmetrical. [...] The reason why the Standard Model is so ugly is that it is obtained by gluing, by brute force, the current theories of the electromagnetic force, the weak force, and the strong force into one theory [37, p. 75].

An aversion to complicated theories and methods not only seems to be a constant fact throughout history, but it also seems that scientists are aware of this fact: the complexity of the Standard Model is not the source of an increased appreciation, but rather, in a fashion analogous to Rota's suggestion, an incentive to search for simpler alternatives, as it is evident in the struggle for achieving simplicity in Great Unification Theories. Mathematicians seem even more aware that aversion to complicated mathematics and predilection for simple theories and methods are timeless classics, so to speak. This is why mathematical ugliness motivates further research.

There is enough evidence that complexity is a historical constant, a negative one. We have seen that the pattern of evolution of positive historical constants poses questions. But that pattern is at least somehow consistent with the aesthetic induction. The evolution of negative historical constants, by contrast, contradicts it directly: complicated mathematical methods have a long track record of success, but their degree of preference has not increased. A similar argument can be substantiated for properties that are the opposites of positive historical constants like symmetry, harmony or unity. The aesthetic induction has no means to deal with negative historical constants satisfactorily, as it treats all properties equally. Furthermore, another consequence of that egalitarian stance is that the aesthetic induction allows predictions such as McAllister's conjecture on computer-assisted proofs, which seems rather implausible, once again, given the historical evidence compiled above.

Computer-assisted proofs are regarded as ugly proofs. As we saw in Chap. 2, in McAllister's view that is merely a historical contingency: the aesthetic merit of computer-assisted proofs might improve as they become acceptable. But our discussion in Chap. 3 and the foregoing one do not support that conjecture. Complicated methods of proof have been accepted by mathematicians ever since Antiquity. This acceptability, however, did not result in an increase in the preference for those methods. Such is the case of proofs by cases. Computer-assisted proofs are instances of proof by cases. Thus, in addition to our analysis in Chap. 3, we have now further reasons to doubt McAllister's conjecture: the history of the method of proof by cases seems to show that the aesthetic induction will do little to improve the aesthetic merit of their contemporary incarnations in computer-assisted proofs.

Now, we have seen that different properties of theories exhibit different patterns of evolution. We focused on historical constants; properties whose degrees of preference seem to remain constant throughout history. Those properties, especially negative historical constants, are problematic for the aesthetic induction. The aesthetic induction predicts an increase in preference for any property associated with theories or methods that enjoy a track record of success. Now, if a property is a historical constant, it very probably enjoys some degree of success; otherwise it would not remain constant in the ever changing landscape of science. The aesthetic induction should increase their aesthetic merit. However, the key characteristic of negative historical constants is that they lack aesthetic merit and they remain that way. The pattern of evolution of properties such as complexity contradicts the aesthetic induction. In general, negative historical constants constitute the clearest type of anomalies in the aesthetic induction, since they exhibit the following characteristics: (1) A long history of presence in science. (2) Their historical track record, due to the ever changing nature of science, includes necessarily some degree of success. (3) Contrary to what the aesthetic induction predicts, their degrees of preference remain small; otherwise they would not be constants.

In the next chapter, I will propose a way to dispose of the anomalies in the aesthetic induction. But right now, let us discuss the second type of problems with McAlister's approach: its theoretical tensions.

4.2 Theoretical Tensions

In addition to the explanatory anomalies, I have identified issues of a more theoretical nature in McAllister's approach. Most of them have to do with McAllister's theoretical assumptions. In this section, I shall discuss four issues with the aesthetic induction: first, the aesthetic induction is not a genuine case of induction. Second, it does not distinguishes between the problem of beauty and the problem of the aesthetic. Third, it incurs in an inconsistency regarding objectivism and projectivism. And fourth, McAllister's aesthetic theory plays no role in accounting for the evolution of aesthetic preferences.

4.2.1 Induction

McAllister sees the aesthetic induction as a special case of inductive projection. However, a simple analysis reveals that is not the case. Induction is a type of inference in which the features of an unobserved instance are predicted based on the features of a finite set of observed instances. More formally, induction is an inference in which we conclude a general or universal proposition from a set of finite instances of it. The general form of induction is:

Given that $a_1, a_2, a_3, \ldots, a_n$ are all *P* that are also *Q*, we conclude that All *P* are *Q*.

There is a variety of induction in which from a finite number of instances we predict the next instance. This variety is called *inductive projection*. Its form is as follows:

 a₁, a₂, a₃, ..., a_n are all P that are also Q,
a_{n+1} is P, We conclude that
a_{n+1} is also Q.

Now, if the aesthetic induction were a special case of projective induction, it would have the following form, which for convenience I label *Idealized Aesthetic Induction* (IAI):

IAI:

1. $a_1, a_2, a_3, \ldots, a_n$ are all A that are also E,

2. a_{n+1} is A,

3. a_{n+1} is also E.

where A is an aesthetic property of scientific theories and E is the property of being empirically adequate.

Now, IAI is adequate to model the *reason* why a scientist chooses a theory based on its aesthetic properties. However, this inference does not model McAllister's conception of the aesthetic induction: "a community attaches to each property of theories a degree of aesthetic value proportional to the degree of empirical success of the theories that have exhibited that property" [62, p. 4, see also p. 78]. In other words, the aesthetic induction is the mechanism that determines the degree of preference W_A associated with the property A. This is very different from what is expressed by IAI. McAllister seems to use the term 'aesthetic induction' to refer to both the *mechanism* that determines the weightings W_A , and the *inference* scientists use to justify their theory choices. Consider, for example, the case in which a scientist chooses a theory S over a competing theory T based not on empirical grounds but on the fact that S is symmetric. Symmetry is preferred over other properties because it possesses a higher degree of preference. In McAllister's model, this degree of preference is the result of the fact that symmetric theories had been empirically adequate in the past. This process somehow resembles inductive projection. However, the act of choosing theory S is not an inductive procedure. Rather, it is merely the result of using the scientist's aesthetic criteria, which is a simple deductive process of comparing degrees of preference W_P and selecting the highest weighting.

IAI expresses something completely different from the foregoing. It expresses that since symmetric theories have been empirically adequate in the past, we can project that a new symmetric theory might also be empirically adequate. The function of IAI is to *justify* that scientists act rationally when they base their theory choices on aesthetic criteria. IAI makes no reference to degrees of preference W_P or to how to determine such degrees. What McAllister calls the aesthetic induction corresponds to a stage prior to the justification of the theory choice. In such prior stage, the degree of preference for certain property is determined by the track record of empirical success of the theories that exhibited such property.

It is clear now that the aesthetic induction as formulated by McAllister is not a special case of induction, but rather a mechanism with at least three discernible stages: a first stage that determines the degrees of preference; a second stage in which those degrees are employed to choose a theory; and a final stage in which inductive projection is used to rationally justify that choice. To clearly see the differences between IAI and McAllister's actual ideas, I present now a more accurate rendering of McAllister's model. For convenience, I label this rendering *Actual Aesthetic Induction* (AAI):

AAI:

(AAI.1) An aesthetic canon is compiled by following this procedure: for every property P there is an associated weighting W_P such that:

$$W_P = CD$$

where:

 W_P : is the weighting associated to property P.

D: is the degree of empirical adequacy as estimated by the history of success of *P*-bearing theories.

C: is a constant of proportionality between the degrees of empirical adequacy and the weightings W_P .

(AAI.2) Given two equally empirically adequate competing theories T and S which exhibit the aesthetic properties A and B respectively, a scientist will choose T over S only if $W_A > W_B$.

(AAI.3) The scientist makes that choice because he believes that IAI is correct; that is, he believes that: (AAI.3.1) $a_1, a_2, a_3, \ldots, a_n$, are all *A* that are also *E*; (AAI.3.2) a_{n+1} is *A*, and (AAI.3.3) a_{n+1} is also *E*.

Where: a_i is a theory, A is an aesthetic property of scientific theories and E is the property of being empirically adequate.

Given the problems with the idea of induction, in the next chapter I shall abandon the idea that the aesthetic induction is a special case of induction, and propose an alternative view.

4.2.2 Beauty and Aesthetic

A central assumption in the idea of the aesthetic induction is that aesthetic evaluations in science are genuinely aesthetic. In McAllister's view, this assumption enables us to distinguish between empirical and aesthetic evaluations and to establish a non-reductive relation between them. As we have seen, McAllister attempts to substantiate the empirical/aesthetic distinction by providing an aesthetic theory that characterizes aesthetic properties. McAllister, however, fails to distinguish between the problem of characterizing the aesthetic and the problem of elucidating the notion of beauty. This is evident when he addresses the issue of aesthetic properties. He addresses aesthetic properties in two separate occasions: the first time, following Hutchenson, he defines aesthetic properties as properties that move the observer to project beauty into an object [62, pp. 32–33]. Thus, aesthetic properties are defined in terms of beauty. McAllister addresses aesthetic properties in a second occasion, while discussing the aesthetic canon. This time, he suggests two criteria for identifying aesthetic properties: the first criterion is that a property is aesthetic if it appears in a public aesthetic expression uttered by a scientist. The second criterion is that "if in virtue of possessing that property, a scientific theory is liable to strike beholders as having a high degree of aptness" [62, p. 37]. In this occasion, McAllister seems to be concerned with the relation between aesthetic properties and aesthetic responses, and with what makes an aesthetic property aesthetic. Although these ways of addressing aesthetic properties do not contradict each other, the way McAllister uses them shows that he does not distinguish between the problem of beauty and the problem of identifying the mark of the aesthetic, or of identifying what makes an aesthetic property aesthetic. These problems are different. Understanding the nature of beauty is one of the central problems of aesthetics, but the problem of finding the mark of *the aesthetic* is much broader and relatively independent from the problem of beauty. The problem of the aesthetic has to do with discerning what things such as aesthetic judgements, aesthetic concepts, aesthetic value, and so forth, have in common; what is that make them all *aesthetic*. The problem of the nature of beauty can be addressed by offering suitable definitions such as Shaftesbury's, Hutchenson's or even Rota's definitions in terms of properties like order, unity, or enlightenment. However, that tactic is useless to explain, for

example, what makes predicates such as 'beautiful' or 'elegant' *aesthetic* predicates. Addressing the problem of the aesthetic needs a completely different strategy. For example, Nick Zangwill [100] starts by defining the notion of aesthetic judgement and then defines the remaining notions in terms of it: aesthetic properties are properties attributed by aesthetic judgements; aesthetic concepts are concepts used in aesthetic judgement; an aesthetic experience is what motivates the passing of an aesthetic judgement; and so forth. McAllister seems to use a mixture of strategies. He defines aesthetic expressions. Now, the issue of characterizing aesthetic properties is a contentious question in aesthetics; we should not expect a definitive answer in this context. But, for that very reason, a more consistent treatment of the problem is desirable.

4.2.3 Objectivism/Projectivism Inconsistency

McAllister's aesthetic theory exhibits some inconsistencies. Recall that the theory rejects objectivism and endorses projectivism. However, in his criterion for identifying aesthetic properties McAllister resorts to an objectivist tactic, since the criterion relies on the property of aptness. This tactic is similar to Shaftesbury's, Hutchenson's and Rota's tactics of explaining beauty in terms of a non-aesthetic property. McAllister endorses projectivism as a way to avoid the metaphysical complications of objectivism. However, he seems tempted to offer objectivist explanations when it seems suitable, as in the case of aptness. Now, projectivism is not the only available way to circumvent metaphysical complications. In the next chapter we shall see that Theo Kuipers, for example, opts for a naturalistic interpretation of McAllister's ideas; an approach that later on I explore and further develop.

4.2.4 Theory and Modelling

The aesthetic induction intends to connect aesthetic and empirical evaluations. But that connection is a little weak as formulated by McAllister. The details of how the aesthetic induction operates are obtained solely by using historical evidence. The aesthetic principles endorsed by McAllister play no role in shaping the aesthetic induction as a model of the mechanism that drives the evolution of aesthetic preferences. This is evident if we examine McAllister's tenet that the aesthetic terms used by scientists to evaluate theories refer to genuine aesthetic from empirical evaluations. Now, the aesthetic induction models the mechanism of evolution of aesthetic preferences. However, modelling such evolution does not require a literal interpretation of aesthetic terms. The model of the aesthetic induction itself, as formulated by McAllister, does not involve things like affective responses, aesthetic pleasure or any of the characteristics usually attributed to aesthetic phenomena. The model depends only on the historical evidence documented by McAllister. In this sense, a perfectly good model of the evolution of preferences can be obtained by attending to McAllister's evidence without resorting to his empirical/aesthetic distinction, since the only thing we need is a much weaker empirical/non-empirical distinction. Thus, McAllister's aesthetic theory is not really necessary for his aesthetic induction. To emphasize his commitment to a literal interpretation of beauty in science, McAllister even attempts to show the existence of the aesthetic induction in the arts, when he draws our attention to the case of cast-iron, steel and concrete structures in architecture. But even if the existence of aesthetic induction in the arts supports McAllister's ideas, this does not give his aesthetic theory a role in the aesthetic induction. A closer relationship between aesthetic theory and preference evolution modelling is desirable if a non-reductivist and *genuinely aesthetic* account of beauty in science and mathematics is to be achieved.

Addressing this and the other issues discussed above shall illuminate some relevant aspects to formulate an articulated aesthetics of mathematics. We shall begin this task in the next chapter.