Chapter 2
Beautiful, Literally

In Chap. 1 I contested a set of reasons to reinterpret mathematical beauty. In this chapter, I examine reasons to embrace a literal interpretation. First, we shall examine a principled reason to reject non-literal interpretations. Next, I shall argue that literal interpretations of beauty are appealing as they can be instrumental in understanding the progress of science: we shall see that in James McAllister's approach a literal interpretation of beauty is the cornerstone in his defense of a rationalist model of scientific development.

2.1 Metaphor and Aesthetic Terms

As we have seen, reinterpreting the term 'beauty' in the context of evaluating scientific and mathematical theories does not help us to safeguard their epistemic or rational character. The reason for this is that any reinterpretation must deal with the subjectivity and idiosyncrasy of the use of terms like 'beautiful', 'ugly', or 'elegant'. Those characteristics seem to be resilient to reinterpretation, as they manifest themselves again at some point in the reinterpretation. Now, that judgements of taste are characteristically subjective has been a tenet of modern aesthetic ever since Kant. Contemporary authors like Rafael De Clerq have explored some semantic aspects of aesthetic terms closely related to their subjectivity. Some of those ideas might help us to explain why attempts to reinterpret mathematical beauty are doomed to failure and, thus, they may spare us the trouble of contesting any more non-literal interpretations.

According to De Clercq, aesthetic terms, terms such as 'beautiful', 'elegant' or 'ugly', possess a salient feature in common:

> their *resistance to metaphorical usage*. In other words, aesthetic terms cannot be turned into metaphors. For instance, it makes no sense to say that something is beautiful "metaphorically speaking." Likewise, it does not make sense to say that something is metaphorically elegant, metaphorically harmonious, or metaphorically sublime [17, p. 27].

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De Clercq suggests an explanation for this feature:

> Aesthetic terms do not have a particular area of application associated with them. There is not a particular *kind* of object to which they are to be applied. As a result, it is not possible to commit something like a "category mistake" with respect to such terms. By contrast, terms for animal species such as 'elephant' and 'crocodile' can be applied only within the animal kingdom: to apply them outside this area is to commit a "category mistake" (which may of course result in a metaphor) [17, p. 27].

If aesthetic terms cannot be used metaphorically, that explain why attempts to reinterpret mathematical beauty fail. Thus, if we intend to safeguard the epistemic or rational character of mathematics, or to understand the meaning and nature of mathematical beauty, we should find a strategy different from reinterpreting mathematical beauty.

Now, De Clercq makes refinements to his view that are further illuminating: some aesthetic terms, 'balanced', for instance, are already metaphors, and some others, such as 'garish', are not universally applicable. De Clercq argues that in the case of already metaphorical terms his characterization in terms of metaphoric resistance is still valid, for these terms cannot be turned into metaphors. There is no such thing, he argues, as a second order metaphor. As for not universally applicable aesthetic terms, De Clercq suggests that we should regard them as "semi-aesthetic" terms [17, pp. 28–29].

Now, even if one finds De Clercq's characterization in terms of metaphorical resilience too strong, for our purposes his weaker assertion that aesthetic terms such as 'beautiful' or 'elegant' can be properly applied to any domain of objects without incurring in a category mistake is still appealing, since it suffices to substantiate a principled scepticism against reinterpreting the term 'beauty'. For example, one might contest De Clercq's view by arguing that instances such as using the expression "a beautiful horse" to mean "a race winning horse" show that aesthetic terms can be turned into metaphors, or at least that they can be used in a non-literal manner.[1] I agree that the usage of expressions like "this horse is beautiful" to refer to the fact that such a horse wins races is non-literal. However, as De Clercq points out, it always makes sense to use terms like 'beautiful' to qualify any kind of entities—including mathematical constructs and entities—in a literal sense. For example, in qualifying racing horses or scientific theories as beautiful, one may encounter some cases in which such expressions are not genuine aesthetic evaluations; nonetheless, such expression can *always* be interpreted in a literal way. The expression "a beautiful something" may mean sometimes *a race winning horse* or *an understanding promoting theory* given the appropriate context. However, without any further information, it is always possible to interpret the expression in a literal way, meaning that the horse or theory elicit a positive aesthetic experience. This is evident in the fact that in order to understand what a "beautiful horse" means in a non-aesthetic sense we need an explanation that tells us what the specialist mean

[1] In fact, some dictionaries includes non-literal definitions like "being advantageous" or "being apt" in their entries for beauty.

by 'beautiful'. But to understand the expression as an aesthetic evaluation, we need no explanation, even if one is not familiar with horses, since we all are familiar with the use of 'beautiful' as a genuine aesthetic term. In order to understand the expression literally there is nothing needed other than acquaintance with language. So, even if we admit that some uses of the term 'beautiful' for evaluating mathematical entities might be intended to convey a non-aesthetic meaning, genuine aesthetic evaluations are always possible in principle. Thus, even if we have a non-literal interpretation of aesthetic evaluations in mathematics acceptable in certain contexts, we still need to deal with the genuine instances of aesthetic evaluations that are possible in principle. Furthermore, the most interesting uses of aesthetic terms in evaluative contexts—such as Hardy's or Russell's, as discussed in the introduction—seem to be genuine aesthetic evaluations. At any rate, non-literal interpretations cannot rule out the possibility of genuine aesthetic evaluations, and thus a thorough analysis of mathematical beauty must address genuine aesthetic evaluations. Any non-literal interpretation is bounded to be insufficient, and it must be supplemented with a way of addressing literal aesthetic evaluations. Now, addressing genuine aesthetic evaluations is not only inevitable, but, as we shall now see, it is also advantageous, since it can provide us with significant insights and tools. Those insights and tools are put to good use by James McAllister.

2.2 McAllister's Approach

In *Beauty and Revolution in Science* [62], by embracing a literal interpretation of aesthetic evaluations in science, James McAllister formulates a rationalist model of scientific change and scientific revolutions. I shall not address the details of that model here; rather I shall concentrate on the issues relevant for our purposes.

McAllister takes an explicit theoretical stance. The results accomplished by that approach illustrates not only that a literal interpretation of beauty can be more coherent and fruitful than a non-literal interpretation, but also that to gain insight into the role of beauty in science a minimal theoretical basis is necessary. McAllister claims that the evolution of scientists' aesthetic preferences for some theories is closely related to the evolution of science itself; that in addition to logical and empirical criteria, aesthetic criteria play a role in the evaluation of scientific theories. So, let us take a closer look.

McAllister abundantly documents the fact that aesthetic evaluations are pervasive in science, which, as we have seen, constitutes a perplexing intrusion of the irrational into science. He also documents the even more perplexing fact that prominent scientists, Dirac, for example [19, p. 10], have endorsed the idea that the beauty of theories plays a significant role in science. In spite of this, McAllister does not opt for reinterpreting beauty; rather he maintains that a rationalist view of science that includes both aesthetic evaluations and scientific revolutions can be defended. Central to McAllister's account is the notion of the aesthetic induction, a mechanism that connects empirical and aesthetic evaluations of theories through an inductive relation.

McAllister shows that throughout the history of science aesthetic evaluations of scientific theories appear not as mere biographical anecdotes or as the result of private idiosyncrasies, but rather as an influential element in the scientists' professional work. In his book [62], McAllister develops a model of scientific change in which the two salient intrusions of irrationality into science, aesthetic evaluations and scientific revolutions, fit coherently into a rational picture of science. To achieve this, McAllister elaborates three closely related theses: first, that the scientists' aesthetic preferences play an actual role in the development of science. Second, that such preferences evolve driven by the aesthetic induction. And third, that scientific revolutions are aesthetic ruptures; that is, episodes in which the set of aesthetic criteria held to by a scientific community is replaced by a new one. We shall concentrate on the second thesis, since that is the most relevant for our purposes.

For McAllister's defence of a rational depiction of science it is essential to establish that the aesthetic evaluations scientists use to choose theories are not irrational. To accomplish this, McAllister argues that there is a non-reductive connection between the scientists' aesthetic evaluations and their empirical evaluations. He argues that scientists come to increase their appreciation for the aesthetic properties of theories that have shown to be empirical adequate in the past because they inductively project that when a new theory exhibits those properties, the theory will be empirically adequate [62, pp. 77–79]. This connection, which he conceptualizes as the aesthetic induction, is the key to give aesthetic evaluations and scientific revolutions a rational basis. Since McAllister's takes the applications of terms such as 'elegant' or 'beautiful' to scientific theories at face value, and that act constitutes the basis for his entire project, a minimal theory of the nature of beauty seems necessary. That theory is our concern here.

2.2.1 McAllister's Aesthetic Theory

McAllister interprets beauty literally; he does not try to find the "true" meaning of beauty in science. Rather, he attempts to gain understanding of, and to some extent systematize, the aesthetic phenomena that affect the way scientific theories are evaluated and chosen by scientists. McAllister's employs the basics of an aesthetic theory to make sense of aesthetic phenomena. However, the degree of rigour with which he discusses the theory is not homogeneous. McAllister utilizes Hutchenson's aesthetic theory to formulate his general approach to the notion of beauty, but he also resorts to more pragmatical and semi-theoretical tactics to address the notion of aesthetic property and to describe the mechanism of evolution of aesthetic preferences.

McAllister endorses some of Francis Hutchenson's ideas to allow him to accommodate scientists' aesthetic judgements in the broader context of aesthetic phenomena. The issue of where value is located and, more specifically, the debate between objectivism, the view that value is available in the world, and

projectivism, the view that value is projected into the world by observers, occupies the central place in McAllister's discussion. Following Hutchenson, McAllister endorses a projectivist approach: beauty is projected into objects by the beholder [62, pp. 31–32]. Moreover, McAllister believes that Hutchenson accounts for the most relevant issues of beauty: Hutchenson tells us what beauty is; an idea caused by the feature of uniformity amidst variety. Hutchenson's ideas also allows us to distinguish between the beauty of a theory and the beauty of other phenomena; for he points out the relevant feature (uniformity amidst variety) that might lead scientists to regard a theory as beautiful. However, McAllister finds Hutchenson's account not completely satisfactory, since McAllister does not believe there is any property that all scientists throughout history recognize as guaranteeing beauty in theories [62, pp. 22–23]. For McAllister, beauty is a dynamic rather than a static concept. Evidence from the history of science supports this idea. Thus, the issue of accounting for the dynamics of the concept of beauty in science becomes central in McAllister's approach. To formulate a model of the evolution of beauty in science McAllister utilizes the evidence provided by the usage of aesthetic evaluations throughout the history of science. His formulation is thus based on the evolution of the scientists' aesthetic preferences. To articulate such formulation, McAllister uses the notions of aesthetic property, aesthetic criteria and aesthetic canon.

2.2.2 Aesthetic Properties, Aesthetic Criteria and Aesthetic Canon

McAllister defines an aesthetic property as "one that evokes aesthetic responses in observers" [62, p. 35] and proposes two criteria to identify which properties of scientific theories are aesthetic.
His first criterion:

> First, I shall judge a property of a theory to be an aesthetic property if scientists in the relevant disciplines react to it publicly as aesthetic, for example by declaring that they attach aesthetic value to it, by citing it in an act of theory evaluation that they describe as aesthetic, or by applying to it standard terms of aesthetic appreciation, such as "beautiful," "elegant," "pleasing," or "ugly." I regard these acts as amounting to aesthetic responses to the property in question, so any property of theories that prompts these acts in scientists satisfies in a straightforward way my definition of aesthetic property [62, p. 36].

His second criterion:

> a property is aesthetic if, in virtue of possessing that property, a scientific theory is liable to strike beholders as having a high degree of aptness. The justification of this criterion is that, in many philosophies of art, the beauty of an object is explicated as its aptness or the aptness of its elements [62, p. 37].

Interestingly enough, McAllister's second criterion covers Weinberg's non-literal interpretation of beauty as referring to the fact that something performs well the task it is expected to perform. McAllister, however, acknowledges that some usages of beauty in that sense are genuinely pseudo-aesthetic. At any rate, as we have

discussed above, the most problematic cases are the genuine usages of aesthetic terms, and we shall concentrate on them. Now, the issue of aesthetic properties is a very contentious one. McAllister's definition is rather pragmatic and, of course, debatable, but I shall assume here that it is sufficient for his purposes; in the next chapters I shall discuss whether or not that is the case.

In McAllister's view, aesthetic properties are essential to explain aesthetic evaluations. McAllister explains aesthetic evaluations in a projectivist way, In such explanation, the links between aesthetic properties and aesthetic evaluations are the aesthetic criteria. In McAllister's projectivist approach, making aesthetic evaluations depends on two factors: the presence of an object—a theory, for instance—bearing aesthetic properties, and the presence of values in the person—a scientist, for instance—observing the object. Scientists are moved to project beauty into a theory as a consequence of their "holding to one or more aesthetic criteria that attach aesthetic value to properties of the theory" [62, pp. 34–35]. Aesthetic criteria are the criteria that attach aesthetic value to specific properties, thus an aesthetic criterion embodies a person's preference, or degree of preference, for certain aesthetic property. For example, let us assume that visual symmetry is a desirable property in things like buildings. McAllister's theory tells us that in that case there is an aesthetic criterion associated to the property of symmetry which is responsible for attributing beauty to symmetrical buildings. Different people hold to different aesthetic criteria, and they do so in different degrees—or with different intensities. This explains the diversity of aesthetic responses in different individuals evoked by the same object. Scientists are not different in this respect: they ascribe beauty to particular scientific theories because they hold to aesthetic criteria that attribute value to the properties of those theories. Now, the aesthetic preferences of an individual or a community change over time. The patters of change of those preferences determine the dynamics of beauty—which is Mcallister's chief interest—and the aesthetic criteria provide a convenient way of modelling those patterns.

2.2.3 The Aesthetic Canon

The aesthetic criteria held by a scientist or a scientific community determine what the scientist or community consider as aesthetically valuable. The exhaustive collection of aesthetic criteria of a scientist or community is called their *aesthetic canon*. In order to model the evolution of aesthetic preferences McAllister expresses the aesthetic canon as an exhaustive, perhaps infinite, list of properties and the numbers representing the intensity of the preference for such properties.

More specifically, McAllister proposes that for every possible property exhibited by a theory, we can conceive a corresponding aesthetic criterion. For example, a property P might have an associated criterion of the type:

If a theory has P, attach more aesthetic value to it than, if other circumstances are equal, it did not.

These aesthetic criteria ground evaluations of theories, since they are guidelines for theory assessment. According to McAllister, such criteria are actually used to choose among theories. Ideally, we can assume that a scientist holds to as many aesthetic criteria as properties to which he responds aesthetically. The scientist's aesthetic canon comprises all such criteria. The different aesthetic criteria possess associated weightings which assess the relative worth the scientist attributes to the property involved in the criterion, and the influence of the criterion in theory choice. Some properties are better regarded than some others; their associated weightings should reflect this fact. In other words, one criterion might weigh or be worth more than another criterion within the canon. To capture these features, McAllister represents aesthetic criteria by means of pairs of items of information: the property P, and its associated weighting W_P. McAllister thus formulate a fully expressed canon as a list of such pairs, as follows:

P, W_P

Q, W_Q

R, W_R

$\vdots$

The aesthetic canon may comprise an infinite number of entries, one for each *possible* property of scientific theories. Most of the criteria will carry a weighting of zero, as scientists typically value only a few properties and are indifferent to the rest. The advantage of this conception of the canon is that any change in aesthetic preferences can be represented simply as a change in the weightings of the criteria [62, pp. 34–35].

All these ideas serve to introduce in an articulated manner McAllister's central notion: the aesthetic induction.

2.2.4 The Aesthetic Induction

Aesthetic preferences are subject to change. That fact is central in McAllister's approach. His key claim is that this change is connected with *empirical* evaluations of theories. In McAllister's view the standard idea that aesthetic phenomena are independent from pragmatic issues is challenged by evidence in the history of art, and, of course, the history of science. McAllister points out that aesthetic preferences do change, evolve, even in the arts. He draws our attention to the case of cast-iron, steel and concrete structures in architecture. These materials were introduced for practical reasons; they were increasingly used in buildings due to their structural advantages. But eventually they gained the appreciation of architects and the public [62, Chap. 9]. So, properties that appeared in buildings due to their practical success, gained in aesthetic preference on its own merit; this influence of pragmatic factors on aesthetic preference is a variety of what McAllister calls the aesthetic induction.

Now, McAllister's goal of defending a rationalistic picture of science depends on showing that the aesthetic evaluations that scientists use to choose theories are not irrational. To accomplish this, McAllister's strategy is to connect the scientists' aesthetic evaluations of some theories with the empirical evaluations they use to choose those theories. Regarding that connection McAllister discusses "two erroneous views of scientists' aesthetic judgments," which he labels *autonomism* and *reductionism* [62, Chap. 4]. Autonomism "regards scientists' aesthetic and empirical evaluations as wholly distinct from and irreducible to one another, whereas reductionism views them as nothing but aspects of one another" [62, p. 61]. McAllister rejects both views, and offers an alternative: scientists come to increase their appreciation for the aesthetic properties that recurrently appear in theories that have shown to be empirical adequate in the past. The reason for this is that they inductively project that when a new theory exhibits those properties, the theory will be empirically adequate [62, p. 77–79]. The change in the scientists' aesthetic preferences is thus the result of the recurrent appearance of certain properties associated with empirically successful theories, this phenomenon is the *aesthetic induction*, and is the link between empirical and aesthetic evaluations which warrants rationality. McAllister describes the aesthetic induction as follows:

> A community compiles its aesthetic canon at a certain date by attaching to each property a weighting proportional to the degree of empirical adequacy then attributed to the set of current and recent theories that have exhibited that property. The degree of empirical adequacy of a theory is, of course, judged by applying the community's empirical criteria for theory evaluation. I name this procedure the aesthetic induction [62, p. 78].

Since there exists a link between empirical and aesthetic evaluations, the possibility that aesthetic properties may be indicators of empirical adequacy cannot be ruled out; and, thus, it is rational to choose theories based on empirical criteria when those theories are empirically equivalent. In this way, aesthetic evaluations are not completely idiosyncratic or subjective. More specifically, in situations where scientists have to choose between empirically equivalent theories, they prefer theories that bear properties with the highest weighting within the aesthetic canon [62, pp. 78–81].

McAllister expands this model of theory choice into a model of scientific change and scientific revolutions by describing different scenarios: the periods in which the aesthetic criteria evolve gradually over time within an aesthetic canon are analogous to what Kuhn calls periods of normal science. The episodes in which an aesthetic canon is replaced by a different one are episodes of aesthetic rupture. McAllister characterize scientific revolutions as episodes of aesthetic rupture. The existence of these scenarios is substantiated and documented by McAllister with a range of historical cases that show the effect of the evolution of aesthetic preferences and episodes of aesthetic rupture in the development of science [62, Chaps. 10 and 11], unfortunately, surveying them is beyond the scope of this book.

To point out an important feature of his model of scientific change, McAllister gives the following illustration of the aesthetic induction at work:

> A scientific community looks back over the recent history of a particular branch of science. It perceives that some theories, which are to a notable degree visualizing (rather

> than abstract) theories, have been empirically very successful, whereas others, which lend themselves to mechanistic analogies, have won little empirical success. Both visualization and tractability by mechanistic analogies are aesthetic properties of theories. In consequence of the empirical success of the visualizing theories, the property of visualization will obtain an increased weighting in the aesthetic canon for theory evaluation that the community will hereafter apply. By contrast, the property of being tractable by mechanistic analogies will receive a lowered weighting in the canon, in virtue of the scarce empirical success of recent theories that displayed this property [62, pp. 78–79].

According to McAllister, "[t]he aesthetic induction is an instance of inductive projection, since it amounts to consulting the properties of past good theories to determine which future theories should be expected to be good" [62, p. 79].

The aesthetic induction induces a bias toward the properties of successful theories:

> By imagining the aesthetic induction in operation, we can infer how a community's set of aesthetic preferences among theories will evolve in particular circumstances. A theory that achieves significant empirical success will cause its community's aesthetic canon to be remodeled to a certain extent, in such a way, that the canon comes to attribute a greater weighting to that theory's aesthetic properties. The canon will therefore acquire a bias in favor of any future theories that exhibit the aesthetic properties of current successful theories. In other words, by their empirical success, theories can predispose the community to choosing future theories with properties similar to their own. A future theory will then win the endorsement from the aesthetic canon in the measure to which it shares the aesthetic properties of current theories that have been attributed high degrees of empirical adequacy. If, on the other hand, a new theory shows properties different from those currently entrenched in the canon. It will be denied endorsement by the aesthetic canon [62, p. 79].

In summary, successful theories greatly contribute to determining which theories will later be welcomed by the scientific community. This type of inductive projection is particular in that the properties involved in past and future theories are not empirical properties, but *aesthetic* properties of theories.

2.2.5 Aesthetic Induction in Mathematics

Although McAllister characterizes the aesthetic induction in terms of empirical adequacy, he argues that a variant of the aesthetic induction influences the development of mathematics [64]. The aesthetic induction operates in mathematics in a fashion similar to how it operates in the empirical sciences:

> [. . .] evidence that conceptions of mathematical beauty evolve under the influence of the aesthetic induction is provided by the gradual acceptance of new classes of numbers in mathematics, such as negative, irrational, and imaginary numbers. Each of these classes of numbers had to undergo a gradual process of acceptance: whereas initially each new class of numbers was regarded with aesthetic revulsion, in due course –as it demonstrated its empirical applicability in mathematical theorizing— it came to be attributed growing aesthetic merit [64, p. 29].

McAllister documents various types of mathematical entities that mathematicians do regard, or have regarded, as beautiful, numbers, theorems, theories; turning, in

the end, to focus on mathematical proofs, as they appear to be specially illuminating. McAllister argues that in Antiquity a prototypical proof was defined as a short, simple series of logical inferences from a set of axioms to the theorem. The series of inferences was required to be sufficiently short and simple that a mathematician could grasp it in a single act of mental apprehension [64, p. 19]. The graspability of a proof is closely related to how well the proof lends itself to being understood:

> Mathematicians' views about beauty in proofs have been influenced by their familiarity with classical proofs. Mathematicians have customarily regarded a proof as beautiful if it conformed to the classical ideals of brevity and simplicity. The most important determinant of a proof's perceived beauty is thus the degree to which it lends itself to being grasped in a single act of mental apprehension [64, p. 22].

McAllister points out that, in recent decades, two new types of proofs have appeared: long proofs, such as Wiles' 108-page-long proof of Fermat's last theorem; and computer-assisted proofs, such as Appel and Haken's proof of the four-colour theorem. These types of proof challenge the classical conception of proof, since they are not graspable in the classical sense, and may even exhibit a logical structure different from the classical proof. McAllister speculates that they might even alter our conception of beautiful proof. In this respect, he focuses on the aesthetic merit of computer-assisted proofs: even if computer-assisted proofs have settled important questions, mathematicians have received them with aesthetic revulsion; but that, McAllister speculates, might change as they become more acceptable.

Since mathematics is not an empirical science, McAllister proposes the acceptability of proofs as the factor involved in the aesthetic induction in mathematics: in the same fashion as the evolution of the beauty of empirical theories depends on their empirical adequacy, the evolution of the beauty of proofs depends on their acceptability. Now, if mathematical beauty indeed evolves driven by the aesthetic induction, the preference for computer-assisted proofs must be driven by it as well:

> [...] the criteria that determine whether a theory is deemed to provide an understanding of phenomena may evolve in response to the empirical success of theories, in accord with the aesthetic induction. If this is true, a deep link exists between the concept of scientific understanding and conceptions of the beauty of scientific theories. On the basis of the reception of computer-assisted proofs, I conjecture that the evolution of aesthetic criteria applied to mathematical proofs is also governed by the aesthetic induction [64, pp. 28–29].

Currently, computer-assisted proofs are regarded as ugly, but in McAllister's view that is merely a contingency. The aesthetic induction allows the possibility that as computer-assisted proofs become more accepted and recurrently succeed in providing results, mathematicians' preferences for them shall evolve, perhaps even to the point of finding them beautiful.

2.2.6 McAllister in Summary

As we have seen, McAllister not only endorses a literal interpretation of beauty in science, but it does so by formulating a sophisticated theory of the nature and role of

beauty in science—although its application to mathematics needs a small variation. That theory can be summarized as follows: (1) Projectivism: McAllister rejects objectivism, which is the view that beauty is an objective property of objects. Beauty is not interpreted as an objective property, but as a value that observers project into objects. A value is something that is considered good, important or desirable. (2) Aesthetic Properties Evoke Aesthetic Responses: objects, including scientific theories, may possess intrinsic properties that evoke aesthetic responses in the observer and lead to project aesthetic value into those objects. These properties are the aesthetic properties. (3) Aesthetic criteria: A person is moved to project beauty into an object when he holds to aesthetic criteria that attribute value to the properties of that object. (3.1) Beauty in science: A scientist, or a mathematician, is moved to project beauty into a theory, or other mathematical entity, when he holds to aesthetic criteria that attribute value to the properties of that object. (4) Aesthetic induction: scientists' aesthetic preferences evolve modulated by an inductive mechanism, the aesthetic induction, in which properties recurrently appearing in empirically adequate, or mathematically acceptable, theories gain in preference. (4.1) Aesthetic induction in Mathematics: mathematicians' aesthetic preferences evolve modulated by the aesthetic induction induced by the acceptability of mathematical entities. (5) Beauty and Scientific Change: scientist often choose between equally empirically adequate theories by opting for theories bearing the properties with the greater degree of aesthetic preference. (6) Beauty and Revolution: Scientific revolutions are episodes of aesthetic rupture [62, pp. 30–34].

Let us conclude this chapter here. In the previous chapter we saw that attempts to eliminate the subjectivity of aesthetic evaluations in science by reinterpreting beauty in science and mathematics have had little success. In this chapter we discussed an argument that should round up and give closure to our discussion of the reasons for embracing a non-literal approach. We encountered reasons to be sceptic about any temptations of reinterpretation. If we take into account ideas like De Clercq's, non-literal interpretations of beauty seem to be hopeless, or at least insufficient in principle. Moreover, McAllister's work gives us further reasons to abandon the search for a non-literal interpretation of beauty in mathematics, since, contrary to any nonliteralist concerns we may have, McAllister's approach shows that a literal interpretation of beauty in science can be used in an articulated and fruitful manner to achieve ambitious goals like defending the rationality of science. McAllister's approach is not only pragmatically appealing, but also very illuminating about the way in which a systematic approach to beauty should be conducted: McAllister's analysis is supported by historical evidence and a proper aesthetic theory. McAllister's work is also illuminating about the possibilities of a coherent approach: it aims to achieve very significant goals, and it is even capable of grounding plausible conjectures. Our goals here are different from McAllister's, but his way of dealing with the subject is inspiring. For that reason, a careful discussion of not only its insights but also its problems is in order. I devote the next two chapters to discuss some problems with McAllister's ideas. Addressing those problems shall prove to be very important for developing an adequate aesthetics of mathematics.