# Chapter 11
# Mathematical Aesthetic Judgements

I endorse a literal approach to mathematical beauty in this book. I thus endorse that mathematical aesthetic judgements are not particularly different from other aesthetic judgements. The notion of aesthetic judgements I have advanced in the previous chapters shall allow us to show that mathematical aesthetic judgements can be characterized in the same terms as other aesthetic judgements. In this brief chapter, I use the aesthetic as process theory to finally account for the leitmotif of our discussion: the term 'mathematical beauty', and mathematical aesthetic judgements in general.

Let us recall Le Lionnais's judgement on Euler's Identity, describing it as insipid. The judgement should be seen as expressing Le Lionnais's subjective state in the context of his aesthetic-process, leading hence to no objective conflict with the mathematicians who judge Euler's Identity the most beautiful theorem in mathematics. For many of the readers of *The Mathematical Intelligencer* the contemplation of Euler's formula elicits a response of pleasure. Since we have assumed that the experience is of the basic type, its pleasure-relation has two possibilities: pleasure and displeasure. The positive affective response is articulated by mapping these possibilities into the schema $\{beautiful, ugly, \ldots\}$, or, even better, $\{beautiful, lovely, ugly, unremarkable, insipid, \ldots\}$ and by reorganizing the domain of mathematical formulae, dividing them into beautiful, ugly, insipid, and so forth. A person who experiences a high degree of pleasure expresses his state by means of the term 'beautiful'. But Le Lionnais, who experiences no response, or even a slightly negative affective response, expresses his state with the term 'insipid'. The object of attention and the modality of the experience in this example is characteristically mathematical. But having characteristic objects and experience modalities is something that is also the case for almost any other type of aesthetic experience. Music, painting or poetry are all very different in terms of the objects with which they present us and the way they engage our attention. In general, aesthetic experience depends on the specifics of each discipline. This is also consistent with my interpretation of aesthetic value, since in that interpretation the value set consist of a wide range of value repositories. In this sense, the idea of a specific

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mathematical value repository makes perfect sense. My conception of aesthetic judgement trivially yields a unified depiction of mathematical aesthetic judgements and the rest of aesthetic judgements, since the process of articulation and the functions of the judgement are the same for mathematical and more traditional aesthetic judgements: mathematical aesthetic judgements express a particular kind of subjective states of mathematicians: the affective evaluation in contemplating a mathematical item.

Now, one of the issues of mathematical beauty is that mathematical aesthetic judgements seem puzzling to a mathematically lay person, since at first sight this person cannot understand how mathematical aesthetic judgements relate to regular, everyday aesthetic judgements. This problem can be easily addressed: the puzzling character of mathematical aesthetic judgements merely manifests the fact that the aesthetic experience in mathematics has a peculiar modality; one that is very dependent on background knowledge. As correctly pointed out by Rota, a great deal of knowledge is necessary to appreciate any piece of mathematics. Now, the modalities of music or painting are also unique in their own way, and, more importantly, they also depend on knowledge: if we know more about music, we hear more things in music; and if we know more about painting we see more things in paintings. In my model, this fact is interpreted as a requirement of the space of mathematical aesthetic intentional objects: in order to be able to appreciate a mathematical item we first must be able to "see" it, that is, we must be able to turn the item into an intentional object. This "seeing" is possible only if we understand the mathematical item. Similarly, in order to enjoy a written poem we must first be able to read it. Mathematical aesthetic judgements are not exceptional: the contents of the experience and the values associated with them are just characteristically mathematical. Mathematical aesthetic judgements constitute one among many classes of aesthetic judgement.

The typical aesthetic-process that grounds the passing of a mathematical aesthetic judgement can be summarized as follows: a literal interpretation of a mathematical aesthetic term $A$ in a mathematical aesthetic judgement '$M$ is $A$', entails that the mathematical item $M$ appears in a locally terminal stage of an aesthetic-process. Such an aesthetic-process is characterized by an experience sub-process in which the content is a mathematical intentional object whose aesthetically relevant dimensions consist of a set of properties "seen" in $M$. The passive and active content of the experience result in an affective response, and, depending on how this occurs, the experience can be categorized as basic, performative or adaptive. The affective response is an affective evaluation that is also involved in a judgement sub-process—described in the previous two chapters. The result of the judgement sub-process is an aesthetic description that expresses the state of the aesthetic-process and that, simultaneously, results in a clarification of the experience—subjective articulation—and the aesthetic-process—process articulation. In addition, mathematical aesthetic judgements carry information that can be used in non-terminal ways, by directly participating in, or encouraging other aesthetic-processes. Mathematical aesthetic terms are terms that participate in encouraging subjective and process articulations in mathematics.

Mathematical aesthetic descriptions are locally terminal, they are bridges between the private and the public aspects of aesthetic-processes. An aesthetic description carries information that can eventually be incorporated in a value repository, thus influencing mathematical aesthetic criteria, future evaluations, and even the work of mathematicians.

We can now give very simple answers to the questions posed at the beginning of this book. Recall that the mathematically lay person is entitled to ask: isn't truth, and not beauty, the goal of mathematics? Or, what is the difference between beautiful and ugly mathematics?

First, truth is the goal of mathematics; beauty comes as an extra, although an important extra that motivates mathematical development. Propositional mathematical knowledge comprises beliefs whose truth is justified by logical means. Truth is a precondition of mathematical knowledge, and this knowledge is in turn a precondition (in the form of a phenomenological space's background-understanding dimension) of mathematical aesthetic experience. A mathematician struggling for beauty, aims at achieving truth *and beyond*, so to speak. Second, the difference between mathematical beauty and ugliness is not simple, as implicit in the existence of composed responses, which leads to the existence of diverse and even overlapping value repositories. This is also manifest in the fact that we need families of terms, rather than isolated terms, to express and clarify aesthetic experiences. However, beauty and ugliness are closely linked in the sense that their expression is often grounded on the fact that they have opposite locations in the same phenomenological space dimension (we like simplicity and dislike complexity, we like harmony and dislike disharmony, and so forth). So, in a sense, the intuitive notion that beauty and ugliness are opposites is valid to a certain extent, even for mathematical beauty and ugliness. Finally, we saw above the reason why the lay person finds strange the use of aesthetic terms by mathematicians: technical knowledge enhances aesthetic experiences in general, but in mathematics knowledge is precondition to even contemplate the object under evaluation.

Now, mathematical aesthetic judgements are intrinsically tied to aesthetic-processes. This means that if we want to analyse a case of mathematical beauty we must be able to produce the appropriate analyses of, at least, experience, value and judgements for mathematical items. The next chapters shall be devoted to present, with some detail, such analyses. I shall apply the theory developed in the previous chapters to three concrete examples of mathematical aesthetic judgements that illustrate a wide enough range of aesthetic terms.