

# A New Measurement of Similarity About Rough Vague Set

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**Abstract** This chapter discussed the concept of rough set and rough Vague (RV) set, the way of representation of knowledge. And also blend rough set with Vague set, described the concept of RV set, gave related concept of rough Vague (RV) value and a new method of measurement of similarity, studied the related property and the method of measurement of similarity for the RV sets.

**Keywords** Rough sets · Vague sets · Rough Vague sets · Similarity measure

## 1 Introduction

The membership function of fuzzy set assigns a number, which is between zero and one for each object as the degree of membership, and it not only includes the proof that the element belongs to the set, but also includes the proof that the element does not belong to the set. For overcoming the insufficiency of the information by the single value description, Zadeh [1] led to go into the interval-valued fuzzy set in 1975, used  $[0, 1]$  of inside closed sub-interval to represent an element how belongs to a set, it descends to carry to order a necessity of means the object belongs to, the top end point the possibility that means the object belongs to. In 1986, Atanassov [2] considered the fuzzy set from a different angle of generalization, he adopted two number to depict a element belonging to the fuzzy set, leading to go into belong to a degree with belonged to one degree concept not, the Atanassov call that definition from here of set for the intuitionistic fuzzy set.

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In 1989, the Atanassov and Gargov [3] point out the interval-valued fuzzy set to intuitionistic fuzzy set, which is the fuzzy set expansion in of two equivalent generalizations. In 1993, the Gau and Buehrer [4] pass “the vote model” to set to carry on understanding to release to the Vague sets. Speak from the essence, the interval-valued fuzzy set, the intuitionistic fuzzy set and the Vague set to have no hypostatic differentiation (see [5]).

At the computer science and its application realm, especially in the artificial intelligence (AI), data mining and knowledge discovery in database (KDD), the theories of rough set have important of physically application, rough set collectively now to description of the thing connection and the whole characteristic, provide important tool to the inside contact of the research thing. The Vague set provides a kind of knowledge to representation of new tool, it is fresh to give people clearly to can know to the thing the degree and scope mean, can carry on a good description to the thing attribute from the form and the top of the contents. Both the rough set theories and Vague set theories study the uncertainty problem in information system, rough set the point of departure that theories solve problem to lie in the knowledge undistinguished in the information system, but the Vague set the fuzzy that theories then is fix attention on in the concept content and person to the concept know of not precision. However, in many situations, the concept is not only misty, and cannot distinguish, cause people’s understanding to the concept also impossibly and completely accurate with overall. According to this because of, need to gather rough theories and the Vague theories carry on blend to make up they are alone it is each while handling an actual problem from of shortage.

In the application study of rough set and Vague set theories, measurement of similarity is an important problem, and it is the application realm of foundation of fuzzy gather, pattern recognition, approximate reasoning, etc. This text owing to this from, studies the problem of similarity measure of RV sets, gives a kind of new measurement method for rough Vague sets.

## 2 The Concept of Rough Set and Vague Set

Let  $U$  be a universe of discourse.  $X \subseteq U$  be an object space, and a Vague set  $V$  defined on  $X \subseteq U$  can use a true membership function  $t_V(x)$  and a false membership function  $f_V(x)$  to mean.  $t_V(x)$  is the membership degree’s bounded to the below from the that the proof of support lead, then  $f_V(x)$  is from the that opposed proof lead of the negation membership degree’s bounded to the below,  $t_V(x)$  and  $f_V(x)$  establish a contact between a real number on  $[0, 1]$  to each one point in  $X$ . i.e.,  $t_V : X \rightarrow [0, 1]$ ;  $f_V : X \rightarrow [0, 1]$ .  $\forall x \in X$ .

The membership degree of  $V$  is denoted by

$$V(x) = [t_V(x), 1 - f_V(x)],$$

where

$$0 \leq t_V(x) + f_V(x) \leq 1.$$

So that, we can noted by  $V = \{(x, t_V(x), f_V(x)) | x \in X\}$ .  $[t_V(x), 1 - f_V(x)]$  be called a Vague value of the point at  $x$  in  $V$ . Noted by  $x_V = [t_V(x), 1 - f_V(x)]$ .

Let  $U = \{x_1, x_2, \dots, x_n\}$  be a limited set,  $R$  be an equivalent relation on  $U$ ,  $U/R$  are all the equivalent classes on  $U$ ;  $[x]_R$  mean containment of  $x$  of  $R$  equivalent class,  $\forall x \in U, \forall X \subseteq U$ .

$$\begin{aligned} \underline{R}(X) &= \{x \in U | [x]_R \subseteq X\}, \\ \overline{R}(X) &= \{x \in U | [x]_R \cap X \neq \emptyset\}. \end{aligned}$$

$\underline{R}(X)$  and  $\overline{R}(X)$  are called low approximate and up approximate. When low approximate and up approximate are not equal,  $X$  is a rough set for  $R$ . Use  $(\underline{R}, \overline{R})$  to represent the rough set in brief;  $Bn_R(X) = \overline{R}(X) - \underline{R}(X)$  is called  $R$  boundary of  $X$ ;  $POS_R(X) = \underline{R}(X)$  is called  $R$  positive area of  $X$ , namely the positive area of rough set;  $Neg_R(X) = U - \underline{R}(X)$  is called the negative area of  $X$ .

### 3 Rough Vague Sets and Rough Vague Value

**Definition 3.1** Let  $U$  be a universe of discourse,  $R$  be an equivalent relation, and  $V$  be a Vague set. The rough Vague sets (RV) are constituted by  $R$  with  $V$  (the RV sets) definition as follows:

$$\begin{aligned} \underline{Rt}(V) &= \inf\{t_V(x) | x \in [x]_R\}; & \overline{Rt}(V) &= \sup\{t_V(x) | x \in [x]_R\}; \\ \underline{Rf}(V) &= \sup\{f_V(x) | x \in [x]_R\}; & \overline{Rf}(V) &= \inf\{f_V(x) | x \in [x]_R\}, \end{aligned}$$

where  $\underline{Rt}$ ,  $\overline{Rt}$  is the least and the biggest value of the true membership degree at same equivalent class and  $\underline{Rf}$ ,  $\overline{Rf}$  is the least and the biggest value of the false membership degree at the same equivalent class. Up and down approximate Vague set mean respectively  $\overline{V} = [\overline{Rt}(V), 1 - \overline{Rf}(V)]$   $\underline{V} = [\underline{Rt}(V), 1 - \underline{Rf}(V)]$ , then  $V = (\underline{V}, \overline{V})$  be called a rough Vague set.

**Definition 3.2** For  $X \subseteq U$ ,  $V = (\underline{V}, \overline{V})$  be a rough Vague set of  $R$  with  $V$  constitute,  $\forall x \in X$ , Record,

$$\overline{V}(x) = [\overline{Rt}(x), 1 - \overline{Rf}(x)], \quad \underline{V}(x) = [\underline{Rt}(x), 1 - \underline{Rf}(x)],$$

$\langle [\underline{Rt}(x), 1 - \underline{Rf}(x)], [\overline{Rt}(x), 1 - \overline{Rf}(x)] \rangle$  be called the rough Vague (RV) value of  $V = (\underline{V}, \overline{V})$ , briefly for  $x$ , i.e.,  $x = \langle [\underline{Rt}(x), 1 - \underline{Rf}(x)], [\overline{Rt}(x), 1 - \overline{Rf}(x)] \rangle$ .

**Definition 3.3** For two rough Vague values

$$x = \langle [\underline{\mathbf{Rt}}(x), 1 - \underline{\mathbf{Rf}}(x)], [\overline{\mathbf{Rt}}(x), 1 - \overline{\mathbf{Rf}}(x)] \rangle$$

and

$$y = \langle [\underline{\mathbf{Rt}}(y), 1 - \underline{\mathbf{Rf}}(y)], [\overline{\mathbf{Rt}}(y), 1 - \overline{\mathbf{Rf}}(y)] \rangle,$$

$x$  with  $y$  be called equal, if and only if  $\underline{\mathbf{Rt}}(x) = \underline{\mathbf{Rt}}(y)$ ,  $\underline{\mathbf{Rf}}(x) = \underline{\mathbf{Rf}}(y)$ , and  $\overline{\mathbf{Rt}}(x) = \overline{\mathbf{Rt}}(y)$ ,  $\overline{\mathbf{Rf}}(x) = \overline{\mathbf{Rf}}(y)$ .

**Definition 3.4** The complement of  $x = \langle [\underline{\mathbf{Rt}}(x), 1 - \underline{\mathbf{Rf}}(x)], [\overline{\mathbf{Rt}}(x), 1 - \overline{\mathbf{Rf}}(x)] \rangle$  is denoted by  $x^c$ , if  $\underline{\mathbf{Rt}}(x^c) = \underline{\mathbf{Rf}}(x)$ ,  $\underline{\mathbf{Rf}}(x^c) = \underline{\mathbf{Rt}}(x)$ ,  $\overline{\mathbf{Rt}}(x^c) = \overline{\mathbf{Rf}}(x)$ ,  $\overline{\mathbf{Rf}}(x^c) = \overline{\mathbf{Rt}}(x)$ .

The complement of the rough Vague set  $V = (\underline{V}, \overline{V})$  is denoted by  $V^c = (\underline{V}^c, \overline{V}^c)$ .

**Definition 3.5**  $(\underline{V}, \overline{V}) \subseteq (\underline{W}, \overline{W})$  if and only if  $\underline{V} \subseteq \underline{W}$  and  $\overline{V} \subseteq \overline{W}$ , i.e.,

$$\underline{\mathbf{Rt}}_V(x) \leq \underline{\mathbf{Rt}}_W(x), \underline{\mathbf{Rf}}_V(x) \geq \underline{\mathbf{Rf}}_W(x) \quad \text{and} \quad \overline{\mathbf{Rt}}_V(x) \leq \overline{\mathbf{Rt}}_W(x), \overline{\mathbf{Rf}}_V(x) \geq \overline{\mathbf{Rf}}_W(x).$$

## 4 Similarity Measurement of Rough Vague Value

In order to study a rough Vague value, we have to first analyze the situation of Vague sets. In the similarity measurement of Vague sets, people often adopt similarity measurement of the Vague value method.

For a Vague value  $x = [t(x), 1 - f(x)]$ , people use  $S(x) = t(x) - f(x)$  record a cent for  $x$ , obviously  $S(x) \in [-1, 1]$ ; we have  $\phi(x) = \frac{1}{2}\{1 - f(x) + t(x)\}$  the middle point of  $x$ , and obviously  $|\phi(x) - \phi(y)| = \frac{1}{2}|S(x) - S(y)|$ ; with  $\pi(x) = 1 - f(x) - t(x)$  as the length (the interval length) of  $x$ , use  $\pi(x)$  to explain Vague's set  $V$  an unknown degree. Correspondently, its known degree can use  $K(x) = 1 - \pi(x) = f(x) + t(x)$  as described and use the degree (affirmation) that its reflection may support.

We know the characteristics of an interval has four important parameters generally, is the left(right) point, the interval length and the middle point. See from the vote model, the Vague value reflected the information of three parts, namely "approve number," "opposed number," and "abstain number." Therefore, we want to measure the similarity of two Vague values and should consider this information and approve a tendency information. For this, the consideration of our comprehensive these aspects, can give the definition to similarity of two Vague values as follows.

**Definition 4.1** Let  $x = [t(x), 1 - f(x)]$  and  $y = [t(y), 1 - f(y)]$  be two Vague values. Then,

$$M(x, y) = 1 - a|t(x) - t(y)| - b|f(x) - f(y)| - c|\pi(x) - \pi(y)| - d|\phi(x) - \phi(y)|$$

is a kind of measurement of the Vague value  $x$  with  $y$ , where  $a, b, c, d \geq 0$  and  $a + b + c + d = 1$ .

So two Vague sets  $V$  and  $W$  defined finite universe of discourse  $X$ , and its measurement of similarity can be given as below:

**Definition 4.2** Let  $X = \{x_1, x_2, \dots, x_n\}$  finite universe of discourse and  $V$  and  $W$  be two Vague sets. Then,

$$M(V, W) = \frac{1}{n} \sum_{i=1}^n \{1 - a|t_V(x_i) - t_W(x_i)| - b|f_V(x_i) - f_W(x_i)| - c|\pi_V(x_i) - \pi_W(x_i)| - d|\phi_V(x_i) - \phi_W(x_i)|\}$$

is a kind of measurement of the Vague sets  $V$  and  $W$ , where  $a, b, c, d \geq 0$  and  $a + b + c + d = 1$ .

According to the above discussion, for  $X \subseteq U$ , a rough Vague  $V = (\underline{V}, \overline{V})$  is constituted by  $R$  with  $V$  set, and its RV value  $x = \langle [\underline{Rt}(x), 1 - \underline{Rf}(x)], [\overline{Rt}(x), 1 - \overline{Rf}(x)] \rangle$  can be written as

$$\underline{S}(x) = \underline{Rt}(x) - \underline{Rf}(x), \quad \overline{S}(x) = \overline{Rt}(x) - \overline{Rf}(x), \quad S(x) = \alpha_1 \underline{S}(x) + \alpha_2 \overline{S}(x),$$

where  $0 \leq \alpha_1, \alpha_2 \leq 1$  and  $\alpha_1 + \alpha_2 = 1$ .

Then,  $S(x)$  can record a cent of a RV value  $x$ . Obviously,  $S(x) \in [-1, 1]$ .

Concerning the middle point of  $x$ , we record

$$\begin{aligned} \phi(x) &= \frac{1}{2} [1 - \underline{Rf}(x) + \underline{Rt}(x)], & \overline{\phi}(x) &= \frac{1}{2} [1 - \overline{Rf}(x) + \overline{Rt}(x)], \\ \phi(x) &= \beta_1 \underline{\phi}(x) + \beta_2 \overline{\phi}(x) \end{aligned}$$

where  $0 \leq \beta_1, \beta_2 \leq 1$  and  $\beta_1 + \beta_2 = 1$ . Record  $\underline{\pi}(x) = 1 - \underline{Rf}(x) - \underline{Rt}(x)$ ,  $\overline{\pi}(x) = 1 - \overline{Rf}(x) - \overline{Rt}(x)$ ,  $\pi(x) = \gamma_1 \underline{\pi}(x) + \gamma_2 \overline{\pi}(x)$ .

$$\underline{K}(x) = \underline{Rt}(x) + \underline{Rf}(x), \quad \overline{K}(x) = \overline{Rt}(x) + \overline{Rf}(x), \quad K(x) = \gamma_1 \underline{K}(x) + \gamma_2 \overline{K}(x),$$

where  $0 \leq \gamma_1, \gamma_2 \leq 1$  and  $\gamma_1 + \gamma_2 = 1$ , then,  $\pi(x)$  can be considered the unknown degree of rough value  $x$  and  $K(x)$  is  $x$  known degree of  $x$ .

Therefore, we can give a following definition to two rough Vague values:

**Definition 4.3** Let two rough Vague values  $x = \langle [\underline{Rt}(x), 1 - \underline{Rf}(x)], [\overline{Rt}(x), 1 - \overline{Rf}(x)] \rangle$  and  $y = \langle [\underline{Rt}(y), 1 - \underline{Rf}(y)], [\overline{Rt}(y), 1 - \overline{Rf}(y)] \rangle$ ,

$$M(x, y) = 1 - \frac{1}{2} \{ a|S(x) - S(y)| + b[|K(x) - K(y)| + |\phi(x) - \phi(y)|] \\ + c[|\underline{Rt}(x) - \underline{Rt}(y)| + |\underline{Rf}(x) - \underline{Rf}(y)|] + d[|\overline{Rt}(x) - \overline{Rt}(y)| + |\overline{Rf}(x) - \overline{Rf}(y)|] \}.$$

Then,  $M(x, y)$  is a kind of similarity measurement for RV value  $x$  and  $y$ , where  $a, b, c, d \geq 0$  and  $a + b + c + d = 1$ , and it reflects the tendency of various information.

So that, we can get a following property by the above definition:

**Theorem 4.1**

1.  $M(x, y) \in [0, 1]$ ;
2.  $M(x, y) = M(y, x)$ ;
3.  $M(x^c, y^c) = M(x, y)$ ;
4. If  $x = y$ , then  $M(x, y) = 1$ .
5. When  $x = \langle [0, 0], [0, 0] \rangle, y = \langle [1, 1], [1, 1] \rangle$   
or  $y = \langle [0, 0], [0, 0] \rangle, x = \langle [1, 1], [1, 1] \rangle, M(x, y) = 0$ .

The (2), (4), (5) in the theorems establish obviously. The one that is underneath proves (1), (3) only.

*Proof (1)* For arbitrarily RV value  $x$ , we can know according to the top definition:  $S(x) \in [-1, 1]$ , then  $|S(x) - S(y)| \in [0, 2]$ ;  $\underline{Rt}(x) \in [0, 1], \underline{Rf}(x) \in [0, 1], \overline{Rt}(x) \in [0, 1], \overline{Rf}(x) \in [0, 1]$ , hence  $|K(x) - K(y)| \in [0, 1], |\phi(x) - \phi(y)| \in [0, 1], |\underline{Rt}(x) - \underline{Rt}(y)| \in [0, 1], |\overline{Rt}(x) - \overline{Rt}(y)| \in [0, 1]$ ,

$$|\underline{Rf}(x) - \underline{Rf}(y)| \in [0, 1], |\overline{Rf}(x) - \overline{Rf}(y)| \in [0, 1], \text{ and } a + b + c + d = 1.$$

Thus,  $M(x, y) \in [0, 1]$ .

*Proof (3)* For arbitrarily RV value  $x$ , since

$$\underline{Rt}(x^c) = \underline{Rf}(x), \quad \underline{Rf}(x^c) = \underline{Rt}(x), \quad \text{and} \quad \overline{Rt}(x^c) = \overline{Rf}(x), \quad \overline{Rf}(x^c) = \overline{Rt}(x).$$

So we have  $S(x^c) = -S(x), |S(x^c) - S(y^c)| = |S(x) - S(y)|$ ;

$$|\underline{Rt}(x^c) - \underline{Rt}(y^c)| = |\underline{Rf}(x) - \underline{Rf}(y)|, \quad |\underline{Rf}(x^c) - \underline{Rf}(y^c)| = |\underline{Rt}(x) - \underline{Rt}(y)|;$$

$$|\overline{Rt}(x^c) - \overline{Rt}(y^c)| = |\overline{Rf}(x) - \overline{Rf}(y)|, \quad |\overline{Rf}(x^c) - \overline{Rf}(y^c)| = |\overline{Rt}(x) - \overline{Rt}(y)|.$$

Hence,  $M(x^c, y^c) = M(x, y)$ .

## 5 Similarity Measurement of Rough Vague Sets

According to the above discussion, let  $X = \{x_1, x_2, \dots, x_n\}$ . For  $\forall x_i \in X$ , two RV sets  $V$  with  $W$  are given by

$$V = (\underline{V}, \overline{V}) = \langle [\underline{Rt}_V(x_i), 1 - \underline{Rf}_V(x_i)], [\overline{Rt}_V(x_i), 1 - \overline{Rf}_V(x_i)] \rangle;$$

$$W = (\underline{W}, \overline{W}) = \langle [\underline{Rt}_W(x_i), 1 - \underline{Rf}_W(x_i)], [\overline{Rt}_W(x_i), 1 - \overline{Rf}_W(x_i)] \rangle.$$

Then, the similarity measurement of  $V = (\underline{V}, \overline{V})$  with  $W = (\underline{W}, \overline{W})$  can be defined as follows.

**Definition 5.1** Let  $X = \{x_1, x_2, \dots, x_n\}$  be finite. The above-mentioned one gives two rough Vague sets, and its similarity measurement can be given as follows:

Let

$$M(V, W) = \frac{1}{n} \sum_{i=1}^n M[V(x_i), W(x_i)]$$

$$= \frac{1}{n} \sum_{i=1}^n \left\{ 1 - \frac{1}{2} [aS_{VW}(x_i) + 2bK_{VW}(x_i) + c\underline{R}_{VW}(x_i) + d\overline{R}_{VW}(x_i)] \right\};$$

where  $S_{VW}(x_i) = |S_V(x_i) - S_W(x_i)|$ ;  $K_{VW}(x_i) = |K_V(x_i) - K_W(x_i)|$ ;

$$\underline{R}_{VW}(x_i) = |\underline{Rt}_V(x_i) - \underline{Rt}_W(x_i)| + |\underline{Rf}_V(x_i) - \underline{Rf}_W(x_i)|;$$

$$\overline{R}_{VW}(x_i) = |\overline{Rt}_V(x_i) - \overline{Rt}_W(x_i)| + |\overline{Rf}_V(x_i) - \overline{Rf}_W(x_i)|; \quad i = 1, 2, \dots, n,$$

$$a, b, c, d \geq 0, \quad \text{and} \quad a + b + c + d = 1.$$

Then,  $M(V, W)$  is a kind of the similarity measurement of rough Vague set  $V$  with  $W$ .

Defining from here can know,  $M(V, W)$  having a following property:

**Theorem 5.1** According to Definition 5.1, the similarity measurement  $M(V, W)$  of rough Vague sets  $V$  and  $W$  has the following property:

1.  $M(V, W) \in [0, 1]$ ;
2.  $M(V, W) = M(W, V)$ ;
3.  $M(V^c, W^c) = M(V, W)$ ;
4. When  $V = (\underline{V}, \overline{V}) = W = (\underline{W}, \overline{W})$ , then  $M(V, W) = 1$ .

*Proof* The one that is underneath proves (1) only.

In fact, since  $S_{VW}(x_i) = |S_V(x_i) - S_W(x_i)| \in [0, 2]$ ;  $K_{VW}(x_i) = |K_V(x_i) - K_W(x_i)| \in [0, 1]$ ;

$$\underline{R}_{VW}(x_i) = |\underline{Rt}_V(x_i) - \underline{Rt}_W(x_i)| + |\underline{Rf}_V(x_i) - \underline{Rf}_W(x_i)| \in [0, 2];$$

$$\overline{R}_{VW}(x_i) = |\overline{Rt}_V(x_i) - \overline{Rt}_W(x_i)| + |\overline{Rf}_V(x_i) - \overline{Rf}_W(x_i)| \in [0, 2]; \quad \text{and} \quad a + b + c + d = 1.$$

So

$$0 \leq aS_{VW}(x_i) + 2bK_{VW}(x_i) + c\underline{R}_{VW}(x_i) + d\overline{R}_{VW}(x_i) \leq 2a + 2b + 2c + 2d = 2.$$

Hence,

$$0 \leq 1 - \frac{1}{2} [aS_{VW}(x_i) + 2bK_{VW}(x_i) + c\underline{R}_{VW}(x_i) + d\overline{R}_{VW}(x_i)] \leq 1, \quad i = 1, 2, \dots, n.$$

Therefore,  $0 \leq M(V, W) \leq 1$ .

## 6 Conclusion

RV sets theories is the mathematics tool of a kind of new processing uncertainty information, very suitable for processing since have the knowledge (concept) that cannot distinguish and have fuzzy. This text synthesizes various circumstance that should consider, giving the new method of the similarity measurement for rough Vague sets, provided the theories foundation for the applied realm that the rough Vague sets.

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