Proficiency of Fuzzy Logic Controller for Stabilization of Rotary Inverted Pendulum based on LQR Mapping

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Abstract. Stabilization of an inverted pendulum is one of the most appealing and conventional problem for control engineering. This system has extremely nonlinear representation and entirely unstable dynamics. The main idea of this research was to design control algorithms for the balancing of rotary inverted pendulum.

Research gives an idea about a convenient approach to implement a real-time control which harmonizes the pendulum in vertical-upright position. Two stabilization controllers, LQR (Linear Quadratic Regulator) and Fuzzy Logic were designed to deal with the non-linear characteristics of the system.

Outcome of both control methods commencing computer simulation are specified to illustrate the efficiency of these controllers. The projected intelligent hybrid controller is evaluated by means of the conventional controller and reliability is demonstrated. The results showed that fuzzy controller exhibit improved performance than LQR near the linearized region.

The paper widened the dynamical representation and initiates the implementation of the considered schemes comparatively.

Keywords: rotary inverted pendulum, stabilization, LQR, fuzzy logic controller, simulink.

1 Introduction

The control of under actuated system is currently a dynamic field of research which is appropriate to the broad application in electromechanical systems like aerospace, robotics and marine vehicles. Pattern of under actuated systems comprised of flexible-link robots, walking robots, acrobatic robots, space robots, helicopters, satellites, under actuated marine vehicles, the pendubot, spacecraft's etc[1]. Under actuated systems comprised into eight classes [2]. The paper demonstrates the control of the rotary inverted pendulum, which belongs to class IIa, as it addresses the tracking problem [3-5]. The rotational configuration is on the whole an amendment of the well-known cart-on-rail pendulum structure.

Compensations of the rotary inverted pendulum system with unhinged poles and non-lowest phase dynamics, nonlinear equations with an uncomplicated arrangement

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direct to choose RIP for testing new control procedure on as a benchmark. As a result engineers like to utilize it for authenticating and estimating the efficiency, robustness, and precision of their recommended control techniques[6].

This is an extremely typical and intellectual nonlinear control dilemma, and numerous techniques previously exist for its explanation [7], for instance, model-based control, fuzzy control, neural network(NN) control, pulse step control, genetic algorithms (GAs)-based control, and so on. On the other hand, the controller was complicated to wholly stabilize a pendulum system within a short period of time[8, 9].

In this paper, distinction of LQR and fuzzy logic control for a rotary-type inverted pendulum system has been identified.

Initially, an LQR was utilized to steady the rotary inverted pendulum in such a way that the pendulum is at all times to retain it upright position and to uphold the arm position in horizontal level surface by making use of a state feedback control to move about unhinged poles of a linear system to steady ones. Accordingly, a Mamdani FIS is deliberated which alleviates the pendulum in the linear region, imitating LQR control just about the stability position. The linear state feedback law is mapped to the system of the fuzzy presumption engine.

2 Mathematical Modeling of Rotary Inverted Pendulum

In this section, the model of the rotary inverted pendulum is established. Rotational inverted pendulum is a nonlinear system of fourth order with a single input variable. The variables relating internal states are as follows: a rotation angle of a base (θ_0), a rotational velocity of a base ($\dot{\theta}_o$), an angle of rotation of the pendulum (θ_1) and its corresponding rotational velocity ($\dot{\theta}_1$).

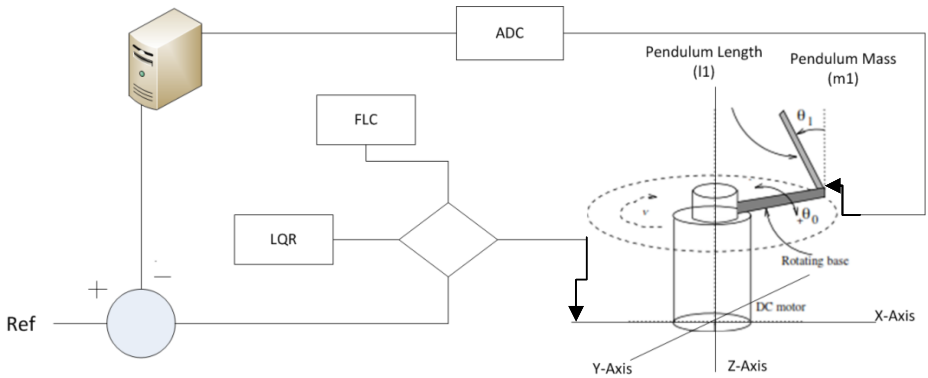

Fig. 1. Orientation and parameters of rotaryinverted pendulum

The input variable for the system is the torque delivered by the motor. The scheme is characterized by two equilibrium points. The steady equilibrium point is attained when the pendulum is leaning upright and pointing downwards. The second equilibrium point is also defined for the vertical orientation, but works for the pendulum pointing upwards[10].

The experimental bed comprised of three prime mechanisms: the plant, digital and analog edge and the digital regulator. The overall scheme is revealed in Fig. 1. The plant embraces of a pendulum and a revolving base type of aluminum rods, an undeviating DC motor to progress the base and two optical encoders as the angular point sensors. While the base swivels all the way through the angleθ_o, the pendulum is liberated to turn around through its angle $\theta_1$prepared with the vertical. Crossing point flanked by the digital controller and the plant comprised of two information possession cards and numerous signal conditioning circuitry[11].

The ordinary differential equations that approximately illustrate the dynamics of the plant are given by:

$$\theta = ap\theta o + Kp\vartheta a \tag{1}$$

$$\ddot{\theta}_1 = -\frac{C_1}{J_1}\dot{\theta}_1 + \frac{m_1 g l_1}{J_1}\sin\dot{\theta}_1 + K_1\ddot{\theta}_o \tag{2}$$

Where:

θ_o = angular displacement of the rotating base
$\dot{\theta}_o$= angular speed of the rotating base
θ_1= angular displacement of the pendulum
$\dot{\theta}_1$= angular speed of the pendulum
ϑ_a= motor armature voltage

Equation (1) and (2) describing the dynamics of the model are extremely nonlinear. Table 1 represents the parameters involved in (1) and (2) of the RIP system:

Table 1. Parameter of Rotary Inverted Pendulum System

Parameter	Description	Value	Unit
K_p	Parameter of DC Motor	74.8903	rad-s^{-2}-v^{-1}
a_p	Parameter of DC Motor	33.0408	s^{-2}
K_1	Torque constant	1.03001×10^{-3}	Kg-m/rad
g	Acceleration due to gravity	9.8006	m/sec^2
m_1	Pendulum mass	0.086184	kg
l_1	Pendulum length	0.113	m
J_1	Pendulum inertia	1.3001×10^{-3}	N-m-s^2
C_1	Friction constant	2.9794×10^{-3}	N-m-s/rad

For the controller synthesis state variable description of pendulum system is required.

This is easily done by defining state variables as:$x_1 = \theta_o$, $x_2 = \dot{\theta}_o$, $x_3 = \theta_1, x_4 = \dot{\theta}_1$ and control signal $u = \vartheta_a$ to get:

$$\dot{x}_1 = x_2 \tag{3}$$

$$\dot{x}_2 = -a_p x_2 + K_p u \tag{4}$$

$$\dot{x}_3 = x_4 \tag{5}$$

$$\dot{x}_4 = -\frac{K_1 a_p}{J_1} a_p + \frac{m_1 g l_1}{J_1} \sin x_3 - \frac{C_1}{J_1} x_4 + \frac{K_1 K_p}{J_1} u \tag{6}$$

Linearization of (3), (4), (5) and (6) about vertical unstable equilibrium position (i.e., $[\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2] = [0, 0, 0, 0]$), results in the linear, time invariant state variable model. By using data in Table 1, linearized model of the rotary inverted pendulum results in:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -33.04 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 49.30 & 73.41 & -2.29 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 74.89 \\ 0 \\ -111.74 \end{bmatrix} u \tag{7}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \tag{8}$$

Equation (7) and (8) is defined by the following equations:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{9}$$

$$y(t) = Cx(t) + Du(t \tag{10}$$

The linearized model in (7) and (8) are not truly represents the physical system, as during the linearization process some of the nonlinearities like motor dynamics, friction , dead-zone and other characteristics are neglected.

3 Full State Feedback (LQR) Design

Linearized model of RIP is completely controllable and observable, therefore linear state-feedback strategies, such as the LQR, are applicable.

In this optimal control technique we try to minimize the defined error as a cost function and the Linear Quadratic Regulator (LQR) method minimizes the cost function (*J*).

The performance index for the LQR is

$$J = \int_0^\infty (x(t)^\intercal Q x(t) + u(t)^\intercal R u(t)) dt \tag{11}$$

subject to

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{12}$$

Q and R in Cost Function represent the weighting matrices of suitable dimension corresponding to the state *x* and input *u*, respectively.

The minimization of *J* results in moving suitable minimum phase poles to stabilize the RIP system immediately with as little controlling force and state deviations as reachable[12]. The control law has the state feedback form

$$u(t) = -\Sigma K_i x_i \tag{13}$$

Given fixed Q and R, the feedback gains *K* in (13) that optimize the function J in (11) can be uniquely determined by solving an algebraic Riccati equation given below:

$$O = G + A^T S + SA - SBR^{-1}B^T S + Q \tag{14}$$

$$K = R^{-1}B^T S \tag{15}$$

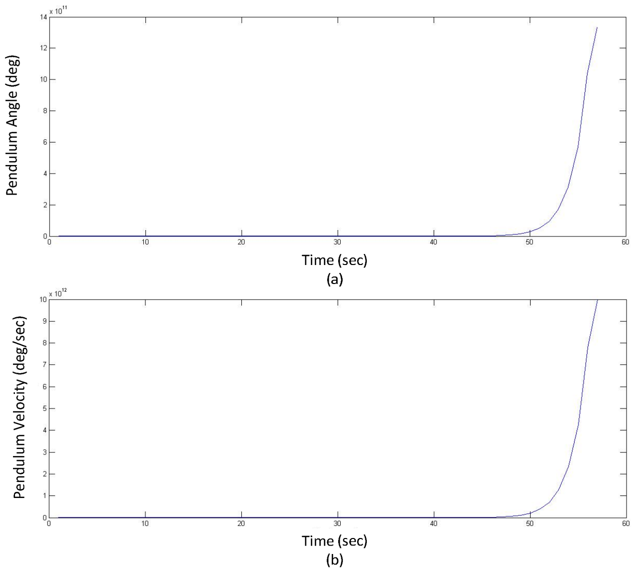

Fig. 2. Open system response with non zero initial condition. (a): simulation result of pendulum angle (θ_1); (b): simulation result of pendulum velocity($\dot{\theta}_1$).

By means of the linearized representation of the system, the subsequent constraints are allocated to devise most favorable gain by LQR technique. Unbolt sphere poles are initiate as 7.4991, -9.7891, 0 and 33.0400. In view of the fact that single pole is lying on the right half of s-plane the system is unbalanced. The unstable response of system with non-zero initial condition is shown in Fig. 2.

By giving the highest priority on controlling θ_1 than regulating the base position, choose the weighting matrices as

$$Q = diag(1,0,5,0) \text{And } R = 1$$

The optimal feedback gains for the controller in (13) corresponding to the weighting matrices Q and R are:

$$K = (-1, -1.191, -9.699, -0.961)$$

On substituting (13) in (12) yields

$$\dot{x}(t) = [A - BK]x(t) \tag{16}$$

The closed loop system poles are -31.84, -14.02, -5.22 and -2.35. They all lie in the left half of s-plane and show the closed loop system is stable.

Applying the control law above, it is observed that the unstable equilibrium point of rotary inverted pendulum remains stable and control performance was found adequate.

The simulation result for stabilization of rotary inverted pendulum by using LQR around unstable equilibrium point with non-zero initial condition is given in Fig. 3.

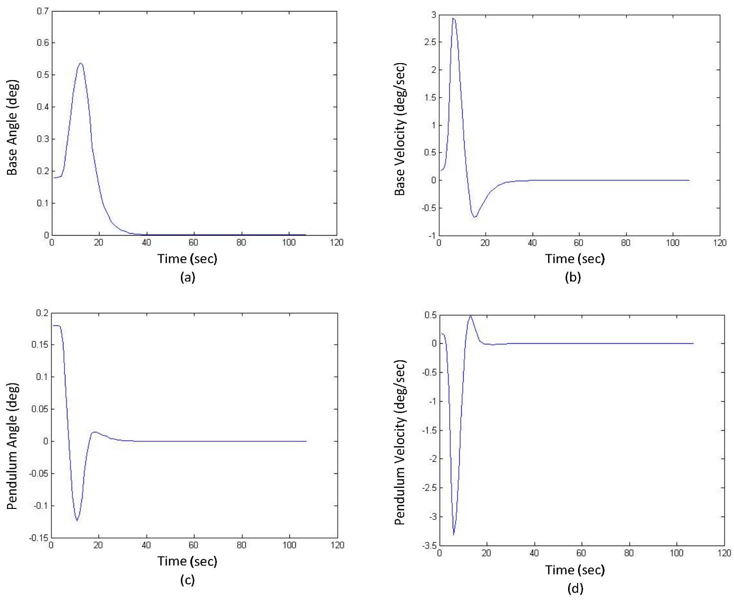

Fig. 3. Rotary inverted pendulum stabilization response by using LQR with non-zero initial condition. (a): simulation result of base angle (θ_o); (b): simulation result of base velocity ($\dot{\theta}_o$); (c): simulation result of pendulum angle (θ_1); (d): simulation result of pendulum velocity($\dot{\theta}_1$).

4 Fuzzy Logic Controller Design

Commencing the realistic point of observation, real-time control necessitates a number of simplifications of the investigational model, and human intrusion is for all time essential for this category of control. In common, a controller based on the understanding of the human machinist is preferred for the realistic function. Fuzzy controllers utilize heuristic information in mounting plan methodologies intended for control of non-linear vibrant systems. This loom eradicates the necessitate for widespread facts and statistical modeling of the system[8]. Within this segment the alleviation of the RIP system by means of FLC through a primary stipulation is presented. Simulink model of fuzzy control system is shown in Fig. 4(a).

The entire numeral of rules is an exponential purpose of the number of contribution and number of association functions. For example for a*n* input system with *N* membership purpose for each input N^n rules are derived.

A four input system through seven connection functions is measured by[13]by means of 2401 rules. Encompassing such a huge amount of rules possibly will grounds difficulties owing to memory restrictions to accumulate the FIS for actual time action using Matlab/Simulink [14]. The instigators of this manuscript originate that for n =2, N =7 and originating 49 rules formulate the assemblage progression too fluctuating but later than additional alteration to the gain these vacillation can be condensed notably.

The two inputs to the fuzzy controller are the position error of the pendulum e_3 and the difference of error e_4. Seven connection functions for every input and output which are uniformly distributed across the universe of discourse are revealed inFig. 4(b), Fig. 4(c) and Fig. 4(d).A Mamdani FIS is deliberated which alleviates the pendulum in the linear zone, imitating LQR control just about the equilibrium position. The linear state feedback law is recorded to the policy of the fuzzy presumption engine. In common, designed for a fuzzy controller by means of *n* inputs and single output, the center of the controller output fuzzy set Y^smembership function would be situated at:

$$(j + k + \cdots l) \times \frac{2}{(N-1)n} \tag{17}$$

Where$s = j + k + \cdots l$ is the index of the output fuzzy set Y^s, $\{j, k, \dots l\}$ are the linguistic-numeric indices of the input fuzzy sets, *N* is the number of connection functions on every input universe of dissertation, and *n* is the number of inputs.

We decide triangular membership functions for these, by means of centers specified by (17) and base widths equal to $\frac{1}{2.5}$.

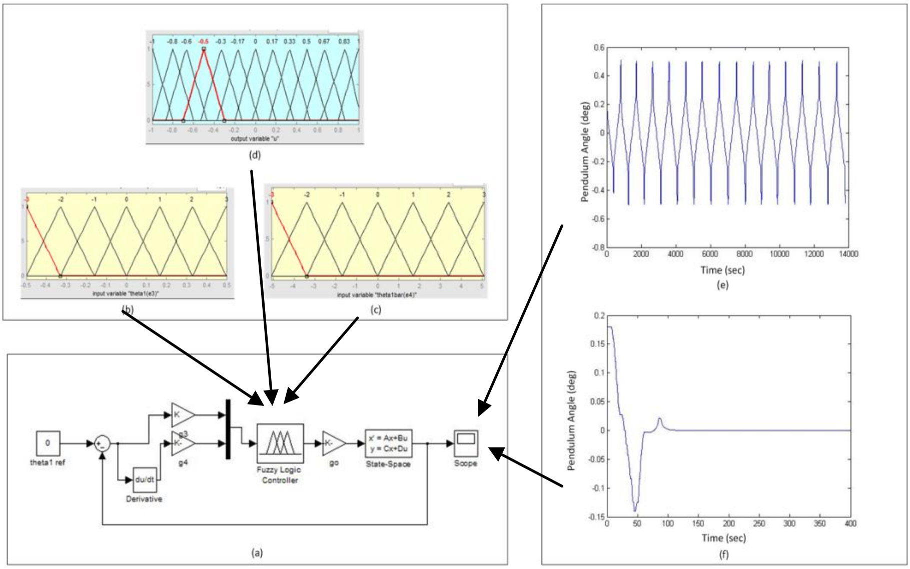

Fig. 4. (a) Simulink model of rotary inverted pendulum with fuzzy logic controller. (b): membership function of input variable (θ_1); (c): membership function of input variable ($\dot{\theta}_1$); (d): Normalized output variable membership function (ϑ_a); (e): simulation response of pendulum angle (θ_1) with derived gain; (f): simulation response of pendulum angle (θ_1) with tuned gains.

The rule-base of RIP system is shown in Table II, where -3, -2, -1, 0, 1, 2 and 3 denote fuzzy linguistic values of negative large, negative medium, negative small, zero, positive large, positive medium and positive small respectively.

Transformation of LQR gains into the scaling gains of fuzzy system is achieved by using following formula

$$g_o g_i = k_i \tag{18}$$

Where k_i are the LQR gains? For $g_o = -4.6$ the fuzzy systems input gains g_1, g_2, g_3 and g_4 are 0.1975, 0.2391, 2 and 0.1957 respectively. Simulation results of RIP system by using FLC with derived and tuned gains are shown in Fig. 8.

5 Results

The simulation results of proposed control system for the rotary inverted pendulum with the SIMULINK in MATLAB 7.0 are shown in Fig. 2, Fig. 3, Fig. 4

and Fig. 5, respectively for conventional Hybrid controller and Intelligent Hybrid Controller.

Fig. 2 shows that rotary inverted pendulum is highly nonlinear model for consideration. To stabilize the system state feedback control technique was used. Fig. 3 shows an adequate stabilization controller response through LQR.

In Fig.4 angle of the pendulum are shown by using FLC. The pendulum shows fluctuating response with calculated gains but after adding tuned gains the response becomes more condensed.

The LQR method for non-zero initial condition couldn't set the pendulum to zero, but fuzzy controller doesn't have this problem[6]. The comparison results of both the controller with non-zero initial condition are shown in Fig.5. It was observed that both FLC and LQR have different steady-state error, settling time and overshoots. Through LQR, pendulum never attained its steady state value to zero. Analysis of obtained results shows that LQR controller relatively gives the fast response and attained its settling state quickly in comparison to FLC, but the pendulum keeps oscillating about its reference position. The proposed fuzzy controller is able to stabilize the pendulum system by tracking the reference signal remarkably, which indicates the disturbance rejection capability of FLC controller

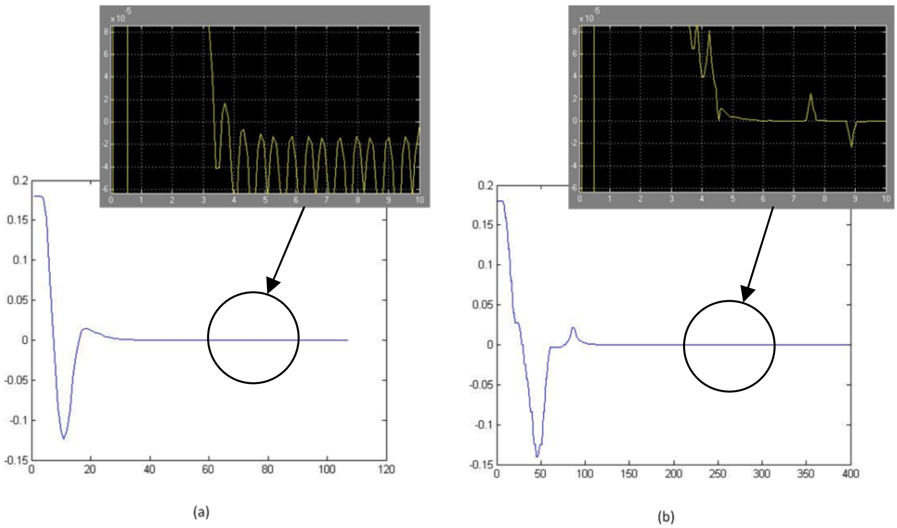

Fig. 5. Comparision results of FLC with LQR for the stabilization of rotary inverted pendulum. (a): simulation result of pendulum angle (θ_1) with LQR; (c): simulation result of pendulum angle (θ_1) with FLC.

Table 2. Inference Rules For Fuzzy Balance Controller

j	k	Y
-3	-3	-1.00
-3	-2	-0.83
-3	-1	-0.67
-3	0	-0.50
-3	1	-0.33
-3	2	-0.17
-3	3	0.00
-2	-3	-0.83
-2	-2	-0.67
-2	-1	-0.50
-2	0	-0.33
-2	1	-0.17
-2	2	0.00
-2	3	0.17
-1	-3	-0.67
-1	-2	-0.50
-1	-1	-0.33
-1	0	-0.17
-1	1	0.00
-1	2	0.17
-1	3	0.33
0	-3	-0.50
0	-2	-0.33
0	-1	-0.17
0	0	0.00
0	1	0.17
0	2	0.33
0	3	0.50
1	-3	-0.33
1	-2	-0.17
1	-1	0.00
1	0	0.17
1	1	0.33
1	2	0.50
1	3	0.67
2	-3	-0.17
2	-2	0.00
2	-1	0.17
2	0	0.33
2	1	0.50
2	2	0.67
2	3	0.83
3	-3	0.00
3	-2	0.17
3	-1	0.33
3	0	0.50
3	1	0.67
3	2	0.83
3	3	1.00

6 Conclusion

The aim of this research was to design a stabilizing controller meant for inverted pendulum and this has been fruitfully attained.

We subsequently compared the performance of the LQR and FLC for a rotary-type inverted pendulum system.

The robustness of both control techniques is verified by running simulation with different initial conditions, which confirms the control efficiency of the method. The results showed that fuzzy controller reveal enhanced performance than LQR near the linearized region.

On the whole, the manuscript presents a relative guide to individuals eager to learn the control laws on such a typical nonlinear and under actuated system.

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