Efficient Global Optimization of a Natural Laminar Airfoil Based on Surrogate Modeling

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Abstract Surrogate-based optimization and efficient global optimization in particular, is considered for aerodynamic design and analysis to deal with some of the drawbacks of classical direct optimization methods. Design of Experiment methods, optimization algorithms, various surrogate modeling methodologies, adaptive sample refinement strategies, multiple criteria for terminating the refinement procedure and several other techniques, are developed and integrated into a practical optimization framework. To search the design space globally and efficiently, several adaptive sample refinement strategies are studied and compared. Two test cases, minimizing the drag of a NLF0416 airfoil with ten design variables and optimizing the performance of a laminar profile with 26 design variables at two design points, are performed. The results indicate that the developed optimization methodology in combination with the adaptive sample refinement strategies features a good balance between global exploration and local exploitation. Additionally, the effect of different design points on the objective function can be automatically considered in the refinement procedure.

1 Introduction

Aerodynamic optimization usually deals with problems with multiple design variables and/or multiple objectives. Thus, the objective function may show highly nonlinear behavior and/or have multiple optima. It is a challenge to find the global optimum for such problems efficiently. Classical direct optimization methods, such as gradient-based methods, are efficient in searching the design space but search locally, while evolutionary algorithms are capable of finding the global optimum at

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the expense of a considerable amount of expensive CFD computations. Surrogate-Based Optimization (SBO) [1], on the other hand, and the Efficient Global Optimization (EGO) [2] method in particular, provides an alternative to overcome some of the drawbacks of the other optimization methods, but relies on additional techniques.

At DLR, a surrogate-based optimization framework has been developed in Python based on an existing framework for classical optimization, called Pyranha [3]. Different enabling techniques, such as DoE methods, sample refinement strategies and criteria for terminating the refinement have been developed and integrated into the framework together with several other modules, including the DLR toolbox for surrogate modeling [4], the in-house tools GenGeo and MegaCADs [5] for geometry parameterization, and the DLR TAU code [6] for flow simulation.

Two test cases, minimizing the drag of a NLF0416 airfoil with ten design variables at constant lift and optimizing the performance of a laminar profile at two design points, were carried out to evaluate and compare different refinement strategies with respect to efficiency and accuracy.

2 Surrogate-based Optimization

As was stated above, surrogate-based optimization depends on several enabling techniques, which are briefly introduced in the following.

2.1 Design of Experiment Methods

Modern Design of Experiment (DoE) methods use certain mathematical algorithms to generate samples within the design space in an optimal way to gain as much information of the objective function as possible from a given number of samples. A certain number of samples are required for constructing an initial surrogate model. The choice of the samples has an obvious effect on the efficiency and accuracy of the surrogate-based optimization chain, because sparsely or badly distributed samples may lead to a surrogate model that is not globally accurate or depicts a fake landscape of the design space, which could result in premature convergence to the region with a local optimum or a huge amount of expensive high-fidelity computations in the sample refinement procedure. The Random Latin Hypercube Sampling [7] (RLHS), the Optimized Latin Hypercube Sampling [8] (OLHS) and the Transport Propagation Latin Hypercube Sampling [9] (TPLHS) were integrated into the framework, because the number of samples is independent of the number of variables, and the random samples can be taken one each time.

2.2 Optimization Algorithms

The optimization methods in the framework consist of gradient-free methods (Simplex/Subplex), three gradient-based optimizers (Conjugate Gradient method (CG), Steepest Descent method (SD), the Newton method), and the Differential Evolution algorithm (DE). Out of those methods, only the DE is a global optimizer.

For classical direct optimization, i.e. by directly evaluating the objective function using a high-fidelity flow solver, gradient-based optimizers are usually the best choice because of their high efficiency, but they search locally. In comparison, the DE is favored for its global search characteristics at the expense of a considerable amount of CFD computations. SBO is a good alternative to overcome some drawbacks of the other optimization methods, because it is much cheaper to approximate the objective(s) on the constructed surrogate model, provided that the surrogate model is accurate enough. The classical optimization methods are also used to select infill samples in order to improve the accuracy of the surrogate model both locally and globally.

2.3 Surrogate Modeling

Surrogate modeling is a mathematical approach to approximate the objective at an unobserved location using observations nearby, or to analyze the trend of the objective function, or to evaluate the correlation between the objective function and the design variables. In SBO, it is used to predict the objective function instead of the high-fidelity solver. Here, the DLR surrogate modeling toolbox was integrated into the framework for SBO, including different Kriging models: Simple Kriging, Ordinary Kriging, Universal Kriging, Regression Kriging and Gradient-Enhanced Kriging. A detailed description of Kriging can be found in the reference [10].

2.4 Sample Refinement Strategies

Adaptive sample refinement is the key technique in SBO due to its decisive effect on the efficiency of SBO and the local and global accuracy of the surrogate model. The EGO algorithm adaptively refines the design space by maximizing the (constrained) Expected Improvement Function (EIF). It evaluates the potential improvement of the objective function on the surrogate model by balancing the probability that the objective at any location in the design space will fall below the current minimum and the model error there. However, in the developed optimization framework, the objective function on the surrogate model, as well as the Kriging error, were also optimized to select infill samples to improve the accuracy of the surrogate model. These are standard sample refinement strategies, called EI-based, Cost-based and KE-based refinement respectively. Besides, different combinations of the three standard refinement strategies were developed to enhance the accuracy of the surrogate model both locally and globally. In addition, multiple infill samples can be selected in each

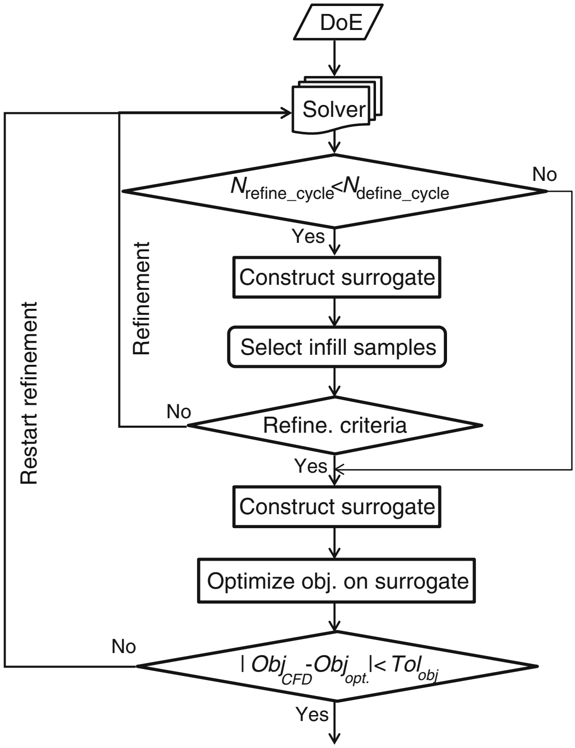

Fig. 1 Flowchart of SBO

refinement cycle, e.g. by running a local optimizer in turn with different starting points when maximizing the EIF. The computation of multiple infill samples per refinement cycle with the flow solver can be performed in parallel to improve the efficiency of the optimization chain.

In case the geometry of an infill sample is infeasible or the CFD computation fails due to convergence, the objective of the infill sample is approximated by interpolating the objective on surrogate model plus considering model error. Thus, the refinement becomes robust, because the surrogate model can always be updated.

To deal with problems with multiple design variables or with multiple optima, for which the convergence of the refinement procedure to the optimum may slow down and the local accuracy of the surrogate model is difficult to be guaranteed, the range of the design space can be decreased after a given number of refinement cycles. This tends to enhance the surrogate model locally in a region where the global optimum may exist and saves computational time by reducing the number of samples used to construct surrogate models. The refinement procedure can be controlled and terminated based on different criteria, e.g. by specifying the maximum number of refinement cycles or a tolerance for the maximum expected improvement on the surrogate model. Properly tuned criteria can guarantee an effective refinement of the design space and a better optimum as well. The flowchart of the efficient global optimization method is shown in Fig. 1.

Table 1 Settings of the refinement strategies

Cases	DoE (RLHS)	Settings of the refinements (The number stands for refinement cycles)	Optimizers for refinement		
			EI	KE	Cost
Pure EI	20	(EI) (0 ∼ 100)	DE		
Hybrid	20	(EI + KE) (0 ∼ 50) + (EI + Cost) (51 ∼ 100)	DE + CG	DE	CG
Enhanced	20	(EI + KE) (0 ∼ 20) + (EI + Cost) (21 ∼ 30) + Start shrinking design space + (EI+Cost)(31 ∼ 35) + (Cost) (36 ∼ 50)	DE + CG (2 start. pts.)	DE	CG (2 start. pts.)

3 Test Cases

3.1 Optimization of a NLF0416 Airfoil with 10 Design Variables

The first test case was performed to evaluate the behavior of different sample refinement strategies. The geometry to be optimized is the NLF0416 airfoil. The flow conditions were a Mach number of 0.1 and a Reynolds number of 2.0e6. The DLR TAU RANS solver combined with the Spalart-Allmaras turbulence model was used and the transition points were predicted using the e^N-method ($N = 8$). The geometry was parameterized with 10 Hicks-Henne bump functions using the in-house parameterization tool GenGeo. During optimization, and hybrid unstructured grid with 40,466 mesh points was deformed using Radial Basis Functions (RBFs). Regression Ordinary Kriging was used to construct surrogate models for its robustness. The objective function is described by

$$\min c_d \quad \text{s.t.} \quad c_{l,\text{target}} = 0.72 \tag{1}$$

The settings of the refinement strategies are shown in Table 1. Different optimization results in Table 2 indicate that SBO is a global optimization method and is more efficient than the Subplex. In addition, the hybrid refinement strategy is able to enhance the local region of the surrogate model more efficiently than the standard EI-based refinement. Although the optimum obtained with the hybrid refinement strategy is slightly better than that using the enhanced hybrid refinement strategy, the latter is more efficient if the infill samples are computed in parallel in each refinement cycle. Figure 2 compares pressure distributions of the optimized geometries with that of the baseline. The Subplex is seen to have extended the laminar extensions on the upper surface, while the SBOs resulted in longer laminar extensions on the upper surfaces.

Table 2 Results of optimizing the NLF0416 airfoil

Optimization strategy	c_l	Opt. (c_d) (d. c.)	c_{dv} (d.c.)	c_{dp} (d.c.)	Trans. pts.		Nr. of cycles	Nr. of CFD comp.	Nr. of proc. used	Wall clock time(s)
					Xtr_U	Xtr_L				
Baseline	0.7193	82.30	53.58	28.72	0.3330	0.5679	×	1	8	ignored
Subplex	0.7209	69.08	44.79	24.29	0.5415	0.5461	500	681	8	327716.8
SBO (Pure EI)	0.7196	64.41	39.35	25.06	0.6833	0.5570	100	122	12	44615.7
SBO (Hybrid)	0.7197	62.97	37.54	25.43	0.7377	0.5461	100	222	24	51137.4
SBO (Enhanced)	0.7201	63.58	37.33	26.25	0.7595	0.5461	50	218	36	24945.7

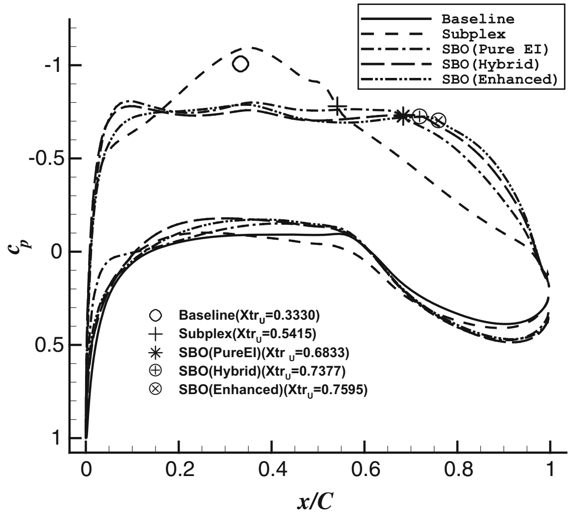

Fig. 2 Comparison of pressure distributions

3.2 Optimization of a Korn Airfoil at Two Design Points

The goal of the second test case is to validate if the SBO framework can efficiently handle multiple design variables and multiple design points. The baseline geometry is the Korn airfoil with the thickness of 15 % of the chord. The freestream conditions at the two design points are a Mach number of 0.66 and a Reynolds number of 1.45e7, and a Mach number of 0.4 and a Reynolds number of 1.2e7, respectively. The DLR TAU RANS solver combined with the Spalart-Allmaras turbulence was used and the transition points were predicted using the e^N-method ($N = 11.5$). The geometry was parameterized with 26 Bezier bumps using the in-house parameterization tool GenGeo. The structured baseline grid with 46,800 mesh points was generated by an in-house structured mesh generator. Regression Ordinary Kriging was chosen to construct surrogate models. To obtain better performance at the maximum cruise velocity as well as at the maximum climb velocity, the objective function was defined as

$$\min\left(c_{d,\text{design point 1}} + c_{d,\text{design point 2}}\right) \text{ s.t. } \begin{array}{l} \text{thickness} \geq 0.150115 \\ c_{l,target,design\ point\ 1} = 0.24 \\ c_{l,target,design\ point\ 2} = 0.44 \end{array} \quad (2)$$

As there was no previous knowledge of the optimum, the optimization was carried out in two steps: in the first step, SBO was performed based on the baseline Korn airfoil; in the second step, SBO with the same settings was performed, using the optimized geometry from the first step as the baseline.

Table 3 Settings of the refinement strategies

Cases	DoE (RLHS)	Settings of the refinements (The number stands for refinement cycles)	Optimizers for refinement		
			EI	KE	Cost
Enhanced	39	(EI+KE+Cost) (0 ∼ 100) Start shrinking design space after 70 cycles	DE+CG (3 start. pts)	DE	CG (3 start. pts)

Table 4 Results for optimizing the Korn airfoil at two design points

Design point 1: $M= 0.66$, $c_{l,target}= 0.24$, $Re= 1.45 \times 10^7$.							
Profile	c_l	c_d(d. c.)	c_{dp}(d. c.)	c_{Dv}(d. c.)	c_m	xtr_U	$\mathbf{xtr}_L$
Korn	0.24	63.18	21.72	41.45	0.13736	0.250	0.248
SBO	0.24	34.35	9.97c	24.39	0.10071	0.635	0.612
LC2	0.24	34.49	10.02	24.88	0.08984	0.584	0.658
Design point 2: $M= 0.40$, $c_{l,target}= 0.44$, $Re= 1.20 \times 10^7$							
Korn	0.44	70.44	21.60	48.82	0.11390	0.130	0.352
SBO	0.44	34.93	9.96	24.97	0.08702	0.659 ∼ 0.647	0.633 ∼ 0.623
LC2	0.44	37.17	10.65	26.52	0.07790	0.599	0.670

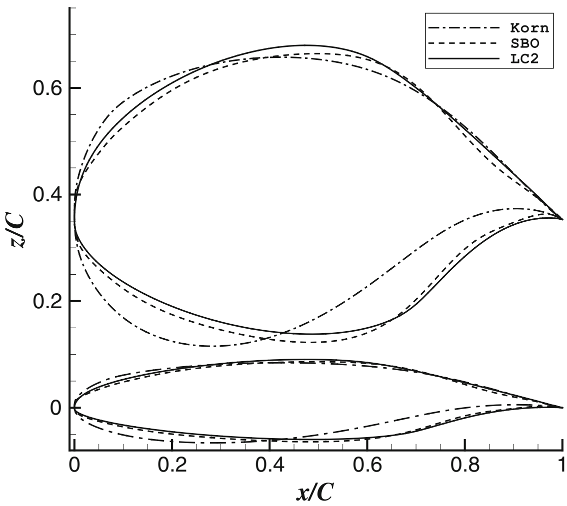

Fig. 3 Comparison of the geometries

The settings for the refinement strategy are shown in Table 3. The results from SBO are compared with those of the baseline geometry from the inverse design (coined LC2 [11]) in Table 4. Figure 4 shows that the thickness distributions of LC2 and the optimized geometry are similar, while the camber distributions are somewhat different. Figure 5 indicates that the optimized geometry features a little longer laminar extension on the upper surface, but a little shorter laminar extension

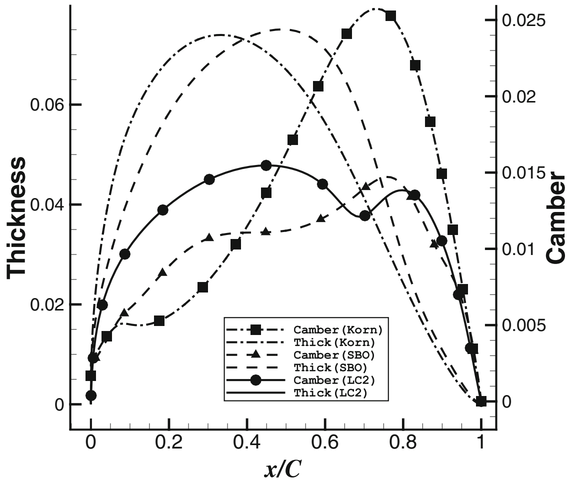

Fig. 4 Comparison of the thicknesses and cambers

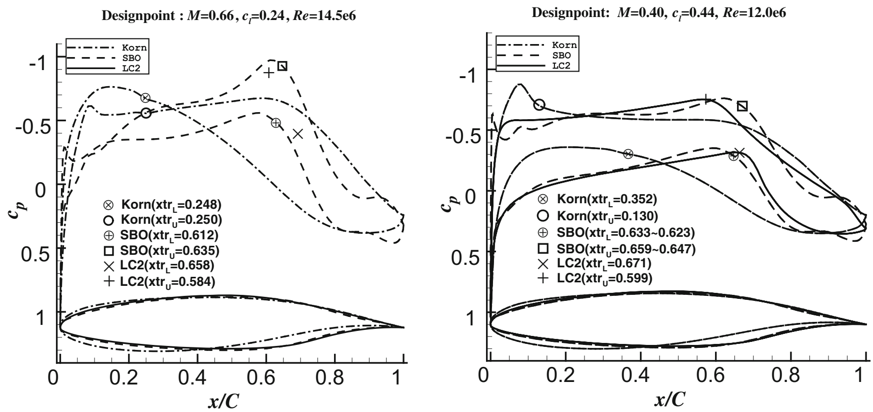

Fig. 5 Comparison of pressure distributions at two design points

on the lower surface in comparison with LC2. Small pressure peaks are seen to arise around the leading edge of the optimized geometry due to the parameterization and the baseline geometry. Figure 6 compares the polars of the baseline, the optimized geometry and the LC2. The optimized geometry has lower drag between the two design points than LC2, while the low drag buckets of LC2 are wider. In the opinion of the author, this is due to the described pressure peaks, which grow fast as the angle of attack increases outside the design range, for it can cause the transition point to quickly move to the leading edge, which results in a sharp increase of the drag.

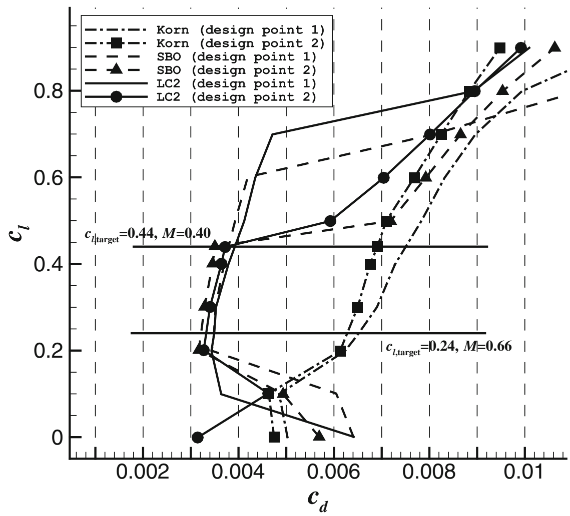

Fig. 6 Comparison of polars of the baseline, the optimized geometry from SBO and the LC2

4 Conclusions

Surrogate-based optimization (SBO) is an efficient global optimization method. Out of different sample refinement strategies, the hybrid refinement strategy can be more efficient in enhancing the accuracy of the surrogate model than the standard individual refinement strategies. The enhanced hybrid sample refinement outperforms others due to its good balance in the local exploitation and the global exploration of the design space. The developed optimization methodology can evaluate the effect of different design points automatically during the refinement.

Despite this, the optimization method shows limitations of setting the design boundaries of the design space in case that no previous knowledge of the optimum is available. Further studies are needed to determine when and how to shrink the design space more effectively.

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