

# Prediction of Transonic Flutter Behavior of a Supercritical Airfoil Using Reduced Order Methods

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**Abstract** The flutter behavior of a supercritical airfoil is investigated using a panel formulation, which solves the subsonic unsteady linearized small-disturbance integral equations. Linear aerodynamic theories provide good predictions for attached moderately subsonic and supersonic flows but break down in the transonic flow conditions due to the nonlinearities inherent in unsteady transonic flow. These nonlinearities dominate the transonic flutter behavior typically resulting in the so-called transonic dip. Time-domain aeroelastic simulations involving Computational Fluid Dynamics (CFD) are computationally very expensive and are not favored when a large number of simulations are required. It is a common practice to correct the unsteady aerodynamics calculated from linear formulations to account for the flow nonlinearities associated with unsteady transonic flows. A Reduced Order Method (ROM) is presented yielding to complex-valued aerodynamic corrections for vibration modes. This ROM is used in linear frequency-domain flutter analyses.

## 1 Introduction

The nonlinearities inherent in unsteady transonic flows make the characteristics of such flows distinctly different from those at low to moderate attached subsonic conditions. The classic reviews of unsteady transonic flows by Tijdeman [1] and Bendiksen [2] reveal that the steady and unsteady flow-fields are essentially coupled in transonic flow. The presence of local supersonic regions terminated by shock-waves (due to the mean steady flow-field) affect the propagation of the unsteady pressure pertur-

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bations. This in turn affects the magnitude and phase of the unsteady pressures on the airfoil/wing. This essential coupling between the steady and unsteady flow-fields is absent in the linear aerodynamic theories. In addition, the unsteady motion of the shock wave causes a peak in the unsteady pressure distribution, which cannot be modeled by linear aerodynamic theories. The above discrepancies are due to the mathematical formulation of the linear governing equations, which assume a constant speed of propagation of the pressure disturbances equal to the free-stream speed of sound [3]. On account of these limitations, linear aerodynamic theories cannot predict unsteady transonic aerodynamic characteristics—magnitude and phase of the unsteady pressures— accurately.

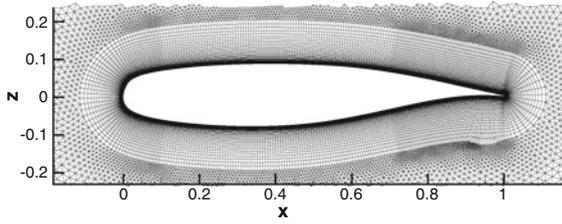
Despite these limitations, linear frequency-domain unsteady aerodynamic theories such as Doublet Lattice Method (DLM) [4, 5] have been in use for decades due to their speed and efficiency. They still are the *de facto* aerospace industry standard for aeroelastic applications. An improvement in the prediction accuracy in the transonic range therefore forms an important step in the calculation of dynamic aeroelastic behavior of airfoils and wings. Several methods have been proposed over the decades. Rodden and Revell [6] used experimentally measured pressures due to steady angle of attack to derive a diagonal correction matrix that would pre-multiply the Aerodynamic Influence Coefficient (AIC) matrix. They assumed that this single matrix based on quasi-steady aerodynamics (frequency = 0) and a single mode (rigid pitch) would be applicable to an arbitrary vibration mode at non-zero reduced frequencies. Bergh and Zwaan [7] extended this method by deriving correction matrices for various reduced frequencies but still based on a single mode. These earliest attempts used experimental pressures as the basis for deriving aerodynamic corrections as accurate numerical predictions of nonlinear unsteady transonic flows were not available. Later on, as the emerging field of CFD matured over the years, numerical calculations were also considered for deriving these corrections. Palacios et al. [8] and Brink-Spalink et al. [9] have reviewed the various aerodynamic correction methodologies developed over the years.

In the present investigation, a CFD based Reduced Order Method (ROM) [10, 11] is presented and implemented in conjunction with a frequency-domain formulation to investigate the flutter behavior of a three degree-of-freedom NLR 7301 airfoil.

## 2 NLR 7301 Supercritical Airfoil

### 2.1 CFD Model of NLR 7301

The configuration considered in this chapter is the 2D supercritical airfoil NLR 7301 [1, 12, 13] at a Reynolds number of  $1.8e6$ . It is a 16% thick airfoil and reaches transonic flow conditions at around Mach 0.7. Figure 1 shows the mesh around the airfoil, which is chosen after a comprehensive grid convergence study. The farfield



**Fig. 1** NLR 7301 nearfield CFD mesh

extension amounts to 100 chords, with 102734 points in total. The  $y^+$  value for the first cell height is kept below 1 in every case.

The CFD calculations are carried out using the TAU code [14] which is based on the Finite-Volume method and solves the time-dependent Reynolds-averaged Navier-Stokes (RANS) equations. The Menter SST [15] turbulence model is applied as well as a central scheme for the discretization of the fluxes. For the temporal discretization, a local timestepping is applied for the steady cases [16], and a dual timestepping [17] for the unsteady calculations.

### 2.2 ZAERO Aerodynamic Model

To demonstrate the applicability of the ROM based aerodynamic corrections, the panel formulation of ZAERO is utilized. ZONA6, the subsonic theory of ZAERO is used to generate the unsteady aerodynamic pressures. ZAERO models wing-like components using a thin sheet of unsteady vortex singularities whereas the body-like components are modeled using a sheet of constant unsteady source singularity to account for the aerodynamic perturbation created by the body volume effects [18].

A box convergence study as well as the *minimum chord length criterion* [18] indicate that 40 chord-wise boxes are sufficient for convergence.

### 2.3 Structural Dynamic Model

The structural dynamic model and the geometrical parameters of the NLR 7301 airfoil are illustrated in Fig. 2. The airfoil consists of three degrees of freedom: plunge ( $h$ ), pitch ( $\alpha$ ), and flap ( $\beta$ ) rotation. The structural parameters are summarized in Table 1.

It is customary to transform the structural dynamic equations of motion from time-domain to the frequency-domain, which in non-dimensional form can be represented as the following matrix equation:

$$\begin{bmatrix} (1 - \omega_{h\alpha}^2 X) & \chi_\alpha & \chi_\beta \\ \chi_\alpha & r_\alpha^2 (1 - X) & (r_\beta^2 + (c_\beta - a_h) \chi_\beta) \\ \chi_\beta & (r_\beta^2 + (c_\beta - a_h) \chi_\beta) & r_\beta^2 (1 - \omega_{\beta\alpha}^2 X) \end{bmatrix} \begin{Bmatrix} h/b \\ \alpha \\ \beta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{1}$$

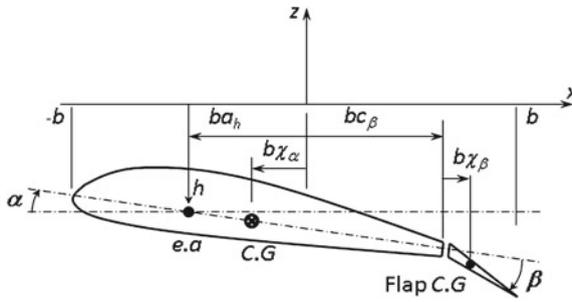


Fig. 2 Airfoil parameters

Table 1 Geometric and structural dynamic parameters [19] of NLR 7301 airfoil

Parameter	$b$	$a_h$	$c_\beta$	$\chi_\alpha$	$\chi_\beta$	$r_\alpha$	$r_\beta$	$\omega_h$	$\omega_\alpha$	$\omega_\beta$	$\mu$
				$\left(\frac{S_\alpha}{mb}\right)$	$\left(\frac{S_\beta}{mb}\right)$	$\sqrt{\frac{K_\alpha}{I_\alpha}}$	$\sqrt{\frac{K_\beta}{I_\beta}}$	$\sqrt{\frac{K_h}{m}}$			$\left(\frac{m}{\pi\rho b^2}\right)$
Units	m	-	-	-	-	-	-	rad/s	rad/s	rad/s	-
Value	0.5	-0.5	0.5	0.086	0.001	0.42	0.143	105.0	140.0	230.0	350.4

where,  $\omega_{h\alpha} = \frac{\omega_h}{\omega_\alpha}$ ;  $\omega_{\beta\alpha} = \frac{\omega_\beta}{\omega_\alpha}$ ;  $X = \frac{\omega_\alpha}{\omega}$ . This is an algebraic eigenvalue problem, whose non-trivial solution yields the mode shapes  $q_i$  and natural frequencies  $\omega_i$  of the airfoil, where,  $i = 1, 2, 3$ . The generalized mass and stiffness matrices along with the aerodynamic model described in Sect. 2.2 form the input to ZAERO.

### 3 Reduced Order Method for Aerodynamic Corrections

With increasing compressibility, the unsteady pressures predicted by linear theories start to depart in comparison to nonlinear formulations such as those based on RANS equations. At transonic conditions, the predictions are grossly incorrect. The purpose of the ROM presented here is to improve the accuracy of the unsteady transonic airloads for all relevant real structural modes and range of reduced frequencies deemed necessary for the flutter computations. In order to restrict the associated computational cost, this has to be done with few CFD computations at each Mach number.

#### 3.1 ROM Description

The various steps in the ROM process are briefly outlined below. For details, refer to [10, 11].

- Step 1: Define suitable synthetic modes.
- Step 2: Compute unsteady pressures due to the harmonic motion of the synthetic and real modes using ZAERO.

Step 3: Compute unsteady pressures due to the harmonic motion of the synthetic modes only using CFD.

The assessment of the flutter behavior in the transonic range requires accurate description of unsteady pressures due to real modes from which the Generalized Aerodynamic Forces (GAFs) are derived. Thus, ZAERO requires a set of correction factors (CF) for each real mode (index  $r$ ) and at each Mach number and reduced frequency. These correction factors are intended to capture the nonlinearities inherent in transonic flows. But, they have to be derived from the available CFD simulations, performed using synthetic modes (index  $s$ ). The correction factors are obtained from the following equation:

$$CF_r = \frac{\Delta p_r^{CFD}}{\Delta p_r^{ZAERO}} = \left\{ 1 + \frac{\sum w_{rs} [\Delta p_s^{CFD} - \Delta p_s^{ZAERO}]}{\Delta p_r^{ZAERO}} \right\} \quad (2)$$

where,  $\Delta p$  represents the pressure difference between the upper and lower surfaces of the airfoil. The weighting coefficients  $w_{rs}$  combine with synthetic modes as a weighted sum to approximate the real structural modes.

### 3.2 Selection of Synthetic Modes

The number and type of synthetic modes used in the ROM depends on the configuration being modeled. The goal is to define a small number of synthetic modes that can accurately describe as many real structural modes as are relevant for the flutter investigation. Synthetic mode sets can include a combination of rigid modes such as pitching and control surface deflection modes and ‘flexible’ modes such as polynomial functions, Legendre polynomials, Tschebyscheff polynomials, or trigonometric functions. Selection of synthetic modes for the NLR 7301 with three degrees of freedom is straightforward. The pure plunge, pitch, and flap deflection modes are considered as the synthetic modes. While the current set-up may seem simplistic, it serves to demonstrate the ROM procedure. However, complex configurations such as 3D swept wings with under-wing nacelles or pods require a careful choice to reduce computational effort.

### 3.3 Unsteady Pressures from CFD

The unsteady pressures for the corrections are obtained using the pulse technique [20]. In this way, the frequency response to each synthetic modeshape can be calculated by only one unsteady calculation. An amplitude of  $0.1^\circ$  for the rotational and 0.01 m for the translational modeshapes are chosen for excitation. The assumption of small perturbation around a steady-state field is still valid.

For the correction process, the corresponding unsteady pressures  $\Delta p_s^{CFD}$  are extracted at discrete reduced frequencies (0, 0.02, 0.04, 0.1, 0.2, 0.4, 0.6, 0.8) at

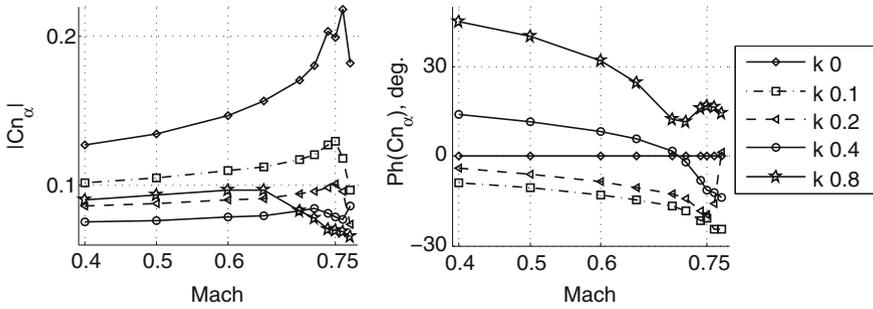


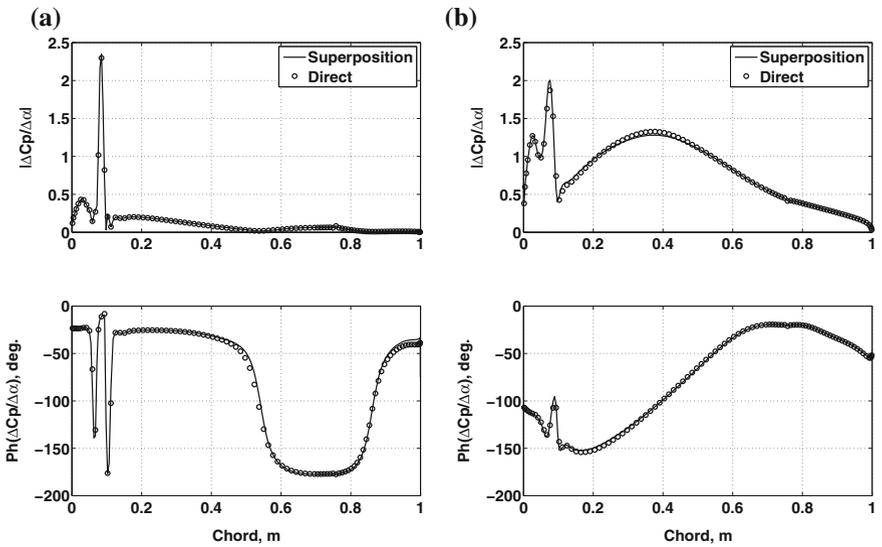
Fig. 3  $Cn_\alpha$  for different reduced frequencies, excitation mode: pitch

each Mach number of interest. These unsteady pressure values are then directly fed into the ROM as discussed in Sect. 3.1. The steady angle of attack is always kept at zero degree. The importance of CFD derived transonic corrections is illustrated in Fig. 3, which shows the complex derivatives of the normal force ( $Cn_\alpha$ ) over Mach number for various reduced frequencies. A pronounced nonlinear behavior is clearly exhibited at transonic conditions ( $Ma > 0.7$ ). These trends may already give a hint to the location of the transonic dip: at a Mach number of about 0.75, the curves show local extrema in magnitude and phase and therefore indicate a high sensitivity of the pressure distribution towards angle of attack.

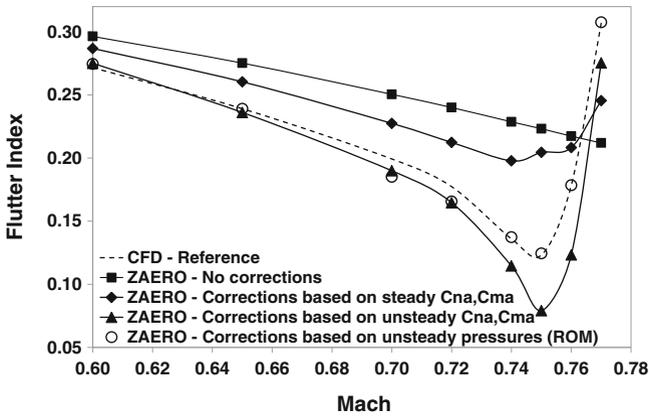
## 4 Results

The validity of the assumption of superposition of the unsteady pressures ( $\Delta C_p$ ) is clearly demonstrated in Fig. 4. The weighted superposition of the unsteady pressures due to synthetic modes gives the same result as the unsteady pressures directly obtained by the perturbation of the real mode.

Figure 5 compares flutter boundaries obtained from various methods. Without any corrections to account for the nonlinear transonic flow effects, ZAERO predicts a flutter boundary without any transonic dip. The reference flutter boundary computed from GAFs that are derived from CFD shows a pronounced transonic dip with a minimum flutter speed at Mach 0.75. When applied to the present configuration, corrections based on steady normal force and moment coefficients due to pitch ( $Cn_\alpha$  and  $Cm_\alpha$  respectively), similar to those derived by Rodden and Revell [6] are unconservative. Corrections based on unsteady  $Cn_\alpha$ ,  $Cm_\alpha$ , similar to those derived by Bergh and Zwaan [7] are conservative. The ROM based corrections, on the other hand, provide the most accurate prediction of the flutter boundary. This implies that it is very important to correct the unsteady aerodynamic pressures of the real structural modes, at least for those relevant for the flutter computations. Corrections based on a single mode (such as pitch) or based only on integrated loads cannot accurately



**Fig. 4** Validation of superposition of unsteady pressures at Mach 0.7. **a** Reduced frequency 0.02 **b** Reduced frequency 0.8



**Fig. 5** Flutter boundaries of NLR 7301 airfoil

capture the phase relationship between the unsteady pressures and the underlying mode shape motion.

## 5 Conclusions and Future Work

A Reduced Order Method is presented and implemented on a 3 degree-of-freedom NLR 7301 airfoil. The ROM provides a means to accurately capture the unsteady aerodynamics of the real modes with the help of synthetic modes. It is important to choose the synthetic modes judiciously: smallest number of synthetic modes that can accurately describe as many real structural modes as are relevant for the flutter investigation. Depending on this number of synthetic modes compared to the real modes, the time saving factor will be different for every configuration. Moreover, unlike the case considered in this chapter, it is possible that the proposed synthetic modes (either the type or the number) may not entirely recreate the real modes, in which case, the accuracy of the flutter prediction is degraded. In the present case, the synthetic modes recreate the real modes perfectly and hence the flutter boundary with the ROM update also recreates the flutter boundary predicted by CFD almost exactly. Also, it is important to individually correct the unsteady aerodynamics (magnitude as well as phase) of each structural mode relevant for flutter. Corrections based only on rigid pitch lead to wrong result and are unconservative if only steady corrections are applied.

Future study involves the validation of the ROM on a complex configuration consisting of a fuselage, 3D swept wing with an under-wing store.

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