Trapezoidal Fuzzy Shortest Path (TFSP) Selection for Green Routing and Scheduling Problems

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Abstract. The routing of vehicles represents an important component of many distribution and transportation systems. Finding the shortest path is one of the fundamental and popular problems. In real life applications, like vehicle green routing and scheduling, transportation, etc. which are related to environmental issues the arc lengths could be uncertain due to the fluctuation with traffic conditions or weather conditions. Therefore finding the exact optimal path in such networks could be challenging. In this paper, we discuss and analyze different approaches for finding the Fuzzy Shortest Path. The shortest path is computed using the ranking methods based on i)Degree of Similarity ii) Acceptable Index, where the arc lengths are expressed as trapezoidal fuzzy numbers. The Decision makers can choose the best path among the various alternatives from the list of rankings by prioritizing the scheduling which facilitates Green Routing.

Keywords: Fuzzy Shortest Path, Bellman's Dynamic Programming, Trapezoidal Fuzzy shortest path, Degree of Similarity, Acceptable Index.

1 Introduction

The vehicle routing and scheduling problems have been studied with much interest within the last four decades. During the last few years, Operations Research (OR) has extended its scope to include environmental applications. In the world, about 73% of the oil is used for transportation purposes. There is a need to design efficient plans for sustainable transportation. Advances in the transportation planning process and efficiency of transportation systems are the key components of the development of sustainable transportation. The routing of vehicles represents an important component of many distribution and transportation systems. The class of problems like routing and scheduling models that relate to environmental issues are known as Green Routing and Scheduling Problems (GRSP), which discuss different problems that relate to sustainable logistics, waste management [1] etc. Some variants of routing and scheduling problems in connection with environmental considerations are : (i) the arc routing problem, which is considered as a major component in waste management, and (ii) the time-dependent vehicle routing problem which allows one

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to indirectly decrease gas emissions involved by transportation activity by avoiding congested routes. Fuzzy logic could be used successfully to model situations in a highly complex environment where a suitable mathematical model could not be provided. For modelling traffic and transportation processes characterized by subjectivity, ambiguity, uncertainty and imprecision, fuzzy logic approach happens to be a very promising mathematical approach. The use of fuzzy logic is advantageous in several situations especially in decision making processes where the description through algorithms is difficult and the associated criteria are multiplied.

The paper is organized as follows: Section 2, discusses the related work. Section 3 provides the prerequisites ie preliminary definitions and concepts required for the computation and analysis of fuzzy numbers. Various algorithms to compute the shortest path in a fuzzy network are demonstrated in Section 4. In Section 5, Experiments using these algorithms are conducted and the results are compared and presented graphically. Section 6 presents conclusions and provides an excellent platform for future work.

2 Related Work

The fuzzy shortest-path problem was first introduced in 1980 [2] , where the authors used Floyd's algorithm and Ford's algorithm to treat the fuzzy shortest path problem. Authors [4] proposed a dynamical programming recursion-based fuzzy algorithm. Authors [5] found the fuzzy shortest path length in a network by means of a fuzzy linear programming approach. Authors of [6] defined a new comparison index between the sums of fuzzy numbers by considering interactivity among fuzzy numbers and presented an algorithm to determine the degree of possibility for each arc on a network. Authors [7] developed two types of Fuzzy Shortest Path network problems, where the first type of Fuzzy Shortest Path Problem uses triangular fuzzy numbers and the second type uses level $(1 - \beta, 1 - \alpha)$ interval valued numbers. The main result from their study was, the Shortest Path in the fuzzy sense correspond to the actual path in the network, and the Fuzzy Shortest Path Problem is an extension of the crisp case. [8] represented each arc length as a triangular fuzzy set and a new algorithm is proposed to deal with the FSPP. [10] studied the Fuzzy Shortest Path Problem Based on Degree of Similarity. Thus numerous papers have been published on the FSPP. In this paper, we discuss and analyze different approaches for finding the Fuzzy Shortest Path and illustrated by considering an example.

3 Pre-requisites

In this section, an overview of the fuzzy-set concepts and definitions are presented.

Definition 1: A *fuzzy number* is a quantity whose value is imprecise, rather than exact as is the case with "ordinary" (single-valued) numbers. Any fuzzy number can be thought of as a function whose domain is a specified set, usually the set of real numbers, and whose range is the span of non-negative real numbers between, and including, 0 and 1. Each numerical value in the domain is assigned a specific "grade of membership" where 0 represents the smallest possible grade, and 1 is the largest possible grade. Let X = {x} be a universe, i.e. the set of all possible (feasible, relevant) elements to be considered. Then a fuzzy set (or a fuzzy subset) A in X is defined as a set of ordered pairs , where A = {x, $\mu_A(x)$), x∈X} is the membership grade or degree of association of x in A, where 0 value indicates non belongingness and 1 indicates full belongingness.

Definition 2: A *fuzzy quantity* is defined as a fuzzy set in the real line R, that is, an R → [0, 1] mapping A. If A is upper semi-continuous, convex, normal, and has bounded support, then it is called a fuzzy number.

4 Methodology

In this section , a detailed description about the method of analysis is presented which comprises the Shortest Path using i) Bellman Dynamic Programming for Crisp Arc Lengths(BDPCL) ii) Bellman Dynamic Programming for fuzzy arc lengths (BDPFL) and iii) Trapezoidal Fuzzy Shortest Path Selection using Degree of Similarity (DS) and Acceptable Index(AI).

4.1 Bellman Dynamic Programming for Crisp Arc Lengths (BDPCL)

Bellman 's Dynamic Programming for the shortest path problem can be formulated as follows: given a network with an acyclic directed graph $G = (V, E)$ with n vertices numbered from 1 to n such that 1 is the source and n is the destination. The shortest path is given by

$$f(n)=0$$

$$f(i) = \min\{d_{i,j} + f(j) \mid \langle i, j\rangle \in E\} \tag{1}$$

Here d_{ij} is the weight of the directed edge i, j ,and f (j) is the length of the shortest path from vertex i to vertex n .

4.2 Bellman Dynamic Programming for Fuzzy Arc Lengths (BDPFL)

Triangular fuzzy number A can be defined by a triplet (a, b, c) defined on R with the membership function defined as

$$\mu A(x) = \begin{cases} (x-a)/(b-a), & a \le x \le b \\ (c-x)/(c-b), & b \le x \le c \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

The edge weight in the network, denoted by d_{ij} and the edge weight should be expressed as triangular fuzzy number.

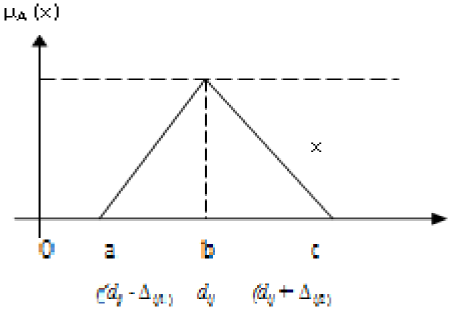

Fig. 1. Fuzzy number d_{ij}

The Signed distance of (a, b) from the origin is given by 1/2(a+b). Similarly, the signed distance of (a, b, c) can be given by $d^* = 1/4(a+2b+c) = d_{ij} + 1/4\,\Delta_{ij}$ (3)
If $\Delta_{ij1} = \Delta_{ij2}$, thus the fuzzy problem becomes crisp.

4.3 Trapezoidal Fuzzy Shortest Path (TFSP)

The trapezoidal shape is originated from the fact that there are several points whose membership function (degree) is maximum (=1). Trapezoidal Fuzzy number A can be defined by a quadruplet (a, b, c, d) defined on R with the membership function defined as

$$\mu A(x) = \begin{cases} (x-a)/(b-a), & a \le x \le b \\ 1, & b \le x \le c \\ (d-x)/(d-c), & c \le x \le d \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

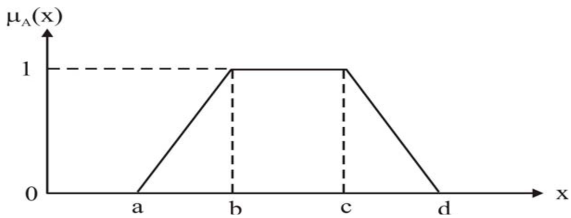

Fig. 2. Trapezoidal fuzzy number

Construct a fuzzy graph (network) G = (V,E) with pure fuzziness, where V is the set of vertices or nodes and E is the set of edges or arcs. Here G is an acyclic digraph. Let the arc length L_{ij} be the trapezoidal fuzzy numbers. Consider all possible paths P_i from the source vertex '1' to the destination vertex 'n' and find the corresponding path lengths Li, i = 1, 2, . . . , n and set Li = (a_i, b_i, c_i, d_i). If $b = c$, the trapezoidal fuzzy number becomes the triangular fuzzy number.

i) Degree of Similarity (DS): Degree of Similarity is the measurement of the nearness degree of two fuzzy sets. Let d_{min} (Fuzzy Shortest Length) = $A1 = (a, b, c, d)$ and L_i(i^{th} fuzzy path length) = $A2 = (a_i, b_i, c_i, d_i)$ be two Triangular Fuzzy Numbers. If $a \leq a_i, b \leq b_i$, $c \leq c_i$ *and* $d < d_i$ then the *Degree of Similarity (DS)* between $A1$ and $A2$ can be calculated as follows:

$$SD(d_{\min}, Li) = \begin{cases} 0 & \text{if } L_i \cap dmin = \Phi \\ 1/2\times\ (d - a_i)^2/((d - c) + (b_i - a_i)) & \\ \quad \text{if } L_i \cap dmin \_\neq \Phi\text{whre } c < x < d,\ a_i < x < b_i & \\ 1/2\times [(d - a_i) + (c - b_i)] \quad \text{if } L_i \cap dmin \neq \Phi\text{here } b_i \leq x \leq c. & \end{cases} \quad (5)$$

ii) Acceptability Index (AI): The *Acceptability Index (AI)* of the proposition L_{min} = (a, b, c, d) is preferred to

L_i =(a_i, b_i, c_i, d_i) is given by

$$AI(Lmin < Li) = \frac{-\ (d\ +\ ai\ )}{(c\ -\ d\ )\ -\ (bi\ -\ ai\ )} \quad (6)$$

Using this Acceptability Index we define the ranking order based on highest Acceptability Index, i.e., in Acceptability Index, $L1 < L2$ iff $AI(Lmin < L1) > AI(Lmin < L2)$.

5 Experiments and Results

In this section , the experiment is carried out to predict the selection of shortest path using a)Crisp values b) Trapezoidal Fuzzy Crisp values and 3) Trapezoidal Fuzzy values. Techniques like like i) Degree of Similarity ii). Acceptable Index. are used to determine the rank of Trapezoidal Fuzzy Shortest Paths by considering the network as shown in the fig 3.

5.1 Bellman Dynamic Programming for Crisp Arc Lengths(BDPCL)

Consider a network Fig 3, Bellman–Ford algorithm computes the shortest paths in weighted directed graphs follows.

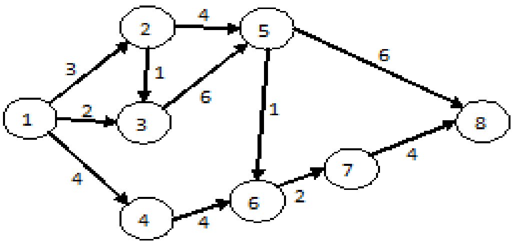

Fig. 3. Network

f(8) = 0

f(7) = min {(d_{7j} + f(j) │<7,j> Є E} = d_{78} = 4

7<

f(5) = min {(d_{5j} + f(j) │<5,j> Є E} = 6

f(6) = min{ d_{67} + f(7)} = 2 + 4 = 6

$f(5) = \min\{d_{58}, d_{56} + f(6)\} = \min(6, 7) = 6$ $f(4) = \min\{(d_{46} + f(6)\} = 4 + 6 = 10$
$f(3) = \min\{(d_{35} + f(5)\} = 12$ $f(2) = \min\{d_{25} + f(5), d_{23} + f(3)\} = 10$
$f(1) = \min\{d_{12} + f(2), d_{13} + f(3), d_{14} + f(4)\} = 13$

Therefore the shortest path (P1->$d_{12} + d_{25} + d_{58}$) Length = 13

5.2 Bellman Dynamic Programming For Fuzzy Arc Lengths (BDPFL)

Considering our network in Figure 3, we get the following inequalities:

Step1: $d_{12} + f(2) < d_{13} + f(3)$
$d_{12} + d_{25} + d_{58} < d_{13} + d_{35} + d_{58}$
Step2: $d_{12} + f(2) < d_{14} + f(4)$
$d_{12} + d_{25} + d_{58} < d_{14} + d_{46} + d_{67} + d_{78}$
Step3: $d_{25} + f(5) < d_{23} + f(3)$
$d_{25} + d_{58} < d_{23} + d_{35} + d_{58}$
Step4: $d_{58} + f(8) < d_{56} + f(6)$
$d_{58} < d_{56} + d_{67} + d_{78}$

To express the uncertainty in edge weights we use trapezoidal fuzzy numbers. The values of parameters are chosen in such a way so as to satisfy the above inequalities and then the fuzzy arc lengths based on equation (3) are calculated:$\Delta_{12} = 0.5$ $\Delta_{23} = 0.9$ $\Delta_{13} = 1$ $\Delta_{67} = 1$ $\Delta 56 = 1.5$ $\Delta_{46} = 1.3$ $\Delta_{58} = 0.8$ $\Delta_{78} = 0.9$ $\Delta_{35} = 1.2$ $\Delta_{14} = 0.8$ $\Delta_{25} = 1$

Defuzzified Arc Lengths: $d^*_{12} = d_{12} + \Delta_{12} = 3.125$; $d^*_{13} = d_{13} + \Delta_{13} = 2.25$; $d^*_{14} = 4.2$; $d^*_{25} = 4.25$; $d^*_{23} = 1.225$; $d^*_{35} = 6.3$; $d^*_{46} = 4.325$; $d^*_{56} = 1.375$; $d^*_{58} = 6.2$; $d^*_{67} = 2.25$; $d^*_{78} = 4.225$; $d^*_{46} = 4.325$.

Table 1. Fuzzy Arc crisp Path Lengths

Paths	**Path Lengths**	**Rank**
*P*1: 1 – *2 – 5 –8*	3.125 + 4.25 + 6.2 = 13.575	1
*P*2 : 1 – *3* – 5 – *8*	2.25 + 6.3 + 6.2 = 14.75	2
*P*3 : 1 – *4* – 6 – *7* – *8*	4.2 + 4.325 + 2.25 + 4.225 = 15	3
*P*4 : 1– *3 –5 –6 –7– 8*	2.25 + 6.3 + 1.375 + 2.25 + 4.225=16.4	5
*P*5 : 1 – *2* – *5* – *6* – *7* – *8*	*3.125+4.25+1.375+2.25+4.225=15.225*	*4*
*P*6 : 1 – *2* – *3*– *5* – *8*	*3.125+1.225+6.3+6.2=16.85*	*6*

5.3 Trapezoidal Fuzzy Shortest Path Length (TFSPL)

Consider the trapezoidal fuzzy numbers for the network are as follows:d'_{12} =(2.8, 3, 3.7, 4.2); d'_{13}=(1.5, 2, 3.5, 6.2) ; d'_{14}=(3.8, 4, 5, 6.5); d'_{25}=(3, 4, 6, 8); d'_{23}=(0.7, 1, 2.2, 3); d'_{35}=(5.7, 6, 7.5, 10);d'_{46} =(3.8, 4, 5.5, 6);d'_{56} =(0.5, 1, 3, 5);d'_{58} =(5.7, 6, 7.1, 7.8;d'_{67} =(1, 2, 4, 6); d'_{78}=(3.8, 4, 5.5, 6.5)

Table 2. The shortest path length procedure is based on the *[4]* method.

Algorithm-1:
Input: Li= (ai',bi',ci',di'),i=1,2,….,n where Li denotes the trapezoidal fuzzy length.
Output: d_{min}=(a,b,c,d) where d_{min} denotes the FSL.
Step 1: Form the set U by sorting Li in ascending orders of bi' and ci'.U={U1,U2,….,Un} where Ui= (ai',bi',ci',di'), i=1,2,….,n
Step 2: Set d_{min}=(a,b,c,d)=U1 = (a1',b1',c1',d1')
Step 3: let i=2
Step 4: Calculate (a,b,c,d)
a = min(a,ai ')

$$b = \begin{cases} b & b < a_i \\ \dfrac{(b \times b_i') - (a \times a_i')}{(b + b_i') - (a + a_i')} & b > a_i' \end{cases}$$

$$c = \begin{cases} c & c < b_i \\ \dfrac{(c \times c_i') - (b \times b_i')}{(c + c_i') - (b + b_i')} & c > b_i' \end{cases}$$

d = min (d , ci ')
Step 5: Set d_{min} = (a,b,c,d)
Step 6: Set i = i + 1
Step 7: i< n+1 goto Step 4
Step 8: Compute the above procedure to get the Fuzzy Shortest Length(FSL)
Step 9: Identify the SP with the highest Similarity Degree or highest Acceptable Index the between FSL and Li,for i= 1 to n.

All the possible paths and the corresponding trapezoidal fuzzy path lengths and the Fuzzy Shortest Length(FSL) are computed by using Algorithm-1and presented in Table 3.

Table 3. Trapezoidal fuzzy Paths Lengths & FSL Computational Procedure

Paths	Path Lengths	FSL
$P1$: 1 – 2 – 5 – 8	(11.5, 13, 16.8, 20)	d_{min} = (11.5, 13, 16.8, 20)
$P2$: 1 – 3 – 5 – 8	(12.9, 14, 18.1, 24)	d_{min} = (11.5, 12.94, 15.45, 18.1)
$P3$: 1 – 4 – 6 – 7 – 8	(12.4, 14, 20, 26)	d_{min} = (11.5,12.68,15,18.1)
$P4$: 1– 3 –5 –6 –7– 8	(12.5, 15, 23.5, 33.7)	d_{min} = (11.5, 12.62, 15, 18.1)
$P5$: 1 – 2 – 5 – 6 – 7 – 8	(11.1, 14, 22.2, 29.7)	d_{min} = (11.1, 12.196, 14.77, 18.1)
$P6$: 1 – 2 – 3– 5 – 8	(14.9, 16, 20.5, 25)	d_{min} =(11.1, 12.196, 14.77, 18.1)

5.4 Results

The shortest path selection can be done by using Degree of Similarity (eqn.5) or Acceptable Index (eqn.6). It has been identified with the highest DS or AI. The ranking given to the paths based on highest DS which helps the Decision Maker to identify the preferable path alternatives. The Degree of Similarity and the Acceptable Index between FSL and Li, for i = 1 to 5 are computed and presented in the Table 4.

Table 4. Results of TFSP using DS and AI

Paths	DS	AI	Rank
P1: 1 – 2 – 5 –8	4.15	1.063	1
P2: 1 – 3 – 5 – 8	2.985	1.025	4
P3: 1 – 4 –6 – 7– 8	3.23	1.026	3
P4: 1–3–5–6–7–8	2.689	0.993	5
P5: 1–2–5–6–7–8	3.885	1.028	2
P6: 1–2–3–5–8	1.155	0.964	6

The shortest path selection will be done on a network with fuzzy arc lengths where the shortest path is identified using the concept of ranking function with regard to the fact that the Decision Maker can choose the best path among various alternatives from the list of rankings. The Rank comparison graph of different approaches i)Crisp Arc Lengths, ii) Fuzzy Crisp Arc Lengths and iii) Trapezoidal Fuzzy Arc Lengths for Shortest Path selection is presented in figure 4.

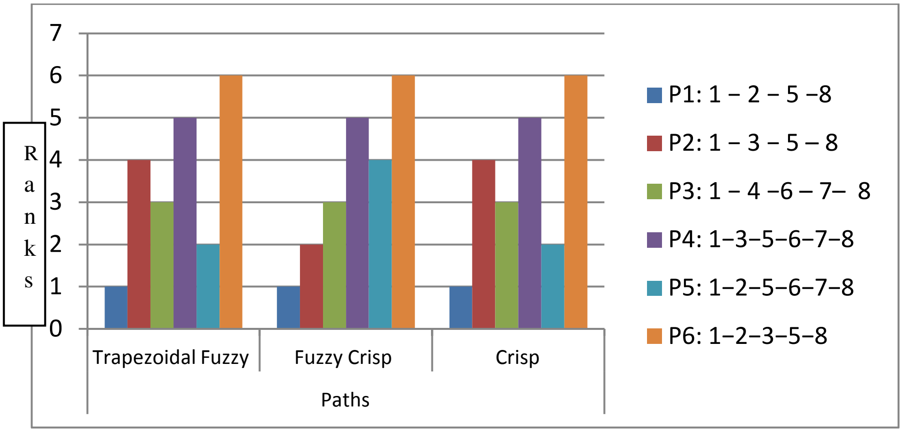

Fig. 4. Ranks VS Paths

6 Conclusions

The study of Green Routing and Scheduling problems is gaining momentum in recent years. We have discussed shortest path algorithms by using a)Crisp values b) Trapezoidal Fuzzy Crisp values 3) Trapezoidal fuzzy values. The rank given to the paths based on these techniques helps the Decision Maker to identify the preferable path alternatives. Prioritizing the scheduling and the selection of the paths automatically results in the reduction in exhaust fumes facilitates Green routing.

Finally, it is concluded that the results of the present investigation would provide an excellent platform for making effective and efficient decisions in the case Green Routing and Scheduling Problems and also to design efficient plans for sustainable transportation.

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