An Interpolation-Free and Fitting-Less Sub-sample Time-Delay Estimation Algorithm

C.A. Teixeira[1], M. Graça Ruano[1,2], and W.C.A. Pereira[3]

[1] Centre for Informatics and Systems, Faculty of Sciences and Technology, University of Coimbra, Coimbra, Portugal
[2] Faculty of Sciences and Technology, University of Algarve, Faro, Portugal
[3] Biomedical Engineering Program - COPPE, Federal University of Rio de Janeiro-UFRJ, Rio de Janeiro, Brazil

Abstract— **In this paper we propose a new algorithm for sub-sample time-delay estimation between two discrete-time signals. The new algorithm operates just over the discrete-time signals without the need for interpolation or fitting. The proposed method is compared with a method based on spline fitting, and that is referred to outperform other methods over a broader set of conditions (Viola & Walker, 2005). The new approach, besides presenting a low computational cost (70% less), seemed to outperforms the spline approach in situations of high noise levels (typically signal-to noise ratios smaller than 20dB).**

Keywords— **Sub-sample time-delay estimation, Non-invasive temperature estimation, Utrasound.**

I. INTRODUCTION

Precise and efficient sub-sample time delay estimation (SS-TDE), i.e., the estimation of the delay between two signals with precision beyond the sample period, is a very important issue in medical ultrasound, being applied for blood velocity estimation [1], elastography [2], non-invasive temperature estimation [3, 4, 5], among others applications. In [1] SS-TDE is used to assess time-shifts which are proportional to the local blood velocity obtained from pulsed-Doppler signals. SS-TDE applied for elastography was used to compute echo-shifts obtained from compression/decompression actions. Different tissues may present different elastic properties which results in different shifts, giving rise to the constructions of non-invasive elastograms [2]. One of the main approaches for non-invasive temperature estimation is based on the fact that as temperature in a given medium changes it expands or contracts and the medium speed-of-sound also changes. When the medium is irradiated by ultrasound pulses, these changes can be viewed in the backscattered echoes as variations on the time-of-flight and that can be measured by SS-TDE methods [3].

SS-TDE can be achieved by interpolating the original signals. Nevertheless, the interpolation of the source signals can result in a high computational cost depending on the interpolation rate. With interpolation TDE precision is improved but still limited by the sampling period of the interpolated signals. Instead of interpolating the original signals, SS-TDE may be achieved by operating on the pattern-matching function, i.e. in the function computed to compare the signals, as for example the cross-correlation function. This approach has the benefit of reduced computational cost and improved time-delay estimates. In this case some samples of the pattern-matching function are used to find the parameters of a continuos-time function. Then by analytical methods a continuos TDE estimate is achieved. The simplest and probably the most used method for continuos time-delay estimation consists on the computation of the correlation function, by fitting a parabola around its peak, and finally by determining the position of the zero of the derivative of the fitted parabola [1, 6]. It is referred that as these methods are based on a pattern-matching function that is sampled at the original frequency, they usually suffer from relatively high bias and variance [7]. Alternatively, SS-TDE can be obtained by a fitting operation over the original signals, instead over the computed pattern-matching function. Viola and Walker [7] proposed a method that produce a continuous-time representation of one of the original signals by applying cubic splines as the fitting function. Using the continuos representation of one of the signals and the discrete version of the second signal it is possible to obtain an analytical pattern-matching function, which is then subjected to analytic operations aiming to find the sub-sample delay estimate. In [7] the pattern-matching function is the sum of the squared errors (SSE). Through simulation results the authors found that the algorithm outperforms other algorithms in terms of jitter and variance over a broad range of conditions. It is claimed that this improved performance was obtained at a reasonable computational cost.

In this paper a new algorithm for SS-TDE is proposed, based only in the auto-correlation and cross-correlation of the involved discrete signals, without the need of any interpolation or fitting.

II. METHOD

In this section we describe the proposed algorithm by considering two identical discrete time signals $s_1[n]$ and $s_2[n]$

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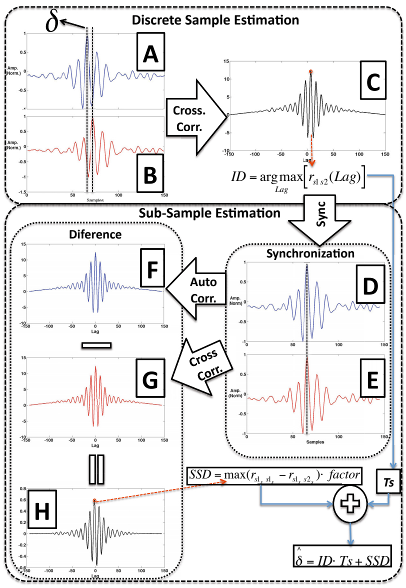

Fig. 1: Proposed method. (A) and (B) original signals. (C) cross-correlation function used to determine the discrete time-delay. (D) and (E) synchronized original signals. (F) autocorrelation of the synchronized s_1 signal. (G) Cross-correlation between the synchronized signals. (H) Difference between the autocorrelation of s_1 and the cross-correlation between s_1 and s_1.

with length N, sampled at $fs = 1/Ts$, and delayed by some amount of time (δ). The first stage of the algorithm is the determination of the discrete (integer) time-delay, as presented in Fig. 1. This is achieved by computing the unnormalized and biased cross-correlation between the original signals ($\hat{r}_{s_1,s_2}(\tau)$):

$$\hat{r}_{s_1,s_2}(\tau) = \begin{cases} \sum_{k=0}^{N-\tau-1} s_1[k+\tau]s_2[k] & \text{if } \tau \geq 0 \\ r^*_{s_2,s_1}(-\tau) & \text{if } \tau < 0 \end{cases} . \quad (1)$$

The integer sample delay estimate (ID) is given by

$$ID = \underset{\tau}{\operatorname{argmax}}\ r_{s_1,s_2}(\tau). \quad (2)$$

Using the integer delay it is possible to align the signals, i.e., to synchronize them (Fig. 1D and E), by eliminating ID samples from both signals. This is the first step of the second phase of the algorithm, which aims to assess the sub-sample delay between the signals. The process to account for the sub-sample delay involves: 1) the computation of the unnormalized and biased autocorrelation of one of the synchronized signals (the reference signal), for example $s_1[k]$, 2) the computation of the unnormalized and biased cross-correlation between the synchronized signals and 3) the difference between these two computed functions (Fig. 1F-H). Mathematically:

$$\hat{r}_{s_1^s,s_1^s}(\tau) = \begin{cases} \sum_{k=0}^{N-ID-\tau-1} s_1^s[k+\tau]s_1^s[k] & \text{if } \tau \geq 0 \\ \hat{r}^*_{s_1^s,s_1^s}(-\tau) & \text{if } \tau < 0 \end{cases} \quad (3)$$

$$\hat{r}_{s_1^s,s_2^s}(\tau) = \begin{cases} \sum_{k=0}^{N-ID-\tau-1} s_1^s[k+\tau]s_2^s[k] & \text{if } \tau \geq 0 \\ \hat{r}^*_{s_2^s,s_1^s}(-\tau) & \text{if } \tau < 0 \end{cases} \quad (4)$$

$$rd(\tau) = \hat{r}_{s_1^s,s_1^s}(\tau) - \hat{r}_{s_1^s,s_2^s}(\tau). \quad (5)$$

Where s_1^s and s_2^s are the synchronized original signals with $N - ID$ samples, and $rd(\tau)$ is the difference between the reference signal autocorrelation ($\hat{r}_{s_1^s,s_1^s}(\tau)$) and the synchronized signals cross-correlation ($\hat{r}_{s_1^s,s_2^s}(\tau)$). It was found that the maximum or the minimum values of $rd(\tau)$ are proportional to the sub-sample delay.

The total estimated delay between the original signals can be found by summing the integer delay (ID) multiplied by the sampling period, with the maximum or the minimum of $rd(\tau)$ multiplied by a scaling factor. In this paper we consider always the maximum:

$$\hat{\delta} = ID * Ts + max(rd(\tau)) * factor. \quad (6)$$

The scaling factor can be computed a-priori by inducing an artificial delay on the reference signal. The process to compute the factor is presented in Fig. 2. In first place the reference signal is artificially delayed by one sample (Fig. 2A and B). Then the autocorrelation of the original reference signal is computed as well as the cross-correlation between the original and delayed reference signal (Fig. 2C and D). The functions are subtracted and the maximum of the difference found (Fig. 2E). As the signals were artificially delayed we know exactly the delay that correspond to the maximum. So, the factor in units of time can be found by dividing the sample period by the maximum. To mention that for multiple delay estimations involving the same reference signal this factor needs to be computed only one time, as an initial step.

III. RESULTS

In order to analyze the performance of the proposed algorithm as compared with the reference method presented in [7] (“Spline method”), simulated signals were applied in a first stage, as described in Sub-Section A.This is important because allows us to access estimation errors. In Sub-Section B the performance of the proposed method is assessed in real-data also as compared with the performance of the Spline method.

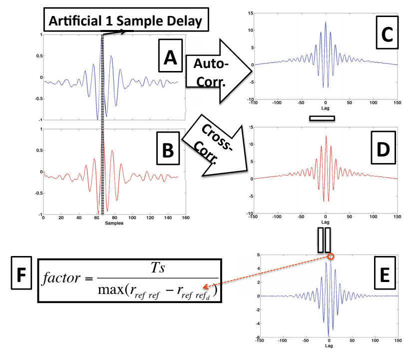

Fig. 2: Method to compute the scaling factor. (A) and (B) original and delayed reference signal. (C) autocorrelation of the original reference signal. (D) Cross-correlation between the original and artificially delayed reference signal. (E) Difference between the autocorrelation and the cross-correlation. (F) The scaling factor is computed by dividing the artificial delay by the maximum of the difference between the functions.

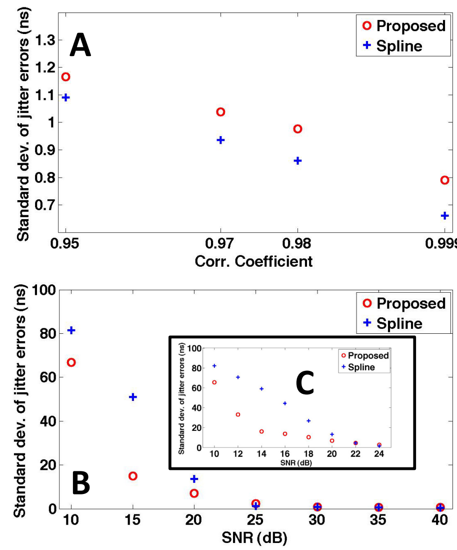

Fig. 3: Results for different correlation coefficients (A) and noise levels (B and C).

A. Simulated Signals

Pairs of reference and delayed signals, representing Gaussian-like echoes, with known noise levels and correlation coefficients, were generated by using the non-deterministic simulation approach described in [8, 9] and used in [7]. In order to evaluate the methods across multiple sub-sample delays, each simulated pair originated 20 signals by delaying one of the signals by [0, 0.05, 0.1,...,0.95] samples. Delayed signals were obtained by imposing a linear phase shift in the Fourier domain. To improve statistics, 1000 repetitions of the same simulation parametrization were performed, meaning that a total of 20000 pairs of signals were used to access the performance of the proposed approach at a given operation scenario.

The performance was evaluated based on the standard deviation of jitter errors. Jitter occurs when signal decorrelation, noise, and finite window lengths cause slight displacements of the true peak of the cross-correlation functions [8]. The standard deviation of the jitter errors is given by:

$$\sigma = \sqrt{\frac{1}{20000}\sum_{i=1}^{20000}(\Delta t_i - \Delta\hat{t}_i)^2}, \qquad (7)$$

where Δt_i is the true delay and $\Delta\hat{t}_i$ is the estimated delay.

Fig. 3 presents the performance of the proposed method as compared with the Spline method, when different correlation coefficients (Fig. 3A), and noise levels (Fig. 3B and C) were considered. The correlation coefficient defines a degree of similarity between signals, and was considered one of the most important factors influencing the estimation errors, as well as the noise level. The results show that the proposed method presents improved performance when the signals were corrupted by high noise levels. For noise levels inferior to 20 dB of SNR the proposed method presents a superior performance. From Fig. 3B and C one can infer that for noise levels inferior to 20 dB the standard deviation of jitter errors presents an exponential decay for the proposed method, while for the Spline method it tends to decrease linearly as SNR increases. Concerning the correlation coefficient one can see a decrease of the standard deviation of jitter errors, for both methods, as the correlation coefficient between signal pairs increases. The spline method presents improved performance, being the maximum difference in performance between the two methods of 0.13 ns (0.0065 samples).

Concerning the computational cost, i.e., the time to compute a time-delay, the proposed method outperforms the spline method, presenting an average execution time of approximately 1.5 ms, while the spline method of approximately 5 ms, i.e. approximately 3 times faster.

B. Real Signals: Temperature-Induced Echo-Shifts

In this section we give an example where ultrasound echo-shifts, induced by temperature changes are assessed. This application was selected as the authors work in the field of non-invasive temperature estimation with ultrasound [4, 5].

In this paper we used data acquired from the experimental setup published in [5], which consisted in a three-layered phantom heated by therapeutic ultrasound. Temperature and backscattered ultrasound signals were collected simultaneously during 35 minutes, including five minutes baseline

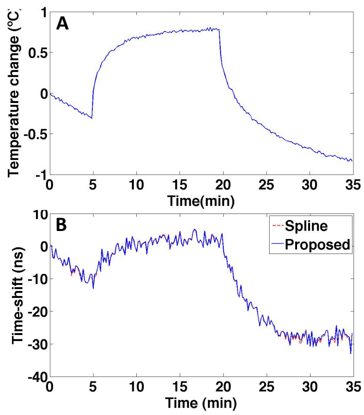

Fig. 4: (A) Temperature change measured in a multilayered phantom and (B) induced temporal echo-shifts.

temperature recording, 15 minutes heating and 15 minutes cooling. In this paper a trial where the therapeutic device was setup to irradiate the medium with a intensity of 0.3 W/cm^2, i.e., a very low intensity, was selected. Small temperature changes allow us to analyze how sensible are the time-shifts computed by the proposed and by the spline methods. The temperature measured by the thermocouple that is located in the middle of the phantom is presented in Fig. 4A. The computed temporal shifts for the echo originated by this thermocouple are presented in Fig. 4B. As expected the temporal shift is monotonic with the temperature. Comparing the result of both methods one can see that they return approximately the same shifts, i.e., small differences are observed.

IV. Discussion and conclusions

In this paper we propose a new method for subsample time-delay estimation. The performance of the new method was compared with the Spline method that is referred to outperform other algorithms concerning jitter and bias over a broader set of conditions [7]. The results showed that the proposed algorithm outperforms the spline method in situations were the signals are highly corrupted by normally distributed white noise. These improvement in performance is because of the difference operation expressed in Eq. 5. Theoretically, the autocorrelation of a white noise sequence is a Dirac function, i.e., a unique component at lag zero. As the correlation is a linear operation, the autocorrelation and cross-correlation computed in the proposed method exhibits two fundamental parts; the components related to the signal contents and a component related to the noise. The noise components are located around the lag zero in both auto- and cross-correlations functions and are reduced by the difference operation. In fact, the remaining noise component in the difference function can be completely removed by forcing the zero lag to be zero. This is a valid operation because for optimal noise free synchronized signals, the difference function will be always null at lag zero.

Although the proposed method does not outperforms the spline method concerning the different correlation coefficients considered, the difference in performance is very small (less than 3% of the sample period). In real signals both methods return approximately the same temporal-echo shifts, validating the proposed method for real-word problems. Besides the good performance, the proposed method is 70% less computational complex due to the interpolation-free and fitting-less approach.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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Author: César A. Teixeira
Institute: Centre for Informatics and Systems
Street: Polo II, Pinhal de Marrocos, City: Coimbra
Country: Portugal
Email: cteixei@dei.uc.pt