A Robust Subset-ICP Method for Point Set Registration

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Abstract. Iterative Closest Point (ICP) is a popular point set registration method often used for rigid registration problems. Because of all points in ICP-based method are processed at each iteration to find their correspondences, the method's performance is bounded by this constraint. This paper introduces an alternative ICP-based method by considering only subset of points whose boundaries are determined by the context of the inputs. These subsets can be used to sufficiently derive spatial mapping of point's correspondences between the source and target set even if points have been missing or modified slightly in the target set. A brief description of this method is followed by a comparative analysis of its performance against two ICP-based methods, followed by some experiments on its subset's sensitivity and robustness against noise.

Keywords: Iterative Closest Point (ICP), Correspondences, Transformation, Registration error, Subset, Expectation Maximization (EM).

1 Introduction

This paper describes an ICP-based method for point set registration by employing partial points instead of the whole set to find point correspondences. First, we examine ICP-based methods currently available in the literature to identify improvements and what weaknesses exist with such ICP-based variants. Next, the method of subset ICP that produces spatial mapping of point's correspondences based only on the partial points is described. By using subset of points to set the boundaries for point registration, we achieve the following merits, (1) the point correspondence computation is reduced due to small mapping space with the order of complexity of $O(TN_1N_2)$ for each iteration, where $T = \frac{N_x}{N_1} = \frac{N_y}{N_2}$, and $N_1 \le N_x$, $N_2 \le N_y$ (N_1, N_2 is the cardinal of subsets X and Y respectively), (2) structural (shape) information is implicitly applied and (3) local minimum trap is avoided because the localization of closest points in the subset provide better estimate than those found in whole set. Such subset can be used to match ordered homologous points between the source and target set even if the points in the target have gone through slight modification or missing in some parts. We demonstrate the subset ICP algorithm, its Mathlab implementation, and briefly analyze its performance using sample data from [15] and [17].

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The algorithm explained in the remainder of this paper was based on our previous work presented in [16]. This paper expanded that work in three areas, first it explains the methods clearly with example, second it provides comparative study against ICP's variants and CPD method, and finally it analyzes subset ICP method effectiveness in term of subset's cardinality sensitivity and it's robustness against noise.

2 Background

Point set registration is an important research topic in computer vision and image processing because of its applications in pattern recognition, shape reconstruction, object tracking and edge detection among others. With only the coordinate information of points, point set registration [1] is defined as to assign correspondences between two sets of points and to recover an optimal spatial mapping that matches one point set to the other so they are as close as possible to each other. Given two finite point sets $X = \{x_1, x_2, \cdots, x_{N_x}\}$ and $Y = \{y_1, y_2, \cdots, y_{N_y}\}$, where $x_i, y_i \in R^n$, N_x, N_y are the cardinalities of set X and Y respectively. The point set registration problem is to find a spatial transformation F, let $\sum \| x - F(y) \|^2 = \varepsilon, \varepsilon \geq 0$. Usually, point set Y is consecutively matched to set X by iterative mappings. The cardinality of set is defined as the number of points in set.

Iterative Closest Point (ICP) proposed in 1992 by Besl & McKay [2] and Chen & Medioni [3] is one of the most popular point set registration methods. It repeats two key steps until convergence is achieved: (1) to search for the correspondences based on the nearest distance criteria; (2) to determine a transformation based on current correspondence sets. However, it is an expensive computational algorithm because the correspondences need to be computed in each of iteration. Furthermore, the convergence is heavily depending on the initialization and it tends to converge to the local optimum.

Some variants of ICP intend to select the most representative points to form data set to reduce the computation complexity. Ezra et. al [4] pointed out that the number of iterations of ICP has a polynomial relationship to the number of input points under the root mean squared (RMS) distance and one-sided Hausdorff distance. A coarse-to-fine multi-resolution combined with the neighbor search technique [5] was introduced into ICP. A hierarchical selection point scheme was added into Picky ICP algorithm[6] that is an extension of ICP algorithm. Only every $2^t th$ point is selected to form "control point set" to perform Picky ICP, where $t+1$ is the number of hierarchical levels. However, local search instead of an exhaustive (global) one is performed to obtain the correspondence pairs, which may overlook some coherent information. Especially, it is difficult to tradeoff between the time and the accuracy requirements.

Other techniques to overcome the limitations of ICP include an evolutionary computation that was used to optimize the initial parameters of ICP. This method mainly motivated by the global optimization nature of evolutionary approaches [7]. Expectation Maximization (EM) algorithm combined ICP named as EM-ICP [8, 9] is used to handle noise and large data clouds. Multiple layer forward-feed neural network [10] is an alternative rigid point set registration method when the correspondences are estimated well. With Lie algebra viewpoint, a generalized ICP was proposed to deal with affine registration [11]. However, this method can neither reduce the computation cost nor avoid the risk of local minima. Rusinkiewicz & Levoy [12] presented an exhaustive and comparative summary for the state-of-the-art ICP variants.

Probabilistic methods such as Gaussian Mixture Model [1, 13], Robust Point Matching (RPM) [14, 15] are developed to solve point set registration problem. The probabilistic methods usually perform better than the conventional ICP, especially, in the presence of noise and outliers. However, the accurate registration results are at the cost of complex procedure and high computation complexity.

3 ICP-Based Registration Methods

In this section, subset ICP method is introduced first, where an example is used to demonstrate clearly its registration process. Next, two ICP-based methods respectively referred to as ICP-1 and ICP-2 are briefly explained. Their description is relevant for comparing the differences in the approach of finding correspondences and later in comparing the results in Section 3.1.

3.1 The Subset-ICP

Subset ICP algorithm starts by partitioning the source set X and the target set Y into multiple disjoints subsets, resulting in k pairs of subsets. The number of subset is determined by the size of the inputs but it is bounded by the minimum of cardinality of set X and Y. The algorithm works by iterating through the number of subset k and in each iteration, the algorithm performs a standard ICP method on each point in subset pair. Then, a spatial mapping in the form of rotation R and translation T are derived from current iteration, before they are applied to the target set Y. This process is repeated until a convergence or terminal condition is met. The registration procedure of subset-ICP is summarized in Fig 1.

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Inputs : point set X with cardinality m and Y with cardinality n
Initialize R0 ← I, T0 ← 0, i ← 1
X is partitioned into k subsets, X = X1 + X2 + … + Xk
Y is partitioned into k subsets, Y = Y1 + Y2 + … + Yk, 1 < k < min(m,n)
While i < k and MSE(X ,Y) > threshold
    For all points in subset Xi and Yi
        Calculate the correspondences based on Euclidean distance in subset Xi and Yi
        Compute and Update transformation Ri and Ti
        Apply Ri and Ti to set Y and all its subsets Y1, Y2, …, Yk
        Accumulate rotation R and translation T
    End ICP-iteration
    Calculate MSE for X and Y
End subset-iteration
Outputs : rotation R and translation T that matches all points in set Y
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Fig. 1. Pseudo-code of subset-ICP algorithm

Fig. 2 illustrates the working of subset ICP for a simple example where only translation is needed to register point set X and Y. Assuming we have an input set X = {1, 2, 3, 4} and Y = {5, 6, 7, 8} with equal cardinality i.e. $m = n = 4$. In this case, both X and Y is uniformly partitioned into two subsets, $k = 2$ such as X_1={1, 2}, X_2={3, 4} and Y_1={5, 6},Y_2={7, 8}. Note that an arbitrary choice of partition can be chosen (as we later show in subsection 3.2), however we should avoided the two worse cases, that is when the subset contains a single point or when there is only a single subset. At first iteration, i.e. k = 1 the closest points of X_1 = {1, 2} to Y_1 are the points {5, 5} respectively hence, the neighbor set is $N_X(Y)$ = {5}. In this example, the translation T is derived from the mean values of the set, thus given their values are 1.5 and 5, T = 1.5-5 = -3.5. Applying translation to set Y we updated the values for subsets Y_1 = {1.5, 2.5} and Y_2 = {3.5, 4.5}. Since the MSE of X and Y is not met, the same process is repeated for subset pair X_2 = {3, 4} and Y_2 = {3.5, 4.5}. Now the neighbor set is $N_X(Y)$ = {3.5, 4.5}, with the mean values of 3.5 and 4, thus the translation is T = 3.5-4 = -0.5.Updating Y with T we get Y_1 = {1, 2} and Y_2 = {3, 4}, which are exactly matching the points in set X. This also denotes that convergence has been met and thus ending the procedure.

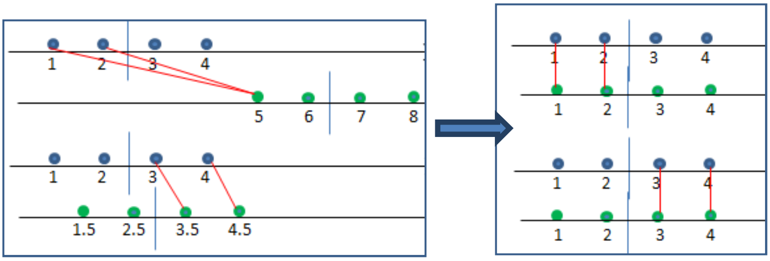

Fig. 2. The registration procedure of subset-ICP on the given data

3.2 Two ICP-Based Algorithms

ICP-1: ICP-1 is the original ICP algorithm from [2]. Two point sets are given which describe the shape of an object. For each point of one set, search for its closest points in another set to form the correspondence set. Based on the first set and its correspondences set, the rotation matrix and translation vector are computed using optimization combined with statistical technique, such as PCA. The correspondences and the transformation are updated iteratively until the stop conditions are satisfied.

ICP-2: ICP-2 is an improved version of ICP as presented in [6]. ICP-2 improves the solution set in ICP-1 by removing redundant points that satisfied the nearest point's condition. It only kept one point with the shortest distance [6]. For instance, if a point b from the source set is the closest point to four other points in template set, says a1, a2, a3, and a4, point a2 is chosen as the winner if it has the shortest distance to point b, i.e $d(\text{a}2,\text{b}) = \min\{d(\text{a}1,\text{b}), d(\text{a}2,\text{b}), d(\text{a}3,\text{b}), d(\text{a}4,\text{b})\}$.

The main difference between ICP-1 and ICP-2 is whether the mapping relation between the template set and the correspondences set is bijection (also known as one-to-one mapping) or not.

4 Experiments

In this section, experiments are carried out on 2D data to analyze subset ICP's performance with respect to its accuracy, robustness and computational time. Elephant shape is taken from part B of CE-Shape-1 [17], whereas the fish and blessing data are taken from [15]. In our experiments, when there are no outliers, the cardinality of the target point set is equal to the cardinality of the template point set. The performance measure is the global mean square error (MSE) on all points instead of the correspondence pairs. We also qualitatively show the results in visual form to complement the quantitative analysis.

4.1 Comparing ICP-Based Point Registration Methods

Fig. 3 shows the registration results of ICP-1, ICP-2 and subset-ICP when tested against three cases of fish data as described below:

- Case 1: the source data (in blue pentagram marks) is synthesized by a slight linear transformation
- Case 2: the source data is produced by the larger rigid transformation.
- Case 3: Gaussian noises added into the source data to deteriorate its shape

The template shape is presented as red circle where its points are kept unchanged in the experiments.

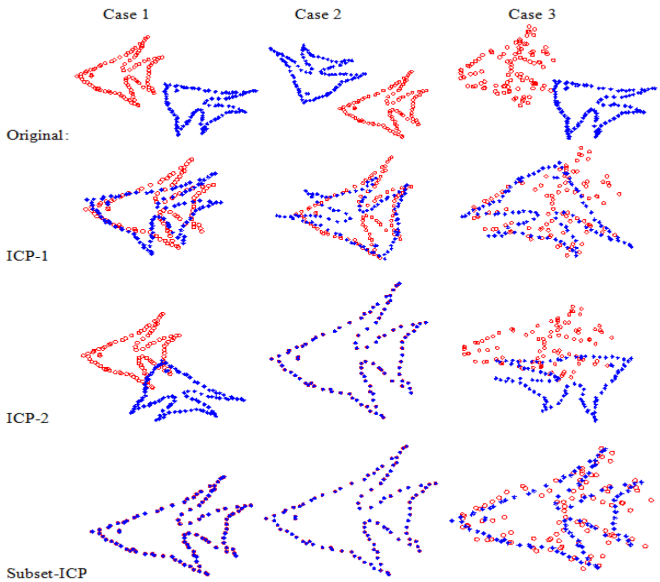

Fig. 3. The registration results on fish data for three ICP-based methods. The original data sets are in the first row. The matching results of ICP-1 are shown in the second row, those of ICP-2 are in the third row, and the last row is the subset-ICP registration results. Note that the transformed source data is still depicted by the blue pentagram marks

Table 1. The Mean Square Errors and execution time of three methods on fish data

	MSE (cm)			Time (seconds)		
	ICP-1	ICP-2	Subset-ICP	ICP-1	ICP-2	Subset-ICP
Case 1	0.0854	0.0224	**4.545e-016**	0.0587	0.0690	**0.0121**
Case 2	0.3691	5.022e-016	**4.459e-016**	0.0294	0.0811	**0.0057**
Case 3	0.3484	0.5188	**0.044**	0.3927	**0.3048**	0.3574

The results in Fig.3 indicate that the subset ICP algorithm achieves significantly better matching accuracy than ICP-1 and ICP-2 algorithms, while it is also superior in term of MSE and execution time (Table 1) over the two algorithms. The higher MSE of the ICP-1 and ICP-2 algorithms are due to them falling into local minima. Subset ICP is also robust against Gaussian noise (Case 3) compare to ICP-1 and ICP-2. In Case 3, ICP-2 took less execution time than the other two algorithms but as shown in Fig.3 the result is only partially matched.

Further tests of the subset ICP algorithm were performed and compared against Coherent Point Drift (CPD) method – an efficient point set registration for both rigid and non-rigid points based on Gaussian Mixture Model [1]. Registration results from the two data sets are shown in Fig.4.

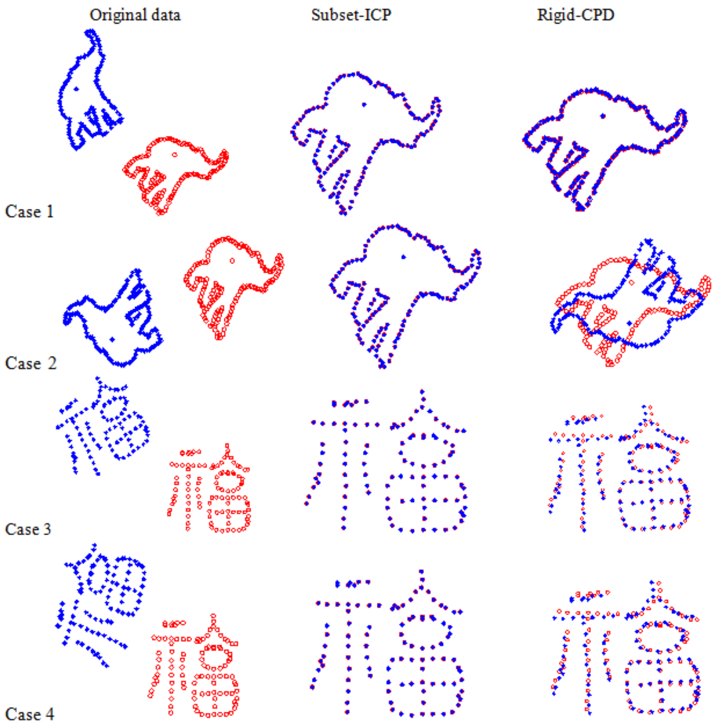

Fig. 4. The matching results of the elephant data and the Chinese's blessing character

For Chinese's blessing character, subset ICP algorithm achieves approximately the same level of accuracy as the rigid-CPD algorithm. However, for Case 2 of Elephant data, where its template has been rotated and translated slightly more than for Case 1 transformation, our method is able to register the template and source set properly.

4.2 Sensitivity Analysis of the Subset's Partition

The subset ICP method relies heavily on subset for registering points in its input set. Thus, it is interesting to study how sensitive is the subset's cardinality in influencing the algorithm's performance. Fig. 5 and Fig. 6 respectively show the MSE and execution time of the algorithm when tested with Elephant data where the cardinality of the subset is varied from 1 to 451.

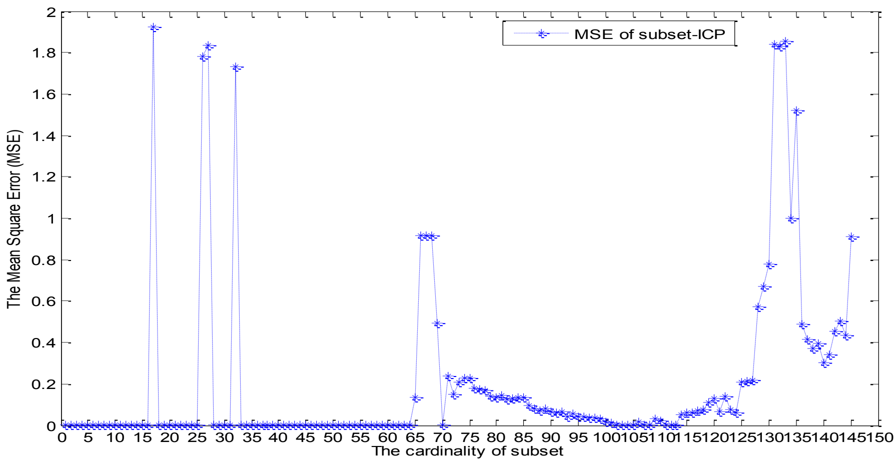

Fig. 5. The MSE of subset-ICP in the elephant data

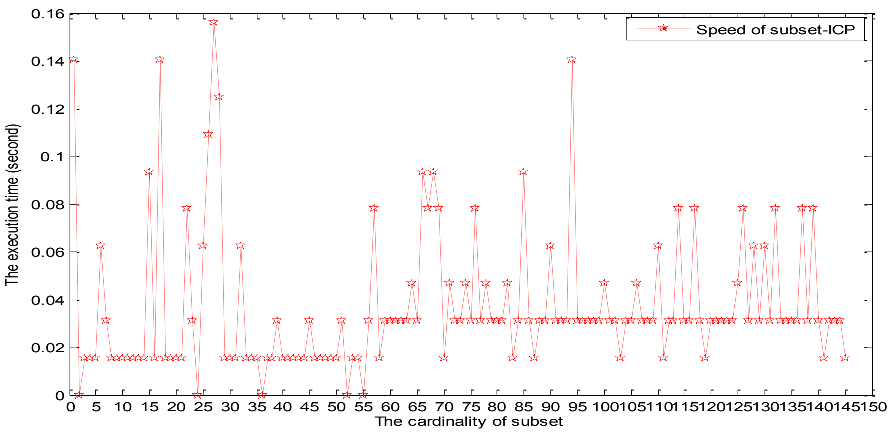

Fig. 6. The execution time of subset-ICP in the elephant data

The results provide the following general observations, first low MSE is recorded when the cardinality of the subset is small as illustrated in the initial graph's plot in Fig.5. In contrast, the opposite trend is exhibited when the size of subset is increased. A few exceptions to this pattern were also evidenced in the plot. For instance, for subset's cardinality 16 and 17 (and also for some other cardinal's values), there is a "spike" in the MSE plot thus nullifying the general pattern observed earlier. Fig.6 provides a confirmation on this observation for Case 2 with cardinal 17, where the two set were incorrectly matched. One possible answer to this flaw is due to the use of absolute orientation algorithm to find the least square distance between corresponding points. That is a larger error distance will have a large effect on the total score, this happen when the source point set is farther away from the template point set. Secondly, the graph plot in Fig.6 does not show any significant pattern between the cardinality of subset and their effect on execution time. This is expected because the convergence rate of the algorithm is predominantly determined by the quality of the selected points rather than by their quantity. Thus we may conclude that the algorithm is robust against the cardinality of subset, if the source point set is closer to the template point set, otherwise it is susceptible and hence sensitive to the cardinality of subset.

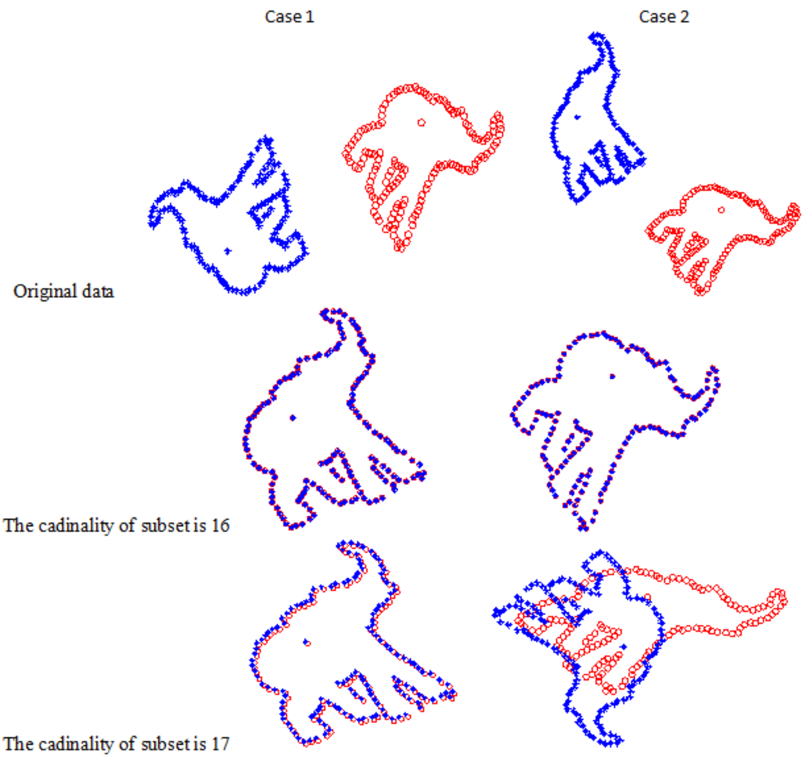

Fig. 7. Registration results of subset-ICP based on different cardinalities of subset

4.3 Noise Robustness

Robustness test evaluates the subset ICP's ability to cope against outlier points (often referred to as noise). In this test, different proportions of Gaussian noise were introduced into the fish's template data. In the experiments we fixed the subset's cardinal, and recorded the registration errors as shown in Table 2. The logarithmic scaling on y-axis allows the behaviour for small MSEs (of case b), to be seen in Fig.8.

Table 2. The MSE on fish data with different ratio Gaussian noise

r	0.02	0.05	0.1	0.15	0.2	0.3	0.4	0.5
a	3.85e-16	5.50e-16	5.37e-16	4.81e-16	6.86e-16	0.023	0.023	5.65e-16
b	6.50e-16	7.76e-16	6.59e-16	5.85e-16	9.62e-16	0.022	4.21e-16	7.60e-16

In Table 2, r is the noise proportion to the number of points on the fish's template data set. The letter a denotes, the noise data were appended at the back of the last point of the template data set. While letter b indicates the noise data were inserted in front of the first point of the template data set. Based on Table 2, we can see that the registration error is kept quite low with the increasing of Gaussian noise proportion.

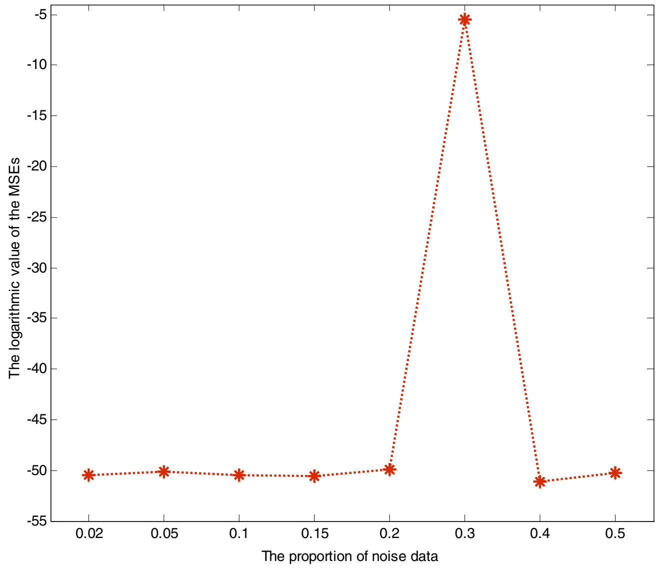

Fig. 8. MSE (in cm) of subset-ICP method as Gaussian noise proportion is varied (for case b)

In order to validate the influence of the outliers to the registration performance, a statistical t-test was performed. The p values of the error and execution time are shown in Table 3. Here the significant level is set to 0.05. If the p value is less than 0.05, this indicates the influence is statistically significant, otherwise, the influence is not statistically significant.

Table 3. The *p* values of t-test

Group	*P* value (for error)	*P* value (for speed)
A	5.92×10^{-11}	0.3026
B	3.3×10^{-6}	0.2647
C	7.14×10^{-7}	0.2683

As summarized in Table 3, the outliers affect significantly the registration error but the computation complexity is not significantly affected. The experimental results verify that for the sample data set used, subset IP method is robust against noise.

5 Conclusion

In this paper, a subset-based ICP method is proposed to handle rigid point set registration, which is an effective method for the larger rotation and less even without overlapping of two point sets. The merits of the proposed method stem from partial points instead of total points to find the correspondences. The registration performances are determined by the cardinality of subset when two point sets are fixed. In other words, the way of partition the entire data set influence heavily the success of the subset-ICP. Experiments are performed to indicate that subset-ICP method is an effective and robust registration method. More works will be done to prove and to explore how we can acquire an optimal partitioning scheme to generate subsets.

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