Exploiting Chaos for Quantum Control

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Abstract The field of *Quantum Chaos* is referred to as the study of quantum behaviors of systems whose corresponding classical dynamics are chaotic, or study of quantum manifestations of classical chaos. Equivalently, it means that quantum behaviors depend on the nature of the classical dynamics, implying that classical chaos can be used to control or manipulate quantum behaviors. We discuss two examples here: using transient chaos to control quantum transport in nanoscale systems and exploiting chaos to regularize relativistic quantum tunneling dynamics in Dirac fermion and graphene systems.

1 Introduction

Controlling chaos in dynamical systems has been studied for more than two decades since the seminal work of Ott, Grebogi, and Yorke [1]. The basic idea was that chaos, while signifying random or irregular behavior, should not be viewed as a nuisance in applications of nonlinear dynamical systems. In particular given a chaotic system, the fact that there are an infinite number of unstable periodic orbits embedded in the underlying chaotic invariant set means that there are an equally infinite number of choices for the operational state of the system depending on need, provided that any such state can be stabilized. Then, the intrinsically sensitive dependence on initial conditions, the hallmark of any chaotic system, implies that it is possible to apply small perturbations to stabilize the system about any desirable state. Controlling chaos has since been studied extensively and examples of successful experimental implementation abound in physical, chemical, biological, and engineering systems

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[2]. The vast literature on controlling chaos, however, has been limited to nonlinear dynamical systems in the classical domain.

Recently, it has been articulated that chaos can be exploited to control or manipulate quantum-mechanical behaviors [3, 4]. For example, in the context of quantum transport through nanostructures, a fundamental characteristic is conductance fluctuations. It has been demonstrated that chaos, more specifically transient chaos, can be effective in modulating the conductance-fluctuation patterns, and it has been suggested [4] that this can be realized experimentally by applying an external gate voltage in a device of suitable geometry to generate classically inaccessible potential barriers. Adjusting the gate voltage allows the characteristics of the dynamical invariant set responsible for transient chaos to be varied in a desirable manner which, in turn, can induce continuous changes in the statistical characteristics of the quantum conductance-fluctuation pattern. In another example, it has been shown that chaos can be utilized to suppress, significantly, the spread in the tunneling rate commonly seen in systems whose classical dynamics are regular, and this is called regularization of quantum tunneling dynamics by chaos [3]. More recently, it has been demonstrated that similar effects arise in relativistic quantum systems [5].

This Brief Review has two purposes: (1) to discuss the two aforementioned examples of exploiting chaos for quantum control, and (2) to argue that the principle of chaos-based quantum control is more general with potentially significant applications in nanoscience.

2 Controlling Quantum Transport by Transient Chaos

A fundamental quantity characterizing the transport of an electron through a nanostructure, such as a quantum dot or a quantum point contact, is quantum transmission probability, or simply quantum transmission. In general, quantum transmission is determined by many electronic and system parameters such as the Fermi energy, the strength of external magnetic field (if there is one), and the details of the geometry of the structure. If the structure is connected through electron waveguides (or leads) to electron reservoirs (i.e., contacts) to form a circuitry, the conductances defined with respect to various voltage biases among the contacts, together with the corresponding currents, will be determined by the quantum transmission [6]. This means that the conductances can also depend sensitively on electronic and geometrical parameters. For example, as the Fermi energy of the electron changes, the conductances can exhibit wild fluctuations and sharp resonances [7–13]. In applications such as the development of electronic circuits and nanoscale sensors, severe conductance fluctuations are undesirable and are to be eliminated so that stable device operation can be achieved. The outstanding question is then, can practical and experimentally feasible schemes be articulated to modulate quantum conductance fluctuations? It has been demonstrated recently and understood theoretically that classical *transient chaos* can be used to effectively modulate conductance-fluctuation patterns associated with quantum transport through nanostructures [4].

Intuitively, the basic principle underlying the transient-chaos based strategy for modulating quantum transport is quantum interference. It has been known that quantum pointer states, which are resonant states of finite but long lifetime formed inside the nanostructure [14–16], can cause sharp conductance fluctuations - a kind of Fano resonance [17, 18]. To give a specific example, consider quantum-dot systems, a paradigm for investigating all kinds of quantum transport phenomena through nanostructures. Such a system typically consists of a finite device region of certain geometrical shape, such as a square, a circle, or a stadium, and a number of leads connected with the device region. For a quantum-dot system whose classical dynamics is either regular or contains a significant regular component, there are stable periodic orbits in the classical limit. If the dot geometry is closed, highly localized states can form around the classically stable periodic orbits as a result of quantum interference. When electronic waveguides (leads) are attached to the quantum dot so that it is open, some periodic orbits can still survive, leading to resonant states, or quantum pointer states. Since the corresponding classical orbits are stable, the resonant states can have long lifetime, so their coupling to the leads is weak. As a result, narrow resonances can form around the energy values that are effectively the eigenenergies for the stable periodic orbits in the corresponding closed system. When the dot geometry is modified so that the underlying classical dynamics becomes fully chaotic, no stable periodic orbits can exist. Although scars can still form around classically unstable periodic orbits in a closed chaotic system [19], the corresponding resonant states in the open system generally will have much shorter lifetimes. This means that these resonant states do couple to the leads more strongly, broadening the narrow resonances in the conductance-fluctuation pattern. Here chaos is transient because the system is open. According to the theory of transient chaos [20], the dynamical invariant sets responsible for transient chaos are non-attracting chaotic sets in the phase space. If the properties of transient chaos can be adjusted experimentally by parameter tuning, the quantum conductance fluctuation-patterns can then be controlled in a desirable manner. For example, one can change the effective geometry of the dot structure continuously so as to enhance the escape rate, a basic quantity characterizing transient chaos, and this could lead to significantly smoother quantum-conductance fluctuations.

To realize quantum control by using chaos, we conceive generating a region about the center of the device or structure with high potential so that it is impenetrable to classical particles. For example, consider a square quantum dot, a prototypical model in semiconductor two-dimensional electron-gas (2DEG) systems. When the dot is closed, the corresponding classical dynamics is integrable so that extremely narrow resonances can arise in the quantum transport dynamics of the corresponding open-dot system. Now imagine applying a gate voltage perpendicular to the device plane to generate a circular, classically forbidden region about the center of the dot, as shown schematically in Fig. 1. In general, the potential profile will be smooth in space. However, qualitatively, the classical scattering behavior is similar to that from an infinite potential well. Thus it is reasonable to impose the infinite potential-well assumption for the central region, which defines a “forbidden” region. Varying the voltage V_0 can change the effective radius R of the forbidden region. Classically, the

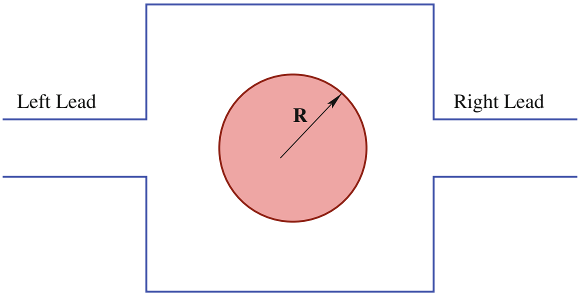

Fig. 1 Illustration of a possible scheme to control transport through a two-dimensional quantum-dot system. When semiconductor materials (e.g., silicon) are used, the system is the traditional two-dimensional electron-gas (2DEG) system described by the Schrödinger equation in non-relativistic quantum mechanics. If the material is graphene, in certain energy regime the system is described by the Dirac equation in relativistic quantum mechanics. By applying a suitable gate voltage perpendicular to the device plane to generate a *circular forbidden region* at the center of the device, the resulting closed system is a Sinai billiard, whose classical dynamics is fully chaotic. Open quantum-dot system can be formed by attaching two leads to the billiard system, one on the *left* and another on the *right side*. The classical dynamics of the device can thus be characterized as chaotic scattering

closed system is thus a Sinai billiard [21], which is fully chaotic, insofar as the radius of the central potential region R is not zero. When leads are connected to the device region so as to open the system, chaos becomes transient. The dynamical characteristics of the underlying chaotic invariant set can be adjusted in a continuous manner by increasing the radius R [22]. Quantum mechanically we thus expect to observe increasingly smooth variations in the conductance with, e.g., the Fermi energy, which has been demonstrated [4] using both semiconductor 2DEG and graphene [23–26] systems.

Insights into why classical chaos can smooth out quantum conductance fluctuations can also be gained from the semiclassical theory of quantum chaotic scattering [27–29]. In particular, in the semiclassical regime, it was established by Blümel and Smilansky that the energy autocorrelation function of the quantum transmission fluctuation is proportional to the Fourier transform of the particle-decay law in the classical limit [27]. For fully developed chaotic transport through a quantum dot, the decay law is exponential with the rate κ. As a result, the quantum energy correlation function decays as a Lorentzian function with the width given by $\hbar\kappa$, where $\hbar$ is the Planck's contant. In the theory of transient chaos [20], κ is the escape rate associated with the underlying non-attracting chaotic set. As the radius of the central potential region is increased, κ also increases. The energy autocorrelation function then decays more slowly, signifying less fluctuations, i.e., less number of *sharp resonances* in the quantum transmission. This semiclassical argument suggests that the degree of quantum transmission fluctuations can be controlled by classical chaos.

Extensive numerical support for the principle of transient-chaos based control of quantum transport and a detailed theoretical analysis can be found in Ref. [4, 30].

3 Effect of Chaos on Quantum Tunneling

The principle of regularization of quantum tunneling by chaos can be understood by considering the prototypical system in Fig. 2, which consists of two symmetrical cavities connected by a one-dimensional potential barrier along the line of symmetry. When the classical dynamics in each cavity is integrable, for sufficiently large energy the tunneling rate can have many values in a wide interval. Choosing the geometry of the cavity such that the classical dynamics become chaotic can greatly enhance and regularize quantum tunneling. Heuristically, this can be understood, as follows. When the potential barrier is infinite, each cavity is a closed system with an infinite set of eigenenergies and eigenstates. Many eigenstates are concentrated on classical periodic orbits, forming quantum scars [19]. For a classically integrable cavity, some stable or marginally stable periodic orbits can persist when the potential barrier becomes finite so that each cavity system is effectively an open quantum system. Many surviving eigenstates correspond to classical periodic orbits whose trajectories do not encounter the potential barrier, generating extremely low tunneling rate even when the energy is comparable with or larger than the height of the potential barrier. The eigenstates corresponding to classical orbits that interact with the potential barrier, however, can lead to relatively strong tunneling. In a small energy interval the quantum tunneling rate can thus spread over a wide range. However, when the classical dynamics is chaotic, isolated orbits that do not interact with the potential barrier are far less likely and, consequently, the states associated with low tunneling rates disappear, effectively suppressing the spread in the tunneling rate.

The idea of using chaos to regularize quantum tunneling dynamics was first conceived and demonstrated in non-relativistic quantum systems governed by the Schrödinger equation [3]. Recently, the question of whether chaos can regularize tunneling in relativistic quantum systems has been addressed [5], where the motion of massless Dirac fermions in the setting of resonant tunneling was investigated to facilitate comparison with the non-relativistic quantum case. In general, it is a chal-

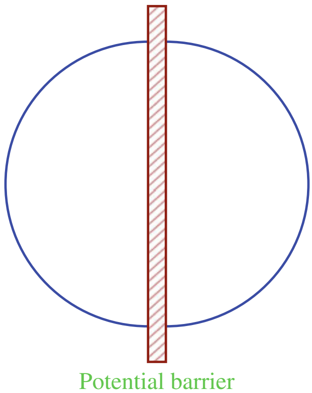

Fig. 2 A closed quantum system of arbitrary shape in two dimensions

lenging task to solve the Dirac equation in a confined geometry, due to the difficulties to incorporate zero-flux boundary conditions and to remove artificial, non-physical effects such as fermion doubling as a result of spatial discretization. A numerical scheme has then been developed [31] to overcome these difficulties, which is based on constructing a physically meaningful, Hermitian Hamiltonian. Extensive computations have revealed unequivocally the existence of surviving eigenstates that lead to extremely low tunneling rates. As for the non-relativistic quantum case, making the cavities classically chaotic can greatly regularize the quantum tunneling dynamics. To explore the practical implications, resonant tunneling devices made entirely of graphene [23] have been studied [5], where the tunneling rates for different energy values have been calculated. Qualitatively similar results have been obtained to those for massless Dirac fermions. One unique feature for both the Dirac fermion and graphene systems, which finds no counterpart in non-relativistic quantum tunneling devices, is the high tunneling rate in the regime where the particle energy is smaller than the height of the potential barrier. This is a manifestation of the Klein-tunneling phenomenon [32–34]. A theory has been developed [5] to explain the numerical findings, which is based on the concept of self energies and the complex energy spectrum of the non-Hermitian Hamiltonian for the “open” cavity.

4 General Thoughts on Chaos-Based Quantum Control

The general principle of chaos-based control of quantum behaviors is motivated by the term *quantum chaos*, which does not mean that there can actually be chaos in quantum mechanical systems but is referred to as the study of *quantum manifestations* of systems whose dynamics in the classical limit exhibit chaos [35, 36]. The basic reason that chaos may be ruled out in quantum systems is that the fundamental governing equations, the Schrödinger equation or the Dirac equation, are linear. At the present, there is tremendous literature on quantum chaos, where various quantum manifestations of classically chaotic systems have been studied. The general result is that distinct classical dynamics, integrable or chaotic, can lead to characteristically different quantum behaviors. Furthermore, different types of chaotic behaviors can generate distinct quantum manifestations. From the point of view of control, all these suggest that quantum behaviors can be manipulated or harnessed for desirable applications by choosing distinct classical dynamical behaviors, in particular chaotic dynamics.

The two examples discussed in this Brief Review, control of quantum transport and quantum tunneling, are based on building chaos into the system. In the transport problem that involves the Sinai billiard type of device structure, the properties of the underlying chaotic set can be modified, for example, by an externally adjustable gate voltage. In the tunneling problem, the geometry of the cavities are deliberately designed to yield chaotic dynamics in the classical limit. Once the structure is fixed, experimentally it may be difficult to change the characteristics of chaos. It is thus nec-

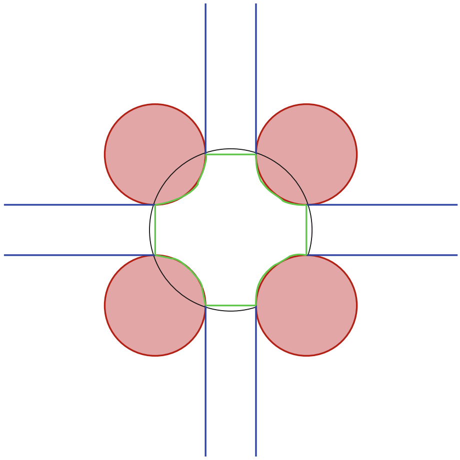

Fig. 3 A four terminal device in the presence of magnetic field. The area enclosed by the *green lines* indicates the region in the physical space where transient chaos can occur. The *black circle marks* the region in which the magnetic field is applied

essary to search for experimentally feasible schemes to modulate the characteristics of the underlying chaotic invariant sets in a continuous fashion.

Figure 3 presents a possible scheme where a single external parameter can be varied to realize chaos-based control of quantum transport. It is a four-terminal device, where four idealized leads join smoothly to form a quantum-dot structure, which has been used widely in the study of, for example, quantum Hall effect [6]. The structure typically exhibits chaotic scattering (transient chaos) in the classical limit. A perpendicular magnetic field can be applied. An earlier work [37] demonstrated that the dynamical invariants of the underlying non-attracting chaotic set can be modified continuously by changing the strength of the magnetic field. It is thus possible to modulate the quantum conductance-fluctuation patterns by simply adjusting the magnetic-field strength [38].

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