Chapter 1
Introduction

Constraints are encountered practically in all real-world control problems. The presence of constraints leads to theoretical and computational challenges. From the conceptual point of view, constraints can have different nature. Basically, there are two types of constraints imposed by physical limitation and/or performance desiderata.

Physical constraints are due to the physical limitations of the mechanical, electrical, biological, etc controlled systems. The input and output constraints must be fulfilled to avoid over-exploitation or damage. In addition, the constraint violation may lead to degraded performance, oscillations or even instability.

Performance constraints are introduced by the designer for guaranteeing performance requirements, e.g. transient time, transient overshoot, etc.

The constrained control problem becomes even more challenging in the presence of model uncertainties which is unavoidable in practice [1, 117]. It is generally accepted that a key reason of using feedback is to diminish the effects of uncertainty which may appear in different forms as disturbances or as other inadequacies in the models used to design the feedback law. Model uncertainty and robustness have been a central theme in the development of the field of automatic control [8].

A straightforward way to stabilize a system with input constraints is to perform the control design disregarding the constraints, then an adaptation of the control law is considered with respect to input saturation. Such an approach is called *anti-windup* [73, 123, 124, 132].

Over the last decades, the research on constrained control topics has developed to the degree that constraints can be taken into account during the synthesis phase. By its principle, model predictive control (MPC) approach shows its importance on dealing with constraints [2, 30, 34, 47, 48, 88, 92, 107]. In MPC, a sequence of predicted optimal control values over a finite prediction horizon is computed for optimizing the performance of the controlled system, expressed in terms of a cost function. Then only the first element of the optimal sequence is actually applied to the system and the entire optimization is repeated at the next time instant with the new state measurement [4, 88, 92].

In MPC, with a linear model, polyhedral constraints, and a quadratic cost, the resulting optimization problem is a quadratic programming (QP) problem [37, 104].

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Solving the QP problem can be computationally costly, specially when the prediction horizon is large, and this has traditionally limited MPC to applications with relatively low complexity/sampling interval ratio [3].

In the last decade, attempts have been made to use predictive control in fast processes. In [20, 100, 101, 127] it was shown that the constrained linear MPC is equivalent to a multi-parametric optimization problem, where the state plays the role of a vector of parameters. The solution is a piecewise affine function of the state over a polyhedral partition of the state space, and the computational effort of MPC is moved off-line. This control law is called *explicit MPC*. However, explicit MPC also has disadvantages. Obtaining the explicit optimal MPC solution requires to solve an off-line parametric optimization problem, which is NP-hard. Although the problem is tractable and practically solvable for several interesting control applications, the off-line computational effort grows *exponentially* fast as the problem size increases [61, 62, 77–79].

In [131], it was shown that the on-line computation is preferable for high dimensional systems where significant reduction of the computational complexity can be achieved by exploiting the particular structure of the optimization problem as well as by early stopping and warm-starting from a solution obtained at the previous time-step. The same reference mentioned that for models of more than five dimensions the explicit solution might be impractical. It is worth mentioning that approximate explicit solutions have been investigated to go beyond this ad-hoc limitation [18, 60, 114].

Note that as its name says, most traditional implicit and explicit MPC approaches are based on mathematical models which invariably present a mismatch with respect to the physical systems. The robust MPC is meant to address both model uncertainty and disturbances. However, the robust MPC presents great conservativeness and/or on-line computational burden [21, 35, 74, 81, 84].

The use of interpolation in constrained control in order to avoid very complex control design procedures is well known in the literature. There is a long line of developments on these topics generally closely related to MPC, see for example [10, 102, 108–110], where interpolation between input sequences, state trajectories, different feedback gains with associated invariant sets can be found.

The vertex control law can be considered also as an interpolation approach based on the admissible control values, assumed to be available for the vertices of a polyhedral positively invariant set C_N in the state space [23, 53]. A weakness of vertex control is that the full control range is exploited only on the border of C_N, and hence the time to regulate the plant to the origin is much longer than e.g. by time-optimal control. A way to overcome this shortcoming is to switch to another, more aggressive, local controller near the origin in the state space, e.g. a state feedback controller $u = Kx$, when the state reaches the Maximal Admissible Set (MAS) of the local controller. The disadvantage of such a switching-based solution is that the control action becomes non-smooth [94].

The aim of this book is to propose an alternative to MPC. The book gives a comprehensive development of the novel Interpolating Control (IC) of the regulation problem for linear, time-invariant, discrete-time uncertain dynamical systems with polyhedral state space, and polyhedral control constraints, with and without disturbances, under state- or output feedback. For output feedback a non-minimal state-space representation is used with old inputs and outputs as state variables.

The book is structured in three parts. The first part includes background material and the theoretical foundation of the work. Beyond a briefly review of the area, some new results on estimating the domain of attraction are provided.

The second part contains the main body of the book, with three chapters on interpolating control that addresses nominal state feedback, robust state feedback and output feedback, respectively. The IC is given in both its implicit form, where at most two LP-problems or one QP-problem or one semi-definite problem are solved on-line at each sampling instant to yield the value of the control variable, and in its explicit form where the control law is shown to be piecewise affine in the state, whereby the state space is partitioned in polyhedral cells and whereby at each sampling interval it has to be determined to which cell the measured state belongs. The interpolation in IC is performed between constraint-aware low-gain feedback strategies in order to respect the constraints, and a user-chosen performance control law in its MAS surrounding the origin.

Thus, IC is composed by an *outer* control law (the interpolated control) near the boundaries of the allowed state set whose purpose is to make the state enter the MAS rapidly (but not necessarily time-optimally) without violating any constraints, and the *inner* user-chosen control law in its MAS whose purpose is performance according to the user's choice.

Novel proofs of recursive feasibility and asymptotic stability of the Vertex Control law, and of the Interpolating Control law are given. Algorithms for Implicit and Explicit IC are presented in such a way that the reader may easily realize them.

Each chapter includes illustrative examples, and comparisons with MPC. It is demonstrated that the computation complexity of IC is considerably lower than that of MPC, in particular for high-order systems, and systems with uncertainty, although the performance is similar except in those cases when an MPC solution cannot be found.

In the last part of the book two high order examples as well as a benchmark problem of robust MPC are reported in order to illustrate some practical aspects of the proposed methods.