Chapter 14
The Direction of Time in Dynamical Systems

Interdisciplinary Perspectives from Cosmology to Brain Research

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Abstract Dynamical systems in classical, relativistic, and quantum physics are ruled by laws with time reversibility. Dynamical systems with time-irreversibility are known from thermodynamics, biological evolution, brain research and historical processes in social sciences. They can also be simulated by computation and information systems. Thus, arrows of time and aging processes are not only subjective experiences or even contradictions to natural laws but can be explained by the dynamics of complex systems.

Keywords Symmetry of time · Reversibility · Cosmic arrow · Complex dynamical system · Time operator · Duration · Aging · Self-organization · Evolutionary time · Social time · Computational time · Unpredictability

1 Time in Classical and Relativistic Dynamics

According to Newton's laws of mechanics, a *dynamical system* is determined by a time-depending equation of motion. Newton distinguished between *relative* and *absolute time*, assuming that all clocks of relative reference systems in the Universe could be synchronized to an absolute world-time of an absolute space. The *symmetry of time* is expressed by changing the sign of the *direction of motion* in an equation of motion [2, 3]. In *classical mechanics*, mechanical laws are preserved (invariant) with respect to all inertial systems moving uniformly relative to one another (Galilean invariance). A consequence of time symmetry is the conservation of energy in a dynamical system. Newton's absolute space can actually be replaced by the class of inertial systems with Galilean invariance. But, according to the Galilean transformation of time, there is still Newton's distinguished absolute time in classical mechanics.

In 1905, Einstein assumed the *principle of special relativity* for all inertial systems satisfying the constancy c of the speed of the light ('Lorentz systems') and derived a common space-time of mechanics, electrodynamics, and optics. Their

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laws are invariant with respect to the Lorentz transformations. Time measurement becomes path-dependent, contrary to Newton's assumption of absolute time. Every inertial system has its *relative* ('*proper*') *time*. An illustration delivers the Twin paradox: In a space-time system, twin brother A remains unaccelerated on his home planet, while twin brother B travels to a star at great speed. The traveling brother is still young upon his return, while the stay-at-home brother has become an old man. But, according to the *symmetry of time*, the twin brothers may also become younger. Thus, relativistic physics cannot explain the *aging of an organism* with direction of time. According to Einstein (1915), gravitational fields of masses and energies cause the *curvature of space-time*. Clocks are affected by gravitational fields: The gravitational red shift of a light beam in a gravitational field depends on its distance to the gravitational source and can be considered as dilatation of time. The effect is confirmed by atomic clocks.

Relativistic cosmology assumes an expanding universe in cosmic time. According to Hubble's law of expansion (1929), no galaxy is distinguished. The *Cosmological Principle* demands that galaxies are distributed spatially homogeneous and isotropic ('maximally symmetric') at any time in the expanding universe. In geometry, homogeneous and isotropic spaces have constant (flat, negative or positive) curvature. In two dimensions, they correspond to an Euclidean plane with flat curvature and infinite content, a negatively curved saddle, or a positively curved surface of a sphere with finite content. With the assumption of the Cosmological Principle and Einstein's theory of gravitation, H.P. Robertson and H.G. Walker derived the three standard models of an expanding universe with open cosmic time in the case of a flat or negative curvature and final collapse and end of time in the case of positive curvature. F. Hoyle's *steady state universe* (1948) without global temporal development can be excluded by overwhelming empirical confirmations of an expanding universe. K. Gödel's *travels in the past* on closed world lines in an anisotropic ('rotating') universe (1949) are excluded by the high confirmation of isotropy in the microwave background radiation.

The beginning and end of time get new impact by the theory of *Black Holes* and *cosmic singularities*. According to the theory of general relativity, a star of great mass will collapse after the consumption of its nuclear energy. During 1965–1970, R. Penrose and S.W. Hawking proved that the collapse of these stars is continued to a point of singularity with infinite density and gravity [1]. Thus, the singularity of a Black Hole is an *absolute end of temporal development*. The Schwarzschild-radius determines the event horizon of a Black Hole. Because of the *symmetry of time*, there might be also 'White Holes' with expanding world lines and exploding matter and energy, starting in a point of singularity. This idea inspired Hawking's theorem of cosmic origin (1970): Under the assumption of the theory of general relativity and the observable distribution of matter, the universe has an *initial temporal singularity* ('*Big Bang*'), even without the additional assumption of the Cosmological Principle. Time is initialized in that point.

From different philosophical points of view, theists or atheists have supported or criticized the idea of an initial point of time, because it seems to suggest a creation of the universe. The mathematical disadvantage is obvious: In singularities of zero extension and infinite densities and potentials, computations must fail. Thus, nothing can be said about the origin of time in relativistic cosmology.

2 Time in Quantum Dynamics

According to Bohr's correspondence principle, a dynamical system of quantum mechanics can be introduced by analogy to a dynamical system of classical (Hamiltonian) mechanics. Classical vectors like position or momentum are replaced by operators satisfying a non-commutative (non-classical) relation depending on Planck's constant h. The dynamics of quantum states is completely determined by time-depending equations (e.g., Schrödinger equation) with *reversibility of time*. The laws of classical physics are invariant with respect to the symmetry transformations of time reversal (T), parity inversion (P), and charge conjugation (C). According to the *PCT-theorem*, the laws of quantum mechanics are invariant with respect to the combination PCT. Thus, in spite of P-violation by weak interaction, the PCT-theorem still holds in quantum field theories. But it is an open question how the observed violations of PC-symmetry and T-symmetry (e.g., decay of kaons) can be explained [2].

An immediate consequence of the non-commutative relations in quantum mechanics is Heisenberg's principle of uncertainty which is satisfied by conjugated quantities such as time and energy: Pairs of virtual particles and antiparticles can spontaneously be generated during a tiny interval of time ('*Planck-time*'), interact and disappear, if the product of the temporal interval and the energy of particles is smaller than Planck's constant. Thus, quantum vacuum as the lowest energetic state of a quantum system is only empty of real particles, but full of virtual particles ('quantum fluctuations').

Furthermore, according to Heisenberg's uncertainty principle, there are no time-depending orbits (trajectories) of quantum systems, depending on precise values of momentum and position like in classical physics. In order to determine the temporal development of a quantum system, R. Feynman suggested to use the sum ('integral') of all its infinitely many possible paths as probability functions. In *quantum cosmology*, the whole universe is considered as quantum system. Thus, Feynman's method of path integral can be applied to the whole universe. In this case, the quantum state (wave function) of the universe is the sum (integral) of all its possible temporal developments (curved space-times). In 1983, J. Hartle and Hawking suggested a class of curved space-times without singularities, in order to avoid the failure of relativistic laws in singularities and to make the cosmic dynamics completely computational.

According to Hawking's hypothesis of an *early universe without beginning*, Feynman's path integral allows different models of temporal expansion which are more or less probable—collapsing universes, critical universes, universes with fast (inflationary) expansion. Hawking uses the (weak) Anthropic Principle to distinguish a universe like ours, enabling the evolution of galaxies, planets, and life, with an early inflation and later retarded expansion of flat curvature [1]. From his hypothesis, R. Laflamme and G. Lyons derived the forecast of tiny fluctuations of the microwave background radiation which was confirmed by the measurements of COBE in 1992. Thus, Hawking's hypothesis of an early universe without temporal beginning has been confirmed (until now), but not explained by an unified theory of quantum and relativistic physics which we still miss.

The temporal development of the universe can be considered as dynamics of phase transitions from an initial quantum state of high density to hot phase states of inflationary expansion and the generation of elementary particles, continued by the retarded expansion of galactic structures. Cosmic time is characterized by the development from a nearly uniform quantum state to more complex states of differing cosmic structures. In this way, we get a *cosmic arrow of time from simplicity to complexity*, which is characterized by a bifurcation scheme of global cosmic dynamics: An initial unified force has been separated step by step into the partial physical forces we can observe today in the universe: gravitation, strong, weak, and electromagnetic interactions with their varieties of elementary particles [2–4].

If in the early universe gravitation and quantum physical forces are assumed to be unified, then we need a *unified theory of relativity and quantum mechanics* with new objects as common building blocks of the familiar elementary particles. The string theory assumes tiny loops of 1-dimensional strings (10^{-35} m) with minimal oscillations generating the elementary particles. In a superstring theory, the unified early state corresponds to a transformation group of *supersymmetry*, which leaves the laws of the unified force invariant. During the cosmic expansion the early symmetry is broken into partial symmetries corresponding to different classes of particles and their interactions. Only three spatial dimensions of the more dimensional superstring theory are 'unfolded' and observable. Today, there are five 10-dimensional string theories and an 11-dimensional theory of supergravitation with common features ('dualities') and identical forecasts of the universe. They are assumed to be unifiable in the so-called M-theory. In this case, the *cosmic arrow of time* could be completely explained by phase transitions from simplicity to complexity.

3 Time in Thermodynamics

In physics, a direction of time was at first assumed in thermodynamical systems. According to R. Clausius, the change of the entropy S of a physical system during the time $\mathrm{d}t$ consists of the change $\mathrm{d_e}S$ of the entropy in the environment and the change $\mathrm{d_i}S$ of the intrinsic entropy in the system itself, i.e. $\mathrm{d}S = \mathrm{d_e}S + \mathrm{d_i}S$. For isolated systems with $\mathrm{d_e}S = 0$, the *second law of thermodynamics* requires $\mathrm{d_i}S \geq 0$ with increasing entropy ($\mathrm{d_i}S > 0$) for *irreversible thermal processes* and $\mathrm{d_i}S = 0$ for *reversible processes* in the case of thermal equilibrium. According to L. Boltzmann, entropy S is a measure of the probable distribution of microstates of elements (e.g., molecules of a gas) of a system, generating a macrostate (e.g., temperature of a gas): $S = k_B \ln W$ with k_B Boltzmann's constant and W number of probable distributions of microstates, generating a macrostate. According to the second law, entropy is a measure of increasing disorder during the temporal development of isolated systems. The reversible process is extremely improbable. For Hawking, the cosmic arrow of the expanding universe from simplicity to complexity, from an initial uniform order to galactic diversity, is the true reason of the second law.

Nevertheless, as the second law is statistical and restricted to isolated systems, it allows the emergence of order from disorder in *complex dynamical systems* which are in energetic or material interaction with their environment (e.g., convection rolls of Bénard-experiment, oscillating patterns of the Belousov–Zhabotinsky-Reaction, weather and climate dynamics) [5]. In general, the development of dissipative systems can be characterized by *pattern formation of attractors* (e.g., fixed point attractor, oscillation, chaos) and *temporal bifurcation trees*. In a critical distance to a point of equilibrium, the thermodynamical branch of minimal production of energy ('linear thermodynamics') becomes instable and bifurcates spontaneously into new locally stable states of order ('symmetry breaking'). Then, the nonlinear thermodynamics of nonequilibrium starts [6]. If the system is driven further and further away from thermal equilibrium, a bifurcation tree with nodes of locally stable states of order is generated. Global pattern formation of complex dynamical systems can be *irreversible*, although the laws of locally interacting elements (e.g., collision laws of molecules in a fluid) are *time-reversible*.

4 Time in Evolutionary Dynamics

Life on Earth is not so special in the universe. In a prebiotic evolution, self-assembling molecular systems become capable of self-replication, metabolism, and mutation in a given set of planetary conditions. It is still a challenge of biochemistry to find the molecular programs of generating life from 'dead' matter. *Darwin's evolution of species*, as far as it is known on Earth, can also be characterized by *temporal bifurcation trees*. Mutations are random fluctuations in the bifurcating nodes of the evolutionary tree, breaking the local stability of a species. Selections are the driving forces of branches, leading to further species with local stability. The distance of sequential species is determined by the number of genetic changes. *Evolutionary time* can be measured on different scaling, e.g., by the distance of sequential species and the number of sequential generations of populations. Its *temporal direction* is given by the order of ancestors and descendants.

As conditions changed in the course of the Earth's history, *complex cellular organisms* have come into existence, while others have died out. Entire populations come to life, mature, and die, and in this they are like individual organisms. But while the sequence of generations surely represents the time arrow of life, many other distinct biological time rhythms are discernable. These rhythms are superimposed in *complex hierarchies of time scales*. They include the temporal rhythms of individual organisms, ranging from biochemical reaction times to heartbeats to jet lag, as well as the geological and cosmic rhythms of ecosystems.

Complex systems that consist of many interacting elements, such as gases and liquids, or organisms and populations, may exhibit separate temporal developments in each of their numerous component systems. The complete state of a complex system is therefore determined by statistical distribution functions of many individual states. It has been proposed by B. Misra, I. Prigogine a.o. that time can be defined as an operator which describes changes in the complete states of complex systems [6]. This *time operator* would then represent the average *age* of the different system components, each in its distinct stage of development. Accordingly, a 50-year-old could have the heart of a 40-year-old, but, as a smoker, have the lungs of a 90-year-old. Organs, arteries, bones, and muscles are in distinct states, each according to its particular condition and genetic predisposition. The time operator is thus intended to indicate the *irreversible aging of a complex system*, its *inner* or *intrinsic time*, not the *external and reversible clock time*.

The *human brain* may also be regarded as a complex system in which many neurons and different regions of the brain interact chemically and are switched among their component states by simple local rules. Our individual experience of "*duration*" and "*aging*" thus reflects the complex-system states of the brain, which are themselves dependent on different sensory stimuli, emotional states, memories, and physiological processes. Hence, our *subjective awareness* of time is not contrary to the laws of science, but is a result of the dynamics of a complex system. This in no way diminishes the intimate subjectively experienced flow of time as described in literature and poetry. Knowing the dynamical laws of the brain does not turn one into a Shakespeare or a Mozart. In this sense, the natural sciences and the humanities remain complementary.

The theory of complex systems also applies to the *temporal dynamics of socio-economic systems* [4]. A city, for example, is a complex residential region in which different districts and buildings have distinct traditions and histories. New York, Brasilia, and Rome are the result of distinct temporal development processes, which are not elucidated by external dates. The time operator of a city refers literally to the average age of many distinct stages and styles of development. Institutions, states, and cultures are similarly subject to growth and aging processes, which external dates can shed little light on. Today, there is the dramatic problem of *aging societies* in western civilization. From the point of view of complex dynamic systems, the discussion of age is not just metaphorical, but offers an explanation in terms of structural dynamics.

5 Time in Computation and Information Dynamics

Modern technical societies depend sensitively on the capacities of computers and information networks. *Computation time* is a measure of the time needed to solve a problem by a computer. As a measure of a problem's complexity, one focuses on the running time and data storage requirements of an algorithm and their dependence on the length of the input. The theory of computational complexity deals with the classification of problem into complexity classes, according to the dependence of running time on input length. It is suspected that appreciably shorter computational times were achievable with computers operating on the basis of quantum mechanics and not according to the principles of classical physics. But, as classical computers are based on classical physics, and quantum computers on quantum mechanics, both kinds of computers are based on the concept of *time reversibility*: The laws of nature under which they operate permit, in principle, their computing processes (other than the act of measurement and reading out in the case of quantum computers) to run backward in time.

This raises the question of whether it might also be possible to use computers to simulate *time-irreversible processes* that are well known from biological evolution and the self-organization of the brain. The emergence of cellular patterns was simulated for the first time in the 1950s by von Neumann's *cellular automata*. Computer experiments show the emergence of patterns that are familiar as the attractors of complex dynamic systems. There are oscillating patterns of reversible automata and irreversible developments from initial states to final patterns. For example, in the case of a fixed point attractor, all developments of a cellular automaton develop to the equilibrium state of a fixed pattern which does not change in the future. As these developments are independent of their initial states, they cannot be reconstructed from the final equilibrium state.

Further on, there are cellular automata *without long-term predictions* of their time-depending pattern formation. These are cellular automata with the property of universal computability. *Universal computation* is a remarkable concept of *computational complexity* which dates back to Alan Turing's universal machine. A universal Turing machine can by definition simulate any Turing machine. According to the Church-Turing thesis, any algorithm or effective procedure can be realized by a Turing machine. Now Turing's famous Halting problem comes in. Following his proof, there is no algorithm which can decide for an arbitrary computer program and initial condition if it will stop or not in the long run. Consequently, for a system with universal computation (in the sense of a universal Turing machine), we cannot predict if it will stop in the long run or not. Assume that we were able to do that. Then, in the case of a universal Turing machine, we could also decide whether any Turing machine (which can be simulated by the universal machine) would stop or not. That is obviously a contradiction to Turing's result of the Halting problem. Thus, systems with universal computation are unpredictable. Unpredictability is obviously a high degree of complexity. It is absolutely amazing that systems with simple rules of behavior like cellular automata which can be understood by any child lead to complex dynamics which is no longer predictable.

There are at least some few cellular automata which definitely are universal Turing machines [7]. It demonstrates a striking *analogy of natural and computational processes* that even with simple initial conditions and locally reversible rules many dynamical systems can produce globally complex processes which cannot be predicted in the long run.

The paradigms of parallelism and connectivity are of current interest to engineers engaged in the design of *neurocomputers* and *neural networks*. They also work with simple rules of neural weighting simulating local connectivity of neurons in living brains. Patterns of neural self-assemblies are correlated with cognitive states. With simple local rules neural networks can produce complex behavior, again. In principle, it cannot be excluded that this approach will result in a technically feasible neural self-organization that leads to systems with consciousness, and specifically with time awareness.

In a technical co-evolution, *global communication networks* of mankind have emerged with similarity to self-organizing neural networks of the brain [4]. Data traffic of the Internet is constructed by data packets with source and destination addresses. Local nodes of the net ('routers') determine the local path of each packet by using weighting tables with cost metrics for neighboring routers. There is no central supervisor, but only local rules of connectivity like in self-assembling neural nets. Buffering, sending, and resending activities of routers can cause high densities of data traffic spreading over the net with patterns of oscillation, congestion, and even chaos. Thus, again, simple local rules produce complex patterns of global behavior.

Global information networks store millions of human information traces. They are *information memories* of human history, reflecting the *aging process of mankind as a complex dynamical system*. What is the future of mankind and its information systems in the universe? Cosmic evolution can also be considered as the aging process of a complex dynamical system. If we are living in a flat universe according to recent measurements, then relativistic cosmology forecasts an infinite expansion into the void with increasing dilution of energy and decay in Black Holes. Does it mean the decay of all information storages and memories of the past, including mankind, an *aging universe* with 'Cosmic Alzheimer' [3]? Or may we believe in the fractal system of a bifurcating multiverse with the birth and recreation of new expanding universes? As far as we know there is a *cosmic arrow of time* in our universe, but it is still open where it is pointing at [8].

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