Fuzzy Linguistic Preference Relations Approach: Evaluation in Quality of Healthcare

Pei-Jung Yang[1] and Tsung-Han Chang[2]

[1] Department of Applied Foreign Languages, Kao-Yuan University,
1821, Jhongshan Road, Lujhu District, Kaohsiung City 821, Taiwan
t00148@cc.kyu.edu.tw

[2] Department of Information Management, Kao-Yuan University,
1821, Jhongshan Road, Lujhu District, Kaohsiung City 821, Taiwan
t90082@cc.kyu.edu.tw

Abstract. This study proposes a linguistic preference relations approach to evaluate the quality of healthcare under a fuzzy environment. Pairwise comparisons are utilized to derive the importance weights of evaluation criteria and to obtain the performance rating of feasible healthcare organizations. The subjectivity and vagueness in the evaluation processes are dealt with linguistic variables parameterized by triangular fuzzy numbers. By calculating the distance of each feasible healthcare organization to the fuzzy positive ideal reference point (FPIRP) and the fuzzy negative ideal reference point (FNIRP) respectively, a closeness coefficient is obtained and utilized to rank the order of all feasible healthcare organizations. A case is simultaneously shown to demonstrate the computational procedures of this proposed approach.

Keywords: Fuzzy linguistic preference relations, pairwise comparison, healthcare service, quality evaluation, fuzzy set.

1 Introduction

Healthcare is a service industry which has the characteristics of highly specialized medical capability and close contact with many persons [1]. From 1995, the inhabitants in Taiwan enjoyed higher quality but relatively cheaper healthcare service compared with the developed countries. The request for healthcare quality is gaining momentum in the world all the time [2]. Hospital is a place where provides several kinds of healthcare services. The goals of managing a hospital successfully are providing good healthcare quality and meeting various requirements for the patients, and then retaining as many customers as possible. To manage these organizations efficiently, comparative healthcare quality evaluation, usually evaluated with itself over time or by comparison with others in the same industry, is taken as an essential resolution of the managerial control function [3]. There are papers discussing the healthcare or medical care quality evaluation problems. Andaleeb [4] investigated the medical care quality perceptions and patients' satisfaction of hospitals in a developing country. Li [6] explored the relationship between healthcare quality management and

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service quality performance for US community hospitals by using a path analytic model. Many critical factors are considered in the healthcare quality evaluation, so this issue is regarded as a multicriteria decision making problem. This study therefore proposes an analytic hierarchy framework based on the fuzzy linguistic preference relations to evaluate the healthcare quality of these healthcare organizations.

2 Fuzzy Linguistic Preference Relations

Wang and Chen [8] initiated the fuzzy linguistic preference relations, with the capability of solving the phenomena that are too difficult to state with traditional quantitative ways, to improve the accuracy, efficiency and effectiveness of analytic hierarchy approach. For the purpose of references, some important definitions and propositions of this method are reviewed as follows.

Definition 1. A fuzzy positive reciprocal multiplicative matrix $\tilde{A} = (\tilde{a}_{ij})$ is consistent if and only if

$$\tilde{a}_{ij} \otimes \tilde{a}_{jk} \cong \tilde{a}_{ki} \tag{1}$$

Proposition 1. For a fuzzy linguistic preference relations matrix $\tilde{P} = (\tilde{p}_{ij})$ is consistent verifies the reciprocal additive transitivity consistency and the following statements are equivalent.

$$Lp_{ji} = \tfrac{j-i+1}{2} - Rp_{i(i+1)} - Rp_{(i+1)(i+2)} - \ldots - Rp_{(j-1)j} \quad \forall i < j \tag{2}$$

$$Mp_{ji} = \tfrac{j-i+1}{2} - Mp_{i(i+1)} - Mp_{(i+1)(i+2)} - \ldots - Mp_{(j-1)j} \quad \forall i < j \tag{3}$$

$$Rp_{ji} = \tfrac{j-i+1}{2} - Lp_{i(i+1)} - Lp_{(i+1)(i+2)} - \ldots - Lp_{(j-1)j} \quad \forall i < j \tag{4}$$

3 Framework to Evaluate the Quality of Healthcare by Using Fuzzy Linguistic Preference Relations

Given t feasible alternatives (A_u, u =1,2,..., t) are compared pairwisely with respect to n evaluation criteria (C_i, i =1,2,..., n) by m evaluators (E_k, k = 1,2,..., m) under a fuzzy environment. The procedures of evaluating the feasible alternatives by utilizing the fuzzy linguistic preference relations are described as follows.

3.1 Determine the Importance Weights of Evaluation Criteria

Providing evaluators linguistic variables to assess the importance weights of evaluation criteria and evaluate the priority ratings of alternatives is critical in fuzzy decision making environments. These linguistic variables are expressed with positive triangular fuzzy numbers as shown in Table 1.

Table 1. Linguistic variables for assessing the importance weights of criteria and priority ratings of alternatives

Linguistic Variables	Linguistic Variables	Corresponding TFNs
Very Very High (VVH)	Very Very Good (VVG)	(0.8 ,0.9 ,1.0)
Very High (VH)	Very Good (VG)	(0.7 ,0.8 ,0.9)
High (H)	Good (G)	(0.6 ,0.7 ,0.8)
Medium High (MH)	Medium Good (MG)	(0.5 ,0.6 ,0.7)
Fair (F)	Fair (F)	(0.5 ,0.5 ,0.5)
Medium Low (ML)	Medium Poor (MP)	(0.3 ,0.4 ,0.5)
Low (L)	Poor (P)	(0.2 ,0.3 ,0.4)
Very Low (VL)	Very Poor (VP)	(0.1 ,0.2 ,0.3)
Very Very Low (VVL)		(0.0 ,0.1 ,0.2)

The procedures to determine the importance weights of evaluation criteria are described as follows.

(1) Construct a pairwise comparison matrix amongst the evaluation criteria C_i ($i=1,2,...,n$). The evaluators E_k ($k=1,2,...,m$) then are asked which is the more important of each two adjoining criteria for a set of $n-1$ preference triangular fuzzy values $\{\tilde{a}_{12}^k, \tilde{a}_{23}^k, ..., \tilde{a}_{n-1n}^k\}$.

(2) Use Eqs. (2)-(6) to derive the unknown triangular fuzzy numbers $\tilde{a}_{ij}^k$.

$$f(Lp_{ij}^k) = \frac{La_{ij}^k + \alpha}{1+2\alpha}, \quad f(Mp_{ij}^k) = \frac{Ma_{ij}^k + \alpha}{1+2\alpha}, \quad f(Rp_{ij}^k) = \frac{Ra_{ij}^k + \alpha}{1+2\alpha} \tag{5}$$

where α is the absolute value of the minimum in this fuzzy linguistic preference relations matrix.

(3) Use the method of average value to integrate the aggregated fuzzy importance weights of criteria. Take $\tilde{p}_{ij}^k$ to denote the transformed fuzzy importance weight of k^{th} evaluator for assessing the criterion C_i and C_j.

$$\tilde{p}_{ij} = \frac{1}{m}(\tilde{p}_{ij}^1 + \tilde{p}_{ij}^2 + ... + \tilde{p}_{ij}^m) \tag{6}$$

where m is the number of evaluators.

(4) Take $\tilde{z}_i$ to denote the averaged fuzzy importance weight of criterion C_i, that is,

$$\tilde{z}_i = \frac{1}{n}\sum_{i=1}^{n} \tilde{p}_{ij} \tag{7}$$

(5) Give $\tilde{w}_i$ to indicate the fuzzy importance weight of criterion C_i, that is,

$$\tilde{w}_i = \frac{\tilde{z}_i}{\sum_{i=1}^{n} \tilde{z}_i} \tag{8}$$

(6) Defuzzify the triangular fuzzy numbers into the best nonfuzzy performance (BNP) values, BNP_{w_i} indicates the BNP value for the triangular fuzzy number $\tilde{w}_i$, that is,

$$BNP_{w_i} = \frac{\left[(Rw_i - Lw_i) + (Mw_i - Lw_i)\right]}{3} + Lw_i \tag{9}$$

3.2 Determine the Priority Ratings of Feasible Alternatives

The procedures to determine the priority ratings of feasible alternatives A_u ($u = 1,2,...,t$) with respect to each evaluation criterion are described as follows.

(1) This study utilizes the method of average value to integrate the fuzzy priority ratings of all alternatives. Give ${}_i\tilde{q}_{uv}$ to denote the aggregated fuzzy priority rating matrix of m evaluator for assessing the alternative A_u and A_v in terms of criterion C_i, that is,

$${}_i\tilde{q}_{uv} = \frac{1}{m}({}_i\tilde{q}_{uv}^1 + {}_i\tilde{q}_{uv}^2 + ... + {}_i\tilde{q}_{uv}^m) \tag{10}$$

(2) Take ${}_i\tilde{\lambda}_u$ to represent the averaged fuzzy priority rating of alternative A_u with respect to criterion C_i, that is,

$${}_i\tilde{\lambda}_u = \frac{1}{t}\sum_{u=1}^{t} {}_i\tilde{q}_{uv} \tag{11}$$

(3) Take ${}_i\tilde{\varphi}_u$ to stand for the weighted fuzzy priority rating of alternative A_u with respect to evaluation criterion C_i, that is,

$${}_i\tilde{\varphi}_u = {}_i\tilde{\lambda}_u \otimes \tilde{w}_i \tag{12}$$

3.3 Rank the Feasible Alternatives

(1) Determine the FPIRP and FNIRP: Because the positive triangular fuzzy numbers are included in the interval $[0,\ 1]$, the fuzzy positive ideal reference point (FPIRP, β^+) and fuzzy negative ideal reference point (FNIRP, β^-) are respectively defined as,

$$\beta^+ = \left(\tilde{\beta}_1^+, \tilde{\beta}_2^+, ..., \tilde{\beta}_n^+\right), \qquad \beta^- = \left(\tilde{\beta}_1^-, \tilde{\beta}_2^-, ..., \tilde{\beta}_n^-\right) \tag{13}$$

where $\beta_i^+ = (1,1,1)$ and $\beta_i^- = (0,0,0)$, $i = 1,2,...,n$

(2) Calculate the distance of alternative to FPIRP and FNIRP: The distance of each feasible alternative to FPIRP and FNIRP can be derived respectively as,

$$D_u^+ = \sum_{i=1}^{n} d({}_i\tilde{\varphi}_u, \tilde{\beta}_i^+), \quad D_u^- = \sum_{i=1}^{n} d({}_i\tilde{\varphi}_u, \tilde{\beta}_i^-) \tag{14}$$

(3) Obtain the closeness coefficient and rank the alternatives: The closeness coefficient (CC_u) of feasible alternative is calculated as,

$$CC_u = \frac{D_u^-}{D_u^+ + D_u^-}, \qquad u = 1,2,...,t \tag{15}$$

4 Empirical Analysis

Four hospitals located in southern Taiwan are evaluated by 6 evaluators according to 5 major evaluation criteria in this study. The fuzzy linguistic preference relations approach is adopted to evaluate the healthcare quality of these sample hospitals. The computations of this proposed framework are described as follows.

4.1 Identify the Evaluation Criteria

The evaluation criteria are derived through literature review [11-17], widespread investigation and consultation with several professors and experts. They are: hospital sanitation and environment (C_1), pharmacy and medical treatment (C_2), service attitude (C_3), professional capability (C_4), and hospital equipment (C_5).

4.2 Weighting Calculation for the Evaluation Criteria

After a series of interviews with 6 evaluators, the importance weights of 5 evaluation criteria are derived.

(1) The pairwise comparison matrices derived from 6 evaluators for a set of 4 adjoining evaluation criteria $\{a_{12}, a_{23}, a_{34}, a_{45}\}$ are listed in Table 2.

Table 2. The linguistic variables for 5 criteria given by 6 evaluators

	E_1	E_2	E_3	E_4	E_5	E_6	
C_1	VVL	MH	VVH	VH	VVL	H	C_2
C_2	F	VL	H	VH	H	MH	C_3
C_3	F	VL	F	H	VL	VH	C_4
C_4	MH	VH	VH	VVH	VH	VH	C_5

(2) The fuzzy importance weights of evaluation criteria are derived by Eqs. (7) - (8), and Eq. (9) is utilized to derive the BNP importance weight of criteria. The results are shown in Table 3.

The results show that the importance weight order of 5 evaluation criteria is: professional capability (0.231) > pharmacy and medical treatment (0.225) > service attitude (0.209) > hospital sanitation and environment (0.200) > hospital equipment (0.168).

Table 3. Importance weight and rank of evaluation criteria

	Averaged TFN	**Fuzzy importance weight**	**BNP**	**Rank**
C_1	(0.37, 0.48, 0.58)	(0.12, 0.19, 0.27)	0.200	4
C_2	(0.47, 0.54, 0.61)	(0.16, 0.21, 0.29)	0.225	2
C_3	(0.45, 0.50, 0.56)	(0.15, 0.20, 0.27)	0.209	3
C_4	(0.49, 0.56, 0.62)	(0.17, 0.22, 0.29)	0.231	1
C_5	(0.29, 0.40, 0.50)	(0.10, 0.16, 0.24)	0.168	5

4.3 Calculation of the Priority Ratings for 4 Hospitals

Six evaluators use the linguistic variables to express their preference about the priority ratings of 4 feasible hospitals with respect to 5 evaluation criteria. The computational procedures are described as follows.

(1) The averaged fuzzy priority ratings of 4 hospitals are derived by using Eq. (10). Then use Eq. (11) to derive the averaged fuzzy priority rating of 4 hospitals with respect to 5 evaluation criteria.
(2) Multiply the fuzzy importance weights of evaluation criteria and the fuzzy priority ratings of 4 hospitals.

4.4 Rank 4 Hospitals

This section describes the processes to rank 4 feasible hospitals, they are described as follows.

(1) Use Eqs. (13) - (14) to measure the distance of 4 hospitals to FPIRP and FNIRP respectively, the results are listed in Table 4.
(2) Uses Eq. (15) to calculate the closeness coefficient of 4 hospitals, the results are listed in Table 4.

Table 4. Rank of 4 hospitals

	D_u^+	D_u^-	CC_u	Rank
A_1	4.427	0.624	0.123705	1
A_2	4.482	0.557	0.110672	3
A_3	4.485	0.555	0.110214	4
A_4	4.468	0.583	0.115565	2

5 Conclusions

Good healthcare quality is a significant factor leads to the success of the health and prosperity of a country. This study extends the fuzzy linguistic preference relation into a multicriteria group decision making environemnt to evaluate the healthcare quality. The proposed approach is based on the reciprocal additive consistent fuzzy preference relations, rather than using conventional multiplicative preference relation. Namely, this method considers only $n-1$ pairwise comparisons, whereas the traditional fuzzy AHP takes $\frac{n(n-1)}{2}$ judgments in a preference matrix with n elements, it is clear that the proposed approach is faster to execute and more efficient than the conventional pairwise comparison methods. According to the importance weights of evaluation criteria, the professional capability and pharmacy and medical treatment are more important than other criteria. Furthermore, this proposed method is suggested to solve the multiple criteria decision making problems.

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References

1. Chang, T.-H.: Hospital service quality evaluation: a fuzzy preference relation approach. In: The IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2011), Taipei (2011)
2. Welch, S.J., Allen, T.L.: Data-driven quality improvement in the Emergency Department at a level one trauma and tertiary care hospital. Journal of Emergency Medicine 30, 269–276 (2006)
3. Chang, H.-H.: Determinants of Hospital Efficiency: the Case of Central Government-owned Hospitals in Taiwan. Omega 26, 307–317 (1998)
4. Andaleeb, S.S.: Service quality perceptions and patient satisfaction: a study of hospitals in a developing country. Social Science & Medicine 52, 1359–1370 (2001)
5. Goldman, L.E., Dudley, R.A.: United States rural hospital quality in the Hospital Compare database–Accounting for hospital characteristics. Health Policy 87, 112–127 (2008)
6. Li, L.X.: Relationships between determinants of hospital quality management and service quality performance–a path analytic model. Omega 25, 535–545 (1997)
7. Tsai, H.-Y., Chang, C.-W., Lin, H.-L.: Fuzzy hierarchy sensitive with Delphi method to evaluate hospital organization performance. Expert Systems with Applications 37, 5533–5541 (2010)
8. Wang, T.-C., Chen, Y.-H.: Applying fuzzy linguistic preference relations to the improvement of consistency of fuzzy AHP. Information Sciences 178, 3755–3765 (2008)
9. Wang, T.-C., Chen, Y.-H.: Fuzzy multi-criteria selection among transportation companies with fuzzy linguistic preference relations. Expert Systems with Applications 38, 11884–11890 (2011)
10. Tan, C., Wu, D.D., Ma, B.: Group decision making with linguistic preference relations with application to supplier selection. Expert Systems with Applications 38, 14382–14389 (2011)

11. Chang, T.-H.: Hospital service quality evaluation: a fuzzy preference relation approach. In: The 2011 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2011), Taipei, Taiwan, pp. 517–522 (2011)
12. Laschinger, H., Hall, L., Pedersen, C., Almost, J.: A psychometric analysis of the patient satisfaction with nursing care quality questionnaire: an actionable approach to measuring patient satisfaction. Journal of Nursing Care Quality 20, 220–230 (2005)
13. Li, L., Benton, W.C.: Hospital capacity management decisions: Emphasis on cost control and quality enhancement. European Journal of Operational Research 146, 596–614 (2003)
14. Lynn, M., McMillen, B., Sidani, S.: Understanding and measuring patients' assessment of the quality of nursing care. Nursing Research 56, 159–166 (2007)
15. Senarat, U., Gunawardena, N.S.: Development of an Instrument to Measure Patient Perception of the Quality of Nursing Care and Related Hospital Services at the National Hospital of Sri Lanka. Asian Nursing Research 5, 71–80 (2011)
16. Shieh, J.-I., Wu, H.-H., Huang, K.-K.: A DEMATEL method in identifying key success factors of hospital service quality. Knowledge-Based Systems 23, 277–282 (2010)
17. Teng, C.-I., Ing, C.-K., Chang, H.-Y., Chung, K.-P.: Development of Service Quality Scale for Surgical Hospitalization. Journal of the Formosan Medical Association 106, 475–484 (2007)