Teachers, Attributions, and Students' Mathematical Work

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An understanding of the relation between teachers' knowledge of students' mathematics and their beliefs about teaching and learning is inherent in the work of mathematics teacher educators seeking to support teachers in learning to make instructional decisions based on students' mathematical thinking. Schoenfeld (2011) proposed that individuals' decision making in well-practiced, knowledge-intensive domains can be fully characterized as a function of three factors: orientations, resources, and goals. Schoenfeld broadly defined orientations to include a myriad of concepts such as dispositions, beliefs, values, tastes, and preferences. He explained that people's orientations shape what they perceive, the meanings they make of these perceptions, the goals they establish for the situation, and the resources they put to use to achieve the established goals. Most importantly, Schoenfeld discussed decision making in relation to teaching and stated that in mathematics classrooms, teachers' orientation toward mathematics, students, learning, and teaching shapes their instruction.

This report is based upon work supported by the National Science Foundation under grant number DRL-1008364. Any opinions, findings, and conclusions or recommendations expressed in this report are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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J.-J. Lo et al. (eds.), *Research Trends in Mathematics Teacher Education,*
Research in Mathematics Education, DOI 10.1007/978-3-319-02562-9_7,

Thompson et al. (1994) used the concept of orientation to describe different approaches to teaching mathematics and included teachers' knowledge, beliefs, and values within this concept. They proposed that orientations molded teachers' images, views, intentions, and goals for mathematics instruction. Similarly, Magnusson et al. (1999) considered that teachers' orientations influenced instructional practice by shaping teachers' knowledge and beliefs about curriculum, students, teaching, and assessment. Philipp (2007) suggested that teachers' orientations were operationalized through language and action.

From our perspective, we consider teachers' discourse about students' mathematical work, in particular the *attributions* that teachers use as they discuss students' mathematics, as one aspect of teachers' orientations toward students. Attributions are perceptions of causality or judgments regarding the occurrence of an incident (Weiner 1972). In the classroom, teachers' attributions refer to the judgments or causal explanations teachers construct to explain students' successes and failures. Teachers' attributions influence their expectations regarding student ability and subsequently impact student performance (Graham 1991). This process of connecting teachers' attributions to students' performance has been documented in various disciplines, including different areas of mathematics (Dobbs and Arnold 2009; Middleton and Spanias 1999). Thus, teachers' attributions in mathematics classrooms are an important aspect of instruction and of concern for mathematics teacher educators working to support teachers in student-centered instruction.

Because our work is in professional development, we extended the role of teachers' orientations and attributions in instruction to professional development settings. Similar to Philipp (2007), we considered that teachers' attributions were operationalized through their discourse. We believe that teachers' attributions play a fundamental role in the conversations teachers have as they engage in professional learning tasks focused on students' mathematical thinking. Therefore, our work examined teachers' attributions by investigating their discourse about students' mathematical successes and failures. We were particularly interested in teachers' discourse within a professional development setting as teachers analyzed students' mathematical work. We explored the following research question: *To what do elementary teachers attribute students' mathematical successes and failures when they consider research results about students' mathematical thinking and learning*?

The results we report are part of a larger design experiment (Cobb et al. 2003) that involves a professional development setting purposefully planned to teach teachers about students' mathematical thinking and learning. Guiding the design and implementation of the professional development was our initial conjecture that learning about students' mathematical thinking would change teachers' discourse by adding new explanations for students' mathematical work to teachers' existing repertoires. Through ongoing analysis, we identified the attributions teachers used throughout the professional development and developed a codebook (DeCuir-Gunby et al. 2011) to investigate teachers' uses of these attributions. The initial phase of the retrospective analysis examined the nature of the attributions and will ultimately characterize changes in teachers' uses of the attributions over time.

In this chapter, we share the attributions that emerged from the ongoing analysis as teachers learned about student mathematical thinking, and we report on the initial phase of the retrospective analysis. We begin by briefly reviewing the literature on attribution theory, focusing particularly on teachers' attributions for students' work. Then, we introduce our research methodology, describing the professional development setting in which we worked, as well as the data collection and analysis processes. We define the attributions identified in our professional development, share examples of how these attributions were present in our work with elementary teachers, and offer findings related to teachers' uses of the identified attributions during the professional development. We conclude with a set of next steps for our research, including a shift in framework to use positioning theory to conceptualize teachers' uses of these attributions as acts of stereotyping.

Teachers' Attributions

Bar-Tal (1978) defined attributions as the inferences made about the causes of one's own or someone else's behaviors. Attribution theory allows for individuals to gain a better understanding of their environments and the determinants of individual behavior (Schunk et al. 2013). The general attribution model (see Weiner 1986, 1992, 2010) consists of the creation of attributions through the *attribution process* and the use of those attributions through the *attributional process*. The following sections describe the various components of the model.

Attribution Process

The attribution process involves understanding the development of attributions. Specifically, it concerns the exploration of antecedent conditions or the causal determinants of behavior. The process of creating attributions considers both environmental factors and personal factors. Environmental factors in the case of academic achievement include issues such as the type of school, the testing environment, and teacher quality, among many others. Personal factors, on the other hand, consist of a variety of features including beliefs about causality, rules used to make attributions, prior knowledge, and individual differences (Schunk et al. 2013). These environmental and personal factors influence the creation of perceived causes to explain behavior.

Antecedent conditions serve as a foundation for understanding the perceived causes of behavior. Early research indicated that ability, effort, task difficulty, and luck were seen as the most common perceived causes of the outcomes of events (Cooper and Burger 1980). Weiner (1986) elaborated this list to delineate explanations for academic success or failure to include ability, skill, stable effort, unstable effort, task difficulty, luck, interest, mood, fatigue, health, and help from others. Once a perceived cause is established, it then impacts the attributional process.

Attributional Process

Attributions may be classified along three dimensions according to their causal structure. The first dimension, locus, establishes whether the source of the outcome is internal or external to the individual. Internal causes are aspects that individuals can control, such as effort spent studying for a test; external causes are beyond the control of individuals, such as luck in answering correctly on the test. The second dimension, stability, explores how consistent the cause is over a period of time. Stability involves understanding a cause as fixed and stable or variable and unstable. For example, intelligence is commonly viewed as a fixed trait and thus stable. The last dimension, controllability, addresses the amount of control a person has over a cause. For instance, effort can be considered controllable because one may put forth more or less effort, whereas ability is often viewed as uncontrollable because one cannot change his or her inherent ability. These attributional dimensions have both psychological and behavioral consequences. They impact expectations for success and emotions associated with achievement. The dimensions also impact specific behaviors including future choices, persistence at engaging in tasks, level of effort placed to complete tasks, and achievement (Schunk et al. 2013). When attempting to infer the causes of another's behavior, the *fundamental attribution error* may result from attributing another's behavior to a personal trait without attending to situational factors (Schunk et al. 2013).

In our work, we take a view of designing professional development to support teachers in learning a framework for students' mathematical thinking as contributing to teachers' attribution and attributional processes. Through learning research results about students' mathematical thinking, teachers engage in the attribution process by considering the antecedent conditions that affect learning and that serve as a foundation for the perceived causes of students' mathematical successes or failures, including environmental factors such as opportunities to learn and personal factors such as previous experiences and current understandings. Our investigation of teachers' attributional processes concerns their perceptions of the causes of students' successes and failures, in particular the fundamental attribution error, which we see impacting teachers' expectations and efficacy. By attributing student failure to an internal, fixed, and/or uncontrollable cause, a teacher may perceive no recourse for teaching, whereas attributing a students' failure to an external, variable, and/or controllable factor suggests that learning may be affected by instruction.

Attributions and Mathematics Education

Attribution theory has been applied to a variety of contexts and tasks within mathematics education. Although research regarding students' attributions of their success and failure exists (e.g., Seegers et al. 2004), a substantial amount of research in this area has focused on the attributions that teachers make regarding students' mathematics learning. For example, Middleton and Spanias (1999) noted that teachers' attributions for their students' successes and failures were reflected in the ways teachers interacted with their students during mathematics instruction. In examining preschool settings, Dobbs and Arnold (2009) claimed that teachers' attributions of students' behavior shaped their behavior toward the child, which in turn often elicited the expected behavior from the child, having a self-fulfilling prophecy effect.

Within mathematics education, studies have indicated that stereotypes related to gender, race, and socioeconomic status can influence teachers' attributions for student success and failure. For instance, Fennema et al. (1990) studied 38 first-grade teachers' attributions for boys' and girls' successes in mathematics. They found that teachers tended to attribute boys' successes and failures to ability while attributing girls' successes and failures to effort. Reyna (2000) found that stereotypes could serve as the foundation for the attributions made regarding the mathematics achievement of students of color. For example, she discussed that whereas some people believe African Americans or Latino/as as a group are lazy, others believe they are underprivileged. As a result, teachers may attribute a students' ability to internal factors, such as effort, or external factors, such as opportunities related to the beliefs they hold about groups of students. Similarly, Reyes and Stanic (1988) examined how students' socioeconomic status impacted teachers' perceptions regarding achievement. They found that teachers' attitudes about students' achievement, as measured through classroom processes, varied based on students' sex, socioeconomic status, and race. However, the causality of these connections had yet to be established. Together, these studies suggest teachers' attributions, such as ability or effort regarding mathematics achievement, may be an extension of social stereotypes.

Methods

The overarching purpose of our research was to understand the ways in which teachers come to learn about students' mathematical thinking in the context of a professional development setting. We used a design experiment methodology within a school-based professional development setting to work toward this purpose. Design experiments are "iterative, situated, and theory-based attempts simultaneously to understand and improve education processes" (diSessa and Cobb 2004, p. 80). They are used to develop "a class of theories about both the process of learning and the means that are designed to support that learning" and they "entail both 'engineering' particular forms of learning and systematically studying those forms of learning within the context defined by the means of supporting them" (Cobb et al. 2003, p. 9).

In line with this methodology, we examined both teacher learning and the set of professional learning tasks that supported their learning experiences. Although we expected teachers' orientations toward students to shape the ways in which they engaged with the professional learning tasks we designed for the study, it was the ways that teachers talked about students' successes and failures that emerged as a major component in their discourse, playing a fundamental role in teachers' engagement with the professional learning tasks and shaping professional conversations around students' mathematics. As the professional development unfolded, we designed tasks that further brought forth various attributions and focused our analysis on exploring these attributions.

Context

Learning Trajectory Based Instruction (LTBI) is a multiyear NSF-funded research project with a strong mathematics professional development component for elementary teachers based on the concept of learning trajectories (LTs). When Simon (1995) coined the expression "hypothetical learning trajectory," he indicated that teachers create representations of the "paths by which learning might proceed" (p. 135) when students progress from their own starting points toward an intended learning goal. He named these trajectories hypothetical because each student's individual learning path was not knowable in advance. However, he suggested that these learning paths represented expected tendencies and that commonalities across students allowed teachers to develop expectations about the progression of learning.

Over time, the concept of LTs developed beyond the notion that teachers have expectations about how learning might proceed to include an empirical search for the highly probable sets of levels through which students progress as their learning of specific mathematics topics evolves. Current work on LTs uses research on student learning from clinical interviews and large-scale assessment trials to seek clarification of the intermediate steps students take as learning proceeds from informal conjectures into sophisticated mathematics. Recently, research on LTs has progressed from an agenda for studying student learning to include an agenda for research on teaching. Daro et al. (2011) called for the translation of LTs into "usable tools for teachers" (p. 57) and indicated the need to make these trajectories available to teachers so that they can guide classroom instruction.

Content Over the course of 1 year, teachers in the LTBI project learned about students' early rational number reasoning through study of the equipartitioning learning trajectory (EPLT). Confrey et al. (2009) defined equipartitioning as the cognitive behaviors that have the goal of producing equal-sized groups or parts as typically encountered by children in constructing "fair shares." The EPLT empirically describes how children begin with informal knowledge of fair sharing, and through instruction, build an understanding of partitive division that unifies ratio reasoning and fractions—see Confrey (2012) for a more detailed description of the EPLT.

The LTBI professional development included both a summer institute and academic-year monthly meetings. These two components of the intervention were designed with different goals in mind. The summer institute offered teachers opportunities to learn about the EPLT and develop an appreciation for the role of the trajectory in understanding student mathematics. In contrast, the academic-year monthly meetings focused on establishing connections between the trajectory and instructional practices. The two components of the professional development totalled 60 h of face-to-face, whole group interactions.

Professional Learning Tasks All tasks developed for use in the professional development were guided by a set of design principles stating that professional learning tasks for LTs: (a) attend mostly to issues of pedagogy, (b) embed opportunities for teachers to examine all facets of their knowledge for teaching, (c) use instructional sequences that begin with practice-based activities that challenge teachers' views of students' mathematics and mathematics learning, and (d) use artifacts similar to the ones researchers used in developing the LT (Wilson et al. 2013). As such, the professional learning tasks incorporated videos of clinical interviews, samples of students' written work, and examinations of teachers' curricular materials. During the summer institute, these artifacts consisted of "anonymous" students' work, whereas during the school year, teachers discussed and examined their own students' work. When possible, teachers brought in their own curricular materials supplied by the school district in which they worked.

One example of a task used in the summer institute was to engage teachers in watching and discussing videos of clinical interviews with students from different grade levels solving similar mathematical problems. Teachers were asked to describe the ways in which each child solved the problem, conjecture about each student reasoning for that particular solution, consider the sophistication of the various strategies, and examine what surprised them about each student's work. In the discussion of the task, despite the facilitator's effort to focus the discussion on what each child did and why, teachers' discourse focused mostly on alignment or deviations from their expectations based on the child's grade level. That is, teachers attributed what the children did to their grade level, and the information about each child's grade level that was offered to teachers as part of the context for the clinical interviews became the center of teachers' subsequent discussion.

Participants

The professional development was offered in partnership with one K–5 elementary school in a mid-size urban area in the southeast USA. The school had approximately 600 students: 35 % Caucasian, 29 % Hispanic, 25 % African American, 7 % Asian, and 4 % other; 54 % of the children qualified for free or reduced-price lunch. Teachers at the school volunteered to participate, and all professional development meetings were conducted at the school at times convenient to the teachers. Of the 24 teachers who started the professional development in July 2010, 22 completed the program 1 year later in June 2011. The initial group of teachers included six kindergarten teachers and three Grade 1, five Grade 2, three Grade 3, two Grade 4, and one Grade 5 teacher. Four teachers taught multiple grade levels.

Data Sources and Analysis

All data were collected by a research team comprising two principal investigators and one graduate student (the first, third, and second authors, respectively). Data sources included the researchers' field notes, 69 video files from the professional development meetings, and 90 transcripts of audio recordings of teachers' small group discussions during the 60 h of summer institute and monthly meetings. Following Cobb (2000), our analysis included both an ongoing and retrospective phase where we used a grounded-theory approach (Strauss and Corbin 1990) to code the data. Open coding was utilized to create concepts from the raw data. These data-drive codes were supplemented with additional codes derived from attribution theory and our research goals. We used the constant comparative method in that we compared various project data sources, including field notes and transcripts as well as research literature (Glaser and Strauss 1967).

In what follows, we describe our data analysis process, focusing on the first two of three stages of the work. The ongoing analysis occurred as the professional development was unfolding. During this time, we generated field notes and used these notes to revise and refine the professional learning tasks for the professional development. The retrospective analysis began after the conclusion of the professional development and was divided into two phases. The first phase included the development of a codebook, data reduction, and analysis of the frequencies of codes by each participant. The second phase consisted of more in-depth analysis of a subset of data to further understand changes in the teachers' discourse over time. In this chapter, we report on the ongoing and first phase of the retrospective analyses.

Ongoing Analysis Teachers' attributions for students' mathematical successes and failures emerged early in our analysis as a fundamental aspect of teachers' discourse, shaping their engagement with the designed professional learning tasks. For example, teachers talked about students not completing a task because they were "low students" or because of the way the task was presented to them. Thus, throughout the first year of the project, we conducted an ongoing analysis to examine the ways in which teachers talked about students' successes and failures in mathematics. We noted emerging attributions in our field notes, and the research team discussed them in regular meetings throughout the implementation of the professional development. From a design perspective, we continued to create and refine professional learning tasks in order to provide new opportunities for teachers to examine the ways in which they talked about their students as mathematics learners as well as their students' successes and failures. For instance, following the example above of teachers talking about students having low ability, we created teaching scenarios in which such vocabulary was used and then asked teachers to discuss these scenarios. We posed teachers' own attribution language back to them for explicit discussion, enabling us to use the design experiment setting to better understand the various attributions that emerged throughout the yearlong professional development. This process assisted us in further eliciting and understanding the teachers' uses of various attributions and the role the learning trajectory played in their discourse.

Retrospective Analysis After the completion of the first implementation of the LTBI professional development, we began a retrospective analysis to understand the attribution process of generating a new explanation for students' mathematical successes and failures based on the EPLT, as well as teachers' uses of the various attributions in their discourse about students' mathematical work. We initially engaged in a grounded theory approach to data analysis (Strauss and Corbin 1990) and coded our field notes generated during the ongoing analysis using open coding. This process enabled us to create concepts from our raw data, offering a first set of attributions that we used as codes for the subsequent analysis. These attributions varied in locus, stability, and controllability and will be elaborated on in the results section.

Building on the concepts identified in the ongoing analysis and refined through our examination of the complete data corpus, we created a codebook through an iterative process between our data and theory (DeCuir-Gunby et al. 2011). In line with the grounded theory approach to data analysis, we used constant comparison methods to compare various project data sources, including field notes and transcripts of small group discussions, and the research literature (Glaser and Strauss 1967) in order to identify and refine our codes. The codebook included definitions and examples of each code, and we revisited the data frequently as we refined our definitions until the codebook was finalized.

Four independent coders were trained to use the codebook to code the transcripts of audio data and the video data with 85% interrater reliability. Because we began with a large data set, we used this process to reduce the data to turns where teachers were explicitly or implicitly talking about students. We defined a turn as one person's statement in a conversation that is not interrupted by another's idea. Coders used the definitions from the codebook to code every turn teachers made in all whole group and small group discussions that were related to students' work, using one or more of the codes identified. In all, 2,868 turns were identified and coded, with 123 turns marked with multiple codes. Each coder coded approximately 40 files and 10% of the files were double coded to maintain reliability and prevent drifting.

Because we were interested in teachers' uses of the language from the EPLT to explain students' mathematical thinking, all turns were then examined a second time for evidence of language from the EPLT and given an additional code of "LT" when evidence of such language was found. For example, when teachers were reviewing students' written work, one teacher commented, "The way the child divided the pizzas was he did benchmarking, so they did the halving first and then they did the radial cuts." Because this teacher is describing what she perceived the student to do mathematically, this turn was coded as "Math." Because she is referencing specific strategies described by the EPLT, in particular benchmarking and radial cuts on a circle, the turn was later also given a code of LT. Because the data were first reduced to turns that concerned students' mathematical work and then were coded based on evidence of the LT, turns coded as LT referenced students' mathematical work in relation to one of the eight identified attributions. Two research team members carried out the coding for LT, discussing any unclear turn until agreement was reached.

Results

Our research sought to identify to what elementary teachers attributed students' mathematical successes and failures when working on professional learning tasks designed to share with them research results about students' mathematical thinking and learning. First, we present the attributions that emerged during our ongoing analysis that informed the development of the codebook. Then, we examine teachers' uses of these attributions during the 60-h professional development from the first phase of our retrospective analysis.

Attributions

We identified eight different attributions that teachers used when explaining students' mathematical work. Common attributions noted by Cooper and Burger (1980) were present in our data, specifically, ability, effort, luck, and difficulty of task. We identified additional attributions of age or grade level, out of school context, teaching, and previous math knowledge. Moreover, these attributions varied in terms of locus of causality, stability, and controllability. In our interpretation, we considered these dimensions from the teachers' perspective, viewing locus of causality and stability in relation to the student and examining whether the teacher has control over a particular attribution. In this section, we present each attribution, beginning with the attributions previously identified in the literature, and then describing additional attributions we identified, along with selections from the data that exemplify how each was presented in teachers' discourse.

Ability Ability was internal to the student, fixed, and an uncontrollable attribution that included the personal traits of students and characteristics that defined fixed qualities related to students' aptitude in mathematics. Often times, teachers used achievement as a proxy to consider students' abilities and attributed their performance to an innate capacity. One example involved a teacher describing her work with a previous student in a discussion during the summer institute. In her comment, she indicated that ability was a fixed characteristic of the student. She stated, "We had evaluated this student, and we were convinced there was a learning disability. The work was really low." Another teacher, also describing past students, expressed a similar explanation related to students' innate abilities. She said, "I had a lot of math geniuses and they can figure things out when they are so young."

Effort Effort was internal to the student, variable, and a controllable attribution that referred to the level of students' attention and engagement with a particular task at a particular moment. This attribution indicated that performance did not always represent a fixed characteristic of the student but depended on how carefully or how speedily that particular student progressed through the work at a particular moment and was thus subject to change. When examining her own students' written work on an assessment during a monthly meeting, one teacher explained a student's incorrect solution by saying, "Well, he just zipped through all this, so, no wonder." In another instance during the same activity, a teacher commented, "He worked on this so carefully." Other teachers explained students' work by speculating about the student's attention during instruction, such as "In my mind, this kid just wasn't paying attention to me while I was teaching and he played connect the dots."

Luck Luck was external to the student, variable, and an uncontrollable attribution that included the idea that what students did had no intentionality behind it. Teachers who attributed students' success or failure to luck implied that students had no real explanation for what they did or why they did it, or questioned whether students knew what they were doing. For instance, during the summer institute, in response to viewing a clinical interview with a child who was equipartitioning a collection of 24 coins among four friends, one teacher remarked, "When questioned how did you know, that is when I realized she really randomly chose to give each one, two pieces. It was not that she had the number fact or she understood." During the same discussion, another teacher commented, "I thought she was just guessing and she was just lucky."

Difficulty of Task Difficulty of task was external to the student, variable, and a controllable attribution for students' work that expressed the notion that students' difficulty was determined by the clarity (or lack thereof) of the question posed to them. It had the embedded idea that there was a perfect way to ask a question so that students would not make a mistake. For example, in the summer institute, when analyzing two video recordings of students sharing a collection of coins among various numbers of people, one teacher said, "The proctor asked her to put things together and then divide them, so, she shared differently [than the first student] because the proctor asked a different question." In another case, teachers were examining written work on equipartitioning assessment items and one teacher commented:

> When we teach a group of students and over half of them make the same mistake, then we have to go back and look at the way we presented it and ask ourselves…"Is it some fault in the way the question was presented?"

Age or Grade Level Age or grade level was internal to the student, fixed, and an uncontrollable attribution that described the expectations teachers had for students' performance given normalized definitions for what a generic student should be able to do at certain points in his or her development. Teachers used grade level to create groups of students at similar developmental levels who should perform in certain expected ways, assessing the quality of a students' work or their mathematical reasoning based on whether it conformed to what is expected of children at that age or grade level. For instance, during the summer institute, one teacher commented on a video recording of a student sharing 24 coins among three friends: "I had expected the third grader to not share dealing it one by one." In response to a similar video of another student, a teacher stated, "I taught Kindergarten, and I would have guessed she would share using one for you, one for you, one for you; what she did was more advanced because she counted two plus two plus two."

Out of School Context Out of school context was external to the student, variable, and an uncontrollable attribution that included out of school understandings and explanations that teachers expected students to generalize to the academic context. This attribute indicated that teachers took into account the experiences students bring with them from their own lives. For instance, when conjecturing in the summer institute about a video of one student's work sharing a collection of coins among two people, a teacher said, "She just shared and she thought, 'It is fair because we each got some,' and that is because of how we use the word 'share' in the real world. She thought, 'We both have some, so we have shared.'" Another teacher remarked about a written task where students were asked to equipartition a rectangular birthday cake among six friends:

> I think that was a problem for a lot of these kids, dishing out the whole birthday cake [to fair share it]. I just wonder if you called it something else besides a birthday cake if they would have seen the whole differently.

These comments indicate that students' out of school experiences are uncontrollable and not necessarily places from which to build instruction, but as justifications for students' mistakes.

Teaching Teaching was external to the student, variable, and a controllable attribution that indicated that students' mathematical work depends on what teachers present to them. Teachers expected students to know or not know a topic depending on whether or not a teacher had already taught the topic. The attribution also indicated that teachers suggested that students had no way of knowing a topic that they were not yet taught. For example, during the summer institute, one teacher stated,

> Sometimes students can say something even when we had not taught it, like, this is 1/2 of 10 so that part has to be 5 as well. It seems simplistic, but I don't know how they would have known that already.

In another case, after examining two tasks related to identifying "one-sixth," one with a circle already partitioned into six equal sized parts and one that asked students to equipartition a circle for six, a teacher remarked, "Don't you think that's kind of hard too? Because like you said, this one's already done for them, and kids have a lot of trouble until you teach them on how to actually divide it I thought." Both of these comments indicate that students' successes can be attributed only to what has been taught to them.

Math Math was internal to the student, variable, and an uncontrollable attribution that described the idea that students' mathematical work can be attributed to their cognitive development based on previous mathematical experiences. It included descriptions of students' mathematical thinking and used mathematical language to talk about students' successes and failures. Given that the nature of the professional development focused on students' mathematical thinking, we expected teachers to use specific language from the EPLT to describe students' mathematical work as they learned about students' mathematics through the EPLT. For example, during the summer institute, teachers were asked to anticipate the way that a child would equipartition a circle into six equal-sized parts. After viewing a clinical interview

Table 1 Frequency of attributions by teacher

	Attribution								
Teacher	Ability	Effort	Luck	Task	Grade	Context	Teaching	Math	Total
A	8	6	3	23	22	5	28	93	188
B	6	4	4	28	6	4	22	100	174
C	12	2	1	11	22	6	20	62	136
D	3	0	2	8	4	2	16	102	137
E	3	1	1	9	7	0	12	22	55
F	8	0	2	15	6	2	32	97	162
G	3	2	5	8	7	4	29	47	105
H	0	3	1	12	2	2	12	48	80
I	3	3	1	15	8	1	25	70	126
J	10	0	0	1	2	2	10	43	68
K	15	4	8	25	18	3	44	135	252
L	4	0	1	19	7	3	24	89	147
M	7	1	5	19	10	0	19	121	182
N	13	1	0	4	12	2	16	68	116
O	6	2	7	24	12	3	52	178	284
P	4	0	0	22	13	9	28	65	141
Q	15	4	5	13	6	3	21	89	156
R	3	3	5	13	1	7	9	52	93
S	4	1	1	11	6	4	20	71	118
T	3	1	0	3	4	2	6	20	39
U	6	1	3	10	4	0	22	67	113
V	8	1	2	12	7	3	19	67	119
Total	144	40	57	305	186	67	486	1706	2991

of the student, one teacher commented, "I thought she would benchmark the half, as she did…. She knew she had to go to sixths, so she drew the diagonal." Because the teacher described what the student did mathematically rather than focusing on other nonmathematical factors, this statement was coded as math. More specifically, the teacher used language from the EPLT to describe what the student did and attributed the student's work on the task to what she knew about common strategies for equipartitioning.

Teachers' Use of the Various Attributions

During the initial phase of the retrospective analysis, we examined the total number of turns coded for the 22 teachers who completed the professional development using the eight attributions identified in our data. As shown in Table 1, most teachers used all of the attributions during the professional development. Column totals showed there was considerable variability in the number of turns coded for each of the attributions. Teachers did not use the attributions of luck and effort as much as ability and age/grade, whereas math was the most used attribution, followed by teaching and task. An examination of the columns also shows that most attributions

Table 2 Frequency and percentage of LT attribution

	Turns coded LT	
Teacher	Frequency	Percentage of teacher turns
A	64	34
B	43	25
C	30	22
D	46	34
E	9	16
F	36	22
G	32	30
H	28	35
I	41	33
J	12	18
K	47	19
L	31	21
M	51	28
N	39	34
O	101	36
P	45	32
Q	37	24
R	11	12
S	29	25
T	17	44
U	48	42
V	22	18
Total	819	27

were used by all teachers, with few zeroes in each column. For instance, only five teachers did not use effort as a way to explain students' mathematical successes and failures, four did not use luck, and three did not use context. In addition, we take the prevalence of turns coded as "Math" to be an indication that the design of the professional development supported teachers in using research-based knowledge to understand students' mathematical thinking. The next phase of the retrospective analysis will address questions of changes in the attributional process over time during the professional development.

During this phase of the retrospective analysis, we also examined teachers' attributions that included the EPLT. Table 2 depicts the number of instances where each teacher made reference to the EPLT within the set of coded turns, that is, the number of turns that were coded as one of the eight attributions in the first round of coding and then later also received an LT code. The table shows that 819 turns referenced the EPLT, indicating that teachers used their learning from the professional development to explain students' mathematical work. This number represents 27 % of the total previously coded turns. The percentage of each attribution that later was double coded as LT ranged from 12 to 44 %.

We conjectured that the LT code would emerge solely within the math attribution, and we considered the math code as a way to capture teachers' emerging use of the LT language. However, as we coded our data, we found that references to the

LT emerged in all of the eight codes, not only mathematics. For example, during one of the last monthly meetings, one teacher discussed her instruction, saying: "We've done things like reallocation with the kids. We present a story and then ask, 'If this kid leaves, how many would each get?'" This turn was coded as teaching because the teacher was describing how she would teach (present a story) in order for students to learn. However, because the teacher is also referencing ideas from the LT (reallocation), this turn was also coded as LT. This example demonstrates how teachers used the LT to talk about students as well as their teaching. Yet, from our field notes and ongoing analysis, our data suggested that teachers also maintained the attributions they had been using to judge or provide causal explanations for students' mathematical work. From this perspective, the LT language neither eliminated nor added to previous attributions. Instead, the various attributions became more complex as teachers used LT language in conjunction with previous language related to other attributions. The next phase of the retrospective analysis will seek to understand the emergence and prominence of the LT attribution over time during the professional development.

In summary, the analysis of teachers' attributional processes showed that the majority of the teachers used all of the eight as causal explanations for student work during the professional development. More specifically, all 22 teachers used at least six of the attributions at least once, and 20 teachers used at least seven of them. Likewise, all teachers made reference to the LT when using these attributions to varying degrees. Together, these observations suggest that teachers do not hold one attribution for students' mathematical work but rather employ a variety when conceptualizing students as mathematics learners. Further, teachers can learn to use students' mathematical thinking as represented in LTs to explain students' successes and failures, adding a new attribution to their repertoire.

Discussion and Next Steps

We sought to identify the attributions that elementary teachers use to discuss students' mathematical successes and failures when working on professional learning tasks designed to share with them research results about students' mathematical thinking and learning. We started with the initial conjecture that as teachers learned about a mathematics LT, they would change their attributional discourse and add new explanations for students' mathematical work to their repertoire. Here, we have reported on the initial phases of our analysis; our ultimate analysis aims to understand the changes in teachers' discourse about students as mathematics learners over the span of the professional development.

Our study documented eight different attributions brought forth in the context of our professional development that teachers used to explain students' mathematics successes or failures when examining student work. These attributions went beyond the traditional attributions of ability, effort, luck, and difficulty of tasks to also include age or grade level, out of school context, teaching, and previous mathematical knowledge. Further, we did not find explicit attributions of gender, race, or socioeconomic status for students' mathematical successes and failures, as previously reported in the literature.

In addition, our work with the teachers around LTs led teachers to include references to students' specific mathematical thinking when discussing their work. As a result of professional development focused on an LT, teachers began to use this research-based knowledge to explain students' mathematical work in their discussions. However, the use of the LT did not substitute or displace the existing attributions teachers used; rather, it added to and was included as part of teachers' previous attributions. Instead of holding one or two particular attributions, teachers used a variety of attributions throughout the professional development to talk about students as mathematics learners. For mathematics teacher educators working to support teachers in learning to make instructional decisions based on students' mathematical thinking, our research suggests that although teachers my acquire such expertise in professional development, they may persist in attributing students' mathematical successes and failures to nonmathematical factors.

Our larger investigation of teacher learning of students' mathematical thinking includes an examination of the relation between teachers' discourse in the professional development and their learning of mathematics LTs. Framing this investigation with positioning theory (van Langenhove and Harré 1999), we are currently examining the ways that teachers position themselves in the discourse of the professional development. Yet, the analysis presented in this chapter highlights that not only do teachers position themselves in discussions in professional development, they also position students through attributional processes. Thus, we are currently reconceptualizing the attributional processes identified in the data as acts of stereotyping students as mathematics learners.

van Langenhove and Harré (1999) questioned the notion of locating stereotypes within the individual, with words and actions being the expression of personally held beliefs. Rather, they considered that stereotypes reside as positions in public collective discourse and that individuals appropriate them in conversation. They defined an act of personal stereotyping as a speech-act that is part of a conversation's storyline and is used to position both the speaker and the object of the stereotyping. Stereotyping draws upon social representations of the stereotyped objects that are available in certain communities. For example, rather than considering one teacher's statement, "I had expected the third grader to not share dealing it one by one" as attributing the student's work to grade level, our reconceptualization suggests that the teacher was calling upon a representation of what a third-grade student should do that is available and accepted in her community.

In continuing our research, we conjecture that the array of personal stereotyping available to teachers within their professional discourse communities is influenced, as teachers learn a framework for students' mathematical thinking. In future analysis, we will seek to understand the ways these stereotypes were called upon over time, the changes in stereotyping as teachers learned about the LT, and the relation between their use and the professional learning tasks designed to support their learning.

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