Overall Commentary: Understanding and Changing Mathematics Teachers

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The main reason for researching mathematics teachers is to understand their mathematical knowledge, practice, and learning, and how to impact them. Despite significant research and progress in these areas over the past few decades, the slow pace of reform in mathematics education suggests that our understandings of teachers are still lacking. Ongoing efforts to reform the teaching of school mathematics suggest the need for continuing efforts to understand teachers and how to help them achieve change or growth in their knowledge, thinking, and practice. The studies reported in this book make a significant contribution to both our understandings of mathematics teachers and ways to support their learning. In particular, the book highlights contributions to three central areas of research in mathematics teacher education: mathematical knowledge for teaching, teacher identity, and tools to facilitate teachers' learning. This chapter discusses the nature of these three areas, highlights specific contributions of the studies in this book, and suggests implications for future research in this field.

Central Themes of the Book

The first section of the book highlights research on mathematical knowledge for teaching. There is a general consensus that teachers need to hold deep content knowledge, as their knowledge affects both what they teach and how they teach it. However, while teachers who do not have strong knowledge of mathematics are likely to be limited in their professional competence, having such knowledge does not guarantee that they will be effective mathematics teachers (e.g., Baumert et al. 2010; Ma 1999). It is not only important what mathematics teachers know but also how they know it and what they are able to mobilize for teaching. As Ball et al. (2008) pointed out, "general mathematical ability does not fully account for the

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knowledge and skills entailed in teaching mathematics" (p. 396). Thus, "a special type of knowledge is needed by teachers that is specifically mathematical, separate from pedagogy and knowledge of students, and not needed in other professional settings" (Chapman 2012, p. 107). This knowledge has become an important basis of the recent research on mathematics teachers' knowledge (Ponte and Chapman in press). However, there is neither a consensus nor a common perspective regarding the nature of this knowledge. For example, Ruthven's (2011) overview of chapters in the book *Mathematical Knowledge in Teaching* (Rowland and Ruthven 2011) distinguished four approaches to subject knowledge for mathematics teaching: subject knowledge differentiated—approaches that categorize knowledge; subject knowledge contextualized—approaches "strongly influenced by material and social contexts" (p. 87); subject knowledge mathematized—approaches concerned with "mathematical modes of enquiry" (p. 91); and subject knowledge interactivated—approaches concerned with "epistemic and interactional processes" (p. 89). While these approaches broaden our understanding of teaching-specific mathematics knowledge, they also illustrate the complex nature of this knowledge, which contributes to the challenges of educating the mathematics teacher.

Despite differences in conceptualization, knowledge specific to teaching is widely valued as important for teaching mathematics. Recent large-scale studies (e.g., Baumert et al. 2010; Hill et al. 2005) have reported positive correlations among this kind of knowledge, teaching quality, and student achievement. Given this situation, a trend in the current research on understanding the mathematics teacher is to investigate the nature of this knowledge that he or she possesses. A review of the recent studies on prospective mathematics teachers (Ponte and Chapman in press) suggested that Ball et al.'s (2008) categories of mathematics knowledge for teaching provided the theoretical perspective for most of these studies, which dealt with topics such as rational numbers (most dominant), functions, reasoning, representation, evaluating students' achievement, and providing explanations. This focus on Ball et al.'s categories is also evident in the studies in this section of the book. These studies add to our understanding of mathematical knowledge for teaching in a variety of ways. For example, they addressed this knowledge in relation to problem solving (Heid, Grady, Jairam, Lee, Freeburn, and Karunakaran), geometry (Herbst and Kosko), proportional relationships (Jacobson and Izsák), and curriculum knowledge (Land and Drake). Thus, they cover content, mathematical processes, and curriculum—all key areas in mathematics education.

The second section of the book focuses on teacher's professional beliefs/identity. There is more to professional practice than mathematics knowledge for teaching. Teachers are engaged in practice not just with their knowledge but also with their whole being. As Palmer (1998) argued, "good teaching cannot be reduced to technique" as it "comes from the identity and integrity of the teacher" (p. 149). In other words, "we teach who we are" (p. 2). The teacher's way of being, his or her identity, is important as a means of understanding the teacher, teaching, and teacher education. For example, identity, as a construct, can inform studies that consider not only what teachers know but also who they are and how they see themselves as teachers, relate to students, deal with problems, reflect on issues, and identify themselves

with the profession. Thus, in recent years, there has been growing interest in identity in educational research (e.g., Beijaard et al. 2004; Gee 2000; Juzwik 2006; Sfard and Prusak 2005). While this interest is reflected in mathematics education (e.g., de Freitas 2008; Hodgen and Askew 2007), studies involving an explicit focus on identity are underrepresented in the literature (Ponte and Chapman 2008). However, given the complex nature and multiple perspectives of identity, the extent of the underrepresentation is not clear-cut because of the overlap between aspects of identity and other constructs (e.g., beliefs, attitude) that could lead to different classifications of related studies. For example, identity has been considered from a sociocultural perspective, as a person's sense of belonging to a group or achieving within the norms of the group or as a function of participation in different communities (e.g., Wenger 1998). Sfard and Prusak (2005) suggested that "identities may be defined as collections of stories about persons or, more specifically, as those narratives about individuals that are *reifying, endorsable,* and *significant*" (p. 16). Identity can also be considered as being made up of personal (psychological) features as well as social (contextual) features, which come together in a construct that encompasses factors such as knowledge, beliefs, image, values, emotions, relationships, contexts, and experiences. Specific to mathematics education, for example, Hodgen (2011) related identity to mathematics knowledge in teaching, while Bjuland et al. (2012) related it to a "teacher's engagement and critical alignment in the community of participants" (p. 405).

In their review of current studies on mathematics teachers, Ponte and Chapman (in press) identified studies that addressed specific aspects of identity associated mainly with a psychological perspective, for example, teachers' confidence, values, efficacy beliefs, views, motivation, and attitudes. The studies in this section of the book contribute to our understanding of the mathematics teacher's identity in a variety of ways. Chao focused on sociocultural aspects of mathematics teachers' identity. Keazer used a narrative perspective focused on teachers' experiences of the change process. Related to a psychological perspective, DePiper considered identity in terms of teachers' positioning in relation to high-stakes accountability teaching contexts, while Wilson et al. focused on teachers' attributions of students' mathematical successes or failures. Together, then, these studies highlight different aspects of teachers' identities in ways that broaden our understanding of mathematics teachers in terms of their personal and professional lives.

The third section of the book focuses on tools and techniques for supporting teachers' learning. The importance of tools in mathematics teacher education is a focus of the international handbook edited by Tirosh and Wood (2008), in which "a range of tools and processes, often used in mathematics teacher education to facilitate various proficiencies needed for teaching mathematics, are described and critically analyzed" (p. 1). Tasks as tools are also presented as significant in mathematics teacher education in edited books at the primary level by Clarke et al. (2009) and at the secondary level by Zaslavsky and Sullivan (2011). The studies in this section of the book contribute to this growing area of research in mathematics teacher education through innovative use of various tools/techniques. For example, to facilitate teachers' learning, Edgington used a mathematics learning trajectory,

Fisher et al. used a framework called *Stages of Early Arithmetic Learning,* Stockero used unedited classroom video with research-like analysis, and Tyminski et al. used three different scaffolding trajectories. These tools/techniques involved meaningful use of research-based constructs as a basis for teacher learning.

The studies in this book, then, are situated in three sections that are representative of areas of research in mathematics teacher education that are both established and growing in importance, and they contribute to these areas in a variety of ways related to understanding and changing mathematics teachers. The next section further highlights some of these contributions.

Themes of Contributions from Across the Studies

Viewed across the three sections of the book, the studies contribute in specific ways to our understanding of mathematics teachers and of facilitating their learning or change. These contributions are considered next in terms of three themes: understanding the teacher, supporting teachers' learning, and research tools.

Understanding the Teacher

The first theme highlighted by the studies in this book involves contributions to understanding the teacher. Given the importance of the relationships between the teacher and teaching and the teacher and reform, understanding the teacher is a central factor in creating twenty-first century mathematics classrooms. The studies in this book provide current insights about the teachers' knowledge and identity that suggest ongoing trends and new considerations in the field of mathematics teacher education, classified here as (a) issues with mathematics knowledge for teaching, (b) sense making of mathematics knowledge for teaching, and (c) personal-professional self.

Issues with Mathematics Knowledge for Teaching Research has consistently raised concerns about teachers' knowledge of mathematics for teaching being problematic in relation to what is considered to be adequate to teach mathematics with depth (Llinares and Krainer 2006; Ponte and Chapman 2006, 2008). Such research findings have been useful to understand the teacher and to inform teacher education of possible issues that need attention. Some of the studies in this book, directly or indirectly, suggest ongoing issues with various aspects of teachers' mathematical knowledge for teaching, thus providing further insights to our understanding of the mathematics teacher. Following is a summary of these issues.

Jacobson and Izsák found that prospective teachers often struggled and misapplied methods when dealing with direct proportions. Herbst and Kosko, in assessing experienced teachers' mathematics knowledge for teaching geometry, found that experienced geometry teachers did much better than their non-experienced

counterparts on items that were most directly connected to what is commonly taught in geometry classrooms. Heid et al. found that their participant seldom enacted mathematical processes like representing and justifying, although she was capable of doing so. This lack of engagement in these processes, along with a tendency to underrepresent the important features of mathematical objects, resulted in a somewhat disconnected treatment of mathematics. Tyminski et al. found that prospective teachers struggled to coordinate attending to student thinking while simultaneously attending to alternate thinking or learning goals. A number of prospective teachers in Land and Drake's study attended more to surface-level, procedural aspects of their curriculum materials than to aspects related to interpreting and assessing student thinking. Collectively, these studies address both content and pedagogical content knowledge and raise awareness of ongoing issues in teacher knowledge that have implications for mathematics teacher education.

Sense Making of Mathematics Knowledge for Teaching In addition to identifying issues, some of the studies draw attention to teachers' sense making (i.e., their meanings, interpretations, or capabilities). Addressing teachers' sense making is important to understand teachers in positive ways that can help to explain their classroom actions and provide a meaningful basis to attend to and build on in facilitating their learning. Examples of teachers' sense making of mathematical knowledge for teaching are reflected in the studies as follows. Land and Drake found that prospective teachers were capable of using curricular supports from within materials to extend beyond the scope of those materials. The prospective teachers in Jacobson and Izsák's study often did not make sense of the situations, but merely applied rote procedures and failed to attend to important mathematical relationships. Edgington's study showed teachers' sense making in using a learning trajectory to plan lessons, ranging from considering accessibility to anticipating student approaches and pitfalls. Finally, Tyminski et al. found that, through intervention, prospective teachers were able to improve their ability to make sense of and evaluate students' thinking strategies in a variety of mathematical contexts. A majority of them were able to attend to student strategies and interpret student thinking. Collectively, these studies provide examples of teacher's sense making for different aspects of mathematical knowledge for teaching that contribute to our understanding of what teachers are able to do with or without intervention.

Personal-Professional Self As discussed above regarding identity, understanding teachers in terms of their personal and professional selves is central to making sense of and reforming mathematics teaching. Some of the studies in this book provide insights into the aspects of teacher identity that show how self-knowledge (as opposed to content or pedagogical knowledge) can impact, for example, the teacher's classroom behavior, process of change, and knowing the students culturally and mathematically. A summary of these ways of understanding the teacher follows.

Chao's study allows us to understand two mathematics teachers through their personal and professional stories. One teacher's personal story involved feelings of isolation and was grounded in quite traumatic experiences. Because of the sensitive nature of this background (which he shared with many of his students), he was

reticent to capitalize on it, even though he recognized that the cultural connection could be beneficial. The other teacher's identity was reflected through how he valued himself and felt valued by others more as a coach than as a mathematics teacher. He felt as if he was much more successful at motivating and inspiring his soccer players than his mathematics students, and he dealt with these feelings of impotence by teaching only those mathematics students who were self-motivated. He too felt ethno-cultural congruence with his students yet could not capitalize on it to his students' benefit. In both cases, the teachers experienced challenges linking an identity valued or meaningful to them with a professional identity as a mathematics teacher, even when they shared an ethno-cultural connection with their students.

Keazer's study provides insights into teachers' professional identity associated with professional change based on their personal experiences in adopting reform-oriented practice. Implied in the findings is how teachers' affective characteristics impacted whether or how they changed. Some teachers became excited and enthusiastic when confronted with the uncertainty of change, while others became frustrated and discouraged. Some grew in confidence and commitment, while others became disappointed and disenchanted. Collectively, the cases of these seven teachers illustrate the relationship between personal attributes and dealing with the complexity of change.

DePiper's study provides insights into prospective elementary teachers' identity in relation to how they viewed themselves teaching mathematics in high-stakes accountability contexts and how this positioning could influence how and what they taught. One teacher doubted her abilities to enact particular teaching practices because of the relationship she perceived between student achievement and her personal reputation, whereas another felt capable and at liberty to enact such practices, but nevertheless uncomfortable in doing so. Collectively, these prospective teachers' positioning also draws attention to how beliefs, implicit or explicit, are important to identity and to shape the teachers they become as opposed to the teachers they want to be.

Wilson et al.'s study allows us to understand the teacher in terms of attributions—"perceptions of causality or judgments regarding…students' successes and failures" (p. 116). The authors identified eight attributions teachers used to explain students' mathematics successes or failures when examining students' work during a professional development involving an equipartitioning learning trajectory: ability, effort, luck, task difficulty, grade/age, out of school context, teaching, and previous mathematics knowledge. Most of the teachers used all eight attributions at one time or another, with prior mathematics knowledge as the most frequently used attribute and luck and effort as the least. The professional development provided the teachers with useful research-based attributions, but they still persisted in employing nonmathematical attributions as well. Thus, the teachers' attributions, as part of their teacher identity (i.e., their ways of perceiving students), seemed well entrenched in their ways of being. Since "teachers' attributions influence their expectations regarding student ability and subsequently impact student performance" (p. 116), this aspect of teacher identity, without appropriate intervention, could negatively impact students' learning.

Finally, Fisher et al.'s study draws attention to teachers' attitudes toward mathematics. They found "significant increase" in pre- and post-assessment "on three of the four factors (enjoyment, self-confidence, and motivation), and in the fourth factor (value) when maximum possible pre-scores [were] removed" (p. 232), suggesting that initially these factors may be of concern for many. These affective factors are important to teachers' professional selves, and this study implies that, without intervention, they may be an issue in supporting meaningful mathematics teaching.

Collectively, these three categories of studies focused on *understanding the mathematics teacher* and allow us to understand the teacher from various perspectives of identity. Across these categories, the studies raised awareness of the ongoing issues in teachers' content and pedagogical content knowledge, provided examples of teacher's sense making for different aspects of mathematical knowledge for teaching, and highlighted the nature of and possible challenges associated with teachers' personal-professional selves**.** They drew attention to challenges both prospective and practicing teachers could experience as a result of their personal or professional identity and the need for providing meaningful support for further development or growth in their professional identity. Given the underrepresentation of published studies on identity in mathematics teacher education, this emphasis on identity is desirable in terms of providing insights to the field and drawing attention to the importance of future research on it, as is discussed later.

Supporting Teachers' Learning and Change

The second theme highlighted by the studies in this book involves contributions to ways of supporting mathematics teachers' learning and change. Given the importance of teachers in reforming the teaching and learning of school mathematics, ongoing efforts to understand learning opportunities that will help them to enhance their knowledge and develop new instructional practices are central to mathematics education. Some of the studies in this book show that a variety of approaches can be used to facilitate or support teachers' learning with positive outcomes. Four important areas in which they contribute insights in understanding teacher learning are learning of content, learning of pedagogy, learning to notice, and changing identity.

Learning of Content Current perspectives of prospective teacher learning of content include engaging them in learning or relearning the mathematics they will teach in ways consistent with current curriculum recommendations, revisiting familiar content to examine it in ways unfamiliar to them, and probing more deeply fundamental mathematical ideas from the school curriculum (Ponte and Chapman 2008). Jacobson and Izsák's study is an example of these views of teacher learning and provides insights into how a course focused on multiplicative relationships and "drawn models of quantities (e.g., number line and area models)" can support prospective teachers' development of an understanding of how such problem-solving strategies "can provide the basis for developing general computation methods" (p. 50).

Learning of Mathematical Pedagogy Current studies of practicing teachers' learning and change suggest a trend that includes teachers working together to improve their practice and embedding professional study within the everyday practice of teaching (e.g., Even and Ball 2009). Edgington's study adds to this view. It shows how a mathematics learning trajectory can be used as a tool to support primary teachers' planning of meaningful student-centered lessons by helping them to become aware of students' mathematical thinking. Participants studied the equipartitioning learning trajectory through a series of professional learning tasks, some of which "were designed to allow teachers to make connections to existing curricula and current practice" (p. 266). For example, teachers experimented with equipartioning-related tasks in their classrooms then came together to reflect on and analyze their lessons. Teacher learning resulting from the experience included using the learning trajectory in their planning to "choose tasks…, specify learning goals…, anticipate students' approaches in a variety of ways" (p. 279), and "consider connections to other mathematical concepts that may emerge during a lesson" (p. 280).

Learning to Notice An emerging body of research related to teachers' noticing supports the importance of it in teaching (e.g., Ainley and Luntley 2007; Mason 2008; Scherrer and Stein 2012; Sherin et al. 2011; Star and Strickland 2008). Noticing involves not only the attention that teachers give to significant classroom actions and interactions, but also their reflections, reasoning, and decisions based on it, i.e., *attention* and *awareness* (Mason 2008). The extent to which a teacher can notice in this way impacts his or her teaching. Many events and interactions occur at once in the classroom (in student-centered classrooms in particular), and a teacher needs to be able to identify key moments that require attention, for example, moments of student thinking that can be used to advance instruction. Thus, helping teachers to enhance their ability to notice is an important goal of teacher education. Some of the studies in this book provide insights about tools and approaches that offer promising directions to accomplish this goal. The following summary highlights these approaches.

Edgington's study was discussed above as offering an intervention for pedagogy. However, its primary goal was to help teachers to learn to notice. Thus, it also shows that a mathematics learning trajectory and the process, as already noted, can be used to help teachers to notice students' mathematical thinking, in particular, conceptions (strategies) and misconceptions. While the focus is on equipartitioning concepts and an equipartitioning learning trajectory, the study illustrates the potential for using learning trajectories to develop a stance of noticing. In general, it suggests that using learning trajectories as representations of student thinking could help teachers to notice and attend to students' mathematical thinking in their planning of lessons.

Stockero showed how activities including "research-like analysis of unedited classroom video and group discussions" supported by a teacher educator early in a teacher education program led to several "transitions in the participants' noticing" (p. 241). The prospective teachers recorded several of their cooperating teachers' mathematics lessons, then analyzed those lessons using a framework that focused on "mathematically important moments that a teacher needs to notice during a lesson"

(p. 244). The approach helped the participants to learn to notice such moments, as well as what individual students were thinking and the effect of teacher–student interactions on learning. Their descriptions of the mathematics of an instance also became more detailed. Thus, the study illustrates how certain mathematics-focused activities can help prospective teachers "learn to attend to important mathematical instances that arise during a lesson" (p. 257).

Fisher et al. showed how an approach based on a framework called *Stages of Early Arithmetic Learning* (SEAL) led to statistically significant growth in professional noticing capabilities of prospective teachers, providing opportunities for prospective teachers to "see mathematics through the lens of a child" and focus "on what children can do conceptually rather than on the procedures of mathematics that children cannot yet do" (p. 232). The intervention used video cases and interviews with children as contexts for developing prospective teachers' attending, interpreting, and deciding skills. The approach helped participants to change in all three of these components of noticing, thus suggesting the importance of being explicit about these components in activities based on children's thinking to guide prospective teachers' noticing.

Tyminski et al. demonstrated the potential success of an approach they developed to engage prospective teachers in professional noticing. The approach was framed in three trajectories of scaffolding (observing to doing, number of concepts, and number choices) with an associated sequence of tasks that "progressed from noticing an expert teacher's task design, to designing a task to address a single mathematical concept, to designing a task that addressed a wide range of student needs" (p. 194). The study suggests, however, that such a scaffolding framework has promising potential to help prospective teachers develop skills of professional noticing of students' thinking.

Finally, Heid et al.'s study, while not intended to be about intervention or noticing, also implied the importance of noticing for oneself as one engages in mathematics and how a restrictive noticing ability could hinder how teachers engage students in doing mathematics. For example, their participant needed to be prompted to notice essential properties of a mathematical object other than local features of the representation relevant for the task at hand and opportunities to incorporate multiple representations. Without the prompting, her noticing ability adversely affected her problem solving and limited her students' mathematical opportunities. These findings suggest that intentional prompting could play a useful role in designing activities to support teachers' noticing in their learning and teaching.

Collectively, these studies add to the growing body of research that indicates the importance of helping teachers to notice students' mathematical thinking in order to teach flexibly and adapt lessons to accommodate students' ideas. They provide further evidence that noticing can be taught and learned and that a variety of approaches can lead to positive outcomes. They suggest the importance of incorporating a specific framework and a structured sequence of activities to help guide teachers' learning to notice and of developing teacher noticing in the context of a specific domain, rather than attending to student thinking in general, as doing so may better support noticing with depth. In particular, such activities are important to

help focus prospective teachers' attention on more complex aspects of teaching and learning, since observing videos alone may not lead them to notice what is intended or to notice productively.

Changing Identity The final area of contribution to our understanding of teachers' learning and change highlighted involves changing identity. Given the perspectives of identity involved, changing a teacher's identity could be a challenging endeavor since it could involve trying to change who the person is. However, constructs such as beliefs and attitudes have been shown to change in response to intervention, and this possibility is supported by two of the studies in this book. Wilson et al. showed that although teachers were able to augment their attributional discourse related to students' mathematical successes and failures, the approach "did not substitute or displace the existing attributions teachers used; rather, it added to and was included as part of [their] previous attributions" (p. 130), suggesting it is easier to impact growth in teachers' identity than to change it. Fisher et al.'s study "revealed the possibility that components of [preservice elementary teachers'] attitudes can improve when experiencing a course where professional noticing skills are explicitly taught, modeled, and reinforced" (p. 232). Based on the attitude scale used, there was a significant increase in their enjoyment, self-confidence, and motivation, and in value when maximum possible prescores were removed. The fact that some prospective teachers had maximum possible prescores on value suggests possible issues related to improving value (a central aspect of identity).

Research Tools

The third theme highlighted by the studies in this book involves contributions to research tools. For research to provide meaningful ways for us to understand teachers and to support their learning, the tools and processes employed are of critical importance. Two important areas in which the studies contribute to this need are in researching mathematics knowledge for teaching and identity.

Mathematics Knowledge for Teaching Appropriate and productive tools are needed to understand teachers' mathematics knowledge for teaching and changes in that knowledge. Some of the studies provide insights about the nature of possible tools that could contribute to research on different components of mathematical content and pedagogical knowledge. Herbst and Kosko focused on developing an instrument to measure mathematical knowledge for teaching high school geometry. They include sample items and describe a process for developing and testing such items. Their study provides insights into the nature of the tool and how it can be used for research. Based on their studies, Land and Drake and Edgington developed trajectories of teachers' learning that provide examples of what such trajectories could look like and tools that can be used to frame further exploration. Land and Drake developed a trajectory of mathematics curriculum knowledge and use for prospective teachers that provides a depiction of the development of expert

curriculum-use knowledge and practices, while Edgington developed a trajectory of teachers' movement from initial to proficient use of a student mathematics learning trajectory as a basis to support teachers' planning of meaningful student-centered lessons. As research tools, these trajectories can provide meaningful starting points to develop more robust trajectories and theories of curriculum use, curricular knowledge, and lesson planning to support teachers' learning.

Identity Researching identity is as challenging as defining it. Narratives/stories are considered most appropriate to study identity, and Chao's study draws attention to a unique and meaningful way of accessing teachers' stories via the use of photographs. The study shows that, as a research tool, teacher-selected photographs could be used as "anchoring structures" (p. 95) to study teacher identity by framing teachers' narratives into professional, personal, and touchstone stories. These photographs provide a visual representation of the narratives of teachers' experiences that unfold during a photo-elicitation/photovoice interview. Chao described how he elicited and analyzed these stories and the aspects of mathematics teacher identity they revealed. He demonstrated how this approach is effective in accessing stories focused on sociocultural aspects of teacher identity and, in particular, how teachers' internal stories can be surfaced through their personal stories and photographs. Thus, the study illustrates a tool with powerful potential to study identity.

Other studies imply that the use of group communication is a meaningful approach to researching aspects of a teachers' identity. For example, in their studies, DePiper used teachers' discussion of mathematics teaching and practices in high-stakes accountability teaching contexts to study teachers' positioning, and Wilson et al. used teachers' discourse about students' mathematical work to study teachers' attributions of students' successes and failures. Each demonstrated how these processes can provide ways of capturing aspects of teachers' identity in a context of actual experience that are less likely to be captured by individual interviews.

Themes for Future Research

We have gained significant insights about mathematics teachers and their learning from the large body of research in the field of mathematics teacher education. But despite the significant progress resulting from it, and given the slow pace of reform in classrooms, there is still much more we need to know to help teachers transform their practice and make a difference to mathematics education. In this section, I focus on some general implications for future research, organized around the three themes of the book: mathematics knowledge for teaching, identity, and noticing.

Mathematics Knowledge for Teaching

The importance of understanding mathematics knowledge for teaching cannot be overstated. While many studies are exploring it in different ways, the complexity of this knowledge (based on, for example, various perspectives for conceptualizing it and various classroom, institutional, and sociocultural contexts that impact aspects of it) makes ongoing research of it necessary. Studies in this book have indicated possible issues with teachers' knowledge that could impact practice, suggesting the need for future research to consider not only the nature of the knowledge but how it is used in actual practice and how it impacts students' learning in actual classrooms (as in the case of Heid et al.'s study). Similarly, as demonstrated in some of the studies, in order to understand the ways teachers hold their knowledge and make sense of content and pedagogy, we need more attention to understanding teacher knowledge from the teacher's perspective and in light of how it informs actual practice.

Some of the studies used or implied approaches that hold promise for producing positive outcomes for teachers' development of mathematics knowledge for teaching. In particular, learning trajectories of students' thinking were shown to be useful in supporting teachers' learning. Research could focus on developing such learning trajectories for different content areas that can be used in teacher education. Finally, measuring or assessing mathematics knowledge for teaching is also an area that deserves more attention. Tools such as those employed by Herbst and Kosko can inform future research in other topics and on exploring the relationship between knowledge in practice and mathematics knowledge for teaching. For example, such tools could inform research on how specific aspects of the actual work of teaching a subject (e.g., Algebra, Geometry) or topic are related to specific mathematics-knowledge-for-teaching demands of teaching that subject or topic.

Identity

As Bjuland et al. (2012) pointed out, "the notion of teacher identity is considered to be a key theme for future directions of research in a sociocultural perspective" (p. 406). However, it is still underrepresented in mathematics teacher education research as an explicit research focus. Identity, depending on how it is defined, can provide a way to connect cognitive, affective, social, and cultural dimensions in considering teachers' knowledge, practices, and development. The studies in this book provided examples of the aspects of these factors that draw attention to the importance of understanding the mathematics teacher from both sociocultural and psychological perspectives and the need for future research to address identity in ways that consider both perspectives. In particular, these studies imply the need for research not only about the nature of identity, but also about, for example, the relationship between identity and actual practice, identity and change in practice,

identity and equitable mathematical instruction for diverse students, and identity and noticing. The studies also suggest ways of accessing identity and approaches that could lead to growth in specific aspects of teacher identity that could be further explored. But more generally, given that the development of a teacher's professional identity is shaped by multiple influences prior to, during, and after teacher education, it is important to understand these influences and ways to explicitly support the development of professional mathematical identity.

Noticing

As previously discussed, noticing is emerging as an important construct in mathematics teacher education research, with particular attention being paid to teachers' noticing of students' mathematical thinking. The studies in this book provided evidence of noticing being a teachable skill, thus suggesting the importance of research to further understand the nature of teachers' noticing and how to support its growth and development. They also imply that such research should investigate teachers' noticing of student thinking for specific mathematical domains to understand what the teachers pay attention to and how they use it to support student learning. In addition, such research should explore the nature of and how to incorporate structured frameworks to help guide teacher noticing and approaches to support and prompt prospective teachers, in focusing their attention on more complex aspects and significant moments of teaching and learning.

In general, these three themes and the studies in this book collectively suggest we need a better grasp regarding how personal, educational, professional, and institutional factors influence teachers' practices and to further explore ways of facilitating teachers' learning and change.

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