

# Chapter 6

## Conclusions

**Abstract** The major goal of the present study is to develop fast and robust algorithms for online damage detection in structural systems. To accomplish this objective, the research study presented in this monograph can contribute to three different research areas: (a) stochastic system identification of multi degrees-of-freedom structural systems via recursive Bayesian inference algorithms, (b) reduced order modeling of multi degrees-of-freedom structural systems through proper orthogonal decomposition; and (c) stochastic system identification of reduced order models of multi degrees of freedom structural systems through recursive Bayesian filters.

### 6.1 Summary of Contributions

The principal contributions and major findings of this research study can be summarized as follows:

- (1) Four state of the art Bayesian filters, namely the extended Kalman filter, the sigma-point Kalman filter, the particle filter and the hybrid extended Kalman particle filter have been adopted. To benchmark the performance of filters and avoid shadowing effects of the structure, the filters have been adopted to recursively identify the parameters of the constitutive model of a single degree-of-freedom dynamical system: an exponential softening, and three bilinear models (linear-hardening, linear-plastic and linear-softening), as possible representatives of initial stages of damage are adopted. The goal is achieved by dual estimation concept, where the parameters of the system are joined to the state vector in order to simultaneously track the state of the system and calibrate the parameters, as new observations become available. Provided that the Jacobian of the evolution equations of the state space model is positive definite and bounded, it is known that the adopted filters are stable and can converge to unbiased estimates; however, such conditions are not

always satisfied in a model featuring softening constitutive law. This fact substantiates numerical assessment of stability and convergence of the studied filters, when applied to the estimation of parameters of a softening constitutive law used to describe damage evolution in the system. The conducted numerical campaign has revealed that while the extended Kalman filter, the unscented Kalman filter and the particle filter all fail to provide unbiased estimates of the sought parameters, the hybrid extended Kalman particle filter performs rather reasonably.

- (2) The extended Kalman filter (because of its computational time efficiency) and the hybrid extended Kalman particle filter (due to its excellent performance when applied to the analysis of single degree-of-freedom nonlinear system) have been adopted for dual estimation of states and constitutive parameters of a multi degrees-of-freedom linear shear building-type structure. The performance of the two filters has been assessed through the estimation of the values of the inter-storey stiffness of the floors of the building. In the simplest case, i.e. a two-storey shear building, both filters furnish quite accurate estimates of the stiffness values; however, moving to a three-storey structure, the performance of both filters is adversely affected. The trend is corroborated by the results in the case of a four storey building: the estimation resulted in a bias up to 50 % of the target values of the parameters. This trend suggests that, when dealing with dual estimation of a multi storey shear building, an increase in the number of storeys rapidly deteriorates accuracy of the parameter estimates. Therefore, this approach will not be an effective damage detection method; thus we the adopted a dual estimation of a reduced order model of the building.
- (3) To manage the curse of dimensionality issue, the method of proper orthogonal decomposition (POD) has been adopted to produce a reduced order model of the vibrating structure. Provided that there exist a set of samples from the response of the system and its members are selected in way that the ensemble contains information on the main dynamic characteristics of the system; thus POD automatically looks for those main characteristics. To accomplish this objective, the POD finds the directions which capture the maximum variation, or equivalently, the maximum energy of the system. Once the relevant directions (called proper orthogonal modes, POMs) in an initial training stage are found, Galerkin projection is employed to project the equations onto the subspace spanned by the computed POMs. The efficiency of the algorithm in terms of speed-up and accuracy of the estimations has been then numerically assessed. The procedure is applied for reduced order modeling of the Pirelli tower located in Milan; prediction capability and speed-up issues are numerically assessed. It is observed that reducing the original 39 degrees-of-freedom structure to a reduced model consisting of four POMs makes the computations 250 times faster; while a reduced model featuring a single POM has a speed-up value of 500. Moreover, robustness of the reduced models, featuring different number of retained POMs, to a change in the source of the external loading has been further analyzed. To produce the samples required

for initial training stage of POD-based reduced model, the Pirelli tower has been assumed to be shaken by the well-known El Centro acceleration time history. The resulted reduced model has then been applied to simulate the response of the structure to the Kobe and Friuli earthquake excitations. It has been shown that the change in the source of excitation does not affect much the prediction capabilities of POD-based reduced models in seismic analysis of the structure.

- (4) Prior to applying the reduced models obtained by the POD in the recursive Bayesian inference algorithms adopted in this monograph, a statistical assessment of the uncertainties induced by reduced order modeling is essential. In this study, all the Bayesian filters adopted are assumed that the uncertainties in the state space model are uncorrelated processes. The null hypothesis of whiteness of the residual error of POD models has been tested by cumulative periodogram-based test of Bartlett (Bartlett 1978). It has been shown that, no matter what the number of the POMs featured by the reduced model is, its residual error is always correlated. However, by an increase in the number of retained POMs, the spectral power of the correlation in the signal decreases. The linear, time-invariant reduced models of the Pirelli tower has been incorporated into a Kalman filter in order to speed-up the calculations. Provided that the noises in the state space equation are white Gaussian processes, it is known that Kalman filter furnishes optimal estimates of state of a linear model. We have shown that the POD-based reduced state space used in this study is not white. That is, when just a single POM is retained in the analysis, residual mean squared error (RMSE) of the POD-Kalman observer is higher than the POD alone; however, as the number of POMs retained in the analysis increases and spectral power of the correlations decrease, POD-Kalman observer performs better, in terms of reducing RMSE of estimates: POD-Kalman observer featuring three and four POMs in its reduced model decrease quality of estimates provided by POD alone. Concerning speed-up gained by introducing POD-based models into Kalman observer, by maintaining a minimal number of POMs, the observer is run up hundreds of times faster.
- (5) Besides its efficiency in model order reduction, the POD has an interesting feature which makes it appropriate for the purpose of damage detection. Proper orthogonal modes which are furnished by the POD have been shown to be sensitive to the severity and location of the damage in the mechanical systems, and they are already used as damage detection tools (Shane and Jha 2011a). These two aspects of POD, namely its efficiency for model order reduction and its capability in identifying the damage, make an ideal candidate for the problem of damage detection in structural systems via reduced order modeling and dual estimation. In this monograph, we have proposed a novel algorithm for dual estimation of a POD-based reduced order model of a time-varying shear model of building. The capability of the algorithm in tracking the state of the system, the parameters of the reduced model and the POMs of the reduced model has been numerically assessed. Our approach has been

employed to detect a variety of damage scenarios in a ten-storey shear building; however, the assessment has been based on pseudo experimental verifications. It has been concluded that the proposed procedure performs accurately.

The major goal of this monograph is to develop robust algorithms for online and real-time detection of the damage in civil structures. The objective of the monograph is perceived by developing a procedure by a synergy of recursive Bayesian inference methods and proper orthogonal decomposition. Therefore, a POD-based reduced model of the structure has been considered: dual estimation concept has been exploited, within a recursive Bayesian framework the state and the parameters of the reduced model are simultaneously estimate based on observational signal which becomes available in discrete time instants. In each recursion, not only the state and the parameters of the reduced model are estimated, but also the proper orthogonal modes employed to construct the reduced model are estimated. It is shown that the POD modes can indicate location and severity of the damage in mechanical systems. The unbiased estimate of the POMs provided by our approach permits robust, online and real-time indication of the damage in a shear type of building.

## 6.2 Suggestions for Future Research

Based on the work presented herein, several research areas have been identified as open to and in need of future work:

- (1) In this monograph, regarding the application of Bayesian filters for dual estimation of states and parameters of the multi-storey shear buildings, we have adopted the family of Kalman filter, particle filter and a combination thereof. However, the use of evolutionary particle filters has not been considered; it is suggested to tackle this problem by utilizing the aforementioned filters as well.
- (2) To construct the POD-based reduced models, the effects of nonlinear mechanisms have been neglected. It is recommended to take those effects into consideration as well.
- (3) The algorithms proposed in this monograph for damage detection via dual estimation of the reduced model and subspace update have been assumed to be fed by displacement response at each floor. The reason is to construct the reduced model POD modes of the displacement response of the structure used for acceleration modes are different from displacement modes; moreover, the accuracy of reproducing accelerations by reduced model is lower than displacements. There are two remedies: one is increasing the number of POMs retained in the reduced model to improve the quality of acceleration reconstruction; hence, this can lead to curse of dimensionality by increasing number

of the parameters to be estimated in the reduced model, and the other option is to compute the displacement response from the acceleration response data. In the literature, there are several methods available to calculate displacement response based on the acceleration (Skolnik et al. 2011). It is recommended to utilize those techniques to verify the algorithms by pseudo experimental data. It is worthy to see if the Bayesian filters can handle the uncertainty introduced by converting the acceleration response into the floor displacements.

- (4) Through this study, the methodologies which were used or developed have been verified via pseudo-experiments. It is recommended to verify the effectiveness of the proposed procedure by utilizing real experiments.
- (5) It has been shown that, dealing with a ten-storey shear building with equal masses and stiffnesses at each floor, there exist an intuitive and clear correlation between damage location and intensity and the POM. However, to quantify the damage index relevant to each floor, it is recommended to utilize artificial neural networks (the standard classification methodologies) in order to provide quantitative damage indexes for each storey based on the POM of the structure; such method has been already adopted to identify damage based on the changes in the coefficients of an auto regressive moving average model of a four storey structure (de Lautour and Omenzetter 2010).

## References

- Bartlett MS (1978) An introduction to stochastic processes with special reference to methods and applications. Cambridge University Press, London
- de Lautour OR, Omenzetter P (2010) Damage classification and estimation in experimental structures using time series analysis and pattern recognition. *Mech Syst Sign Proces* 24:1556–1569
- Shane C, Jha R (2011a) Proper orthogonal decomposition based algorithm for detecting damage location and severity in composite beams. *Mech Syst Sign Proces* 25:1062–1072
- Skolnik DA, Nigbor RL, Wallace JW (2011) A quantitative basis for building instrumentation specifications. *Earthquake Spectra* 27:133–152