Chapter 5
Dual Estimation and Reduced Order Modeling of Damaging Structures

Abstract In this chapter, the dual estimation and reduced order modeling of a damaging structure is studied. In this regard, proper orthogonal decomposition is considered for reduced order modeling in order to find a subspace which optimally captures the dynamics of the system. Through a Galerkin projection, the equations governing the dynamics of the system are projected onto the subspace provided by the proper orthogonal decomposition technique. It is proven that the subspace established by application of the proper orthogonal decomposition is sensitive to changes of the parameters; therefore, it can be profited in the algorithms for estimation of the damage incidence. As for the dual estimation goal, the extended Kalman filter and extended Kalman particle filter are adopted; both filters, in their so-called update stage, make a comparison between the latest observation and the prediction of the state of the system to quantify the required adjustment in the estimation of the state and parameters. In the case of the reduced order modeling, for realization of such a comparison, reconstruction of full state of the system is required, which is obviously possible only if the subspace is known. In this chapter, an adjustment of the dual estimation concept has led to an online estimation of the proper orthogonal modes, components of the reduced stiffness matrix and the states of the structure. This novelty can intuitively help to detect the damage in the structure, locate it and potentially identify its intensity.

5.1 Introduction

To detect changes in the mechanical properties of structural members, it can be assumed as a method to monitor their health. In many cases, to identify the damage in the structure, one can considered it as a reduction of the stiffness (Yang and Lin 2005). This may be caused due to failure of a member to sustain further action, or it can be due to degradation in its material properties. That means that damage detection in a structure can be modeled as a system identification problem. To deal with a linear structure, offline identification of system matrices can be carried out

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via several robust algorithms; as for output only techniques, data driven stochastic subspace identification (SSI) algorithm is the de facto standard stochastic system identification method (Van Overschee and De Moor 1996), however, in recent years the research on developing new techniques e.g. blind source separation (Abazarsa et al. 2013a, b; Ghahari et al. 2013c). The aforementioned method is successfully applied to identify the modal parameters of multi-storey buildings (Ghahari et al. 2013a, b) and modal identification of long span bridges (Ghahari et al. 2013d). Moreover, subspace identification algorithm is instead extensively applied to identify deterministic input–output systems (Loh et al. 2011). The aforementioned methodologies include singular value decomposition (SVD) and QR decomposition techniques (Moaveni et al. 2011). Extension of such methodologies to online system identification is normally perceived via setting a fixed length moving time window; as new observations become available, new subspace identification is perceived. Computational costs associated with SVD and QR prevent real-time application of such methods. Several methods were proposed to reduce the computational burden of the SVD and QR operations based on updating SVD and QR decomposed matrices; moreover, they are made suitable for near real-time applications (Loh et al. 2011). In this study, damage detection has been tackled via dual estimation of state and stiffness parameters by utilizing recursive Bayesian filters in an online means. We have shown in Chap. 2 that, as the number of DOFs of the space model of the structure increases, biases frequently affect the estimates furnished by the filters. To manage this problem, dual estimation of state and parameters of a reduced model of the structure are employed as the last resort.

Nevertheless, dissimilar to the identification of the full model of the system, to estimate components of the reduced stiffness yield no precise information concerning the intensity and location of the damage. It is a well-known fact that appropriate orthogonal modes of the structures include information regarding location and intensity of the damage (Ruotolo and Surace 1999; Vanlanduit et al. 2005; Galvanetto and Violaris 2007; Shane and Jha 2011). Hence, this feature of POMs can potentially resolve deficiencies of parameter estimation of a reduced model as an indicator of damage location and severity. To accomplish this objective, an algorithm for dual estimation of state and parameters of a reduced model, accompanied by an online estimation of the POMs of the structure is suggested. The proposed procedure utilizes appropriate orthogonal decomposition for model order reduction; afterwards, it exploits Bayesian filters for dual estimation of the full state and reduced parameters of the system. At each recursion, Kalman filter is adopted to update the subspace spanned by the POMs retained in the reduced model. This method can effectively detect, locate and identify the severity of the damage in shear building type structures. The efficiency of the methodology is testified through pseudo experimental data obtained by employing direct analyses.

The proceeding sections of this chapter are organized as follows. In Sect. 5.2 the state space formulation of shear buildings is reexamined; moreover, key features of the reduced order state space model of the system are highlighted in Sect. 5.3. In Sect. 5.4 the peculiarities of dual estimation and reduced order modeling of a

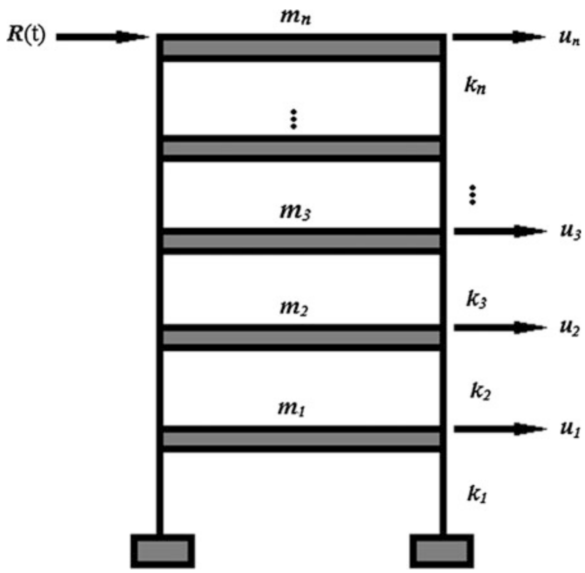

Fig. 5.1 Schematic view of a shear building

damaging structure are presented and discussed, and we define our proposal as how to tackle the problem. Finally, efficiency of our proposed method is numerically testified in Sect. 5.5.

5.2 State Space Formulation of Shear Building-Type Structural Systems

In this study, it is aimed to develop an algorithm for multi-storey buildings and to investigate shear buildings, i.e. models obtained by lumped mass assumption for each story, see Fig. 5.1.

Representing storey displacements, velocities and accelerations by $\boldsymbol{u}$, $\dot{\boldsymbol{u}}$ and $\ddot{\boldsymbol{u}}$ respectively, the governing equation of motion of the building reads:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{D}\dot{\boldsymbol{u}} + \boldsymbol{K}(t)\boldsymbol{u} = \boldsymbol{R}(t) \tag{5.1}$$

where $\boldsymbol{M}$ is the stationary mass matrix, $\boldsymbol{D}$ denotes time invariant damping matrix and $\boldsymbol{K}(t)$ stands for time varying stiffness matrix, whose variation in time is due to possible damage phenomena and is usually unpredictable; $\boldsymbol{R}(t)$ is the loading vector:

$$\boldsymbol{M} = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix} \tag{5.2}$$

$$\boldsymbol{K}(t) = \begin{bmatrix} k_1(t)+k_2(t) & -k_2(t) & & & \\ -k_2(t) & k_2(t)+k_3(t) & & & \\ & & \ddots & & \\ & & & k_{n-1}(t)+k_n(t) & -k_n(t) \\ & & & -k_n(t) & k_n(t) \end{bmatrix} \quad (5.3)$$

In general, $\boldsymbol{R}(t)$ can be any kind of loading; however, in this study, we assume that it is a harmonic force applied to the top floor:

$$\boldsymbol{R}(t) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ a \ \sin \omega t \end{bmatrix} \quad (5.4)$$

where a and ω are the amplitude and frequency of excitation, respectively. For the sake of simplicity, in this study, we neglect damping effects.

To numerically solve the set of ordinary differential equations, Newmark explicit integrator is employed. To write the equations in the discrete state-space form, we introduce an extended state, $\boldsymbol{z}$, that at each time instant t_k includes $\boldsymbol{u}$, $\dot{\boldsymbol{u}}$ and $\ddot{\boldsymbol{u}}$ according to:

$$\boldsymbol{z}_k = \begin{bmatrix} \boldsymbol{u}_k \\ \dot{\boldsymbol{u}}_k \\ \ddot{\boldsymbol{u}}_k \end{bmatrix}. \quad (5.5)$$

Then state-space form of Eq. (5.1) is written as:

$$\boldsymbol{z}_k = \boldsymbol{A}_k \boldsymbol{z}_{k-1} + \boldsymbol{B}_k \quad (5.6)$$

where:

$$\boldsymbol{A}_k = \begin{bmatrix} \boldsymbol{I} - \beta\Delta t^2 \boldsymbol{K}_k \boldsymbol{M}^{-1} & \Delta t \boldsymbol{I} - \beta \Delta t^2 \boldsymbol{M}^{-1}(\boldsymbol{D} + \Delta t \boldsymbol{K}_k) & -\beta\Delta t^2 \boldsymbol{M}^{-1}\left(\Delta t^2\left(1/2 - \beta\right)\boldsymbol{K}_k + \Delta t(1-\gamma)\boldsymbol{D}\right) + \Delta t^2\left(1/2 - \beta\right)\boldsymbol{I} \\ -\gamma\Delta t \boldsymbol{K}_k \boldsymbol{M}^{-1} & \boldsymbol{I} - \gamma\Delta t \boldsymbol{M}^{-1}(\boldsymbol{D} + \Delta t \boldsymbol{K}_k) & -\gamma\Delta t \boldsymbol{M}^{-1}\left(\Delta t^2\left(1/2 - \beta\right)\boldsymbol{K}_k + \Delta t(1-\gamma)\boldsymbol{D}\right) + \Delta t(1-\gamma)\boldsymbol{I} \\ -\boldsymbol{K}_k \boldsymbol{M}^{-1} & -\boldsymbol{M}^{-1}(\boldsymbol{D} + \Delta t \boldsymbol{K}_k) & -\boldsymbol{M}^{-1}\left(\Delta t^2\left(1/2 - \beta\right)\boldsymbol{K}_k + \Delta t(1-\gamma)\boldsymbol{D}\right) \end{bmatrix} \quad (5.7)$$

and:

$$\boldsymbol{B}_k = \begin{bmatrix} \beta\Delta t^2 \boldsymbol{M}^{-1}\boldsymbol{R}_k \\ \gamma\Delta t \boldsymbol{M}^{-1}\boldsymbol{R}_k \\ \boldsymbol{M}^{-1}\boldsymbol{R}_k \end{bmatrix} \quad (5.8)$$

β and γ are parameters of the Newmark algorithm, for details see Sect. 2.6.

Concerning the observation process, it is assumed that a part of state vector is directly observable; hence, observation equation is expressed as:

$$\boldsymbol{y}_k = \boldsymbol{H}\boldsymbol{z}_k + \boldsymbol{w}_k \quad (5.9)$$

where $\boldsymbol{H}$ denotes a Boolean matrix of appropriate dimension which links the states of the system to observation process, and $\boldsymbol{w}_k$ denotes associated measurement noise.

5.3 Reduced Order Modeling of Structural Systems

A detailed study of the application of POD for model order reduction of structural system has been presented in Chap. 3. However, to keep this chapter self-contained, in this Section, we review key features of the procedure. Let us assume that the displacement field $\boldsymbol{u} \in \mathbb{R}^m$ of the system can be written in a separable form, according to:

$$\boldsymbol{u}(\boldsymbol{x}, t) = \sum\nolimits_{i=1}^{m} \boldsymbol{\varphi}_i(x)\alpha_i(t) \tag{5.10}$$

where $\boldsymbol{\varphi}_i(x)$ are a set of orthonormal vectors that satisfy proper orthogonal decomposition (POD) requirements and, α_i are temporal functions. Dealing with structural problems with high dimensional state vectors, the main variation in the data is usually occurring in a rather small subspace; consequently, it is frequently possible to approximate the state of the system by keeping just a few, say l proper orthogonal modes, with $l \ll m$:

$$\begin{aligned} \boldsymbol{u}(\boldsymbol{x}, t) &\approx \sum\nolimits_{i=1}^{m} \boldsymbol{\varphi}_i(x)\alpha_i(t) \\ &= \boldsymbol{\Phi}_l\, \boldsymbol{\alpha} \end{aligned} \tag{5.11}$$

where $\boldsymbol{\Phi}_l$ denotes the matrix containing the retained l POMs of the system.

Substituting (5.11) into (5.1), and applying Galerkin projection yield the reduced dynamic model of the system:

$$\boldsymbol{M}_l\ddot{\boldsymbol{\alpha}} + \boldsymbol{D}_l\dot{\boldsymbol{\alpha}} + \boldsymbol{K}_l\boldsymbol{\alpha} = \boldsymbol{R}_l(t) \tag{5.12}$$

where:

$$\boldsymbol{M}_l = \boldsymbol{\Phi}_l^T\boldsymbol{M}\boldsymbol{\Phi}_l,\ \boldsymbol{D}_l = \boldsymbol{\Phi}_l^T\boldsymbol{D}\boldsymbol{\Phi}_l,\ \boldsymbol{K}_l = \boldsymbol{\Phi}_l^T\boldsymbol{K}\boldsymbol{\Phi}_l,\ \boldsymbol{R}_l(t) = \boldsymbol{\Phi}_l^T\boldsymbol{R}(t). \tag{5.13}$$

The reduced dynamic model in state-space form then reads:

$$\boldsymbol{z}_{r,k} = \boldsymbol{A}_k\boldsymbol{z}_{r,k} + \boldsymbol{B}_k + \boldsymbol{v}_k^z \tag{5.14}$$

$$\boldsymbol{y}_k = \boldsymbol{H}\boldsymbol{C}\boldsymbol{z}_{r,k} + \boldsymbol{w}_k \tag{5.15}$$

where the reduced order state includes the temporal coefficient, its first and second time derivatives:

$$\boldsymbol{z}_{r,k} = \begin{bmatrix} \boldsymbol{\alpha}_k \\ \dot{\boldsymbol{\alpha}}_k \\ \ddot{\boldsymbol{\alpha}}_k \end{bmatrix}. \tag{5.16}$$

In (5.14):

$$\boldsymbol{A}_k = \begin{bmatrix} \boldsymbol{I} - \beta\Delta t^2 \boldsymbol{M}_l^{-1}\boldsymbol{K}_{l,k} & \Delta t\boldsymbol{I} - \beta\Delta t^2 \boldsymbol{M}_l^{-1}(\boldsymbol{D}_l + \Delta t\boldsymbol{K}_{l,k}) & -\beta\Delta t^2 \boldsymbol{M}_l^{-1}\left(\Delta t^2\left(1/2 - \beta\right)\boldsymbol{K}_{l,k} + \Delta t(1-\gamma)\boldsymbol{D}_l\right) + \Delta t^2\left(1/2 - \beta\right)\boldsymbol{I} \\ -\gamma\Delta t\boldsymbol{M}_l^{-1}\boldsymbol{K}_{l,k} & \boldsymbol{I} - \gamma\Delta t\boldsymbol{M}_l^{-1}(\boldsymbol{D}_l + \Delta t\boldsymbol{K}_{l,k}) & -\gamma\Delta t\boldsymbol{M}_l^{-1}\left(\Delta t^2\left(1/2 - \beta\right)\boldsymbol{K}_{l,k} + \Delta t(1-\gamma)\boldsymbol{D}_l\right) + \Delta t(1-\gamma)\boldsymbol{I} \\ -\boldsymbol{M}_l^{-1}\boldsymbol{K}_{l,k} & -\boldsymbol{M}_l^{-1}(\boldsymbol{D}_l + \Delta t\boldsymbol{K}_{l,k}) & -\boldsymbol{M}_l^{-1}\left(\Delta t^2\left(1/2 - \beta\right)\boldsymbol{K}_{l,k} + \Delta t(1-\gamma)\boldsymbol{D}_l\right) \end{bmatrix} \tag{5.17}$$

$$\boldsymbol{B}_{l,k} = \begin{bmatrix} \beta\Delta t^2 \boldsymbol{M}_l^{-1}\boldsymbol{R}_{l,k} \\ \gamma\Delta t\boldsymbol{M}_l^{-1}\boldsymbol{R}_{l,k} \\ \boldsymbol{M}_l^{-1}\boldsymbol{R}_{l,k} \end{bmatrix} \tag{5.18}$$

and, in (5.15):

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{\Phi}_l & & \\ & \boldsymbol{\Phi}_l & \\ & & \boldsymbol{\Phi}_l \end{bmatrix} \tag{5.19}$$

Throughout the paper, whenever two indexes are used to denote a variable, the first subscript (r) refers to a property associated with reduced order model, while the second subscript refers to the time instant at which variable is considered.

In (5.14) and (5.15), $\boldsymbol{v}_k^z$ and $\boldsymbol{w}_k$ are the process and measurement noises, respectively. The former uncertainty stems from the loss of accuracy due to the reduced modeling which needs to be further assessed in order to determine its probability distribution and verify the correlation structure in it. In Chap. 4, we have tested the whiteness of the residual error signal of POD-based reduced model of Pirelli tower; it has been shown that, by an increase in the number of POMs retained in the analysis, a reduction occurs in the amplitude of the noise signal and its spectral power. As a consequence, the effect of the non-white uncertainty in the Kalman-POD observer becomes negligible. Hence in this chapter, we assume that the noises satisfy the requirements of the family of recursive Bayesian inference algorithms.

To tackle the dual estimation problem, we now augment the parameters of the reduced model into the state vector, to comply with the state space form. Subsequently, we introduce the augmented state vector $\boldsymbol{x}_{r,k}$, that at any time t_k encompasses both states and parameters of the system $\boldsymbol{x}_{r,k} = \begin{bmatrix} \boldsymbol{z}_{r,k} & \boldsymbol{\vartheta}_{r,k} \end{bmatrix}^T$. In Sect. 2.2, it is shown that dual estimation of states and parameters of a linear system leads to a nonlinear state-space model. The new state space equation is written as:

$$\boldsymbol{x}_{r,k} = \boldsymbol{f}_{r,k}\left(\boldsymbol{x}_{r,k-1}\right) + \boldsymbol{v}_k \tag{5.20}$$

$$\boldsymbol{y}_k = \boldsymbol{H}\,\boldsymbol{L}\boldsymbol{x}_{r,k} + \boldsymbol{w}_k \tag{5.21}$$

$$L = \begin{bmatrix} C & \\ & \mathbf{0} \end{bmatrix} \tag{5.22}$$

where: $\mathbf{0}$ in L is a null matrix of appropriate dimension to annihilate the effects in the observation mapping of parameters in the augmented state vector; $f_{r,k}(.)$ maps the state of the system in time and H denotes the correlation between states and observables of the system; L links the reduced states of the system to the full state; whereas v_k and w_k stand for the zero mean white Gaussian processes with associated covariance matrices V and W. Likewise previous Chapters, $\vartheta_{r,k}$ includes the parameters of the reduced state space model that should be estimated, namely the components of the reduced stiffness matrix $K_{l,k}$.

5.4 Dual Estimation of Reduced States and Parameters of a Damaging Structure

Dual estimation problem for a non-damaging (elastic) structure can be pursued via the estimation of reduced state and parameters since there will not be changes in the subspace of the problem. On the contrary, subspace of a damaging structure varies in time: for instance, a change in a story stiffness can lead to a change in the POMs. As a consequence, dual estimation of the reduced state and parameters of a damaging structure not only includes tracking of the reduced state and estimation of the reduced parameters of the system, but also needs online update of the relevant subspace of the structure.

In this section, we introduce a novel approach for simultaneous state and parameter estimation, accompanied by an online subspace update in order to obtain an estimate of the full state. In this regard, we adopt recursive Bayesian filters: the extended Kalman filter (EKF) and the extended Kalman particle filter (EK-PF). They have been discussed in Chap. 2, and used for dual estimation. A Kalman filter is instead used to update the subspace furnished by POD. Likewise all recursive Bayesian inference algorithms, the iterations start by an initial guess; next, within each time interval $[t_{k-1} t_k]$, provided that at t_{k-1} estimations of state, parameters and subspace of the system are available, the state $z_{r,k}$ and parameters in $K_{l,k}$ are simultaneously estimated. Let us consider the following state space model:

$$x_{r,k} = f_{r,k}\left(x_{r,k-1}\right) + v_k \tag{5.23}$$

$$y_k = HL_k x_{r,k} + w_k \tag{5.24}$$

where:

$$L_k = \begin{bmatrix} \Phi_{l,k} & & & \\ & \Phi_{l,k} & & \\ & & \Phi_{l,k} & \\ & & & \mathbf{0} \end{bmatrix}. \tag{5.25}$$

Along with Eqs. (5.23) and (5.24), an additional equation should be introduced in order to permit time variation and update of $\boldsymbol{\Phi}_l$, similar to the trick used for dual estimation of states and parameters. The following equation is introduced to allow the subspace to vary over time, and use the data in observation in order to adapt to the possible changes:

$$\boldsymbol{\Phi}_{l,k} = \boldsymbol{\Phi}_{l,k-1} + \boldsymbol{\upsilon} \tag{5.26}$$

where $\boldsymbol{\upsilon}$ denotes a fictitious zero mean, white Gaussian noise with associated covariance υ that needs to be obviously tuned to obtain unbiased estimates of the subspace vectors.

To recursively update the subspace, Eqs. (5.26) and (5.24) are assumed as the state-space model for subspace evolution. The former equation governs the evolution of the subspace, and the latter one links the observation to the subspace. In Eqs. (5.26) and (5.24), it is assumed that $\boldsymbol{x}_{r,k}$ remains independent of $\boldsymbol{\Phi}_{l,k}$. The observation Eq. (5.24), when used for subspace update can be rewritten as:

$$\boldsymbol{y}_k = \boldsymbol{H}_{ss}\boldsymbol{\Phi}_{l,k} + \boldsymbol{w}_k \tag{5.27}$$

where $\boldsymbol{H}_{ss}$ is a stationary matrix which links the observation process to the subspace spanned by the POMs, and can be computed by manipulating Eq. (5.26). Equation (5.27) establishes a linear relationship between the observation $\boldsymbol{y}_k$ and the subspace $\boldsymbol{\Phi}_{l,k}$, whose linearity allows us to use the Kalman filter (the optimal estimator for linear state-space models) for the estimation of the subspace.

In Tables 5.1 and 5.2, an algorithmic description of the procedure is reported; the EKF and the EK-PF are used for dual estimation. In the Table 5.1, $\nabla_x \boldsymbol{f}_{r,k}(x)|_{\boldsymbol{x}=\widehat{\boldsymbol{x}}_{k-1}}$ denotes Jacobian of $\boldsymbol{f}_{r,k}(\blacksquare)$, at $\boldsymbol{x}_r = \boldsymbol{x}_{r,k}^-$.

As seen in Table 5.1, the algorithm has two main stages of prediction and update. In the prediction stage, the evolution equations are used to map in time the reduced state $\boldsymbol{x}_{r,k-1}$ along with its covariance. In the update stage, first the reduced state and parameters and their associated covariances are corrected by incorporating the information contained in the latest observation (steps 1 and 2); next, the Kalman filter is exploited to update the subspace $\boldsymbol{\Phi}_l$. Step 3 in the prediction stage of dual estimation algorithm is in fact the predictor stage of the Kalman filter to update the subspace. In step 4, Kalman gain is computed and is used in step 5 to update the estimate of the subspace by taking the latest observation into account.

Concerning the use of EK-PF for dual estimation, according to previous Chap. 2, combined with the Kalman filter for subspace update, similar to the procedure used by EKF-KF algorithm, the reader is referred to Table 5.2. In the Table 5.2, $\boldsymbol{F}_{r,k}^{(i)}$ is:

$$\nabla_x \boldsymbol{f}_{r,k}(\boldsymbol{x})|_{\boldsymbol{x}=\widehat{\boldsymbol{x}}_{k-1}} \tag{5.28}$$

where it denotes Jacobian of the reduced evolution $\boldsymbol{f}_{r,k}(\boldsymbol{x}_r)$ at $\boldsymbol{x}_r = \boldsymbol{x}_{r,k}^{(i)-}$.

Table 5.1 EKF-KF algorithm for dual estimation of the reduced model and subspace update

- Initialization at time t_0

$$\widehat{\boldsymbol{x}}_{r,0} = \boldsymbol{L}_0^{\mathrm{T}}\mathbb{E}[\boldsymbol{x}_0] \qquad \boldsymbol{P}_{r,0} = \boldsymbol{L}_0^{\mathrm{T}}\mathbb{E}\left[(\boldsymbol{x}_0 - \widehat{\boldsymbol{x}}_0)(\boldsymbol{x}_0 - \widehat{\boldsymbol{x}}_0)^{\mathrm{T}}\right]\boldsymbol{L}_0$$

$$\widehat{\boldsymbol{\Phi}}_{l,0} = \mathbb{E}\left[\boldsymbol{\Phi}_{l,0}\right] \quad \boldsymbol{P}_{ss,0} = \mathbb{E}\left[\left(\boldsymbol{\Phi}_{l,0} - \widehat{\boldsymbol{\Phi}}_{l,0}\right)\left(\boldsymbol{\Phi}_{l,0} - \widehat{\boldsymbol{\Phi}}_{l,0}\right)^{\mathrm{T}}\right]$$

- At time t_k, for $k = 1, \ldots, N_t$
  - Prediction stage
    1. Computing process model Jacobian

$$\boldsymbol{F}_{r,k} = \nabla_x \boldsymbol{f}_{r,k}(\boldsymbol{x})|_{\boldsymbol{x}=\widehat{\boldsymbol{x}}_{k-1}}$$

    2. Evolution of state and prediction of covariance

$$\boldsymbol{x}_{r,k}^{-} = \boldsymbol{f}_{r,k}\left(\boldsymbol{x}_{r,k-1}\right)$$
$$\boldsymbol{P}_{r,k}^{-} = \boldsymbol{F}_{r,k}\boldsymbol{P}_{r,k-1}^{-}\boldsymbol{F}_{r,k}^{T} + \boldsymbol{V}$$

  - Update stage
    1. Use $\boldsymbol{\Phi}_{l,k-1}$ to estimated $\boldsymbol{L}_k$ and Kalman gain

$$\boldsymbol{G}_k = \boldsymbol{P}_{r,k}^{-}\boldsymbol{L}_k^{\mathrm{T}}\boldsymbol{H}^{\mathrm{T}}\left(\boldsymbol{H}\boldsymbol{L}_k\boldsymbol{P}_{r,k}^{-}\boldsymbol{L}_k^{\mathrm{T}}\boldsymbol{H}^{\mathrm{T}} + \boldsymbol{W}\right)^{-1}$$

    2. Update state and covariance

$$\boldsymbol{x}_{r,k} = \boldsymbol{x}_{r,k}^{-} + \boldsymbol{G}_k\left(\boldsymbol{y}_k - \boldsymbol{H}\boldsymbol{L}_k\boldsymbol{x}_{r,k}^{-}\right)$$
$$\boldsymbol{P}_{r,k} = \boldsymbol{P}_{r,k}^{-} - \boldsymbol{G}_k\boldsymbol{H}\boldsymbol{L}_k\boldsymbol{P}_{r,k}^{-}$$

    3. Predict subspace and its associated covariance

$$\boldsymbol{\Phi}_{l,k}^{-} = \boldsymbol{\Phi}_{l,k-1}$$
$$\boldsymbol{P}_{ss,k}^{-} = \boldsymbol{P}_{ss,k-1} + \Upsilon$$

    4. Calculate Kalman gain for updating subspace

$$\boldsymbol{G}_{ss,k} = \boldsymbol{P}_{ss,k}^{-}\boldsymbol{H}_{ss}^{\mathrm{T}}\left(\boldsymbol{H}_{ss}\boldsymbol{P}_{ss,k}^{-}\boldsymbol{H}_{ss}^{\mathrm{T}} + \boldsymbol{W}\right)^{-1}$$

    5. Calculate Kalman gain for updating subspace

$$\boldsymbol{\Phi}_{l,k} = \boldsymbol{\Phi}_{l,k}^{-} + \boldsymbol{G}_{ss,k}\left(\boldsymbol{y}_k - \boldsymbol{H}_{ss}\boldsymbol{\Phi}_{l,k}^{-}\right)$$
$$\boldsymbol{P}_{ss,k} = \boldsymbol{P}_{ss,k}^{-} - \boldsymbol{G}_{ss,k}\boldsymbol{H}_{ss}\boldsymbol{P}_{ss,k}^{-}$$

5.5 Numerical Results: Damage Detection in a Ten Storey Shear Building

This section deals with the numerical assessment of the proposed algorithm to detect damage in a 10-storey shear building. To deal with the damage scenarios, it is not straight forward to use the model of Pirelli tower, due to the fact that a static condensation has been carried out to derive matrices of lumped mass system of the Pirelli towers. For the sake of simplicity, in the numerical example, it is assumed that all the floors have equal mass and inter-storey stiffness, i.e. $m_i = 20$ Kg and $k_i = 300$ Kg/m where $i = 1, 2, \ldots, 10$, and the damping effect is neglected. It the analysis, the external load shaking the structure, is a sinusoidal load applied to the last floor (roof) of the building, varying according to:

$$R(t) = a_m \sin 2\pi\omega t \tag{5.29}$$

Table 5.2 EK-PF-KF algorithm for dual estimation of the reduced model and subspace update

• Initialization at time t_0

$$\widehat{\boldsymbol{x}}_{r,0} = \boldsymbol{L}_0^{\mathrm{T}}\mathbb{E}[\boldsymbol{x}_0] \qquad \boldsymbol{P}_{r,0} = \boldsymbol{L}_0^{\mathrm{T}}\mathbb{E}\left[(\boldsymbol{x}_0 - \widehat{\boldsymbol{x}}_0)(\boldsymbol{x}_0 - \widehat{\boldsymbol{x}}_0)^{\mathrm{T}}\right]\boldsymbol{L}_0$$

$$\widehat{\boldsymbol{\Phi}}_{l,0} = \mathbb{E}\left[\boldsymbol{\Phi}_{l,0}\right] \quad \boldsymbol{P}_{ss,0} = \mathbb{E}\left[\left(\boldsymbol{\Phi}_{l,0} - \widehat{\boldsymbol{\Phi}}_{l,0}\right)\left(\boldsymbol{\Phi}_{l,0} - \widehat{\boldsymbol{\Phi}}_{l,0}\right)^{\mathrm{T}}\right]$$

$$\boldsymbol{x}_{r,0}^{(i)} = \widehat{\boldsymbol{x}}_0 \qquad \omega_0^{(i)} = p\left(\boldsymbol{y}_0|\boldsymbol{x}_{r,0}\right),\ i = 1,\ldots,N_{\mathrm{P}}$$

• At time t_k, for $k = 1,\ldots,N_t$

- Prediction stage

1. Draw particles

$$\boldsymbol{x}_{r,k}^{(i)-} \sim p\left(\boldsymbol{x}_{r,k}|\boldsymbol{x}_{r,k-1}^{(i)}\right) \quad i = 1,\ldots,N_{\mathrm{P}}$$

2. Push the particles toward the region of high probability through an EKF

$$\boldsymbol{P}_{r,k}^{(i)-} = \boldsymbol{F}_{r,k}^{(i)}\boldsymbol{P}_{r,k-1}^{(i)}\boldsymbol{F}_{r,k}^{(i)T} + \boldsymbol{V}$$

$$\boldsymbol{G}_k^{(i)} = \boldsymbol{P}_{r,k}^{(i)-}\boldsymbol{L}_{k-1}^{\mathrm{T}}\boldsymbol{H}_k^{\mathrm{T}}\left(\boldsymbol{H}\boldsymbol{L}_{k-1}\boldsymbol{P}_{r,k}^{(i)-}\boldsymbol{L}_{k-1}^{\mathrm{T}}\boldsymbol{H}^{\mathrm{T}} + \boldsymbol{W}\right)^{-1}$$

$$\left.\begin{aligned}\boldsymbol{x}_{r,k}^{(i)} &= \boldsymbol{x}_{r,k}^{(i)-} + \boldsymbol{G}_k^{(i)}\left(\boldsymbol{y}_k - \boldsymbol{H}\boldsymbol{L}_{k-1}\boldsymbol{x}_{r,k}^{(i)-}\right)\\ \boldsymbol{P}_{r,k}^{(i)} &= \boldsymbol{P}_{r,k}^{(i)-} - \boldsymbol{G}_k^{(i)}\boldsymbol{H}\boldsymbol{L}_{k-1}\boldsymbol{P}_{r,k}^{(i)-}\end{aligned}\right. \quad i = 1,\ldots,N_{\mathrm{P}}$$

- Update stage

1. Evolve weights

$$\omega_k^{(i)} = \omega_{k-1}^{(i)}p\left(\boldsymbol{y}_k|\boldsymbol{x}_{r,k}^{(i)}\right) \quad i = 1,\ldots,N_{\mathrm{P}}$$

2. Resampling, see Table 2.5.
3. Compute expected value or other required statistics

$$\widehat{\boldsymbol{x}}_{r,k} = \sum_{i=1}^{N_{\mathrm{P}}}\omega_k^{(i)}\boldsymbol{x}_{r,k}^{(i)}$$

4. Predict subspace and its associated covariance

$$\boldsymbol{\Phi}_{l,k}^{-} = \boldsymbol{\Phi}_{l,k-1}$$
$$\boldsymbol{P}_{ss,k}^{-} = \boldsymbol{P}_{ss,k-1} + \Upsilon$$

5. Calculate Kalman gain for updating subspace

$$\boldsymbol{G}_{ss,k} = \boldsymbol{P}_{ss,k}^{-}\boldsymbol{H}_{ss}^{\mathrm{T}}\left(\boldsymbol{H}_{ss}\boldsymbol{P}_{ss,k}^{-}\boldsymbol{H}_{ss}^{\mathrm{T}} + \boldsymbol{W}\right)^{-1}$$

6. Update subspace and its associated covariance

$$\boldsymbol{\Phi}_{l,k} = \boldsymbol{\Phi}_{l,k}^{-} + \boldsymbol{G}_{ss,k}\left(\boldsymbol{y}_k - \boldsymbol{H}_{ss}\boldsymbol{\Phi}_{l,k}^{-}\right)$$
$$\boldsymbol{P}_{ss,k} = \boldsymbol{P}_{ss,k}^{-} - \boldsymbol{G}_{ss,k}\boldsymbol{H}_{ss}\boldsymbol{P}_{ss,k}^{-}$$

where $a_m = 10\,\mathrm{N}$ and $\omega = 0.01\,\mathrm{Hz}$.

Consider a case in which a stiffness reduction equal to 50 % has occurred at the 5th floor. The POMs of the structure, before and after damage occurrence, are computed and presented in the Fig. 5.2. To compute these POMs of the healthy and damaged cases, two direct analyses have been carried out to assemble the so-called snapshot matrices. Looking at Fig. 5.2, it can be seen that the ten POMs of the structure are affected by the stiffness reduction at the 5th floor. The effect of the

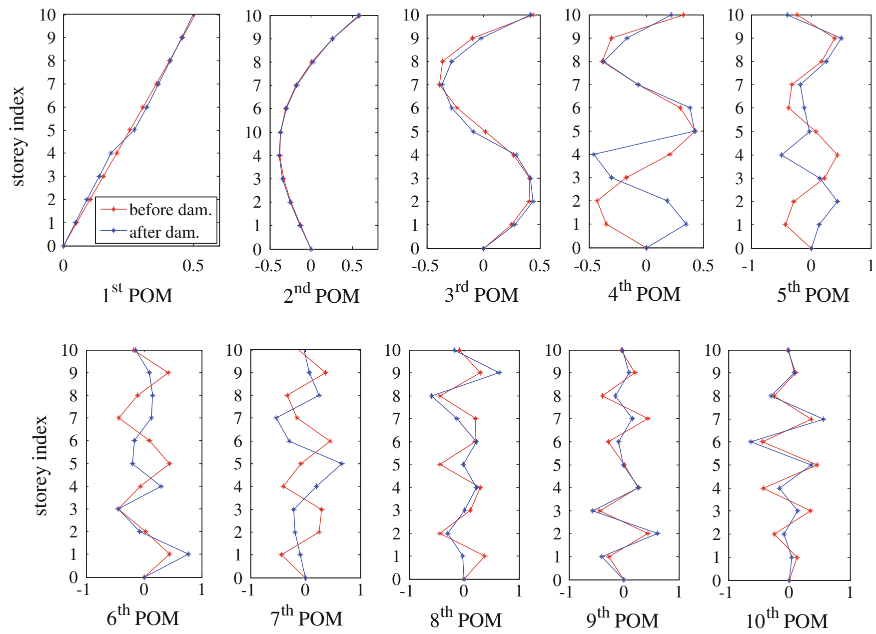

Fig. 5.2 Proper orthogonal modes of a 10 storey shear building before and after damage

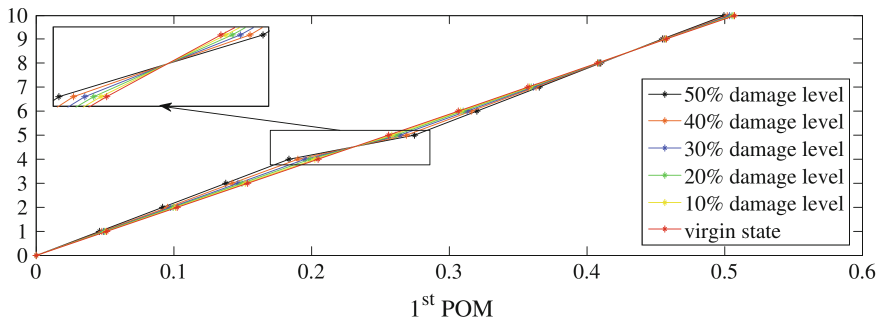

Fig. 5.3 1st POM of the 10 storey shear building subject to different levels of damage at 5th floor

damage in the first POM is quite visible, the usefulness of such sensitivity to damage, even in the first POM, helps tracking the evolution of damage in a single DOF reduced model.

Figure 5.3 compares the first POM of the structure when the 5th floor of the structure suffers a damage of varying intensity; the close-up in the graph allows us to compare the shape of the POM in the vicinity of the damage location. Obviously, the intensity of damage leads to an increase in the deviation of the POM

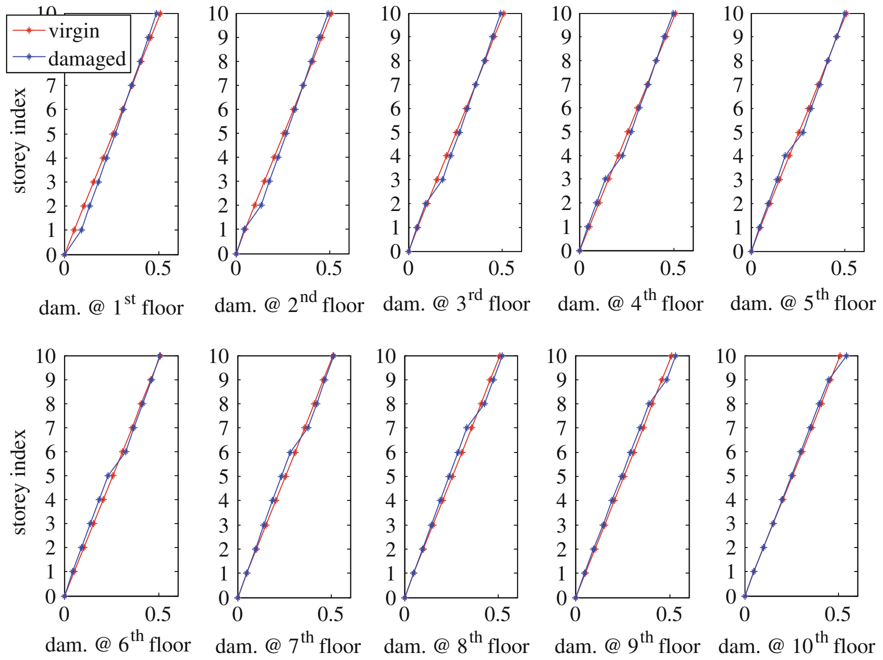

Fig. 5.4 1st POM of a ten storey shear building for a damage occurring at different storeys of the building

relevant to the damaged state with respect to the healthy state of the structure. To highlight the sensitivity of the 1st POM to damage location, in Fig. 5.4 the first POM of the damaged state is compared with healthy state, when damage occurs at different floors. The imposed level of the damage in all the cases is equal to a 50 % reduction of the stiffness of the relevant floor.

Now that the link between the first POM of the structure and the location and severity of the damage is established, we move to the problem of the recursive estimation of the state, parameters and POMs of the reduced model of the structure. To detect the damage, the POMs of healthy and current state of the structure are compared; thus information concerning the healthy state of the structure is needed. In this study, the case in which the reduced models retain one or two POMs are assessed, the latter case is mainly reported to verify the performance of the algorithm in case of the higher number of parameters to be estimated: dual estimation of reduced models which retain more POMs includes calibrating a high number of parameters, and can therefore potentially pose the problem of curse of dimensionality, as discussed in Chap. 2.

First, we deal with the reduced model constructed through a single POM. Pseudo-experimental data for evaluation of the methodology have been created by running direct analysis, to compute the response of the structure, and then adding zero mean white Gaussian noise to allow for uncertainties in measuring the

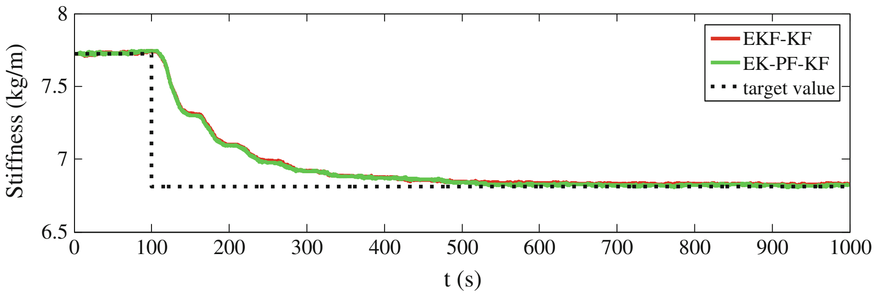

Fig. 5.5 Estimation of the reduced via EKF-KF and EK-PF-KF algorithms

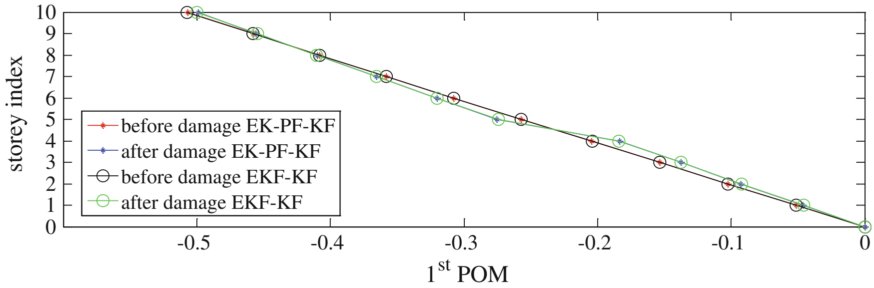

Fig. 5.6 1st POM of the structure estimated EKF-KF and EK-PF-KF algorithms

response of the structure. The covariance of the added noise to all the pseudo experimental data considered in this section is set to 10^{-4} m^2 to simulate a high level of measurement uncertainty. The duration of the analysis is set to 1,000 s, in order to allow the estimates of the algorithms to converge to a steady state value. The damage scenario is once again a reduction of 50 % in the stiffness of the 5th floor, which occurs at $t = 100$ s. Other damage scenarios, featuring severities ranging from 10 to 40 % in the reduction of the stiffness of other floors has been assessed; the algorithms show similar performance dealing with those scenarios; thus the results are not presented for the sake of brevity.

Since the goal of this Section is the damage identification, the results concerning the estimation of the state are not discussed. Figure 5.5 shows the time history of the estimated stiffness of the reduced system when compared with its target value. It is observed that before damage occurs, the estimation coincides with the target value; however, after damage occurs, it takes almost 400 s for the algorithm to make its estimate to converge to the target value. Figure 5.6 shows the estimated POMs of the building before and after damage: the POM concerning the healthy state is related to $t = 50$ s, and the POM concerning the damaged state is related to $t = 1{,}000$ s. To compare the performance of the algorithm in tracking the POM of the system over time, Fig. 5.7 shows time history of the estimated POM, compared with its target value. It is observed that the estimations of the

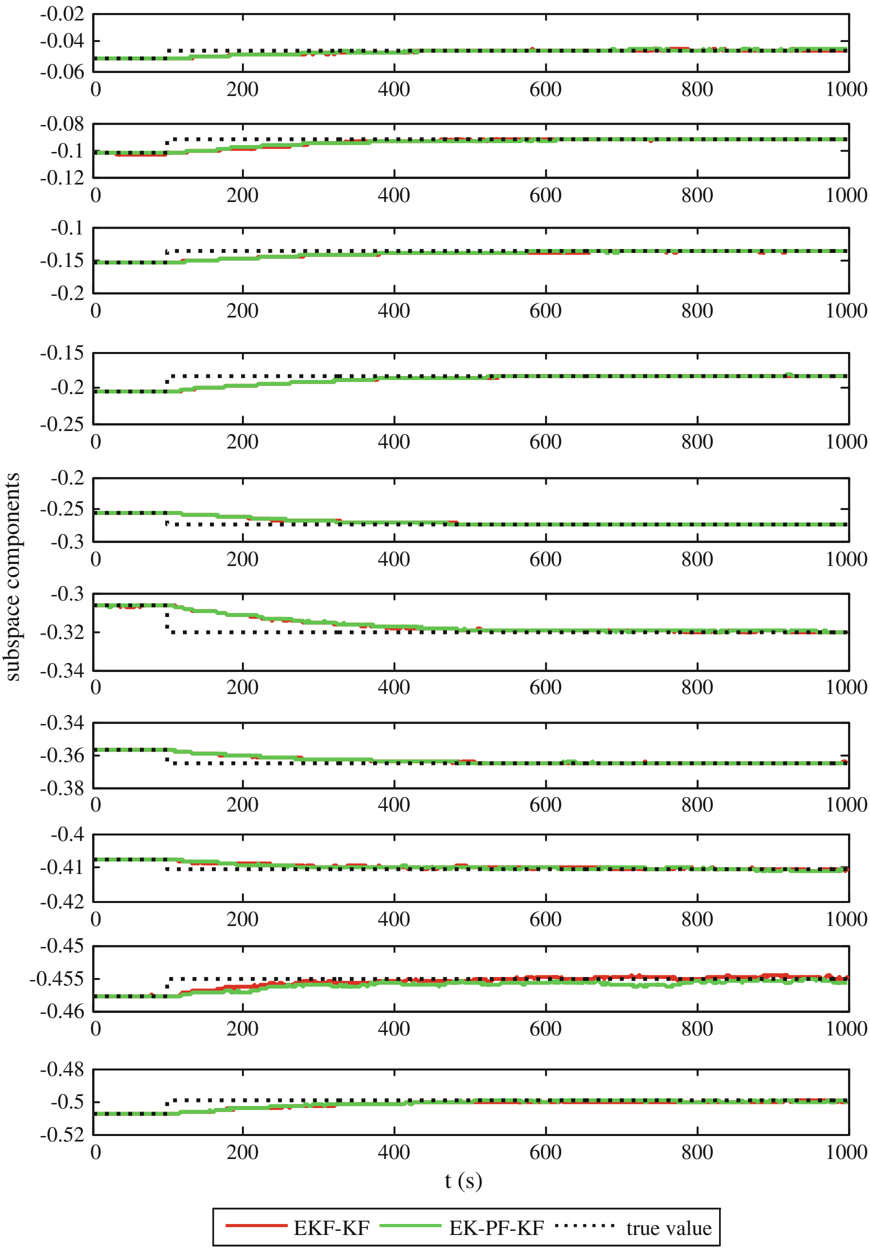

Fig. 5.7 Time histories of the components of the 1st POM of the structure, from *top* to *bottom* respectively corresponds to first to last component of the POM vector (time histories of entries of POM)

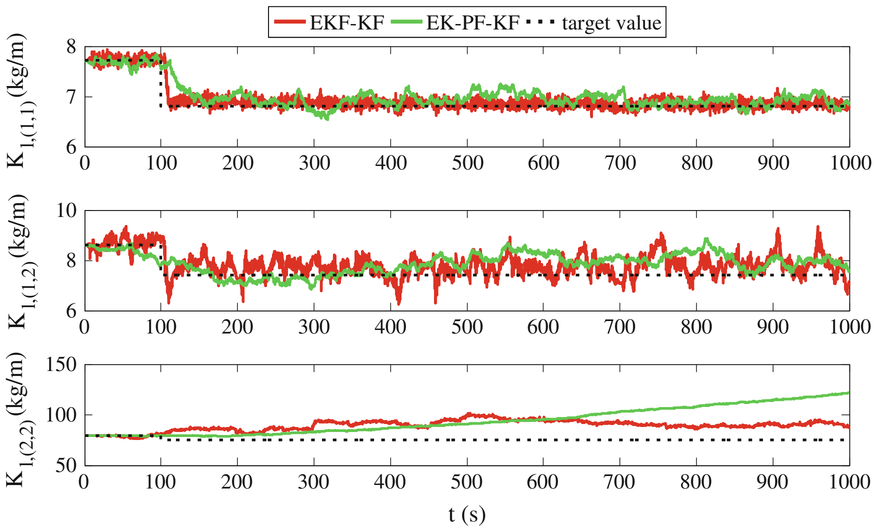

Fig. 5.8 Time histories of the parameter estimation of the reduced model via EK-PF-KF and EKF-KF algorithms: $K_{l,(1,1)}$, $K_{l,(1,2)}$ and $K_{l,(2,2)}$from *top* to *bottom*, respectively

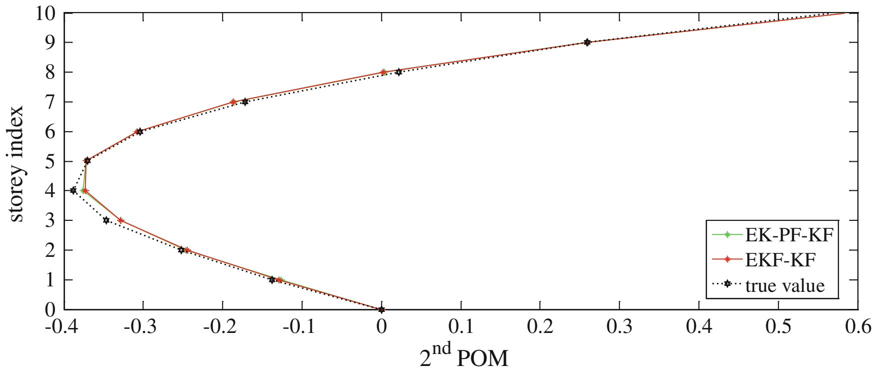

Fig. 5.9 Results concerning estimation of the second POM of the shear building after damage occurs

POM components before damage occurrence coincide with the true value; after damage occurs, the algorithm needs nearly 400 s, similar to parameter estimates, to reach steady state. EK-PF, when dealing with several problems discussed in Chap. 2 outperforms the EKF; hence, it is used to verify if its convergence rate would be better than EKF's one. However, it is seen in Fig. 5.5 that the quality of estimation of the reduced stiffness and the 1st POM of the structure do not change, when either EKF-KF or EK-PF-KF are used for dual estimation and reduced order modeling of the damaging shear building.

Now, let us move to a case in which there are two POMs retained in the reduced order model of the system. In this case, taking advantage of the symmetry of the stiffness matrix, the reduced stiffness matrix $\boldsymbol{K}_l$ has three components to estimate. Figure 5.8 shows the results of the reduced stiffness matrix estimation via the EK-PF-KF and EKF-KF algorithms. It is observed that both algorithms are able to calibrate two of the components of the reduced stiffness matrix, while the $K_{l,(2,2)}$ component is failed to be estimated. The reason for such failure can be the insensitivity of the observations to the sought parameter.

Figure 5.9 shows the results of the estimation of the 2nd POM of the structure by utilizing both the proposed algorithms. It is observed that, they fail in furnishing an estimate of the 2nd POM; this failure can be due to the small contribution of the second POM in the response of the structure.

5.6 Summary and Conclusion

In this chapter, we consider dual estimation and reduced order modeling of a damaging structure. Moreover, proper orthogonal decomposition has been considered for reduced order modeling in order to find a subspace which optimally captures the dynamics of the system. Through a Galerkin projection, the equations governing the dynamics of the system are projected onto the subspace provided by the proper orthogonal decomposition algorithm. As for the dual estimation goal, the extended Kalman filter and extended Kalman particle filter have been adopted; both filters, in their so-called update stage, make a comparison between the latest observation and the prediction of the state of the system to estimate the quantity of correction which is needed in estimation of the state. In the case of the reduced order modeling, for realization of such a comparison, reconstruction of full state of the system is required, which is obviously possible only if the subspace is known. It is established that the subspace found by proper orthogonal decomposition is not robust to changes of the parameters; therefore, we have proposed algorithms for online estimation of the subspace spanned by proper orthogonal modes retained in the reduced order model of the system. Such an online estimation of the proper orthogonal modes of the structure makes it possible to detect the damage in the structure, locate it and potentially identify its intensity.

References

Abazarsa F, Ghahari SF, Nateghi F, Taciroglu E (2013a) Response-only modal identification of structures using limited sensors. Struct Control Health Monit 20:987–1006

Abazarsa F, Nateghi F, Ghahari SF, Taciroglu E (2013b) Blind modal identification of non-classically damped systems from free or ambient vibration records. Earthq Spectra. doi:http://dx.doi.org/10.1193/031712EQS093M (in press)

Galvanetto U, Violaris G (2007) Numerical investigation of a new damage detection method based on proper orthogonal decomposition. Mech Syst Signal Process 21:1346–1361

Ghahari SF, Abazarsa F, Ghannad MA, Celebi M, Taciroglu E (2013a) Blind modal identification of structures from spatially sparse seismic response signals. Struct Control Health Monit. doi:10.1002/stc.1593 (in press)

Ghahari SF, Abazarsa F, Ghannad MA, Taciroglu E (2013b) Response-only modal identification of structures using strong motion data. Earthquake Eng Struct Dynam 42:1221–1242

Ghahari SF, Ghannad MA, Taciroglu E (2013c) Blind identification of soil-structure system. Soil Dynamics Earthq Eng 45:56–69

Ghahari SF, Ghannad MA, Norman J, Crewe A, Abazarsa F, Taciroglu E (2013d) Considering wave passage effects in blind identification of long-span bridges. Topics in model validation and uncertainty quantification, vol 5. Springer, New York, pp 53–66

Loh C, Weng J, Liu Y, Lin P, Huang S (2011) Structural damage diagnosis based on on-line recursive stochastic subspace identification. Smart Mater Struct 20:055004

Moaveni B, He X, Conte JP, Restrepo JI, Panagiotou M (2011) System identification study of a 7-story full-scale building slice tested on the UCSD-NEES shake table. J Struct Eng 137:705–717

Ruotolo R, Surace C (1999) Using SVD to detect damage in structures with different operational conditions. J Sound Vib 226:425–439

Shane C, Jha R (2011) Proper orthogonal decomposition based algorithm for detecting damage location and severity in composite beams. Mech Syst Signal Process 25:1062–1072

Van Overschee P, De Moor B (1996) Subspace identification for linear systems. Kluwer Academic, Dordrecht

Vanlanduit S, Parloo E, Cauberghe B, Guillaume P, Verboven P (2005) A robust singular value decomposition for damage detection under changing operating conditions and structural uncertainties. J Sound Vib 284:1033–1050

Yang JN, Lin S (2005) Identification of parametric variations of structures based on least squares estimation and adaptive tracking technique. J Eng Mech 131:290–298