Chapter 9
Visuospatial Reasoning in Contexts with Digital Technology

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The Challenge

The ecocultural perspective on visuospatial reasoning was established by considering Indigenous communities and their practices and appropriate schooling and other diverse ecocultural practices illustrating visuospatial reasoning. However, today is the digital age so does research on visuospatial reasoning support this ecocultural perspective. Since most of the research on visuospatial reasoning has been focussed on dynamic computer-generated images, it is important to consider digital technological facilities as an ecological context. How can an ecocultural perspective of visuospatial reasoning enhance our understanding and valuing of visuospatial reasoning? In this chapter we consider how a computer-facilitated learning age influences an ecocultural identity and both self-regulation and visuospatial reasoning. It is then important to consider how these personal dispositions impact on mathematical identity.

This chapter focuses on prior-to-school and elementary or primary schooling and the impact of the digital age on visuospatial reasoning. In particular, it will consider how students are reasoning visuospatially in the context of hand-held robots in early childhood (Highfield et al., 2008). Highfield showed that children were capable of reasoning and learning concepts in mathematics through the use of robots. Analogies will be drawn with the use of diagrams for reasoning.

Sections of the chapter cover the importance of dynamic geometry softwares in the way that students reason visuospatially. Considering that there are many research articles in this area, several will be selected, especially those that look at the use of ICTs in different cultural groups and with primary and middle school students. The importance of reasoning in a visuospatial environment (Jones, 2000) and the importance in design of software (Christou, Pittalis, Mousoulides, & Jones, 2006; Jonassen, 1999) as it impacts on visuospatial reasoning, self-regulation and sociocultural identity will be explored.

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There is also an increasing interest in computers as tools in modelling (Goos et al., 2003) and in terms of this book in ethnocomputing (Eglash, 2007). However, this modelling approach can be more broadly interpreted in the modelling sense that is developed from a particular cultural group (Rosa & Orey, 2012). This strong support for valuing cultural mathematics joins with ecocultural perspectives in visuospatial reasoning and hence developing mathematical thinking identity that moves beyond the western-dominated perspective.

The argument continues with important reasons in today's society for visuospatial reasoning. A discussion of the importance of ecocultural perspectives for appreciating geographical studies and the mathematical understandings necessary for such studies will illustrate the importance of visuospatial reasoning and the impact that ecology and culture have on this development and thinking.

Ecocultural Perspective of Measurement in Changing Worlds

An historical look at geometry from Fibonacci to the twentieth century shows that diagrams and practical mathematics with measurement was commonplace and proofs such as Euclid's were not always centre stage (Menghini, 2012). Is it possible that the digital era built on this background of measurement and experiment? Even Fibonacci gave a way of calculating the volume of a heap of wheat in a corner by measuring lengths horizontal to the floor on either side, multiplying and dividing by 2. Perhaps the PNG communities who use lengths for assessing volumes are not so different. Measuring is not uncommon in digital dynamic geometry experiences. Nevertheless, the geometric or spatial reasoning needs to link to more theoretical ways of thinking if the technology is to be considered mathematics.

Wassmann (1994) noted that the Yupno of Papua New Guinea employed three different ways of spatial perception and not just the one western way which is egocentric. They used object-centred locations such as relative positioning, absolute positioning (east, west, south, north), and anthropomorphic description to locate themselves depending on the time and context. However, it is possible to explore visuospatial reasoning in a digital environment despite Turkle and Papert's (1990) suggestion that the multiple modes of thinking cannot be known. The ecocultural perspective and the model of mathematical identity within context assist us to understand the process of visuospatial reasoning in the digital context.

We are challenged by Lévi-Strauss's (1968) view that visuospatial reasoning is only of the "primitive mind". While we recognise this as a strength that should not be lost to Indigenous cultures, we can better understand visuospatial reasoning and improve learning in the digital age by realising the importance of visuospatial reasoning in the context of the "human-with-media" (Borba & Villarrea, 2005). The place of visuospatial reasoning within the model of identity assists us to appreciate how

> the computer, with its graphics, its sounds, its text, and its animation, can provide a port of entry for people whose chief ways of relating to the world are through movement, intuition, and visual impression (Turkle & Papert, 1990, p. 131)

and enhance learning for all students through media. Despite

> discrimination in the computer culture (that) takes the form of discrimination against approaches to knowledge, most strikingly against … an approach we call “bricolage”. (Turkle & Papert, 1990, p. 135),

there are various students who, for one reason or another, do not want to do “black box” programming. The creative, visual approach is illustrated by a student Anne

> Instead of thinking of compound objects as a way of getting a picture to be bigger, she thinks of compound objects as a way of getting sprites to exhibit a greater complexity of behavior, an altogether more subtle concept. Thus, Anne’s level of technical expertise is as dazzling in its manipulation of ideas as in its visual effects … ., her path into this technical knowledge is not through structural design, but through the pleasures of letting effects emerge. (Turkle & Papert, 1990, p. 139)

Other students produced unexpected solutions. Thus Turkle and Papert argue that visuospatial reasoning is to be respected as much as formal abstract reasoning, that working with objects is also to be valued. As Sternberg (1987) put it, one of the intelligences is that of practical.

Because young children can form rules and properties that are incomplete, some children may not do as well from the more abstract approach as a child who has “a tendency to see things in terms of relationships rather than properties, access to a style of reasoning that allowed them to imagine themselves ‘inside the system’” (Turkle & Papert, 1990, p. 144). They used a relationship to the gears to help them think through a problem but they presented their results in a more formal way.

Furthermore, the characteristics of media and their engagement of students have led to a number of studies connecting visuospatial reasoning to ecocultural contexts. The use of census databases is just one way (e.g. Australian Bureau of Statistics, 2013). Graphing programmes such as Tinkerplot also allow students to move and select and so both physically and visually engage with visuospatial reasoning to support a growing understanding of visuospatial displays of data and statistics. Dynamic, visual software and movies were used by Dalin (2013) to create a powerful means of students becoming mathematicians. He set about to create

> teaching and learning school mathematics in a human environment and through a human learning process. It can be done by translating the mathematical language into graphic, visual-dynamic-quantitative representation and providing the needed tools for active learning through self-experience and exploration. (Dalin, 2013)

Thus learning is understood in terms of the model in Chap. 1 in which ecocultural contexts are significant in the development of the mathematical learner through using visuospatial reasoning as well as other cognitive processes with motivation and self-regulation.

The Role of Digital Media in Developing Self-Regulation for Learning

Self-regulation especially in terms of motivation and self-assessing of actions has generally been assessed by observation and self-reports but the use of computers allows for some monitoring. In young children this is a possible important step forward for self-regulation in problem solving (de la Fuente Arias & Díaz, 2010). Furthermore, we can see the visuospatial reasoning that young children are undertaking.

Highfield's (Highfield, Mulligan, & Hedberg, 2008; Highfield, 2010, 2013) work with young children suggests that simple robotics may provide opportunities for young mathematics learners to engage in self-regulation including metacognitive and problem-solving strategies. This work suggests that the act of planning, programming, and observing the robots movement can act as a catalyst for engagement in a range of mathematical concepts and processes as well as prompt reflection and revision of plans. While this cyclic engagement in problem solving (see Fig. 2.17) highlights the potential of these tools—the context of learning, the child's engagement and responsiveness, multiple representational modes, and the role of a community of learners are key in this process. Figure 1.2 emphasises the context and self-regulation in the cycles.

The role of the teacher, however, may be critical in how well the computer tool and the students' use of it and their collaboration. Laborde, Kynoigos, Hollebrands, and Strässer (2006) note that all papers presented at the PME conferences mentioned that in the dynamic geometry software environment "the notion of dependency is difficult for students and not understood initially" (p. 286). Furthermore, the role of teacher seemed crucial in assisting students to move from the visualisation to another more substantial form of understanding. Laborde et al. emphasised the movement from graphic to mental back to graphic and then to mental activity. This switching, reminiscent of code switching, is an important aspect of both self-regulation and visuospatial reasoning. A strength of dynamic geometry software is the availability of numerical and figural cues and the ability to produce and refine objects to find a solution (Love, 1996). Key to understanding how students find a general solution is a recognition that examples and attention to features is dependent on "a deep, personal, situated structure" (Goldenberg & Mason, 2008, p. 138).

A study by Goos, Galbraith, Renshaw, and Geiger (2003) highlights how the idea of an ecocultural perspective on visuospatial reasoning can occur within the ecocultural context of classrooms with digital media. The tools were becoming extensions of the students' thinking especially under the prompting of the teacher. The teacher intervened on a few occasions encouraging the students to find a solution using an alternative digital means, by seeing what other students were doing, and he also encouraged the group to share their findings. The classroom approach as the teacher portrayed was critical in this ecocultural perspective.

Students' attention and so persistence and self-regulation were the visuospatial representation of the graph that indicated three intersection points. It was then a matter for the group to verify these points. Again the teacher "encouraged the group to use the technology as a *partner* to re-organise their thinking" by using the graphics calculator and spreadsheet on the computer simultaneously. Then when they were still struggling, he encouraged them to see what other groups were doing and the technology became a partner to mediate mathematical discussion between students, resulting in the group coming back together and working out how to set up the spreadsheet to find all solutions. Their hastily prepared presentation was improved by questioning from students and the teacher helping to draw attention to salient aspects of the task and how the different technologies created different representations of the task.

> Mathematical and communications technologies were thus seamlessly integrated to share and support argumentations on behalf of the group of students, suggesting that technology became an extension of *self* for members of this group. … "we were doing it ourselves, not just listening to the teacher. And seeing something visual helped our understanding." … The students' recollections of this experience hint at the sense of autonomy and power associated with appropriating technology into one's personal repertoire of mathematical practice, that is, as an extension of self. (Goos et al., 2003)

Reasoning about the nature of the graph, the use of the spreadsheet and the algebraic representation was parallel to the way in which Indigenous communities were reasoning about the visuospatial representations in their ecocultural context. "Tools"—computers and calculators producing graphs, spreadsheets, equations on the one hand; buildings, carvings, paintings, weaving, dancing, navigating, and rituals creating visuospatial representations on the other hand—were used with reasoning and manipulation. Alternative and "hidden" meanings were understood from the mathematical context and ways of thinking and reasoning about the visuospatial representations. Both required technical expertise. Both required knowledge of the mathematical visuospatial ways of reasoning.

Furthermore, both achieved self-regulation, goal setting, cognitive processing with visualisation. Both involved communication with others within an ecocultural context and both resulted in a sense of achievement and belonging. Both resulted in being a member of the community of practice. Both connected the members to a sense of autonomy and power associated with appropriating technology. One was seen as mathematical but was also ecocultural; the other was mathematical but seen as ecocultural. Both were visuospatial reasoning from an ecocultural perspective.

Dynamic geometry software has been shown to encourage internalisation of motion from the visual screen that students manipulate. The dragging and trace tools in the dynamic geometry software are seen and manipulated being transformed into psychological tools supporting students' reasoning.

> From the combination of observation and action students grasped variability as motion, while the idea of covariation, incorporated in the coordinated movement of points on the screen, was experienced through the coordination between eyes and hands. In most of the cases, students' formulations reflect the asymmetrical nature of the independent and dependent variables and the twofold meaning of trajectory. (Falcade et al., 2007, p. 331)

These researchers showed how the classroom conversations encouraged the abstraction and recognition of the meaning of trajectory at a point and as a "journey" illustrating how the ecocultural context (a communicating classroom with DGS facility) was significant in students' visuospatial reasoning and sense of creating the mathematical notions associated with functions.

Rivera (2011) established the importance of the role of technology for visuospatial reasoning. Computers as "servants" in Goos et al.'s terminology, not only "produce such static displays (i.e. the concrete objects) quickly and easily, but in addition it then becomes straightforward to create rotation and morphing animations that can bring the known mathematical landscape to life in unprecedented ways" but they also allow users to “obtain fresh insights concerning complex and poorly understood mathematical objects” (Palais, 1999, pp. 647–648) as illustrated by Goos et al. above. Rivera particularly notes the evolving processes that occur with animation and how interaction with the computer and others assists the development of relations and theory implied by diagrams or codes that model a structure and display the relationship or concept. The tool becomes an extension of thinking.

In another example, the Singaporean use of representational rectangles that are manipulated, for example in fraction work, provide a strong visual analogy giving meaning through the classroom and curriculum culture of labelling the components and defining the spatial relations among the components, and thus becoming visuospatial reasoning. Similarly, diagrams that represent geometric relationships, often as a theorem, involve culturally accepted ways of marking vertices and segments, and a classroom accepted way of understanding the diagrams. Rivera (2011) established the existence and importance of visuospatial reasoning associated with the world of computer technology in education. In each case, the technology is “an extension of self”, a position established as part of visuospatial reasoning in an ecocultural context in the earlier chapters and again by Goos et al. (2003).

If we turn to the younger age group, there are benefits of virtual manipulatives for visuospatial reasoning. For example, virtual Pattern Blocks have colours that can be changed, they can be “snapped” into position, unlike concrete material and they “stay where they’re put” (Clements, 1999, p. 51). The development of simple repetition, and transformation skills such as reflection, rotation, and scaling are enhanced through on-screen manipulations. Virtual Pattern Blocks and dynamic interactive software can provide representations of concrete manipulatives that allow children to experiment with a broader range of patterns with ease and flexibility. Moyer, Niezgoda, and Stanley (2005) found that children’s patterns were more creative, complex, and prolific using virtual manipulatives compared with patterns formed with concrete materials. Highfield and Mulligan (2007) found technological tools allowed ease of representation, with children using virtual manipulatives consistently engaged in increased experimental patterning producing a broader range of patterns, and edited or deleted them before completion. In part, this could be attributed to the “delete tools” that held “novelty value”, with the children enjoying “rubbing out” and “chucking” things in the “bin”. However, from observations, this can mean children fail to stop and reflect on their pattern making thus requiring teacher intervention to encourage greater self-regulation in the problem solving. Interestingly, children not only use colour but can also orient blocks to form their pattern as illustrated in Fig. 9.1. Transformations are also explored for fun as captured in the following conversation associated with Fig. 9.1b, c:

Nicholas: Oh he’s really big now. He’s really, really big. Wee … Oh … Big … Fat (*scaling the lion, enlarging it*)
Yvette: Make him long (*pointing to the seals*).
Nicholas: Flat (*after shearing the seal*).
Yvette: They’re both flat (*pointing to the seals*).

The fact that virtual objects can be cloned and repeated also allows for measurement units. However, with young children, using the mouse and accidentally

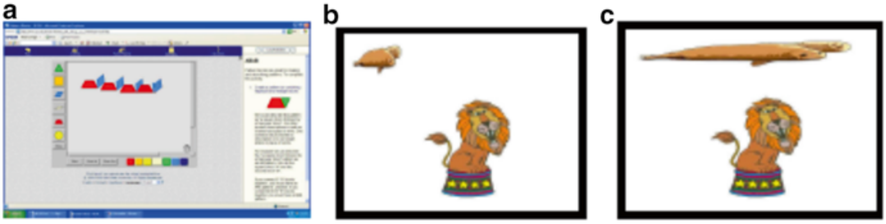

Fig. 9.1 Children's visuospatial reasoning when playing by manipulating objects on computer screens. (**a**) Block patterning. (**b**) Enlarged lion. (**c**) Lengthened seals

clicking on the wrong icon can prevent good construction requiring some guidance. The use of touch screens overcomes some of the difficulties of using the mouse. In some instances, the screen manipulation can prevent good conversations and interactions between the children, again requiring some guidance. In this respect, adult support can be important in the context of play with technology. Further, it is found that children are more likely to click to be entertained and to choose books read to them rather than interactive programmes with non-routine problems. Hence the immediate class context as well as the digital age cultural context influences learning.

Particularly effective for problem solving with technology is the flexibility of multiple strategies (Sarama & Clements, 2009; Siegler, 1999), including: identification of mathematical relationships, inference, generalisation, representation, analogy, recursive cycles of trial and error, and verification (Greenes et al., 2003). However, as stated by Sophian (1999), successful problem solving is "more than the aggregate of the strategies they use; they also know something about the goal" and structure of problems and responses (p.18). Thus self-regulation assists with ensuring the cognitive approaches are effective in students' responsiveness (Fig. 1.2).

Visuospatial Reasoning in Geometry and Measurement Learning Through Digital Technology

When we consider visuospatial reasoning in geometry and measurement, research predominantly focuses on older children and on screen based tools. The focus on older children is likely due in part to curriculum based expectations—with older children encountering formalised geometry and thus this content area being given increased prevalence in research. However, it could be argued that this focus on older children is misplaced with essential measurement and geometry understandings developing at a much younger age (Clements & Sarama, 2007a). The focus on screen based tools is also key here and while this research (as outlined above) provides insight into a range of tools for use in developing geometry and measurement learning there are a range of alternate digital tools that also have potential for learning.

Besides the studies above and studies on digital technology for area and three dimensional stacking for young children, research has mainly focused on the use of Logo programming but there have been differences in the success and usefulness of Logo. Some research has not always found Logo to be effective for young children. The dynamic representation of angle was found to cause confusion for some children. The pathway that the turtle moved through and the angle of turn were not always easily interpreted even when laser beams illustrated the direction of the turtle (Kieran, 1986). Cope and Simmons (1994) also suggested that the immediate feedback obtained from Logo programming may inhibit the development of angle concepts. Their research with students aged 9–11 years, indicated that some learners utilised trial-and-error strategies rather than moving to more advanced, higher level understandings of angle and rotation. Nevertheless, Clements and Battista (1989) and Noss (1987) describe children's increase in understanding of angle concepts when using Logo. Misconceptions may in part be mitigated by appropriate pedagogic structures (Sarama & Clements, 2004). Lehrer and Littlefield (1991) proposed mediated instruction, including structured teaching of Logo skills, as beneficial for children in mastering Logo. Clements and Battista (1991) also recommended tasks that are carefully planned to encourage comparison and avoid misconceptions. To this end, Lehrer, Jacobson, Thoyre, Kemeny, Strom, Horvath et al. (1998) espoused potential benefits of sequenced tasks and inquiry-based learning with Logo.

As a child plans and programmes the turtle's movement in Logo their actions are inherently linked with spatial and geometric concepts, including shapes and angles, directionality, linear measurement, location and position and pathways (Clements, Battista, Sarama, Swaminathan, & McMillen, 1997). Clements and colleagues found that children's engagement with shape construction in Logo enables children to progress quickly in geometric understandings (Battista & Clements, 1991; Clements, 1998). Children's active construction of shapes in Logo facilitated the noticing of properties, verbal descriptions, and integration of geometric understandings. Butler and Close (1989) also found that work in Logo enabled children to develop understanding of two-dimensional shapes. The construction of shapes in a dynamic environment pushed children beyond the static representations they would normally view in traditional representations of geometric concepts. Similar findings are supported by research with children in primary school (Battista & Clements, 1991; Hoyles & Noss, 2003; Lehrer & Littlefield, 1991) and high schools (Khasawneh, 2009).

Simple easily programmable robotics engages students and avoids some of the issues of Logo on screen. Lack of interest partly results from other aspects of the digital age, namely fast moving, noise-producing manipulative screens. Spatial issues are also reduced with the floor turtle. Children are in the same three dimensional space and can face in the same direction as the turtle whereas a vertical screen made some tasks, especially on angles, difficult for students. In Highfield's (2012) study, evidence of visuospatial reasoning is demonstrated not only by the children's activities and conversations captured on video but also by their drawings. Highfield classified the drawings made by 30 children (4 three-year olds, 6 four-year olds, and 20 year 1, around age 6) as idiosyncratic/non mathematical, emergent spatial struc-

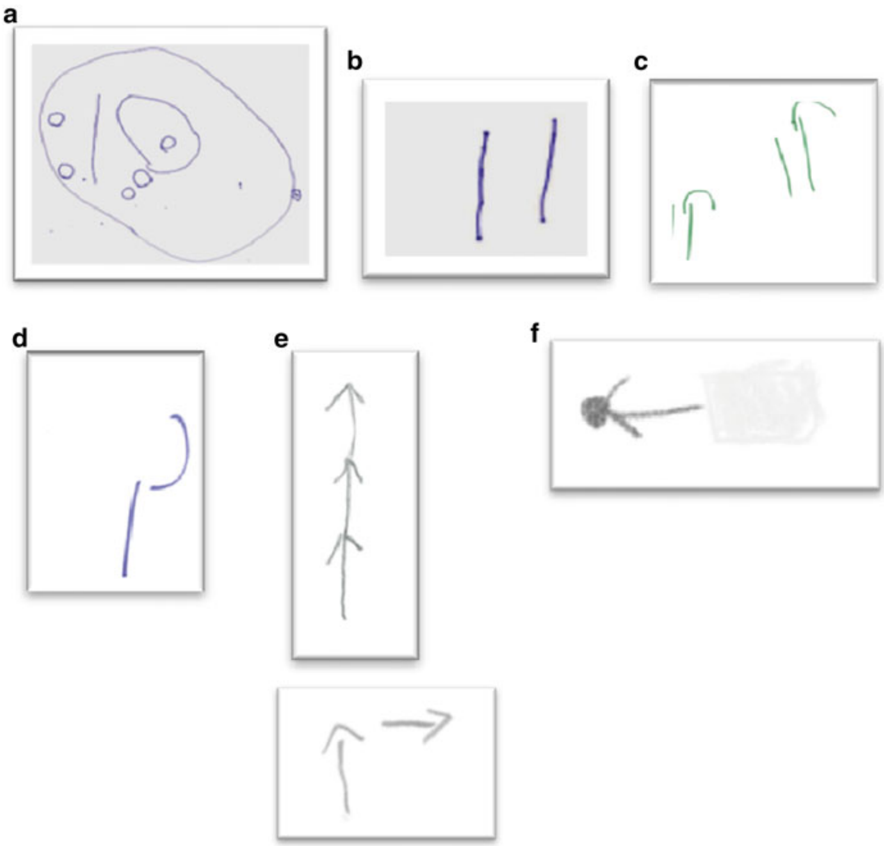

Fig. 9.2 Drawings by children of the movement of their robot. (**a**) Pictorial idiosyncratic. (**b**) Emergent spatial. (**c**) Symbolic emergent spatial. (**d**) Symbolic partial-spatial. (**e**) Symbolic spatial. (**f**) Integrated spatial

ture, symbolic non-spatial structure, symbolic partial-spatial structure, symbolic spatial structure, and integrated symbolic spatial structure. Although only the four youngest children (3 year olds) produced the first two of these categories, nevertheless, one child appeared to be drawing the Beebot (commercial robotic toy) (Fig. 9.2a) with reasonable spatial arrangement and some movement by a line and two additional circles. The drawings of two lines labelled emergent spatial structure (see an example in Fig. 9.2b) may have shown the child's dynamic visuospatial reasoning by the physical order in which they were drawn. It should be remembered that the child's ability to draw to represent their thoughts might lag behind their spatial thinking as found in the Count Me into Space project where a child could verbalise the structure of four squares covering a larger square but could not draw it and knew it did not represent the image in his mind (see also other examples in Chap. 2).

One 4-year-old child's drawing was classified as symbolic emergent spatial structure. His representation indicated the number of steps and a turn but placed these symbols side-by-side so that there was no indication of the robot's movement path (Fig. 9.2c). The most common type of drawing for 4 year olds (5 of the 6) and 6 year olds (11 of 19 actually drawn) was classified as symbolic partial-spatial structure. These demonstrate an understanding of the robot's movement and the use of symbols to demonstrate movement. In Fig. 9.2d, the child uses the symbols of a straight line to indicate movement forward and a curve line to indicate a rotation. Another child has used arrows to indicate three steps forward, with the number of arrows indicating the number of steps taken. The children's use of spatial structure is classified as partial as the step length and the angle of rotation is indicative of the movement rather than structured with a measured or good estimate of the angle of rotation.

The next two drawings classified as symbolic spatial structure represent sufficient information in themselves to convey the movement and direction that the robot took with evidence of the programme steps (Fig. 9.2e). These were produced only by 5 year 1 students (around 6 years old). The last category of representation is integrated spatial structure produced by three children of this older group. They show evidence of integrating a representation of programming (for example correct number of equally sized steps and correct direction for turns) and incorporating programming elements in a coherent manner with the use of both symbols and spatial structure (see Fig. 9.2g). Interestingly these children integrated their symbol for turn (like the dot) into other representations during the project.

Like any study attempting to classify diagrams into a fixed set of structures, some drawings were not easily classified. This indicates the diversity of visuospatial reasoning in their robot action, their thinking, and their drawing. Nevertheless, the study indicates that young children are reasoning visuospatially. More interesting are the videotapes of children problem solving in specific ways from easy movements such as make the robot move backwards to make the robot move forward three steps and to move forward and rotate. The children could respond to these tasks confidently and without need for multiple attempts or using metacognitive strategies.

The more complex tasks of programming the robot to move in a square and to move through "house" tasks which required the child to move from the home position in specified ways (see Highfield & Mulligan, 2009 for further information) presented an opportunity to observe children's use of problem-solving strategies and tools. Frequently children required multiple attempts and they used embodied action and gesture to problem solve such as using hands to indicate the steps they were considering, using the toy to model the planned movement, using whole body action to act out the steps for the toy to move or using symbols such as arrows to plan steps or movement. For example, there were 20 examples of children pointing to a position on the mat, 29 instances of them using their hand to iterate steps and plan length, and 40 instances of children moving their hand in an arc or sliding motion to indicate general movement or rotation without indicating distinct steps. These were embodiments of visuospatial reasoning. In 56 examples, either the eyes or head were used to "point" or indicate steps of movement also indicating visualising reminiscent of the way children learn to count and count on in their heads for early addition. In eight cases children used the toy to act out and plan movement.

One interesting occurrence in the classroom play with robots happened when 4-year-old children communicated in their pairs by observing and recording on their diagram a hook shape similar to that used by another pair. This was reminiscent of the problem solving that occurred in Owens study (Chap. 2) in which groups would use what other children were doing to assist their heuristic of assessing their own work or as Goos recorded above when students were working with dynamic algebra systems or the architecture students (Chap. 5) said that they asked opinions of their friends or looked at what others were trying to do to assist their visuospatial reasoning to get started with problem solving or consider the aesthetics of their paper sculpture. In the last case, they noted that others used cultural practices, and they subsequently, used cultural practices. In a similar way, children observing others in a digital technological classroom illustrate a cultural affinity to that kind of classroom, an ecocultural context.

Yelland and Masters (2007) articulate three types of collaborative scaffolding: cognitive, technical, and affective, and demonstrate that children who were scaffolded using these techniques demonstrated more sophisticated strategies in solving problems. Effective teacher cognitive scaffolding includes ensuring that the children have understood the task and are utilising and articulating the specific strategies, intervening at appropriate times to assist students with a difficulty for which they need a little piece of information, and in larger more formal classrooms encouraging children to share at different steps in the inquiry (McCosker & Diezmann, 2009; Williams, 2008). Thus the role of the teacher in the ecocultural classroom in the digital age has an important role just as Elders in an Indigenous community.

Highfield and Mulligan (2009) provided a list of ways in which robots could be used to establish processes and concepts in geometry and measurement in particular. These are shown in Table 9.1. The use of the robot can facilitate children's visuospatial reasoning and learning.

Highfield and Mulligan note:

> In this project it is significant that the children engaged in multiple mathematical processes concurrently and sequentially; and they demonstrated perseverance, motivation and responsiveness to these tasks that would not usually be evident in their regular programs. (Highfield & Mulligan, 2009, p. 27)

Educational robotic application (ERA) principles (Catlin & Balmires, 2010) for effective learning are grouped into three areas pertaining to technology, student, and teacher.

- Technology should demonstrate a range of intelligent behaviours, interact through a range of semiotic systems and use embodiment, enable the student to learn through meaningful interactions situated in space and time.
- Students should have engagement fostered, be able to engage in sustainable and long-term learning and be able to personalise the robotic learning experience.
- Teachers should be able to access and demonstrate effective pedagogy, present tasks that intersect with curriculum and assessment opportunities, ensure equitable access to the technology, meeting the practical needs of organising and delivering educational opportunities.

Table 9.1 Uses of robots to develop visuospatial reasoning for concepts

Spatial concepts	Capacity: Creating and measuring space that is large enough for the toy to move through (such as a tunnel) or fit inside (such as a garage) Angle of rotation: Exploring the rotation of the toy as a pre-set 90° angle, creating pathways that utilise a 90° angle Directionality: Examining concepts such as forward, backward, rotate, left, right, and positional language Position on a plane: Using increasingly complex language, "over there" becomes "in the far left corner". Using terms such as over, under, beside, through, near, and far Transformational geometry: Exploring concepts such as rotation and linear motion
Measurement	Informal and formal units: Using informal units, such as hands, counters, blocks, or the toy's length, and formal units such as measuring tapes to ascertain distances and assist in creating programmes Identification and iteration of a unit of measure: Using the toy's pre-set step as a unit of measure, when moving the toy; using hand and eye gestures as place holders in measuring distance Direct comparison: Using the toy's length to compare directly the distances needed to complete a pathway
Structure	Grid: Developing and using grids showing the toy's step length to assist in planning and developing programmes Gesture and movement: Using gestures and body movement to indicate and imagine the structure of regular steps, For example, when asked how she knew what the programme required, a child responded "I imagined where the steps would be"
Number	Perceptual and figurative counting: Engaging in both perceptual and figurative counting to ascertain the number of steps required to complete a given pathway Comparison of number: When comparing programmes or movement pathways the children frequently compared number; for example: "I went eight forward and you only went six forward and so mine went further"
Problem solving	Estimation: Predicting and estimating the number of steps required to complete a pathway; examining the estimation to assess reasonableness before programming Reflecting: Observing a programme, reflecting on attempts, and making the changes required Trial and error: developing confidence to trial a programme, even if incorrect and identifying errors Recall of prior knowledge: recalling prior knowledge and skills to apply in programmes Investigating multiple solutions: Predicting and developing multiple solutions to tasks; for example, travelling clockwise, or anti-clockwise Evaluating solutions: Examining the efficiency of a programme to decide if it was most effective
Representation	Semiotic understanding of symbols: In order to programme the robot to move the children needed to develop an understanding of what each symbol meant. The forward arrow meaning one step forward, arrows to the left or right meaning rotation (not movement to the right) Constructing and recording programmes using symbols: After completing a programme the children represented what they had done in the "robot diaries". This required learners to develop a symbol system representing their programme

Note: Source—Highfield and Mulligan (2009), p. 26

Although not yet widely adopted Catlin and colleague's ERA principles are most relevant to the design and pedagogic affordances of a broad range of robotics and robotic toys.

Thus we find that children's ecocultural context in the digital age influences not only their ecocultural identity but their self-regulation in terms of both affective and cognitive processes and hence responsiveness. This responsiveness is assisting in establishing their ecocultural (digital) mathematical identity. In turn they were influencing each other in the classroom. The role of the teacher who has also established an ecocultural (digital) mathematical identity in this digital context is also critical.

Visuospatial Reasoning in the Digital Age Taking Account of Ecocultural Contexts

It is no wonder then that a number of researchers have used computer technology to engage Indigenous and disenfranchised students. Eglash (2007) has prepared a number of different programmes to engage students with pleasing results. Brown (2008) has also carried out a study in Australia emphasising

> Mathematics programs that accentuate Aboriginal students' life experiences and contexts bring relevance to their learning, thus providing purpose and in turn increased levels of motivation and engagement. Mathematical modelling and problem solving can inject curiosity into what is sometimes considered by students to be a boring subject: when the two are properly combined, they can improve students' attitudes towards mathematics (Falsetti & Rodríguez, 2005; Brown, 2008, p. 95)

Brown's study involved urban Indigenous Grade 4–7 students (primary school) in Queensland where cyclones are becoming more prevalent and have always been a concern. She utilised visual and written texts including graphs about cyclones and chocolate. Students in groups participated well saying they had a job to do and the mathematics was genuinely useful, and some shared the work, but the mathematics they were utilising, they did not necessarily recognise as themselves doing mathematics at the time.

> Students are offered a variety of modes to deliver their findings and indeed some students have requested to formulate their own. It is this level of student interest that indicates that mathematical modelling can be perceived by students to be a productive and worthy enterprise. (Brown, 2008, p. 97)

Thus we see a sense of self-regulation, ownership, and identity with the requirements of the task, not necessarily seeing it as mathematical. Interpreting the visuospatial representations was given a context of relevance to the students in their ecocultural environment.

Simulations are a digital age tool that can encourage visuospatial reasoning but also empathy for the tools and for the sources of content (Holton, 2010).

Holton reminds us that learners' views about learning from the computer and beliefs about control are critical in self-regulation and working with digital media. Jonassen (Jonassen, 1999; Jonassen et al., 1999) whose work was critical for establishing the model in Chap. 1 (Fig. 1.2) noted the importance of information and computer technologies facilitating meaningful learning experiences that were active, constructive, collaborative, intentional, complex, contextual, conversational, and reflective. These aspects all interact with each other. Thus students will create visuospatial representations but also discuss these so they have shared meanings. Contextual experiences take account of the ecocultural contexts.

Eglash's work on culturally situated design tools (Eglash & Rensselaer Polytechnic Institute, 2003) including the VBL (virtual bead loom) is established on a careful discussion of the ecocultural background from which the digitized designs are linked. So, for example, he discussed the extensive use of four-fold symmetry in Native American cultures for the VBL included on the webpage.

> Before reading the text, teachers can ask students to look at the designs and describe them; such discussions offer opportunities to introduce symmetry as a term and concept. The text describes, (as he does in the paper), how four-fold symmetry is a deep design theme in many Native American cultures, and is evident not only in a wide variety of native arts, but also indigenous knowledge systems such as base four counting, four-quadrant architecture, the "four directions" healing practice, etc. A second web page shows how such structures are analogous to the Cartesian coordinate system. Finally, the webpage introduces the Native American bead loom as another example in which we find an analogue to the Cartesian grid. (Eglash, 2009)

Not only can students create given designs but from their creative design on the virtual tool, then can recreate a real example on a bead loom. Eglash noted

> There are three pedagogical frameworks that can be used with VBL. In application/reinforcement we start students with the task of simulating one of the original beadwork designs. Teachers have reported success in using this software for teaching Cartesian coordinates, reflection symmetry and its relation to Cartesian values, numeric aspects of translation, and other subjects. In structured inquiry specific math challenges can be proposed by teachers: developing rules for the reflection of polygons about the axis, numeric descriptions for color sequences, etc. For example, teacher Kristine Hansen at the Shoshone-Bannock reservation school had students create a rectangle in quadrant I (the positive-positive quadrant), and then apply the following:
>
> 1. Reflect your rectangle into quadrant II with the following transformation (x,y) → (-x,-y)
>   Students then created transformation rules to place the rectangle in other quadrants. Doing this with asymmetric triangles might be even more effective since it would help visualize the reflections. Another exercise carried out by Hansen:
> 2. Program a green isosceles triangle at the bottom of the screen. Use the transformation (x,y) → (x,y+5) to translate your triangle up 5 units. Continue to iterate this translation by translating your last triangle up 5 units until you reach the top of the grid.
>   This was assigned in early December; she reports that she had intended that the students create a Christmas tree, but to her surprise the students modified the assignment and closely overlaid the triangles using a multitude of colors, creating what she describes as "the feathered bead pattern we see in a lot of the beadwork here on the reservation." This indicates that one advantage to this more open-ended approach to ethnomath is that it lends itself better to "appropriation" (Eglash et al., 2004), thus offering a more constructivist-based learning environment in which students' cultural sensibilities can be used as a bridge to math education.

> Finally there is guided inquiry, in which students chose their own challenges. For example, one student of Puerto Rican heritage decided to create a beadwork image of the Puerto Rican flag, which includes an equilateral triangle. At first he tried to create an equilateral triangle by having the same number of beads on each side, but that did not work because the beads along the diagonal are spaced farther apart than the beads along the vertical or horizontal. He finally arrived at a solution by using the ratios of a 30-60-90 triangle to arrive at a discrete approximation (Fig. 8); a challenge that he might have balked at had it simply been assigned to him. (Eglash, 2009)

Adam (2010) went back and discussed possible food covers with the weavers. Eglash (2009) went back to the Shoshone-Bannock to find the algorithm they used and built that into his programme. "Using iterative rules—e.g. "subtract three beads from the left each time you move up one row." It worked better than the standard computer algorithm.

Moving Forward

This chapter has outlined some research that has considered the value of digital technology in encouraging visuospatial reasoning in problem solving. The digital age provides digital tools that can be engaged to enhance visuospatial reasoning as students learn mathematical processes and concepts. The ecocultural background is significant for the students of today and influences the self-regulating student in terms of affective and cognitive strategies. The impact of the classroom context is evident. Furthermore, Eglash and others have shown how there can be a synergy between ecocultural Indigenous contexts and ecocultural digital-aged contexts. There is evidence to show that both engage the students' self-regulation and visuospatial reasoning.

The last chapter encapsulates the arguments presented throughout the book providing a synthesis of research from across the world, across time, and across paradigms of psychology, anthropology, and psychological education and critical philosophical approaches to education.