Chapter 8
The Importance of an Ecocultural Perspective for Indigenous and Transcultural Education

Kay Owens, Marcos Cherinda and Ravi Jawahir

Give a child a set of mathematical problems to be solved in ten minutes and graded for accuracy against the work of others, and the resulting performance may be dismal. Put the same child in a situation in which the problems are made meaningful, the same mathematics is used, and the solutions matter, and the child's performance can soar.

(Kilpatrick & Silver, 2000)

In the mathematics classroom…an ethnographic approach can give valuable insights into education and knowledge technology… towards how knowledge of pattern is generated and reproduced as carrier of thought of a particular kind.

(Were, 2003, pp. 25–26)

The Challenge

While there has been considerable work on visualising to assist children to learn early arithmetic (NSW Department of Education and Training, 1998; Wright et al., 2006), and some work on using patterns to begin early arithmetic for Indigenous Australians (Warren, Cole, & Devries, 2009), there is still a significant challenge for this book. An argument is now presented that revolves especially around the ecocultural perspective of education for visuospatial reasoning in geometry and measurement and how this reasoning assists students in problem solving and in maintaining a cultural identity as part of their mathematical identity.

Lehrer, Jacobson et al. (1998) suggested that quality instructional design required input from researched models of student thinking, classroom-based collaborative research, parents as partners, professional development workshops, and teacher

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authoring. Parents who are enabled to recognise their ecocultural mathematics will influence curriculum if they become partners. In their study,

> teachers went from posing tasks in isolation to developing sequences of tasks that provided children with opportunities for progressive elaboration of core concepts … on wayfinding, mapmaking, and Logo … In particular teachers increasingly emphasized representational fluency; children invented or appropriated multiple forms of representation. (Lehrer, Jacobson et al., 1998, p. 176)

Children reasoned from physical movements to recognise equivalent transformations and to generalise from their actions. Such questions as "do you think this is true all the time?" encouraged students to visualise and reason visuospatially. For example, in deciding whether a particular rectangle was larger than another, one child folded the thinner rectangle (6×2) in half and measured the other one (4×3). From this, she generalised to claim the 1×12 rectangle could go into the first one too and proceeded to fold it into four and show how the strips covered the 4×3 rectangle. From professional learning to developing substantial resources for teaching from a cultural basis, a strong identity and community of practice developed in a series of workshops in which student reasoning was the focus. The keys were building on children's informal knowledge, promoting their invention, having classroom conversations for understanding, and teachers orchestrating the curriculum tasks and tools. These young children's visuospatial reasoning was an entrée to conjecture and proof.

However, the challenge is to provide examples related to visuospatial reasoning from various cultural practices. The weaving boards developed by Cherinda are a well-researched example of an ethnomathematical activity in practice (Cherinda, 2001) along with many examples from origami (but not covered in this book). I have provided several papers on using just paper and string to teach mathematics as a result of my transcultural experiences (Owens, 1996a, 1998b, 1999a, 2001b). There are other examples within the realm of paper-folding which link to other African practices (Gerdes, 1999). Other examples will come from object making, travel and position, and various topological visuospatial reasoning activities.

In effect, much of the change towards an ecocultural perspective comes from face-to-face discussions between teachers and community members in order to develop a hybridity of thinking that will authentically link the ways of thinking of the community and the ways of thinking for a global education (González, Moll, & Amanti, 2005). In the future, students will be facing new and challenging problems for which a maintenance of ecocultural ways of thinking can provide a sound basis.

Continuities in Education Between Community and School

Many studies have considered the transitions and continuities between the contexts of learning mathematics (Owens et al., 2012) but the forces of globalisation work to homogenise mathematics curricula and negate the differences (Atweh, Barton, & Borba, 2007). Nevertheless, there are studies that have recorded mathematical activities incorporating ethnomathematics into formal mathematics (Eglash, 1997, 2007) ensuring that diverse mathematical thinking is not lost in school. These include studies by Cherinda (2001, 2002), the University of Goroka coursework projects (1995–2008) (some of which are referred to in this book under their authors' names), Gorgorió, Planas, and Vilella (2002), McMurchy-Pilkington and Bartholomew (2009), Meaney and Fairhall (2003), and RADMASTE (1998). Some of these studies have resulted in actual curricula recommendations (Jannok Nutti, 2008; Lipka & Adams, 2004; Litteral, 2001; Pinxten, van Dooren, & Harvey, 1983).

Providing a culturally appropriate curriculum is a challenge for small communities as found in PNG (Litteral, 2001). Nevertheless, some efforts are being made to ensure that mathematics of small communities are used in education as determined by the education reform (Matang, 2008; Matang & Owens, 2006; Owens & Kaleva, 2008a, 2008b; Paraide, 2003). Similarly in Alaska, funding has been provided for the establishment of culturally relevant mathematics units. Lipka and the Yup'ik community have established units of work that begin in the activities of the community. Students might build models of fish racks or discuss the importance of locating eggs on the island. They have shown that students' attainment on mathematics has increased significantly (Lipka & Adams, 2004).

According to Clarkson and Kaleva (1993) one crucial reason for failures in implementing curricula that consider cultural aspects is an unwillingness by many experienced teachers to change their ways of presenting mathematics lessons that have become routine for them. Hence change in the classroom is difficult, leaving aside the bigger political difficulties inherent in any educational change. Successful implementation of any curriculum reform in the classroom focusing on culture will depend at least on (a) the role of the teacher under the reform, (b) teacher beliefs and values about the new curriculum reform, (c) teacher background knowledge in mathematics (Matang, 1996), and (d) involvement of community in the school curriculum and learning activities (Department of Education Employment Workplace Relations, 2009; Yunkaporta & McGinty, 2009). We would argue that these four points must be addressed.

One way to meet this change is for teachers to begin to utilise students' mathematical spatial experiences, which will be set within their own cultural experiences (ethnomathematics), gained from everyday encounters (their informal education) as the basis to teach school mathematics (Trudgen, 2000). We believe this will result in students becoming active participants of the information-sharing process within their classroom. Clearly in this scenario the responsibility of the teacher changes from one that is imparting knowledge in an authoritarian manner to one that seeks to create a learning environment that promotes meaningful and interactive mathematical discussions not only between teacher and students, but also between the students themselves. This no doubt will include the students telling the rest of the class mathematically related family stories (Matang, 2003; Matang & Owens, 2006; Owens, 2000b; Trudgen, 2000; Yunkaporta & McGinty, 2009). Similar suggestions which could be followed have been made in relation to mathematics and language (Clarkson, 2009).

In a comparative study of transcultural education in four countries: Sweden, Australia, PNG, and Yemen, I concluded that variation in the contexts highlighted the following themes in teaching:

- Aspects of cultural context relevant to mathematics
- Meeting language differences in different ways
- Maintaining culture in different ways
- Teaching in a cultural context
- Teaching mathematics in a cultural context
- Having an emphasis on national values
- Using national language appropriately
- Developing context-specific strategies for diversity (Owens, 2008)

If the impact of students' informal education outside the classroom, and the notions of ethnomathematics are to be taken seriously, then teacher education programmes must also change. Pre-service teachers will need to be given the opportunity of in-depth investigative studies of the mathematics content knowledge that they will teach. This mathematical background will give them confidence to approach the teaching of mathematics within the immediate socio- and ecocultural contexts of school students, as has occurred in the Luléa University programme in Sweden (Johansson, 2008), the secondary education projects in PNG (University of Goroka Students SMAC351, 1998–2007), and the elementary school project on improving mathematics education (Bino, Owens, Tau, Avosa, & Kull, 2013).

Language-Based Activities in Multicultural Classrooms

Children in multicultural preschools assist each other very slowly to speak in English but factors such as the table mix of languages, the amount of English understood and spoken at that stage, the child's choice of playmates, the children's personalities, and non-linear rates of learning, influence the progress made by children (Fassler, 2003). One recommendation developed from this study was that teachers need to spend time talking with each group of children.

There are a number of language issues that arise in terms of visuospatial and geometric concepts. Words (morphemes) like verbs for actions and words like nouns for observable objects such as blocks are best used so students may learn to compare, estimate, and measure before they appreciate attributes of shapes. Even so, words that sound similar need careful pronunciation and experience e.g. for English "side" and "size"; "estimation" and "evaluation", "triangle" and "rectangle". Words may have two meanings—mathematical and general e.g. English "area" means a place in general usage but the measurable space inside a 2D boundary in mathematics (Owens, 1996b). Difficulties arise with what accompanies the idea implicitly e.g. length when referring to volume (Owens & Kaleva, 2008b), not dissimilar to the linear scale on a measuring jug but requiring considerably more visuospatial reasoning from experiences. Some concepts have many constructs and representations. For example angle and fraction require considerable language and experience for students to grasp. Difficulties arise with prepositions as discussed in Chap. 4 on deixis but numerous hidden meanings in mathematical expressions may be overcome by using multiple language patterns (Davis, 2009).

Oral work by both teacher and students needs to be slow, purposefully repeated but not rapidly or loudly repeated. Oral work is often facilitated if teachers use whole class discussion, followed by paired or triple group discussions before individual thinking or work. This should then be followed by further sharing in the small group and then in slightly larger groups prior to any further whole class discussion so that all children have a chance to listen, talk, read, and write. Teacher's and other students' sentences need to be simple. Students should be allowed to code (language) switch. Code switching may occur between every day and more precise mathematical language; between dialects; between lingua francas and home languages and various national languages. Switching languages assists learning if knowledge is constructed rather than kept as unrelated ideas. Students of the same language group may talk in their own language to explain to each other while students from other groups can try to explain as their gestures are often beneficial in helping another student. Students help each other and speak more mathematics. Gaining understanding by playing with words and using bilingual facility will assist in constructing meaning. For example, a grade 2 student with English as a second language was able to develop the meaning of "bigger angle" by realising it did not mean "sharper" but the opposite "opening wide" (Owens, 1996b). The words were switched for the different visuospatial imageries through reasoning and listening to and watching the teachers' explanation and the actions of other students.

Visuospatial Reasoning in Metaphors: Selection and Relevance

Many metaphors are used in mathematics and teachers need to explain the metaphor. Teachers should avoid using western colloquial metaphors if these are not used in everyday talk in the community. However, there are often rich metaphors in cultures that can be used in mathematics. We have already shown that the slope of a roof is a good introduction to work on angles. Visualisation (images in the mind from action on objects and pictures) provides a major context for oral learning. It requires students to make explicit in words what their learning is in addition to their learning by observation.

Difficulties with a new language do not mean students have a lower ability to reason and furthermore the metaphors and experiences that can be drawn on by students are all the more critical for mathematics education. Students should be exposed to a range of tasks and not just easy ones and they should problem solve and speak about mathematical concepts to enhance the transference of skills and metaphorical language. Visuospatial representations either in drawings, gestures, or metaphors are important. Language games assist students to speak and listen to mathematical terms e.g. drawing what is read in the school language or speaking this language to explain the drawing. For example, Murray provides a range of activities for teaching children with English as a further language (Murray, 2011).

Some mathematical areas are often stronger in other cultures e.g. circle geometry in Arabic art than in a western school culture, topology in Navajo or string designs (Vandendriessche, 2007). Furthermore, estimation skills (not necessarily matched by language) are often strong in Indigenous, subsistence cultures, and among artisans (Millroy, 1992; Owens & Kaleva, 2008a). These can be important for the planning of curriculum. However, teacher education also requires addressing.

PNG Secondary Teachers' Ethnomathematics Studies

In Chap. 5, I discussed the mathematical thinking involved in different bridge-making experiences in PNG (Fig. 5.7). In Yambi 's UoG (2004) project, he drew a bridge (Fig. 8.1a) and related a number of school mathematics problems to the bridge. In particular, Yambi related the making of the bridge and the bridge diagram to assist students to visualise the context, the triangles, and the trigonometry required to solve the story problems, thus assisting the students visuospatial reasoning for trigonometry (see Owens, 2014). Yambi also used the wave of the vines holding the platform to the hand rails as an example of the sine wave. While this might not quite fit a sine wave in practice, it was a wonderful way of modelling a sine wave (see Fig. 8.1a). Another teacher, Imasa from Pindiu, Morobe, also linked the sine wave to the curving of the rope used to bind split bamboo to two sticks to form a platform for a bandicoot trap.

Another teacher from Hela Province, PNG, noted that the framework for making a wig as shown in Fig. 8.1b was similar to a parabola (Piru, 2005). This allowed Piru to encourage students to plot points and explore parabola on Cartesian coordinates. Piru emphasised the importance of accuracy in measuring so the wig was a snug fit for the dancer's head. They used a small unit of length called the *ki*, related to the width of the finger. (See a wig from another cultural group in Fig. 5.21c.)

Martin who speaks Magi (Mailu), Central Province, PNG, illustrated the connection between the traditional making of fishing nets from bush string and the tying of knots to form squares (Fig. 8.1c). He noted measurement of both length and area (in Chap. 5 we noted that the idea of an area unit was not common but there were examples in culture of area units that could be used). The squares could be larger for different fish. He also mentioned other links to geometry such as parallel and perpendicular lines, upper and lower limits (lines), cylinders, and two-dimensions to three-dimensions in making a net for the weights (mostly stones). Paraide (2010) discussed Indigenous knowledge in terms of currents, swells, and fishing weights for baskets and nets and links between culture and school mathematics.

In the five stones game (called knuckles in some countries) played in Finschaffen, Morobe Province, PNG, and many other areas of PNG, the stones are seen in a visuospatial arrangement by David (2007) although scattered. From the arrange-

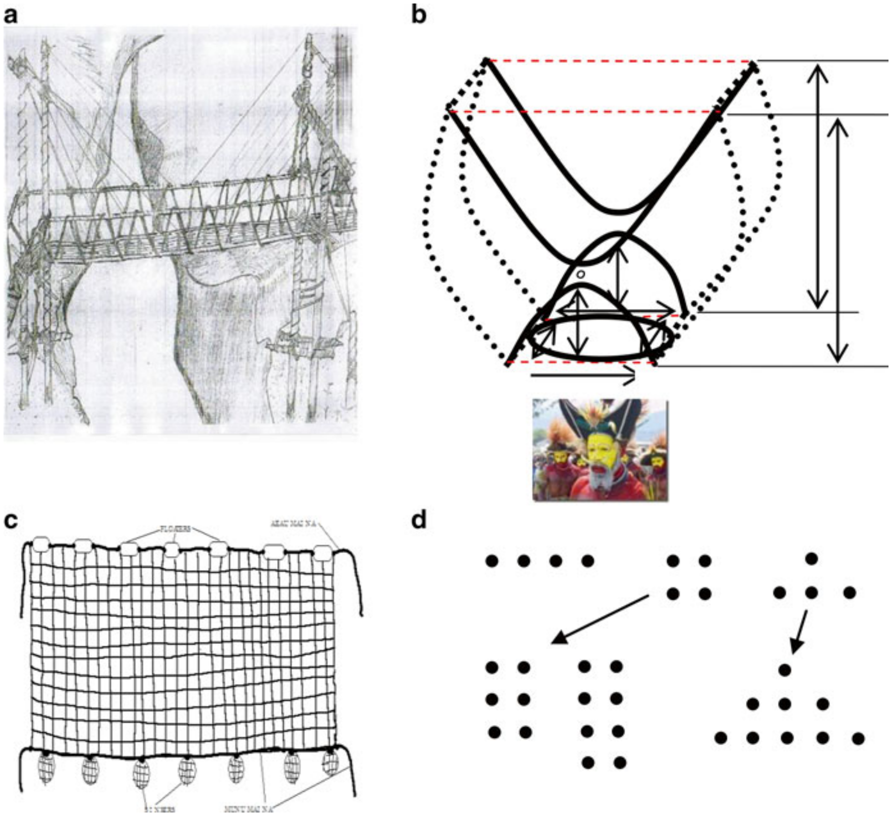


Fig. 8.1 Examples of visuospatial reasoning in ethnomathematics for secondary schools. (**a**) The vines from the handrail to the walking platform provides a metaphor for sine wave (Yambi, 2004). (**b**) The wig shape is outlined by the *parabolas* and *dotted lines*. The *head circle* is marked (Piru, 2005). (**c**) Fishing net ecocultural visuospatial representations (Martin, 2007). (**d**) Visuospatial arrangements in the five stone game: first line illustrates how stones are to be swept up (singles, twos, three); second line illustrates how rectangle and triangle numbers can be extended to give new patterns (David, 2007)

ment of the stones in pairs and a triple further number patterns can be generated (Fig. 8.1d). There can be a step-by-step increase in the size of the square (as well as rectangle) and a triangle formed. She provided questions such as:

1. What shapes can be formed with four stones?
2. What other mathematical properties can be formed with four stones?
3. When a square is formed. We can increase the size of this square by adding more stones. How many more extra stones do you add to increase the size of the square to its next biggest size?
4. What do you get after adding those extra stones in Question 3?

and she encouraged students to draw tables to see the patterns and then to form algebraic statements.

Figure 8.2 based on John's (2007) report illustrates how the teachers were linking ecocultural mathematics and visuospatial reasoning to school mathematics. Each of the points provided by John in this table was supported by example problems for the students. He chose the making of an Asaro mudmen mask (see Fig. 5.21a). While the syllabus and textbooks gave some examples of the topics related to typical PNG experiences, the project encouraged teachers to be creative and use visuospatial reasoning themselves but related to ecocultural contexts. As Kono (2007) said,

> this project only serves to give some clues to practicing or prospective mathematics teachers to be resourceful by incorporating cultural activities as concrete examples in teaching mathematics rather than abstract borrowed western ideas.

Julius (2007) is typical of the way in which the ecocultural aspects were integrated into their mathematical presentations (Chap. 5, Figs. 5.13c and 5.14a, c give background details). Figure 8.3 is from Julius' report. Further examples are found in Owens (2014).

These teachers were perhaps for the first time posing problems and finding solutions. They were really motivated to achieve a high standard over a period of time. Thus, as portrayed in Fig. 1.2 and discussed in Chap. 7, the context which is not only the culture and environment but also the project encouraged the teachers to provide a transition between cultural mathematics and school mathematics. The ecocultural context influenced the self-regulating learner which in turn developed their visuospatial reasoning, diagrams, and other representations. In turn, identity as a mathematical thinker was beginning to develop. The analytical, rote-learnt school working out was not always accurate in the examples but in the high majority of cases this was adequate and diagrams were frequent (Owens, 2014).

Furthermore, the ecocultural context was a realistic focus for learning. de Corte, Verschaffel, and Eyende (2000) showed the importance of structuring the problem solving in mathematics by encouraging heuristics such as drawing a diagram, planning, and checking progress with the problem. The ecocultural context for these teachers were real world requiring similar ways of problem solving but not in somewhat contrived questions but in the daily real lives of people in a subsistence culture closely related to neighbours and the land. Teachers spontaneously represented their visuospatial reasoning mostly through diagrams but also through description of dynamic action in story form providing their mental visuospatial imagery and reasoning.

Thus consideration of visuospatial reasoning from an ecocultural perspective is important for extending our understanding of the self-regulating learner. These projects illustrated how self-regulation was more than metacognition but also realistic, involving, requiring guidance in structuring the problem context and intrinsically motivating as cultural connections for mathematics became evident.

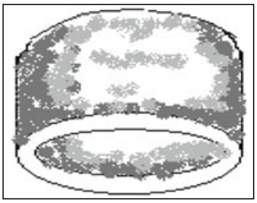
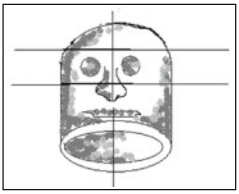
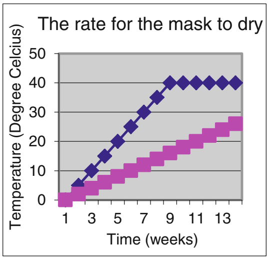
Stage of Cultural Activity	Mathematical Ideas/topics Derived from the Cultural Activity	
Clay and water 	* Ratio and Rates - Ratio Simplifying Using proportion - Rates Time and rates Rates graph	* Circle and Volume - Circle Circumference Area of circle - Volume (prisms) Volume of cylinder * Surface Area
Semi-oval bamboo skeletal shape 		* Measurement Length * Circle Circumference Area of circle * Geometry Shapes
Semi-oval clay head 		* Measurement - Area Length Width * Circle and Volume - Circle Circumference Area of circle Surface Area of solids - Volume (prisms) Volume of cylinder
Face of the mask 		* Symmetry and Construction - Symmetry Line symmetry Parallel lines * Geometry - Construction Bisecting a straight line Angle of 90° Perpendicular from a point on a straight line.
Drying in sun (small dots) or heated house (larger square dots)		* Measurement Time * Rates Time and rates Rates graph

Fig. 8.2 Links between ecocultural mathematics and school mathematics (John, 2007)

This project confronts the idea of Eurocentrism-i.e. the widespread prejudice about mathematics as being predominantly of European origin, which can be used and manipulated as deemed best according to their interpretation. This write-up gives a light by way of illustrations and proofs using cultural designs of pitpit wall, as well as establishing a common understanding of 'cultural conceptualization' in the context of teaching in classroom. … Within the diversity of rich cultural heritage is embedded the complex and varied sets of mathematical knowledge that, I would say, are not fully exploited to this very day. … We look at these mystique (sic) geometric forms and patterns of such traditional activities and posed the question; why do these materials or products possess the form they have?' We are all part and puzzle (sic) of these cultural activities and it reflects some sort of mathematical knowledge, experience and wisdom. … I will use the designs of pitpit wall pattern as illustration to determine the sum of the interior angle of polygons. Special reference will be given to exterior angle of a triangle and sum of interior angle of a quadrilateral. … Traditionally, these designs are performed by skilled and experienced person(s).There are, in fact, eight different types of designs identified in the Sinasina area namely; X-shape, rending, zigzag, modified, basic pentagonal, hexagonal shape, starry, and octagonal shape. I will use the last four patterns as examples to determine interior and exterior angles of polygons (i.e. triangle & quadrilateral), as well as showing the proof of the formula of the angle sum of polygons. … traditionally termed; *kewah, egleh, gamlageh, bongeh*, for starry, pentagonal, hexagonal and octagonal shapes, respectively.

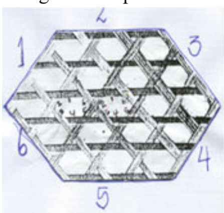
a) If we consider the starry shape it seems to look like a quadrilateral. By drawing lines on the edges, as illustrated below, we would come up with the kite

* Starric (sic) pattern from traditional Sinasina *'kewah'*

b) Inscribed in the basic pentagonal weave are three shapes, namely triangle, rhombus (sic, means kite) and pentagon

* Basic pentagonal weave from Sinasina *'egleh'*

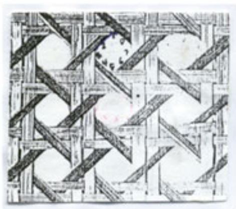
c) Either looking at the small patterns inside the design or the sides derive the octagonal shape



'Gamlageh'

* Hexagonal shape from Sinasina

d) If we count the sides of the pattern inscribed in this pattern we would identify eight sides.



'Bongeh'

* Octagonal shape from traditional Sinasina

Fig. 8.3 Tabare weaving shapes from Julius' (2007) project

PNG Elementary School Teacher Education Study

Having noted the impact on the secondary teachers through this project, it was important to see how elementary school teachers could incorporate an ecocultural perspective to mathematics and how that might impact on their visuospatial reasoning. Elementary schools are built and maintained by the community. Teachers are grade 10 graduates who speak the language of the community. There is a transition from local language to English during elementary schools. Figure 8.4 shows typical rural schools in PNG. In 2013–2014, a design-based study in PNG, has elaborated several principles for teachers that can be applied across PNG's 850 cultures and languages and ecologies, remoteness, and experience. It applies no matter what the use of Tok Pisin or the use of Tok Ples provides. Although Muke (Muke & Clarkson, 2011) illustrated that teachers will use all three languages including English to explain concepts in class (see also Setati & Adler, 2000), there was a much more complex picture emerging in practice. "Cultural Mathematics" required other principles in teachers' knowledge.

Figure 8.5 provides the model of principles used to guide workshops. After the first workshop, it was found the learning experiences appropriate for children to learn mathematics needed to be expounded further so the middle principle was elaborated particularly based on research in mathematics education but also incorporating what we had learnt from our studies of PNG cultural ways of thinking mathematically. The principle on the nature of mathematical thinking deliberately moved away from learning facts and is inclusive of PNG mathematics. We were able to supply analyses of language from previous research. Early childhood principles needed enunciated as many teachers were teaching as they had been taught in

Fig. 8.4 Schools in Papua New Guinea. (**a**) Typical bush material school, Morobe, PNG, ~ 1984. (**b**) Group work in Tsigimil Primary School grade 3. (**c**) Elementary school, Atzera language, Binimap, Morobe, 2006

Mathematical ways of thinking
- Links to school mathematics;
- Mathematics is thinking e.g.
  - problem solving
  - Reasoning
  - Fluently applying concepts and procedures
  - Patterning and abstracting relationships

Early mathematical thinking
- Patterning
- Sorting,
- Ordering
- Comparing
- One-to-one matching
- Symmetry
- Recognising equality
- Noticing attributes of groups and shapes
- Trialling ideas
- Posing questions

Activities appropriate for children
- Learning experiences 'in but outside' school;
- Early childhood emphasis: play and inquiry

Learning experiences to promote children's efficient mathematical thinking
- Counting
- See and know
- Visualising
- Recognising pairs and groups
- Describing and classifying
- Arithmetic in language
- Group counting
- Measuring informally
- Locating
- Explaining
- Investigating
- Enjoying challenges

What they learn?
Assess, report, plan

Language resources
- Rich diversity
- Patterns in counting
- Gestures
- Decision-making
- Grammar and meaning

Cultural activities
Extend cultural mathematics
- patterns,
- groupings,
- arithmetic inside counting,
- representation,
- measuring,
- ratio,
- spatial relations ,
- traversing the land and sea
- language for location,
- space and place
- constructing and designing
- building relationships

Cultural capacity and partnerships
- Values: need to preserve cultures and languages of people
- Elders roles for community cohesion

Fig. 8.5 Design of key principles for teacher professional development in *Cultural Mathematics*

primary school and few assessed their children's learning. Finally, the role of Elders in the school was yet to be realised and applied to *Cultural Mathematics*.

A manual outlined all of these principles together with an inquiry model of planning and examples. A stand-alone website was developed with the manual information; example lessons; video examples of cultural activities, children learning and assessment tasks, and teachers' sharing; and other workshop ideas. This was loaded onto touchscreen computers where possible and these effectively engaged the teachers. The workshop began with a welcome activity setting the scene about mathematics and talking mathematics. Then Elders from a similar ecology were shown explaining their cultural activity and mathematics (see Chap. 5 for examples). This stimulated teachers in small groups to discuss some of their own cultural activities and tease out the mathematics involved. The systematic ways of thinking were supported by activity and discussion but systematic ways tend to be visuospatial reasoning about measurement, space, and geometry.

Weekly Learning Plan for Mathematics

Purpose: Children are expected to think and do mathematics through activities linked to cultural practices. Children are expected to have a sense of belonging with the new ideas in culture and school through a good transition that links cultural ways of thinking with school ways of thinking.
Key Ideas: e.g. What is the new pattern and relationship? How does the thinking lead to problem solving?
Prior knowledge: What do they know? How do they think and feel?
Resources: Places to visit; materials for exploring, com paring, measuring, recording, modelling; game cards, spinners, Elders who know the cultural activity
Assessment: Observing ways children try things, what they say, how they problem solve, what they write, what they ask to make clear or to extend their exploring

Day 1
- **Tuning In**
  - Children are motivated, have real world experience e.g. outdoor; listen and participate in a story.
- **Planning to find out**
- **Finding Out**
  - Children observe, notice, compare, measure, discuss mathematical patterns

Day 2
- **Sorting Out**
  - Children discuss, model, compare, make a table, draw a diagram, find same and different,

Day 3
- **Going Further**
  - Children apply to other numbers or another situation, read and discuss the maths book, use symbols, play a game, solve an open problem,

Day 4
- **Making Connections**
  - Children summarise the mathematics, whole class discussion, or story writing,
- **Taking Action**
  - Share at home, solve a real problem.

Day 5
- **Sharing, discussing, and reflecting**
  - Children explain the mathematics, write a maths story, write their own summary, say what new mathematics they have learnt.
  - Teacher reviews and decides what to teach next based on assessment, cultural activities, and syllabus guides.

Fig. 8.6 Design of inquiry method for *Cultural Mathematics* based on Murdoch (1998)

Cultures are quite different across PNG and we deliberately targeted three ecologies for the research: highlands, coastal, and inland in coastal provinces. Starting with culture was a clear way of engaging teachers who are proud of their cultures. The workshop then covered good teaching approaches for young children, what is known from research about how children best learn about arithmetic, measurement, and space and the key concepts in each. A key for bringing together each of these principles in planning learning experiences was the use of an inquiry method. To make it easy, it was suggested that the topic was covered over a week. Although days were allocated to each step, it was made clear that this need not be strictly followed. The inquiry method based on Murdoch (1998) and applied to *Cultural Mathematics* is shown in Fig. 8.6.

Teachers were given example learning plans (see Appendix F for an example) and early readers that related to mathematics topics. Murdoch's steps proved to be a bonus for teachers to bridge the gap between culture and school mathematics. They were able to use it to extend their children's thinking mathematically and to encourage practice of concepts and skills. Since the cultures are rich in visual examples these were linked to research on early arithmetic, measurement, and geometry (Moschkovich, 1996).

Some examples of the teachers preparing learning experiences will help to explain the importance of the visuospatial reasoning within the mathematical thinking of the teachers. The first three examples come from the coastal village of Tubusereia, National Capital District, PNG where Motu is the home language, often with English. In the first example, the teachers of Elementary 2 were referring to the cultural practice of sharing fish. They set up the task of sharing 24 fish between four families (Fig. 8.7e). They knew each family would get fish according to family size. However, their school mathematics at first made them decide it had to be equal shares like division. Through discussion, teachers realised that an open-ended problem that was actually more like reality, was to decide all different ways of sharing the fish. The children responded well to deciding numbers and how they could find other ways of sharing the fish. They began to systematically record. They practiced how to add numbers. All the time, the children were either using the fish they had made from cardboard to explain to each other or they were using the empty number line (which was new to them) to jump to the next number being added on and then decide on the last number to make 24. Note that in practice, the sharing often starts with the larger fish to each family, then the smaller ones (Odobu, 2007). Children used various strategies to make up new sets of numbers such as realising that if they gave 10 and 6 fish, then they needed another 2 numbers that added to 8.

In another class, Elementary 1, the teacher took the tears tattoo, 2, 1, 2 pattern modelled it with steps (large)—2 forward, turn left, 1 forward, turn right, 2 forward; with strips of paper; and with dots on a sheet of paper. The ratio impact was evident to the children for the various activities using the same pattern. The results of visuospatial reasoning of Elementary 2 children who copied a design are shown in Chap. 5, Fig. 5.4. Another teacher asked Elders to make a model garden for yam expecting it to be in rows of two mounds but instead the Elders used the triangular pattern (Fig. 5.6). Nevertheless, the children in Elementary Prep not only asked questions of the Elders (the teacher had prompted children by giving some example questions) but also showed how the centres of the mounds were equidistant using a stick. They also compared the lengths of various yams and put them in order. They showed different ways of comparing and measuring. The open question approach to teaching reduced copying from the board and meant children were using visuospatial reasoning more and more. Questions asked of children after the lessons indicated that they understood well the meaning of half as big again and how to measure informally and to use a smaller unit to measure the remaining length and to give more than one pair of numbers that added to 13, explaining how they reasoned mentally to get another answer. Gestures indicated that they had used the idea, for example, of taking away one visually from one number and adding one to the other number.

In another area where teachers came from a number of different language groups in the mountains and along the Rai Coast of Madang Province, PNG, one group of teachers visually considered how they planted dry land rice and demonstrated using feet lengths (Fig. 8.7c). Then they went further discussing other measuring units for

Fig. 8.7 Bringing ecocultural experiences into the elementary classroom. (**a**) Preparing a dictionary of mathematical terms. (**b**) Trying out the lesson idea of using the body to make shapes following the squares (*diamond*) in the weaving. (**c**) Simulating using steps to measure when planting rice. (**d**) Sharing the making of rope for a bilum and making a bilum relating it to mathematics. (**e**) Sharing fish between families according to need. Recording of children's solutions explaining how they worked from one addition to another with visual supports

length, with both straight lines and curves. Another group first wove patterns including the *diamond* pattern. In going further, while sitting on the ground with crossed legs, they made diamonds with their knees at right angles and used their feet with the man opposite to make another smaller square. Then they discussed the properties of the squares. Other groups of teachers made model houses, models of different kinds of fences, and models of bilums (Fig. 8.7d) but each time discussing the equal lengths of fence posts or different heights of posts for the house for the roof slope, making the right angles, and using the radius to form a circle. One discussion was about the height of the bow and arrows and how to compare them and people's heights, and how to informally measure them. Each time, key ideas about measuring were being emphasised within the activity. Teachers were getting small groups to represent and discuss the key ideas based on their visuospatial representational and mental reasoning. Much of this work was being said in their Tok Ples which was translated into Tok Pisin for me or spoken in Tok Pisin.

The significance of this approach to teaching mathematics is the impact that the design principles have on establishing both teachers and children as mathematical thinkers and identifying not only with cultural mathematics but also school mathematics. The ecocultural mathematical context (see Fig. 1.2) is evident in the mathematical problems, models of cultural practice, cultural tools of measuring and drawing, and valuing cultural practice. The cultural ways of thinking especially about ratio and pattern and size are not being lost. In terms of the model of identity, we see how the visuospatial reasoning within the more open questioning classroom was allowing children to be self-regulated, an important aspect in their mathematical identity but also to use a range of cognitive and affective aspects of learning as indicated also by the model in Fig. 2.17. Teachers and children were goal setting, reasoning (including visuospatial reasoning), planning, self-evaluating by explaining to their peers, reviewing by reflection on the learning, using drawn and mental models, and working out how to present results or their thinking. There was a strong sense of ownership. Furthermore the social interactions and having a go at responding (instead of copying) were relatively new experiences. Thus we can see the beginnings of not only a cultural mathematical identity developing but one related to school. In the case of the teachers they were engaging in mathematical thinking and engaging others while some of the children were doing the same in explaining to peers.

In the workshops, after preparing and where possible trying out their planned learning experiences, the teachers discussed the issues of language and emphasised how language can have real treasures that can be used for explaining the mathematical concepts to children. Teachers considered a number of English words used in mathematics to try to decide on some appropriate Tok Ples words. This was not an easy task and again visuospatial reasoning was used to help make decisions (Fig. 8.6a). Finally we looked at assessment and learning stories. We also introduced a reflective teacher's questionnaire and an interview schedule that teachers could use with a couple of children in their own class. They practiced it on each other and then children. This interview schedule reinforced much of what was being discussed throughout the workshop. Visualisation strategies were assessed as they were in Count Me In Too Schedule for Early Numeracy Assessment (NSW Department of Education and Training, 1998) and Count Me into Space (NSW Department of Education and Training Curriculum Support and Development, 2000). Matang (Matang & Owens, 2014) had modified some of the former in his study and we used simpler and shorter schedules than Matang's but asked some questions on position, shapes, and measurement and questions specifically linked to PNG cultural practices.

Working with Mental and Physical Visuospatial Representations in Africa

For cultural weaving activities to be introduced into school, weaving boards of card with slits and loose cardboard strips were used by Cherinda (2001, 2002). Initially students were asked to copy the pattern and continue it. Children could feel, both tactually and mentally, how twill weaving developed from a cultural experience into visuospatial reasoning related to school mathematics. Questions, such as the following, have been used to stimulate their thinking about weaving designs such as in Fig. 8.8b.

Observe the path of strip L1 (first horizontal *loose* strip). What is the number of the next L-strip with the same path as the path of L1?

Then, the path for L6 is the same as for L2. Consider the set L1–L4 as section A, and the set L5–L8 as section B. What do you say about the appearance of the two sections?

By the already verified repetition of the paths of the L-strips, students can see that the two sections, A and B, are the same. The section B appears as an image of section A when this section is moved down. In such cases the "movement" from A to B (or vice-versa) is called a translation. Students are then encouraged to weave another pattern that shows vertical symmetry. For example, Fig. 8.8b, d makes an attractive design with vertical symmetry or two axial symmetries.

Students are then encouraged to repeat the set which produces a square design (Fig. 8.8b finished) and they can see both axial symmetries through the centre and by turning the board they can note there is a coloured square in each corner. This provides an example of rotational symmetry. For the design in Fig. 8.8d, with a horizontal line of symmetry, students were asked to reflect the design in the weaving. The diagonal pattern in Fig. 8.8c is continued to provide a rotational symmetry design. Each diagonal will have three squares with the zigzag in between. The board can be rotated 180° to show the rotational symmetry.

Then students are asked to find the fundamental block or unit for the weaving and this is then discussed in terms of squares, for example the four columns by five lines in Fig. 8.8d. Then other fundamental blocks are provided and these are used to create intricate repeated designs.

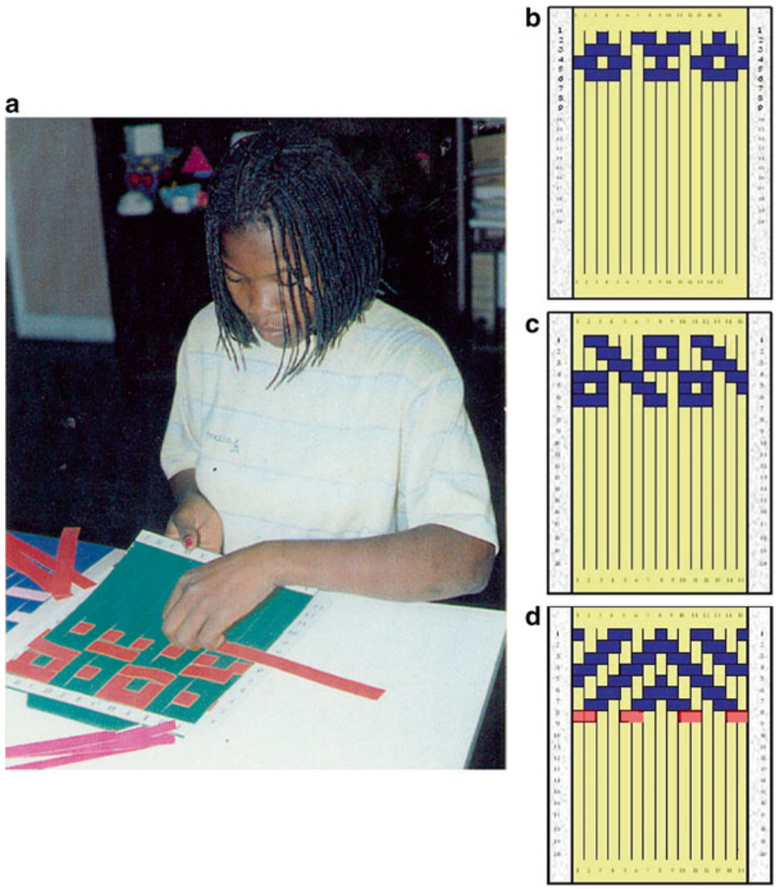


Fig. 8.8 Weaving in Mozambique to encourage visuospatial reasoning in geometry. (**a**) Student motivated to learn in Mozambique. (**b**) Translation experience. (**c**) Repeated to produce rotational symmetry. (**d**) Weaving a reflection pattern

One important finding from Cherinda's work is that weaving boards (WB) provided students with interaction between various representations of concepts. Furthermore, they were able to express their thinking.

> The fact that the subjects have manipulated the WB to certain extent and then continued thinking without it, at least physically (using numbers only, or reproducing woven pattern on squared paper to facilitate the reasoning) reveal the different representational systems that the learners used in attempting the least complex way to acquire and develop mathematical knowledge. (Cherinda, 2012, p. 942)

Cherinda (2002) found that students were motivated to learn (Fig. 8.7a) and the active problem solving created this interaction between representations encouraging mental imagery and reasoning. Both these aspects were found by myself in PNG (Owens, 1999b, 2012a) and Australia (see Chap. 2; Owens, 1993).

Teachers need to be shown how to carry out and incorporate the weaving activities into mathematics. The students did not always copy as expected and needed to be specifically directed to observe the repeated lines. Similarly for students to see the symmetry patterns specific questions were required. Cherinda encouraged the students to draw up number patterns, presenting the line associated with each repeated set of lines. In higher years of school, more complex examples were used. For example, they used two blue vertical colours followed by two yellow vertical strips on the weaving board to make much more complex patterns such as the herringbone pattern.

Mental Mapping of the Navajo, USA

In Chap. 6, we looked at the Navajo's visuospatial reasoning and it is appropriate to note the importance of considering ethnomathematics as the overarching mathematics in which academic mathematics is a part (Owens, 2013a). What works might become just as important as formal proof. There is no reason to doubt what Pinxten and François (2011) suggest could occur for any of the mathematical knowledges in "multimathemacy". Building on Pinxten's earlier work (Pinxten et al., 1983; Pinxten, van Dooren, & Soberon, 1987), their example was that:

> Chee's [Navajo youth] mental mapping and measuring be recognized and generalized to make it a powerful nonwestern geometry which has possibly more potential than we ever guessed, because it would offer problems and solutions about movement through space while starting from an intrinsically dynamic spatial understanding which is typically 'Navajo Indian' and hence beyond the normal scope of the western academic mathematician. (Pinxten & François, 2011)

Quotations and discussions in Chap. 6 justify the need for such comments to be illuminary in mathematics education.

In Chap. 2, mental mapping was raised in discussing its role in attention, intention, and responsiveness. Ecocultural context has a prime role in mental mapping and should not be just regarded as background as other authors have suggested. Language as mentioned in Chap. 4 is a significant part of the ecocultural perspective. For example, as mentioned in Chap. 4, in many PNG cultures (Capell, 1969; Matang & Owens, 2014) and Navajo (Pinxten et al., 1983), the emphasis is on verbs and not nouns so objects, parts, and wholes are not as significant as verbs related to moving rather than being.

> The child pictures the landscape by means of 'significant' rocks (i.e., with particular shapes), the adjacency of dips and water sources, the movements of the sun throughout the day, eventual greenery at different places, the entry of the hogan one finds along the path (i.e., the door of the hogan or dwelling place will always point to the east), the changes of color of the air throughout the day (going from white in the morning, over blue and yellow to black), and so on. All of these elements co-define distance in the Navajo view.
>
> …we start from the preschool and outside of school knowledge, make the concepts and intuitions explicit and label them in the native language. E.g., a line is conceptualized as the result of the uninterrupted movement of the child through a landscape. It corresponds more

> to a path and is closely tied to the moving body. Or it can be understood as the result of two movements in a plane, one rock plateau spreading towards a different moving progression of what we call a river. Where the two movements meet we speak about a line. When looking at the world of experience of the Navajo child, one notes that 'a river' is not a continuous thing … we can then go on and label 'line' in Navajo as a particular movement. Next, we explore the characteristics of the line explicitly and only then move to the lemmas and problems defined in the Euclidean geometry system. In this way, multimathemacy is shaped in the classroom practice. (François, Pinxten, & Mesquita, 2013. p. 30–31)

Thus we find language and visuospatial reasoning are significant in an ecocultural perspective. Curricula that take account of the ecocultural context encourage a sense of ownership but also provide for Indigenous knowledge processes to be used in reasoning especially visuospatially, planning, reviewing, and structuring the learning environment. Thus learner mathematical identity is encouraged (Fig. 1.2).

Yup'ik Mathematics Education

Like in PNG, there are many cultural practices that use proportional reasoning and some common body measurements that are used in a range of activities. However, a first point should be made about realising how Elders who make garments are able to visually consider a person and then cut out a garment that will fit using mental visualisation and reasoning based on experience. In another activity during the fishing period a net of fish is loaded into wheelbarrows and taken to the drying racks providing typical volume units and proportional reasoning. People can estimate the number required and then the amount of fish on the rack becomes a volume unit for filling the smokehouse. Finally another volume unit is used to distribute the fish to families. Berries are picked into specific containers, amalgamated and then placed in plastic bags for the freezer providing further volume units and proportional reasoning between quantities.

In terms of measures of length, the various body parts are used for the kayak, for building the fish racks, and even for houses. A fish rack is five half-fathoms by four elbow to elbow, hands clenched at the centre of body. Children build these to illustrate measurement units, how to measure and how the rack is utilised. Interestingly, the various body measures allow for a range of equivalent fractions as two of the smaller units equals the next larger unit. One unit equals three of the double hand-span—little finger knuckle to little finger knuckle, thumbs touching naturally. Measures are also used in determining heights of stars, direction for travel from the star position, and estimated distances by angle of movement across the sky to signify an hour of time passing and distance likely to be travelled. Part of the reasoning relates to the fact that body parts are always available and objects fit the person better if their own body measurements are used. An important point is that children who undertook these cultural mathematics units did significantly better on a test of mathematics compared to the comparable control group (Lipka, Mohatt, & The Ciulistet Group 1998).

Thoughts on an Ecocultural Curriculum

These studies suggest that an alternative approach to curriculum for mathematics education in geometry and measurement might be needed. It is not easy to provide teachers with a curriculum that emphasises processes (see the UK move away from the curriculum *Man*: *A course of study* in the 1980s). However, early childhood education in mathematics often emphasises processes such as sorting, comparing, matching, one-to-one correspondence, describing, drawing, and ordering. These are processes so it is not unexpected that a curriculum that notes the mathematical processes of an Indigenous group would also have such a list.

Comparing as measure: many of the cultural activities of PNG use comparing as a skill, generally as a visual skill supported by group discussions. Hence the curriculum should contain a development of this skill. Comparing lengths is a beginning first by sight and then by culturally used units such as steps and hand-spans. This would develop for comparing large areas such as gardens or floor space. This might include the number of mounds or people that could fit into the area as an estimate prior to considering in terms of an informal area unit. Volume would also come as a comparison of items such as food like yams and piles of food. Small baskets or wraps of food could be used as informal volume units to supplement the capacity units such as pots or bamboo lengths.

Comparing as ratio: many activities compare two sizes at once. It might be the giving of packets of biscuits or bundle of sago to each child and the total number all together. It might be the floor of one house and the floor of another (half as much again), it might be noting how big a garden will need to be for a bigger family, how large an area of kunai or how large a sago plant to obtain roofing material for two different-sized houses. Comparing how many people can sit on a number of mats compared to another number of mats or how many people are provided for by one basket or bundle of food compared to several baskets or bundles. These can form ready reckoners for the idea of ratio or comparison. In school mathematics these will lead to multiplication and fractions and to ratio including trigonometry ratio but that is much later in school. The important point is to keep these ratio ideas as a strength for learning.

Ordering: many things might be ordered. Size or number may be ordered. Order might represent status. Position in relation to other objects or people in space might also be denoted. Order is often associated with value and this could be in terms of gifts or money or for other relationships. Order might not just be linear in the complex web of family relations. Time can also be ordered.

Sorting for classifying: In geometry, we name shapes with certain properties and we relate shapes according to properties. We analyse problems using these properties and derived information. In schools, these tend to be around Euclidean shapes predominantly 2D shapes. Sorting designs and shapes and objects according to their properties can be around curved designs, related either according to common use or symbolism use in creating a complex design, or by the way they are created. Hence different types of lines, particularly curves, and shapes or designs can be connected to each other. By incorporating shapes other than Euclidean shapes, the curriculum can maintain the richness of traditional understanding of lines and shapes. Classifying is also evident in language such as classifiers for counting. Objects can be grouped and the shapes discussed in both Tok Ples and English.

One-to-one correspondence: There are communities where counting is not carried out, but there may be both one-to-one and one-to-many matches as well as the use of suffixes to indicate number. It is also apparent in body part tally systems. Corresponding matching is associated with distribution, providing an introduction to sharing and division (often by pairs) as well as multiples for groups. It might be a different approach to introduce numbers.

Brazilian Initiatives

Rosa and Orey (2012) suggest that ethnomodelling is an important pedagogy for acknowledging the cultural origins of mathematics for different groups of students. Ethnomodelling encourages students to find out how a particular practice is carried out in the community and then to summarise it in a sequence that provides a mathematical ethnomodel. In Brazil, a number of studies have considered the mathematics of different cultural groups. For the areas involving the Landless Movement, the students can consider the methods for calculating area of irregular quadrilaterals by multiplying the averages of the opposite sides (Knijnik, 2002) or by squaring the result of dividing the sum of the sides by 4 (Flemming, Flemming Luz, & de Mello 2005) cited in Rosa and Orey (2012). In areas where Italian immigrants brought wine making or the German immigrants brought strong European mathematical ways of thinking, alternative ethnomodels could be likely for the students based on their ecocultural context. Rosa and Orey saliently reminded their readers that it is important to recognise the changing nature of cultures in undertaking ethnomodelling.

However, the language and complexity of the classroom illustrates how power dominates the situation in terms of language available to students and the ways in which ethnomodelling is expressed in language (Knijnik, 2002). Nevertheless, by focussing on visuospatial reasoning, teachers like the researchers

> explore the idea of miniature cycles of learning actions to focus on the mathematical learning that is taking place. We describe the dynamics and the complexity of the ongoing activity in the calculation of areas; and, how drawings form a part, and show their influence, in it. We argue that part of this influence was associated with the contradiction between abstract mathematical ideas and their empirical representations, revealed by the tensions perceived in the activities analysed; and, simultaneously, that we could see as an impelling force for the learning of the rules and norms which regulate the use of visual representations in school mathematics. (David & Tomaz, 2012)

The small cycles of learning in the classroom incident are closely linked to the cycle in Chap. 2 that arose from my study in primary schools in Australia (Owens, 1993) but also formed the basis of the establishing of a mathematical learning identity that takes account of the ecocultural context summarised in Chap. 1.

Language and Inquiry for Visuospatial Reasoning in Geometry: Mauritius

This research work was carried out with upper primary level pupils (fourth and fifth graders) from four primary schools in Mauritius. To involve schools of different categories in the study, one high-performing school (School 1 with 213 students), two average-performing schools (Schools 2 and 3 with 367 students), and one low-performing school (School 4 with 165 students) were randomly selected. A quasi-experimental design was selected to test the acquisition of geometric skills (visual, logical, applied, drawing, and verbal) after experimental teaching with inquiry-based teaching with the use of manipulatives and/or local language Creole. The students from each school were classified into four groups: Group1 (traditional teaching with textbook and English), Group 2 (traditional teaching with textbook and Creole), Group 3 (inquiry-based teaching with manipulatives and English), and Group 4 (inquiry-based teaching with manipulatives and Creole). After the experimental teaching, a posttest in the form of a multiple-choice paper-and-pencil questionnaire (MCQ) was collectively administered in all the groups for both fourth and fifth graders. It contained 31 multiple-choice items and there were 4 options for every item and only 1 response was correct. The same posttest was again conducted after 6 or 7 weeks as a retention test. The aim was to measure how well pupils from different groups were able to perform on the MCQ geometry questionnaire and how their performances were affected with the passage of time.

It might be argued that direct drill and practice of the names of the 2D shapes and their properties with visual representation and practice exercises might be more effective at the recognition level and even at the analysis level for each of the geometric skills. However, this research study has shown that the use of inquiry-based methods when combined with manipulative materials encouraged the pupils to become more adept at using the higher order thinking skill of analysis and to acquire the geometric skills with the additional use of Creole as language of instruction. Visual skill at both recognition and analysis levels were significantly influenced by the use of inquiry-based methods with manipulative materials. This shows that acquisition of visual skill at the recognition level was generally within the ability of these primary level pupils. In addition, the use of inquiry-based teaching with manipulatives in Creole had promoted the acquisition of visual skill at the analysis level (basic property analysis of van Hiele level 2). In particular, in four of the eight items in the MCQ questionnaire where there were high success rates (greater than 67.4 %), the pupils from group 4 outperformed pupils from groups 1 and 2. However, the other four items were difficult tasks with low average success rates of 43.3 % in the posttest and 42.9 % in the retention test. In the retention test only, the pupils irrespective of the teaching strategies used, had consistent poor performances in these four items whereas a few significant differences in the posttest were favouring pupils from groups 3 and 4. It must be borne in mind that visual skill is crucial in the acquisition of geometric skills and also helps in the understanding of other abstract mathematical concepts and thinking (Clements, Battista, & Sarama 1998;

Jones, 1998; Lean & Clements, 1981; Wheatley & Cobb, 1990). It is the visuospatial skill that helps pupils to acquire the "geometrical eye" essential in mathematics as stated by Godfrey (1910; Jones & Mooney, 2003). Hence, the acquisition of visuospatial skill is important and the use of inquiry-based teaching methods with manipulatives and Creole is significantly helping in achieving this skill at both recognition and analysis levels.

Logical skill at recognition level is mainly about realising that there are differences and similarities among figures and that figures conserve their shapes in various positions. There were hardly any significant discrepancies in the performances of the pupils from the four groups. Logical skill at analysis level is mainly about using properties to distinguish figures and understand that figures can be classified into different types. Despite the low success rates in both posttest and retention test, pupils from group 4 had performed significantly better.

Applied skill at recognition level is mainly about identifying geometric shapes in physical shapes. There were a few significant discrepancies which mostly favoured pupils from groups 3 and 4. Applied skill at analysis level is mainly concerned with the recognition of geometric properties of physical objects and there was only one item in MCQ questionnaire testing the acquisition of applied skill at analysis level. Despite the low success rates in both posttest and retention test, pupils from group 4 performed significantly better.

Drawing skill is an essential skill required for the acquisition of geometric thinking and it is a crucial part of the geometry curriculum. Its acquisition at the recognition level requires the making of sketches of figures accurately labelling given parts and its acquisition at analysis level requires the translation of given verbal information into picture and the use of given properties of figures to draw or construct the figures. The drawing skill requires a significant amount of visuospatial reasoning. There were more pupils who acquired van Hiele level 2 skill in drawing the equilateral and isosceles triangles than the two quadrilaterals: parallelogram and rhombus in both posttest and retention test. Thus, drawing triangles was easier than the quadrilaterals for the primary-level pupils. Concerning group-wise performances, no significant difference was observed in the performances of the pupils in the drawing of the parallelogram, isosceles triangle, square, and rectangle. However, pupils from group 4 performed significantly better than pupils from the other three groups in drawing the rhombus and equilateral triangle. Consequently, the use of different teaching strategies had little impact on the acquisition of drawing skill. Using isosceles right-angled triangles to construct quadrilaterals seemed to be a relatively difficult task for the pupils (success rate below 52.8 %) and in general, an overall average percentage of 21.5 % had acquired van Hiele level 1 skill and 18.9 % had acquired van Hiele level 2 skill in manipulating the triangles to construct the quadrilaterals. The use of the different teaching strategies had little impact on improving drawing skill. It is observed that the teaching strategies had hardly influenced the acquisition of drawing skill in both the high-scoring and low-scoring items. Perhaps, the applied and practical nature of the items especially the use of identical triangles to construct quadrilaterals had equal impact on the pupils.

It might have been thought that the use of English in the textbook, in class for instruction, and in the test may have assisted students with their verbal skill. However, for the seven items in the questionnaire on verbal skill, it was found that overall the success rate in all groups in both the posttest and retention test were low. The item which required completing a half-drawn rhombus with a line of symmetry needed both verbal and drawing skills and the average success rates were 66.1 % in the posttest and 66.2 % in the retention test. Definitely the given diagram had helped all the pupils in their attempts. Otherwise for the other six items requiring only verbal skill, the success rates were low in both posttest and retention test (success rates 26.1–57.0 %). Generally, the pupils from groups 3 and 4 significantly outperformed their counterparts from groups 1 and 2 in all the seven items in the posttest and in three items in the retention test. Table 8.1 shows results. Writing the properties of some shapes in their own words proved to be a difficult task for the primary level pupils in both posttest and retention test and the low percentages of success confirmed the fact. Concerning group-wise performances, generally pupils from group 4 performed significantly better than the other groups mainly groups 1 and 2 in the tasks in both posttest and retention test. Thus, despite the low success rates, pupils taught using inquiry-based methods with manipulatives had acquired verbal skill at analysis level significantly better than the others. The use of Creole had further improved the acquisition.

The use of the local language can act as a bridge to learning geometry. As mentioned by Khisty (1995), native language is a resource for learning because pupils are more successful when they continue to develop their native language skills

Table 8.1 Performance of pupils in items requiring verbal skill

		Posttest		Retention test	
		Group 4 significantly better than groups listed[a]	Percentage of pupils acquiring the required skill in group 4	Group 4 significantly better than groups listed[a]	Percentage of pupils acquiring the required skill in group 4
1	Properties of sides of an isosceles triangle	1, 2	62.3	1, 2	56.6
2	Identifying a statement which was not true about a rectangle	1, 2	48.7	1, 2	47.5
3	Properties of a square	1, 2	74.7	1, 2	66.9
4	Completing a half-drawn rhombus following a given statement about its symmetry	1, 2	77.4	2	71.9
5	Properties of a rhombus	1, 2	35.5	1, 2	40.1
6	Identifying a statement which was not true about a parallelogram	1, 2	44.2	1, 2	43.8
7	Comparing an angle with a right angle	1, 2	38.5	1	32.7

[a]Based on the partitioning Chi-square test with $p<0.05$

Fig. 8.9 Children learning from manipulative activities in Mauritius. (**a**) Noting angle size (Group 4). (**b**) Different shapes and sizes (Group 3). (**c**) Different quadrilaterals (Group 4). (**d**) Halving a rectangle (Group 4)

rather than focusing exclusively on learning in English. Teachers and pupils must be encouraged to use their native languages to communicate especially at the basic schooling level. This creates a learning environment in which pupils feel more comfortable and have a greater sense of ownership of mathematics. In addition, pupils are able to acquire the practices of mathematics while at the same time maintaining their cultural and linguistic identities. When pupils present their ideas in their local language, they bring a great deal of thinking resources from their lives and habits which are helpful in the learning endeavour. Thus, the inclusion of local language Creole can be viewed as a step in providing a language-sensitive framework for constructing and reviewing content area assessments.

Figure 8.9a shows two pupils who were collaborating to check the equality of the three angles of the equilateral triangle using a bent pipe cleaner as angle-tester. In the following excerpt, a conversation between the two girls is presented.

Pupil 1: Shall we bend the pipe cleaner and then place on the angles?

Pupil 2: Maybe it is better to place the pipe cleaner on one angle and get a measure (just like an angle-tester) …then check for the other two angles… what do you say?

Pupil 1: It's a good idea…ok let's try it. Here is the angle-tester [she bends the pipe cleaner on one of the angles in the triangle]. Now you check whether it fits the other two angles.

Pupil 2: Yes, see…it is fitting the other two angles also…. Let's tell the teacher that we found that all the angles are equal.

This small conversation illustrates how the two girls were able to construct the angle-property of the equilateral triangle while collaborating and using concrete materials. Their collaboration seemed to boost their confidence to explore the angle-property. They used verbal, visual, and logical skills in carrying out the task.

It was very encouraging to see how pupils, especially the low-ability ones, were able to spot the isosceles triangles on their own. Figure 8.9b illustrates students' enthusiasm and achievement. The isosceles triangles were the most commonly constructed triangles and it was observed that pupils from all the four schools were having an instinctive habit of choosing two equal length straws and one different length straw for their construction of triangles. As a result, all the pairs of pupils had at least one isosceles triangle constructed. The kinesthetic actions and semiotic activity had facilitated the learning of different types of triangles.

Figure 8.9c shows the collaboration of pupils to construct parallelograms with elastic bands on geoboards. It was observed that the pair of pupils was constructing their parallelograms separately on the same geoboard then discussing whether the shapes were correct parallelograms. Then they removed or reshaped the ones they doubted as examples of parallelograms. Many pupils were very creative as they first drew squares and rectangles and then stretched one pair of opposite sides to get their parallelograms.

Figure 8.9d shows the pupils' active involvement in drawing the diagonals of a green rectangular paper cut-out shape. Students were physically involved in the activity of drawing diagonals on a range of shapes and then discussing their results. Activity assisted the connection between emotional engagement and learning that was central to the creation of the learning environment in the experimental classes with manipulatives. It not only guided the learning activities in the classroom but also sharpened the creation of the classroom culture and maintained a supportive and emotionally stable classroom environment in which all felt comfortable to take risks and to explore ideas in new ways.

Beside visuospatial reasoning, the classes with manipulatives have led the pupils to achieve experience of the aesthetic. Greene (2001) defined aesthetic learning as an "initiation into new ways of seeing, hearing, feeling, moving, reaching out for meanings and learning to learn integral to the development of persons—to their cognitive, perceptual, emotional, and imaginative development". Thus, it is found that the use of inquiry-based teaching and investigative methods with manipulative materials is promoting cognitive, perceptual, emotional, and imaginative development in the pupils which are in fact the main aims of teaching. Undoubtedly, these strategies will be beneficial to the teaching of other mathematics content area beside geometry.

Moving Forward

This chapter took up the challenge of why an ecocultural perspective on visuospatial reasoning was important for Indigenous communities in transcultural situations. The chapter showed how intuitive or incidental experiences within an ecocultural context strengthen our understanding of mathematics and mathematics education for Indigenous communities. The chapter also provided an example of visuospatial reasoning in classrooms where everyday language differed from the formal school language and the impact that language has on visuospatial learning and reasoning. An ecocultural perspective sheds new light on how gazing, noticing important features, selectively attending, and interpreting visuospatial representations occur in visuospatial reasoning in problem solving in cultural contexts. This extends and in one sense re-interprets recent research on diagrams or other external visual imagery (e.g. Lowrie, Diezmann, & Logan, 2012; Mason, 2003) taking account of the issues for different groups of students especially those undertaking national or state paper-and-pencil tests.

The chapter illustrated how teacher education that takes account of ethnomathematics provides teachers with a means of incorporating the strengths of visuospatial reasoning in an ecocultural context into the learning of students. In particular, the chapter considered geometry and measurement and extends the arguments from earlier chapters on the importance of taking an ecocultural perspective to extend our understand of visuospatial reasoning and the role it plays in problem solving, learning about geometry and measurement, and encouraging a stronger cultural and mathematical identity.

It provided examples of what Castagno and Brayboy claim as important for Indigenous youth, "a more central and explicit focus on sovereignty and self-determination, racism and Indigenous epistemologies in future work on CRS [culturally responsive schooling] for Indigenous youth" (Castagno & Brayboy, 2008, p. 941). The examples tap into at least some of the technologies, worldviews, relativities, place bases, and responsibilities to community, self and the use of power of the learner. However, the holistic perspective must be maintained. It is anticipated that students will have the opportunity to use Indigenous knowledge and language to meet both local and western education goals. In particular, recognition of visuospatial reasoning is in line with a stronger emphasis on holistic learning and seeing the whole picture as a critical aspect of Indigenous ways of learning. An ecocultural perspective of visuospatial reasoning as the examples show is in line with the Alaska Native Knowledge Network principles for CRS:

- A culturally-responsive curriculum reinforces the integrity of the cultural knowledge that students bring with them.
- A culturally-responsive curriculum recognizes cultural knowledge as part of a living and constantly adapting system that is grounded in the past, but continues to grow through the present and into the future.
- A culturally-responsive curriculum uses the local language and cultural knowledge as a foundation for the rest of the curriculum.

- A culturally-responsive curriculum fosters a complementary relationship across knowledge derived from diverse knowledge systems.
- A culturally-responsive curriculum situates local knowledge and actions in a global context. (Alaskan Native Knowledge Network, 1998)

At the same time, can such an ecocultural perspective developed in the last few chapters and considered in this chapter as place-based education with Indigenous or transcultural communities apply to students in a digital age? This is the focus of the next chapter.